Resummation in $t\bar{t}$ production at hadron colliders

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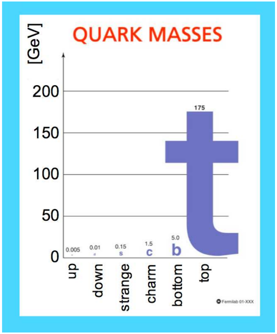
General motivation: the top quark is special



The top-quark tends to stick out...

- almost as heavy as a gold atom
- couples to new particles in many models of new physics
- producing top quarks and studying their properties is a main goal of present hadron colliders

Theory motivation: resummation for $t\bar{t}$ prod.

- quantify at two loops structure of IR divergences in scattering amplitudes involving both massive and massless particles
  [Ferroglia, Neubert, BP, Yang '09]
- implement NNLL resummation for processes with four partons (non-trivial matrix structure in color space)
  [Ahrens, Ferroglia, Neubert, BP, Yang '10, '11]
- understand soft-gluon resummation for very boosted production (e.g. $M_{t\bar{t}} \gg m_t$) [Ferroglia, BP, Yang '12]
- phenomenology

Outline

- factorization and fixed order calculations
- resummation for top pair invariant mass distribution
  - the techniques
  - three issues
- total cross section
  - different approaches to resummation and recent NNLO results

Factorization for inclusive production

Factorization for $h_1 h_2 \to t\bar{t}X$:

$$d\sigma_{h_1,h_2}^{t\bar{t}X} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F) d\hat{\sigma}_{ij}\left(\hat{s}, m_t, \ldots, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

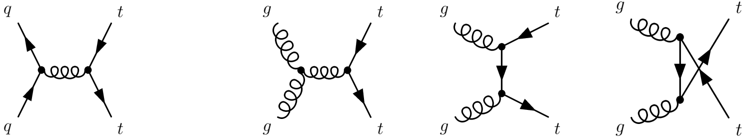
$$s = \left(p_{h_1} + p_{h_2}\right)^2 \,,\, \hat{s} = x_1 x_2 s$$

Strategy:

- take PDFs from data (PDF set collaborations)
- calculate partonic cross sections $d\hat{\sigma}_{ij}$ in QCD (Feynman diagrams)

$$d\hat{\sigma}_{ij} = \alpha_s^2 d\hat{\sigma}_{ij}^{(0)} + \alpha_s^3 d\hat{\sigma}_{ij}^{(1)} + \ldots$$

Feynman diagrams for $d\hat{\sigma}_{ij}$



- $q\bar{q}$ dominant at Tevatron ($\sim$ 90% of cross section)
- gg dominant at LHC ($\sim$ 75% of cross section at 7 TeV)

Higher-order corrections:

- virtual corrections and real emission
- $(qg, \bar{q}g) \to t\bar{t}X$ (numerically small)

NLO known for 20 years, but would like to go beyond

Total cross section at NNLO in fixed order

$$\hat{\sigma}^{\rm NNLO}_{t\bar{t}+X} = \hat{\sigma}^{\rm VV} + \hat{\sigma}^{\rm RV} + \hat{\sigma}^{\rm RR}$$

Many partial results in fixed order

- $\hat{\sigma}^{\rm VV}$: Czakon, Mitov, Moch; Bonciani, Ferroglia, Gehrmann, Maitre, Manteuffel, Studerus; Kniehl, Korner, Merebashvili, Rogal ...
- $\hat{\sigma}^{\rm RV}$ (1-loop $t\bar{t}+j$): Dittmaier, Uwer, Weinzierl '07; Bevilacqua,Czakon, Papadolpoulos, Worek '10; Melnikov, Schulze '10; Gehrmann-De Ridder, Glover, Pires '11
- $\hat{\sigma}^{\rm RR}$: Czakon '11; Abelof, Gehrmann-De Ridder '11

Recent paper completed NNLO calculation in $q\bar{q} \to t\bar{t}X$ channel (!)
[Baernreuther, Czakon, Mitov '12]

Differential cross sections?

When fixed order is not enough . . .

Example: differential cross section at large pair invariant mass $M_{t\bar{t}}$ ($M_{t\bar{t}} = \sqrt{(p_t + p_{\bar{t}})^2}$)

$$\frac{d\sigma}{dM_{t\bar{t}}} = \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z} \, ff_{ij}(\tau/z, \mu_f) \, \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}}$$

$$\sim \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z} \, ff_{ij}(\tau/z, \mu_f) \left[\delta(1-z) C_0^{ij} + \sum_{m\geq 1} \sum_{n \leq 2m-1} \alpha_s^{m+2} d_{mn}^{ij} \left[\frac{\ln^n(1-z)}{1-z} \right]_+ + \dots \right]$$

- when $\tau = M_{t\bar{t}}^2/s \to 1$, logs give large corrections, fixed order expansion fails
- large logs in $z = M_{t\bar{t}}^2/\hat{s} \to 1$ limit related to soft gluon emission and can be resummed using factorization and RG techniques (in SCET)

$ff_{ij}(y, \mu_f) = \int_y^1 \frac{dx}{x} \, f_{i/h_1}(x, \mu_f) \, f_{j/h_2}(y/x, \mu_f)$ are parton luminosities

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Factorization in the soft limit

Momentum scales at $z = M_{t\bar{t}}^2/\hat{s} \longrightarrow 1$:

$$\hat{s}, M_{t\bar{t}}^2, m_t^2 \gg E_s^2 \sim \hat{s}(1-z)^2 \gg \Lambda_{\rm QCD}^2$$

Factorization:

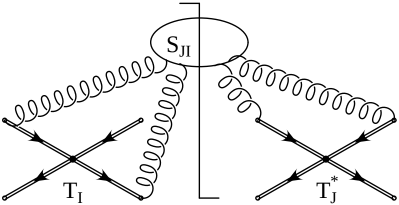
$$\frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}d\cos\theta} = \mathrm{Tr}\left[\mathbf{H}_{ij}(M_{t\bar{t}}, m_t, \cos\theta, \mu_f)\,\mathbf{S}_{ij}\left(\frac{2E_s(z)}{\mu_f}, v_i\cdot v_j, \ldots, \alpha_s(\mu_f)\right)\right] + \mathcal{O}(1-z)$$

Kidonakis, Sterman ('97)

- $\mathbf{H}_{ij}$ are matrices related to hard virtual corrections
- $\mathbf{S}_{ij}$ are matrices related to soft real emission. They depend on z through $\delta(1-z)$ or

$$\alpha_s^n\left[\frac{\ln^m(2E_s(z)/\mu)}{1-z}\right]_+; \quad m = 0, \cdots, 2n-1$$

Hard-soft factorization (quark channel)



(b)

$$T_I \sim C_I(M, m_t, \dots) \times \bar{\chi}_{\bar{n}}^{a_2}\, \chi_n^{a_1}\; \bar{h}_{v_3}^{a_3} h_{v_4}^{a_4} \times c_I^{\{a\}}$$

$$\left(c_1^{q\bar{q}}\right)_{\{a\}} = \delta_{a_1 a_2}\delta_{a_3 a_4}\,, \quad \left(c_2^{q\bar{q}}\right)_{\{a\}} = t^c_{a_2 a_1} t^c_{a_3 a_4}\,,$$

$$d\hat{\sigma} \sim \sum_{I,J} C_I S_{IJ} C_J^* \times |\langle t\bar{t}|\bar{\chi}_{\bar{n}}\, \chi_n\; \bar{h}_{v_3} h_{v_4}|q\bar{q}\rangle|^2 \equiv \mathrm{Tr}[\mathbf{H}\,\mathbf{S}]$$

RG equations: Hard function

RG equation: (recall $\mathbf{H}_{IJ} \sim C_I C_J^*$)

$$\frac{d}{d\ln\mu}\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{\Gamma}(M, m_t, \cos\theta, \mu)\,\mathbf{H}(M, m_t, \cos\theta, \mu) + \mathbf{H}(M, m_t, \cos\theta, \mu)\,\mathbf{\Gamma}^\dagger(M, m_t, \cos\theta, \mu)$$

- anomalous dimension $\mathbf{\Gamma}$ related to IR poles in on-shell scattering amplitudes (IR in QCD = UV in SCET)

Solution:

$$\mathbf{H}(M, m_t, \cos\theta, \mu) = \mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu)\,\mathbf{H}(M, m_t, \cos\theta, \mu_h)\,\mathbf{U}^\dagger(M, m_t, \cos\theta, \mu_h, \mu)$$

$$\mathbf{U}(M, m_t, \cos\theta, \mu_h, \mu) = \mathcal{P}\exp\int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\,\mathbf{\Gamma}(M, m_t, \cos\theta, \mu')$$

- should choose $\mu_h \sim M$ to avoid large logs in $\mathbf{H}$

Universal soft anomalous dimension to NNLL

- General structure (any scattering amplitude!!) [Mitov, Sterman, Sung; Becher, Neubert 2009]:

$$\begin{aligned}\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu) &= \sum_{(i,j)} \frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\,\gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ &- \sum_{(I,J)} \frac{\mathbf{T}_I\cdot\mathbf{T}_J}{2}\,\gamma_{\text{cusp}}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) \\ &+ \sum_{I,j} \mathbf{T}_I\cdot\mathbf{T}_j\,\gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{m_I\mu}{-s_{Ij}} \\ &+ \sum_{(I,J,K)} i f^{abc}\,\mathbf{T}_I^a\,\mathbf{T}_J^b\,\mathbf{T}_K^c\;F_1(\beta_{IJ},\beta_{JK},\beta_{KI}) \\ &+ \sum_{(I,J)}\sum_k i f^{abc}\,\mathbf{T}_I^a\,\mathbf{T}_J^b\,\mathbf{T}_k^c\;f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big)\,.\end{aligned}$$

- two parton correlations known from on-shell form factors
- F_1 and f_2 are three-parton correlations, known at two loops [Ferroglia, Neubert, BP, Yang '09]

Two loop anomalous dimension in top production

$$\mathbf{\Gamma}_{q\bar{q}} = \left[C_F\,\gamma_{\text{cusp}}(\alpha_s)\,\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + C_F\,\gamma_{\text{cusp}}(\beta_{34},\alpha_s) + 2\gamma^q(\alpha_s) + 2\gamma^Q(\alpha_s)\right]\mathbf{1}$$
$$+ \frac{N}{2}\left[\gamma_{\text{cusp}}(\alpha_s)\,\left(\ln\frac{t_1^2}{M^2 m_t^2} + i\pi\right) - \gamma_{\text{cusp}}(\beta_{34},\alpha_s)\right]\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$+ \gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{t_1^2}{u_1^2}\left[\begin{pmatrix} 0 & \frac{C_F}{2N} \\ 1 & -\frac{1}{N} \end{pmatrix} + \frac{\alpha_s}{4\pi}\,g(\beta_{34})\begin{pmatrix} 0 & \frac{C_F}{2} \\ -N & 0 \end{pmatrix}\right]$$

$$\mathbf{\Gamma}_{gg} = \left[N\,\gamma_{\text{cusp}}(\alpha_s)\,\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + C_F\,\gamma_{\text{cusp}}(\beta_{34},\alpha_s) + 2\gamma^g(\alpha_s) + 2\gamma^Q(\alpha_s)\right]\mathbf{1}$$
$$+ \frac{N}{2}\left[\gamma_{\text{cusp}}(\alpha_s)\,\left(\ln\frac{t_1^2}{M^2 m_t^2} + i\pi\right) - \gamma_{\text{cusp}}(\beta_{34},\alpha_s)\right]\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$+ \gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{t_1^2}{u_1^2}\left[\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -\frac{N}{4} & \frac{N^2-4}{4N} \\ 0 & \frac{N}{4} & -\frac{N}{4} \end{pmatrix} + \frac{\alpha_s}{4\pi}\,g(\beta_{34})\begin{pmatrix} 0 & \frac{N}{2} & 0 \\ -N & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

RG equations: Soft function

- Laplace-transform:

$$\tilde{\mathbf{s}}(L, M, m_t, \cos\theta, \mu) = \frac{1}{\sqrt{\hat{s}}} \int_0^\infty d\omega \, \exp\left(-\frac{\omega}{e^{\gamma_E} \mu e^{L/2}}\right) \mathbf{S}(\omega, M, m_t, \cos\theta, \mu)$$

- RG equation:

$$\frac{d}{d\ln\mu} \tilde{\mathbf{s}}\left(\ln\frac{M^2}{\mu^2}, M, m_t, \cos\theta, \mu\right) =$$
$$- \left[\mathbf{\Gamma}(M, m_t, \cos\theta, \mu) + \gamma_\phi\right] \tilde{\mathbf{s}}\left(\ln\frac{M^2}{\mu^2}, M, m_t, \cos\theta, \mu\right)$$
$$- \tilde{\mathbf{s}}\left(\ln\frac{M^2}{\mu^2}, M, m_t, \cos\theta, \mu\right) \left[\mathbf{\Gamma}(M, m_t, \cos\theta, \mu) + \gamma_\phi\right]$$

- Momentum-space solution [Becher, Neubert '06]

$$\mathbf{S}(\omega, M, m_t, \cos\theta, \mu_f) = \sqrt{\hat{s}} \, \exp\left[-4S(\mu_s, \mu_f) + 4a_{\gamma^\phi}(\mu_s, \mu_f)\right]$$
$$\times \, \mathbf{u}^\dagger(M, m_t, \cos\theta, \mu_f, \mu_s) \, \tilde{\mathbf{s}}(\partial_\eta, M, m_t, \cos\theta, \mu_s) \, \mathbf{u}(M, m_t, \cos\theta, \mu_f, \mu_s) \, \frac{1}{\omega} \left(\frac{\omega}{\mu_s}\right)^{2\eta} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

Resummed partonic cross section

All-orders resummation formula ($\lambda \sim 1 - z$):

$$\frac{d\hat{\sigma}}{dM_{t\bar{t}}} \sim \exp\left[4a_{\gamma^\phi}(\mu_s, \mu_f)\right] \mathrm{Tr}\Big[\mathbf{U}_\lambda(\mu_h, \mu_s)\,\mathbf{H}(\mu_h)$$
$$\mathbf{U}_\lambda^\dagger(\mu_h, \mu_s)\,\tilde{\mathbf{s}}_\lambda(\partial_\eta, \alpha_s(\mu_s))\Big] \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \left[\frac{1}{\lambda}\left(\frac{M_{t\bar{t}}\lambda}{\mu_s}\right)^{2\eta}\right]_+$$
$$+\mathcal{O}(\lambda)$$

Ingredients for NNLL

γ_{cusp}	$\mathbf{\Gamma}$	$\mathbf{H}$, $\tilde{\mathbf{s}}_\lambda$
3-loop	2-loop	1-loop

Ahrens, Ferroglia, Neubert, BP, Yang ('09, '10)

Three questions:

- is this also useful if $M_{t\bar{t}}$ is not large?
- if $M_{t\bar{t}}$ is large, shouldn't one use $M_{t\bar{t}} \gg m_t$?
- how to choose μ_s?

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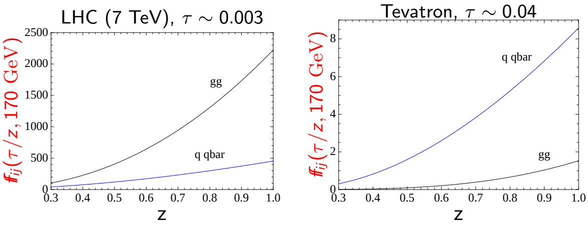
Dynamical threshold enhancement

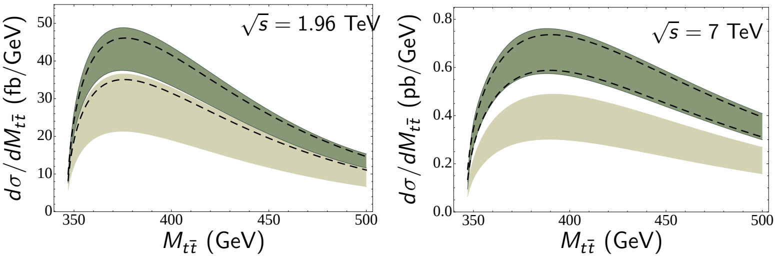
$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_{i,j} \int_\tau^1 \frac{dz}{z}\, ff_{ij}(\tau/z,\mu_f) \left[\delta(1-z) C_0^{ij} + \sum_{m\geq 1} \sum_{n \leq 2m-1} \alpha_s^{m+2} d_{mn}^{ij} \left[\frac{\ln^n(1-z)}{1-z} \right]_+ + \ldots \right]$$

Leading terms in $z \to 1$ limit dominant if:

- $\tau = M_{t\bar{t}}^2/s \to 1$ (high invariant mass)
- $ff_{ij}(\tau/z,\mu)$ largest as $z \to 1$, even if τ not close to 1 ("dynamical threshold enhancement")



DOMINANCE OF SOFT GLUON CORRECTIONS AT NLO



- green band = exact fixed order at NLO ($\mu_f = 200, 800$ GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200, 800$ GeV)

Soft gluon corrections dominate cross section at all $M_{t\bar{t}}$
(at NNLL, where logarithms *and* constants determined)

$$d\widetilde{\hat{\sigma}} \sim 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots$$

Three questions:

- is this also useful if $M_{t\bar{t}}$ is not large?
- if $M_{t\bar{t}}$ is large, shouldn't one use $M_{t\bar{t}} \gg m_t$?
- how to choose μ_s?

Resummation for boosted production

When $M \gg m_t$: [Ferroglia, BP, Yang '12]

$$\begin{aligned}\frac{d\hat{\sigma}}{dM} &\sim \mathbf{H}_m(m_t, M)\mathbf{S}_m(m_t, M(1-z)) + \mathcal{O}(1-z) \\ &\to \mathbf{H}(M)\mathbf{S}(M(1-z)) \otimes D^2_{t/t}(m_t, m_t(1-z)) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t}{M}\right) \\ &= \left[C_D^2(m_t)\mathbf{H}(M)\right]\left[\mathbf{S}(M(1-z)) \otimes S_D^2(m_t(1-z))\right] + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t}{M}\right)\end{aligned}$$

- m_t dependence is in heavy-quark fragmentation functions $D_{t/t}$
- M dependence is in hard and soft functions with $m_t = 0$ from beginning
- can use RG equations to resum soft logs and logs of m_t/M
- can also calculate NNLO matching coefficents to determine δ-function correction in limit $m_t \gg M$ (only NNLO soft function unknown [Ferroglia, BP, Yang, in progress])

Size of NLO and NNLO corrections

$$d\hat{\hat{\sigma}} \sim 1 + \alpha_s \left(\sum_{n=1}^{2} c_n(M, m_t) L^n + c_0(M, m_t) \right) + \alpha_s^2 \left(\sum_{n=1}^{4} d_n(M, m_t) L^n + d_0(M, m_t) \right) + \ldots$$

	$M = 500$ GeV	$M = 1500$ GeV	$M = 3000$ GeV
$c_n L^n$ terms, exact in m_t	0.074	1.04×10^{-4}	0.70×10^{-7}
$c_n L^n + \ln(m_t/M)$ terms in c_0	0.085	1.35×10^{-4}	0.94×10^{-7}
$c_n L^n + c_0$ for $m_t \to 0$	0.126	1.79×10^{-4}	1.19×10^{-7}
$c_n L^n + c_0$ exact in m_t	0.154	1.86×10^{-4}	1.20×10^{-7}

Table: NLO corrections to the differential cross section $d\sigma/dM$ (in pb/GeV) with $\mu_f = M$ at LHC with $\sqrt{s} = 7$ TeV.

	$M = 500$ GeV	$M = 1500$ GeV	$M = 3000$ GeV
$d_n L^n$ terms, exact in m_t	5.67×10^{-2}	1.22×10^{-4}	1.11×10^{-7}
$d_n L^n + \ln(m_t/M)$ terms in d_0	6.35×10^{-2}	1.40×10^{-4}	1.26×10^{-7}

Table: NNLO corrections.

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- if $M_{t\bar{t}}$ is large, shouldn't one use $M_{t\bar{t}} \gg m_t$?
- how to choose μ_s?

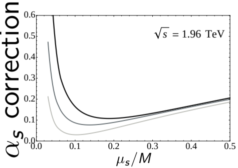
Choice of μ_s ("QCD vs. SCET")

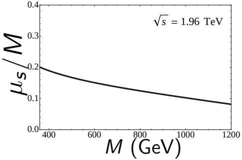
$$\mathbf{S}(\omega, M, m_t, \cos\theta, \mu_f) = \sqrt{\hat{s}}\,\exp\left[-4S(\mu_s, \mu_f) + 4a_{\gamma^\phi}(\mu_s, \mu_f)\right]$$

$$\times\,\mathbf{u}^\dagger(M, m_t, \cos\theta, \mu_f, \mu_s)\,\tilde{\mathbf{s}}(\partial_\eta, M, m_t, \cos\theta, \mu_s)\,\mathbf{u}(M, m_t, \cos\theta, \mu_f, \mu_s)\,\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

1) "QCD": choose scale μ_s in Laplace or Mellin space so that partonic logs vanish (must deal with Landau pole but exponentiate partonic logarithms)

2) "SCET": choose $\mu_s = \mu_s(M)$ to minimize corrections to hadronic cross section (no Landau pole but don't exponentiate all partonic logarithms)





$M = 400$ GeV (dark), $M = 700$ GeV (medium), and $M = 1000$ GeV (light)

Different philosophies, not clear (to me) that one is better than the other (see discussions in [Ahrens et. al. 'll] and especially [Bonvini, Forte, Ghezzi, Ridolfi '12])

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Resummation in three soft limits

Name	Observable	Soft limit
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}}dy$	$(1-z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = \hat{s} + \hat{t}_1 + \hat{u}_1 \to 0$
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$

$$d\sigma = PDFs \otimes \left[d\hat{\sigma}_{\rm NNLL}(\ln\lambda) + \mathcal{O}(1,\lambda)\right]; \quad \lambda \in \{\beta,\, s_4,\, 1-z\}$$

- dynamical threshold enhancement and structure of $\mathcal{O}(1,\lambda)$ corrections is different in each limit, although $\beta \to 0$ is a special case of the other two

NNLL calculations for total cross section

1) production threshold ($\beta \to 0$, total cross section only)

- Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09; Beneke, Czakon, Falgari, Mitov, Schwinn '09
- method for joint resummation of soft gluon and Coulomb terms ($\sim \ln^m \beta/\beta^n$) developed and implemented in Beneke, Falgari, Schwinn '09; Beneke, Falgari, Klein, Schwinn '11
- Mellin-space NNLL in Cacciari, Czakon, Mangano, Mitov, Nason '11

2) PIM kinematics (integral of $M_{t\bar{t}}$ and y distributions)

- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '10

3) 1PI kinematics (integral of p_T and y distributions)

- Kidonakis '10
- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '11

NNLL and approximate NNLO

In Laplace space:

$$d\tilde{\hat{\sigma}} \sim \exp\Bigl(\overbrace{\overbrace{Lg_1(\alpha_s L) + g_2(\alpha_s L)}^{NLL} + \alpha_s g_3(\alpha_s L) + \cdots}^{NNLL}\Bigr) \underbrace{C(\alpha_s)}_{\text{constants}} + \mathcal{O}(\lambda)$$

$$\sim \underbrace{\underbrace{1 + \alpha_s(L^2 + L + 1 + \mathcal{O}(\lambda))}_{NLO} + \alpha_s^2(L^4 + L^3 + L^2 + L}_{approx.NNLO} + 1 + \mathcal{O}(\lambda)) + \mathcal{O}(\alpha_s^3)$$

- resummed formulas (NLL, NNLL, etc.) exponentiate large logs
- approximate NNLO formulas keep logarithmic parts of full correction
- some freedom in what to add to $\mathcal{O}(1)$ and $\mathcal{O}(\lambda)$ terms

Approximate NNLO implementations of NNLL

$$\hat{\sigma}^{(2)\,\text{approx p.t.}}_{ij=q\bar{q},\,gg} = \sum_{m=1}^{4} d^{\text{p.t.}}_{2m} \ln^m \beta + \frac{1}{\beta}\left(c^{\text{p.t.}}_{22} \ln^2 \beta + c^{\text{p.t.}}_{11} \ln \beta + c^{\text{p.t.}}_{10}\right) + \frac{c^{\text{p.t.}}_{20}}{\beta^2} + \hat{R}'^{\,\text{p.t.}}(\beta)$$

$$\hat{\sigma}^{(2)\,\text{approx. 1PI}}_{ij=q\bar{q},gg} = \int d\hat{t}_1 ds_4 \left\{ \sum_{m=0}^{3} d^{\text{1PI}}_m \left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4}\right]_+ + c^{\text{1PI}}\delta(s_4) + \hat{R}'^{\,\text{1PI}}(s_4) \right\}$$

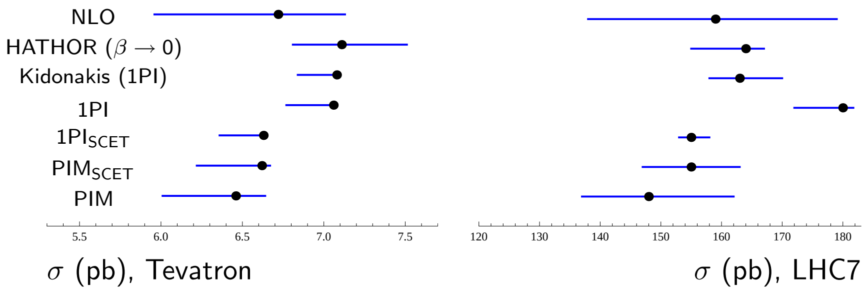
- pieces in blue determined exactly from NNLL (or Coulomb res. for p.t.)
- two schemes appear in literature for 1PI (and PIM) kinematics:

	scheme	extra terms in R	"damping factor"
Ahrens et al	1PI_{SCET}: $2E_s(s_4) = \frac{s_4}{\sqrt{m_t^2+s_4}}$	$\frac{1}{s_4}\ln^n(1+s_4/m_t^2)$	——
Kidonakis	1PI: $2E_s(s_4) \approx s_4/m_t$	——	$\hat{\sigma}^{(2)} \to \hat{\sigma}^{(2)} \times 2m_t/\sqrt{\hat{s}} = \hat{\sigma}^{(2)} \times \sqrt{1-\beta^2}$

- different groups also include different structure of $\ln^n m_t/\mu$ terms (HATHOR includes all, Kidonakis includes $\delta(s_4)$, SCET includes some of $\delta(s_4)$ but not all)

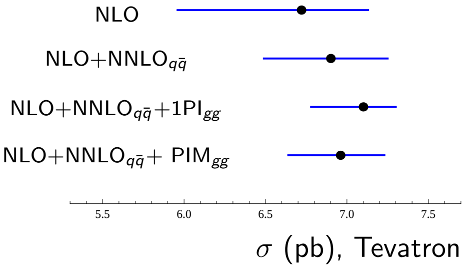
NNLO approximations for total cross section

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 PDFs



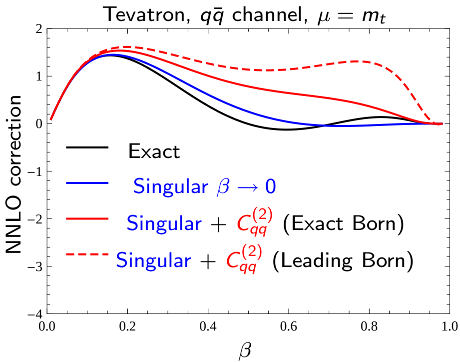
- scale variation not necessarily good indication of uncertainties from subleading terms in a given soft limit
- various efforts to include more sources of uncertainty within certain limits exist, but ultimately only the NNLO calculation will settle this!

TEVATRON CROSS SECTION WITH NNLO IN $q\bar{q}$ CHANNEL $\left(m_t = 173.1 \text{ GeV}, m_t/2 < \mu_f = \mu_r < 2m_t, \text{ MSTW2008 PDFs}\right)$



- NNLO$_{q\bar{q}}$ is exact result from [Baernreuther, Czakon, Mitov]
- even using most discrepant NNLO approximations in gg channel (the PIM and 1PI) makes little difference for cross section
- can further reduce scale dependence by resummation at NNLO+NNLL in $\beta \to 0$ limit (see [Baernreuther, Czakon, Mitov] for numbers)

NNLO corrections in $\beta \to 0$ limit vs. exact



$$\sigma_{q\bar{q}}^{(2)} = \sigma_{q\bar{q}}^{(0)} \left[\frac{3.60774}{\beta^2} + \frac{1}{\beta}\Big(-140.368 \ln^2\beta + 32.106 \ln\beta + 3.95105 \Big) \right.$$

$$\left. +910.222 \ln^4\beta - 1315.53 \ln^3\beta + 592.292 \ln^2\beta + 528.557 \ln\beta + C_{qq}^{(2)} \right] + \mathcal{O}(\beta)$$

$$\sigma_{q\bar{q}}^{(0)} \sim \pi\beta\rho(2+\rho); \quad \rho = 1 - \beta^2$$

Power corrections to $\beta \to 0$ limit are large

Summary

- Two routes beyond NLO in top-pair production:
  - fixed order to NNLO
  - soft gluon resummation to NNLL

- For total cross section, exact NNLO is known in $q\bar{q}$ channel. Approximations are still used in gg channel, but (at least to me) there is no motivation for threshold resummation.

- For differential cross sections such as $d\sigma/dM_{t\bar{t}}$ at large $M_{t\bar{t}}$, the best approach is still debatable. Exact NNLO and improved results in $m_t \ll M_{t\bar{t}}$ limit will shed light on this.

- Can also study other differential quantities, such as $A_{\rm FB}$ (backup slides) or p_T and rapidity distributions.

backup slides

Outline

- factorization and fixed order calculations
- resummation for top pair invariant mass distribution
  - the techniques
  - three issues
- total cross section
  - different approaches to resummation and recent NNLO results

Forward-backward asymmetry

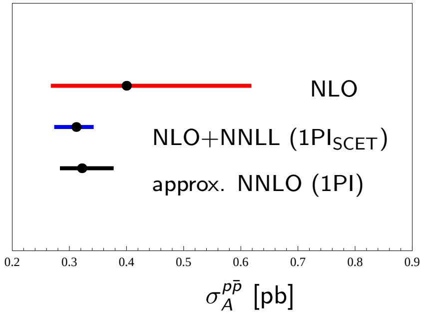
$$A_{\mathrm{FB}}^{i} = \frac{N_t(y^i > 0) - N_t(y^i < 0)}{N_t(y^i > 0) + N_t(y^i < 0)} = \frac{\sigma_A^i}{\sigma_S^i} = \alpha_s A_{\mathrm{FB}}^{(0)} + \dots$$

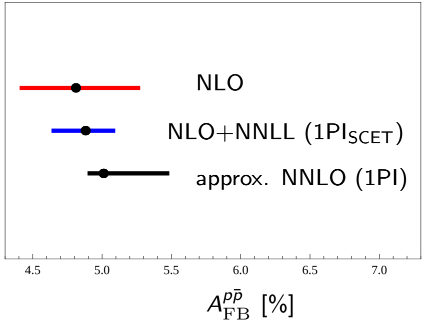
Tevatron:

- total asymmetry measured in $i = p\bar{p}$ or $t\bar{t}$ rest frame
- also with cuts on $M_{t\bar{t}}$ and $\Delta y = y_t - y_{\bar{t}}$ in $t\bar{t}$ frame
- NLO+NNLL calculation in [Ahrens, Ferroglia, Neubert, BP, Yang '11]

Tevatron FB asymmetry in lab frame

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008

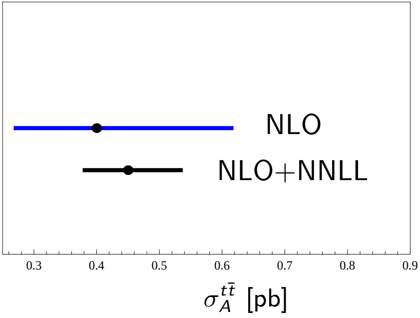


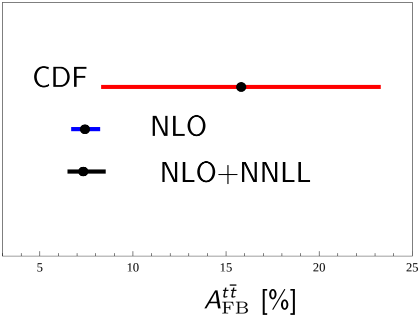


- approximate NNLO in 1PI nearly gives exact NNLO $q\bar{q}$ cross section at $\mu = m_t$ (and is about 8% larger that NLO+NNLL), but only slightly increases A_{FB} at that scale

Tevatron FB asymmetry in $t\bar{t}$ frame

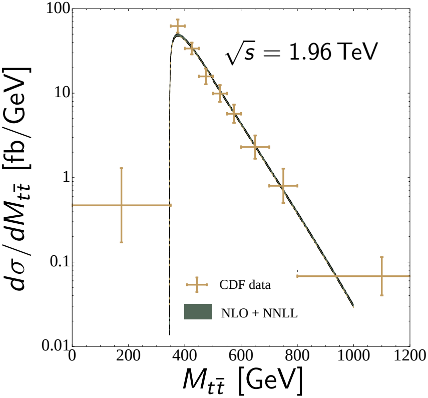
$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL

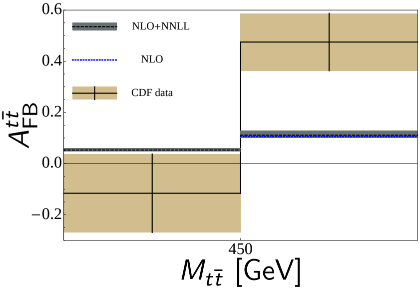




- resummation roughly halves scale dependence in σ_A and σ compared to NLO, but scale dependence of $A_{\rm FB}$ somewhat larger $\Delta\sigma_{\rm FB}$
- if approximate NNLO (1PI) is "boosted" to $t\bar{t}$ frame, it gives $A_{FB}^{t\bar{t}}$ within errors of NLO+NNLL (which is PIM_{SCET})
- EW corrections enhance $A^{\rm FB}$ by about 1.2 [Hollik, Pagani '11]

CROSS SECTION AND A_{FB} AS FUNCTION OF $M_{t\bar{t}}$





- good agreement of NLO+NNLL theory for pair-invariant mass distribution hard to reconcile with $A_{\rm FB}$ at $M_{t\bar{t}} > 450$ GeV from CDF (less so with D0)
- EW corrections enhance $A^{\rm FB}$ by about 1.2 both in upper and low bin [Hollik, Pagani '11]

Momentum-space resummation: NLO+NNLL

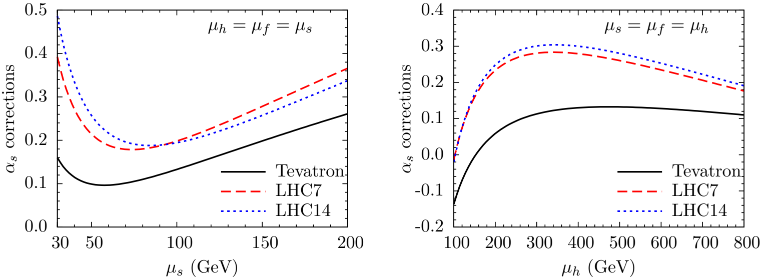
Philosophy: choose μ_s and μ_h such that there are no large logs in matching functions, run to μ_f using RG

Complication is with soft function (ignoring matrix structure)

$$S(\omega,\mu_f) = e^{-4S(\mu_s,\mu_f)+2a_{\gamma^s}(\mu_s,\mu_f)}\,\tilde{s}\left(\partial_\eta,\mu_s\right)\left[\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\right]_+ \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

- logs vanish for $\mu_s \sim \omega$, but ω is integrated down to $\omega = 0$, so hit Landau pole
- choose μ_s *after* integration over ω, such that correction from soft function is minimized
- keep parametric counting $\mu_s \sim \omega$, even though it is a *number*

Hard and soft scales for total cross section in 1PI kinematics



- left: NLO correction (%) from soft function $\Rightarrow \mu_s \sim 50 - 90$ GeV
- right: NLO correction (%) from hard function $\Rightarrow \mu_h \sim 400$ GeV

Approximate NNLO

Philosophy: plus distributions give dominant contributions at a given order, but corrections small enough for series to converge

In approximate formulas re-expand in limit $\mu_s \to \mu_f$ $(\eta \to 0)$

$$S(\omega, \mu_f) = \tilde{s}\left(\partial_\eta, \mu_s\right) \left[\frac{1}{\omega}\left(\frac{\omega}{\mu_f}\right)^{2\eta}\right]_+ \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}\bigg|_{\eta\to 0}$$

$$= \delta(\omega) + \sum_{m=1}\left(\frac{\alpha_s}{4\pi}\right)^m \left[D^{(m)}\delta(1-z) + \sum_{k=0}^{2m-1} S^{(m,k)} P_m(\omega)\right]; \quad P_m(\omega) = \left[\frac{1}{\omega}\ln^m\frac{\omega}{\mu_f}\right]_+$$

- RG equations determine all two loop plus distributions in terms of one-loop matching function and two-loop anomalous dimensions
  - in form above, must derive two-loop logs in $\tilde{s}(L, \mu_s)$ separately and add on to NNLL solution, which only has one loop matching
- two-loop delta-function term undetermined

Resummation vs. approximate formulas

The two approaches are *different* in structure of plus distributions, at a given order in α_s.

Example: approximate NLO (use 1-loop anomalous dimensions and tree matching)

$$\tilde{s}(L, \alpha_s(\mu)) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[\frac{\Gamma_0}{2}L^2 + L\gamma_0^s + s^{(1,0)}\right]$$

$$\Rightarrow S_{\mathrm{NLO_{approx}}} = 1 + \frac{\alpha_s(\mu_f)}{4\pi}\left[4\Gamma_0 P_1(\omega) + 2\gamma_0^s P_0(\omega) - \frac{\pi^2}{3}\Gamma_0\,\delta(\omega)\right]$$

Compare with $\mathrm{NLL}|_{\mathrm{NLO}}$. To expand resummed formula, use $\alpha_s(\mu_s) = \alpha_s(\mu_f)[1 - \beta_0\alpha_s(\mu_f)/(4\pi)\ln(\mu_s^2/\mu_f^2) + \ldots]$

$$S_{\mathrm{NLL}|_{\mathrm{NLO}}} = 1 + \frac{\alpha_s(\mu_f)}{4\pi}\left[2\Gamma_0 L_s P_0(\omega) + \left(\frac{1}{2}\Gamma_0 L_s^2 + \gamma_0^s L_s\right)\delta(\omega)\right]; \quad L_s = \ln\left(\frac{\mu_s^2}{\mu_f^2}\right)$$

All orders exponentiation

In momentum space, can exponentiate the distributions using

$$S(\omega,\mu_f) = \left\{ \left[e^{-4S(\mu_s,\mu_f)+2a_{\gamma^s}(\mu_s,\mu_f)} \tilde{s}\left(0,\alpha_s(\mu_s)\right) \right] \Big|_{\ln(\mu_s^2/\mu_f^2)\to\partial\eta} \right\} \frac{1}{\omega} \left(\frac{\omega}{\mu_f} \right)^{2\eta} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \Big|_{\eta\to 0}$$

where

$$\frac{\alpha_s(\mu_s)}{4\pi} = \frac{\alpha_s(\mu_f)}{4\pi}\frac{1}{X} - \frac{\alpha_s(\mu_f)^2}{(4\pi)^2}\frac{1}{X^2}\frac{\beta_1}{\beta_0}\ln X + \ldots \, ; \qquad X = 1 + \beta_0 \frac{\alpha_s(\mu_f)}{4\pi} \ln\frac{\mu_s^2}{\mu_f^2}$$

But this doesn't match the "philosophy" of RG-improved perturbation theory