Resummation for N-Jet Processes using SCET

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Workshop on Event Generators and Resummation
DESY, May 2012

Outline:

- Introduction to Soft Collinear Effective Theory

- Cross Sections with Jets
  - $pp \to H + 0\,\mathrm{jets}$ NNLL+NNLO

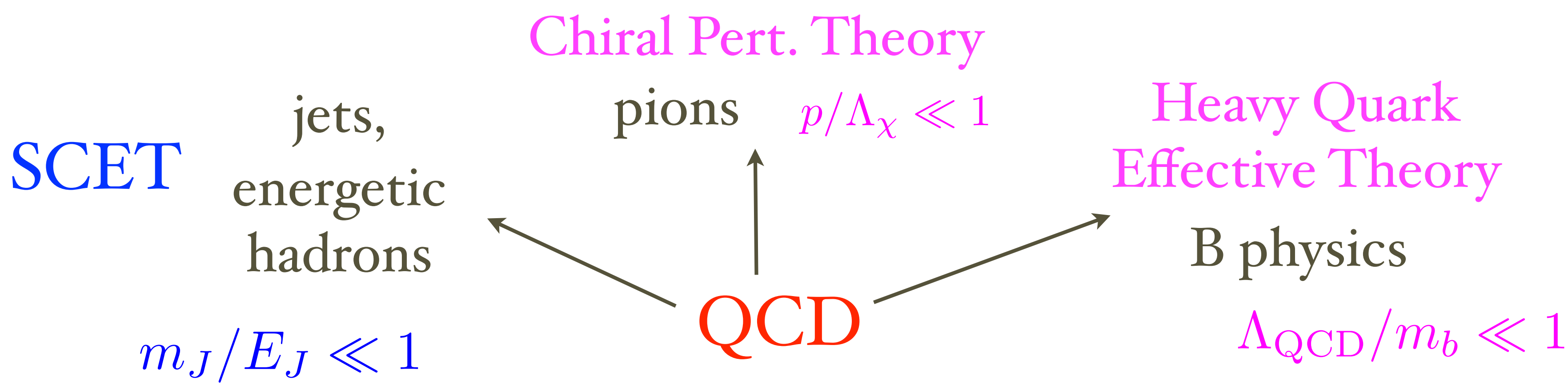
    scales, including fixed order, profiles, theory uncertainties, power corrections
  - N-Jettiness event shape & $pp \to H + 1\,\mathrm{jet}$ NNLL

- Jets Nearby in phase space

- Jet Substructure

Soft Collinear Effective Theory (SCET) is limit of QCD

Bauer, Fleming, Pirjol, IS



- Results derived with SCET must be equivalent to results derived directly from QCD. And SCET results are results about QCD.

- Goals:

  * organize calculations around treatment of scales, exploit field theory
  * simplify treatment of factorization (new formulas, extensions, ...)
  * systematic expansion (factorize power corrections, estimate theory errors)
  * sum logs with higher precision (NNLL, N^3LL)

$pp \quad e^+e-$

Soft Collinear Effective Theory (SCET) is limit of QCD

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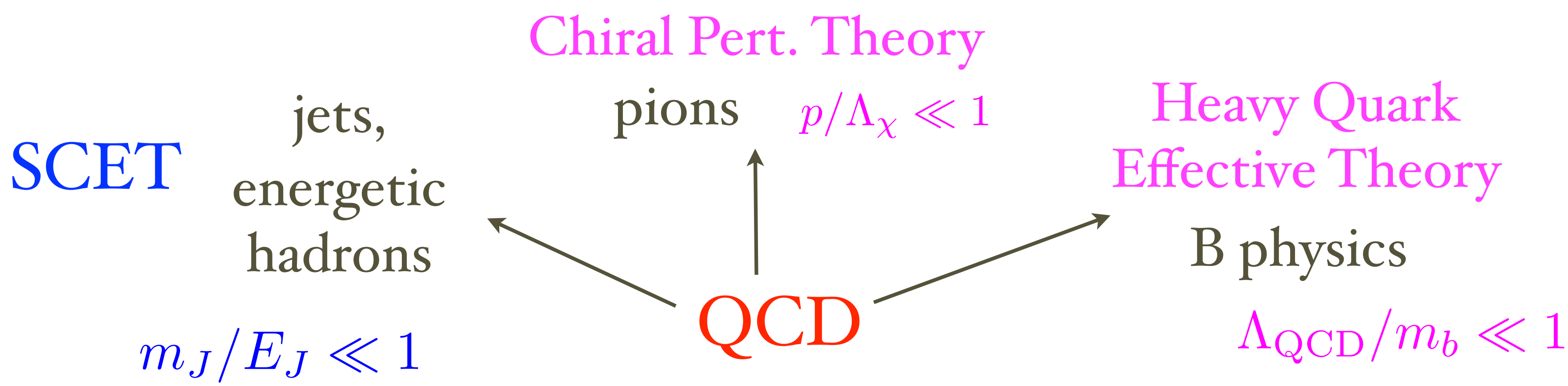
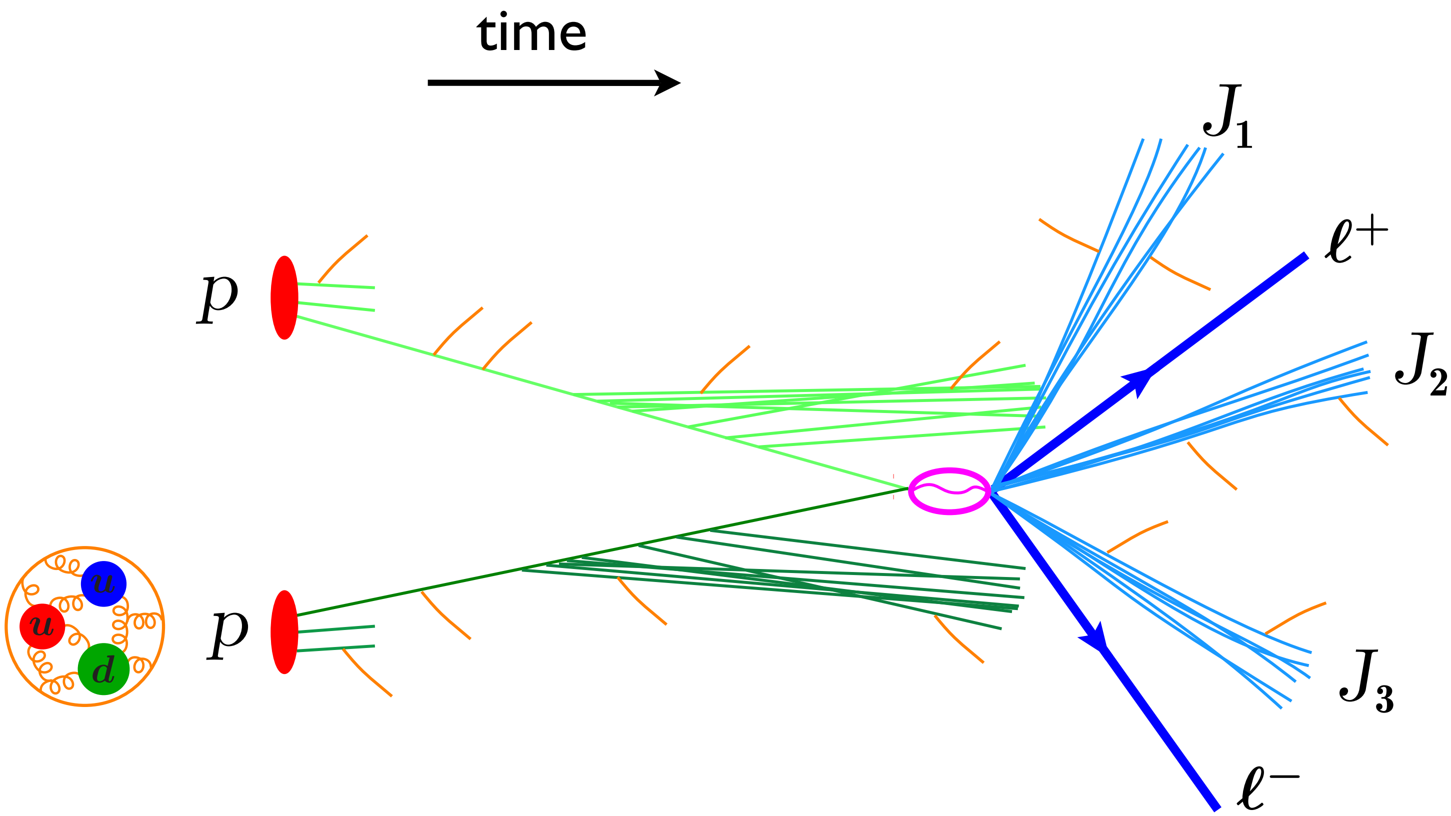


- Applications for Event Generators / Shower MC:

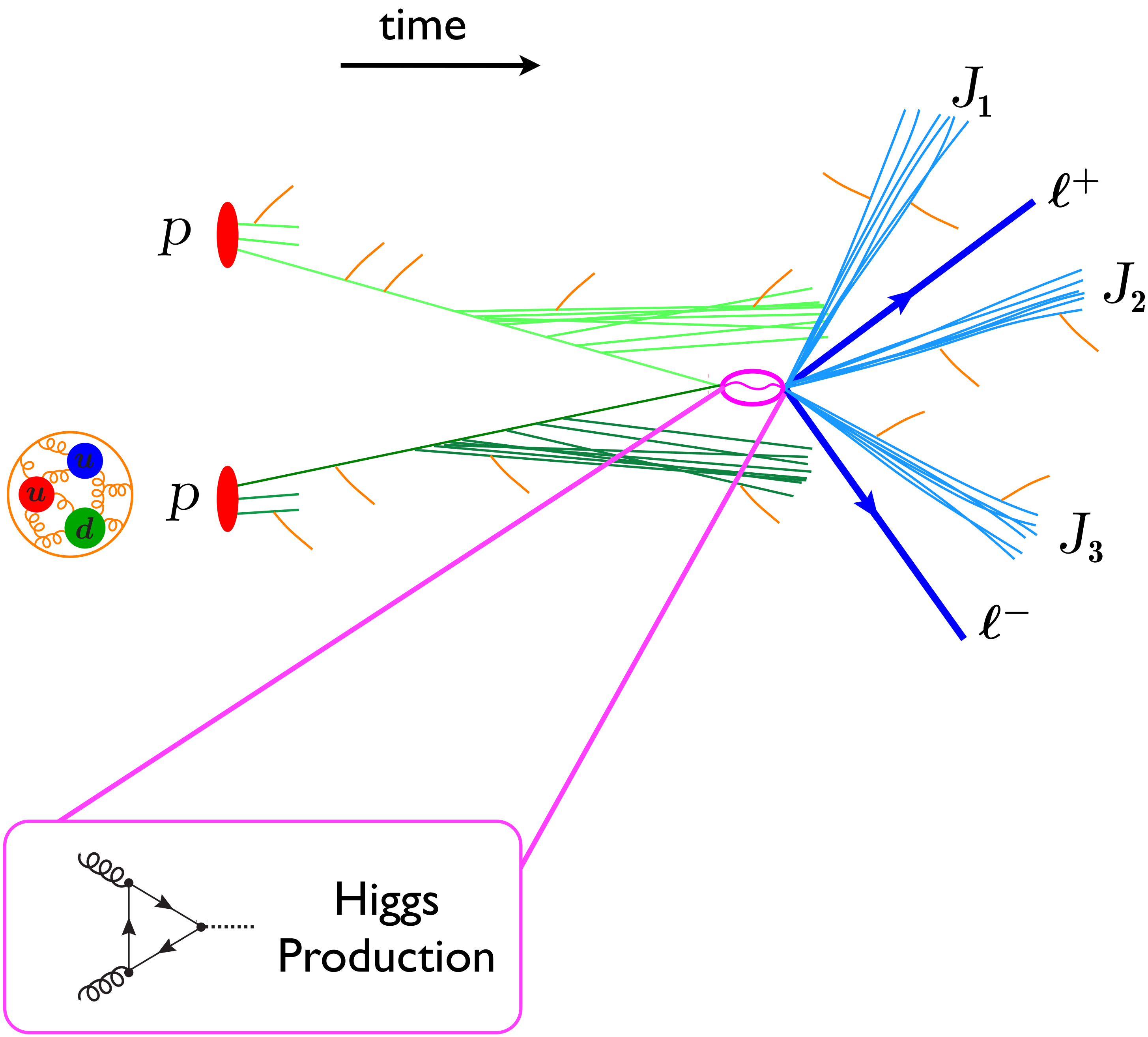
  * test MC against (higher order) resummed calculations with uncertainties

  * provide theory ingredients to improve accuracy of shower
    (as in Geneva, see talks by Bauer & Vermilion)

Exclusive Jet Production with a Hard Interaction:



Exclusive Jet Production with a Hard Interaction:



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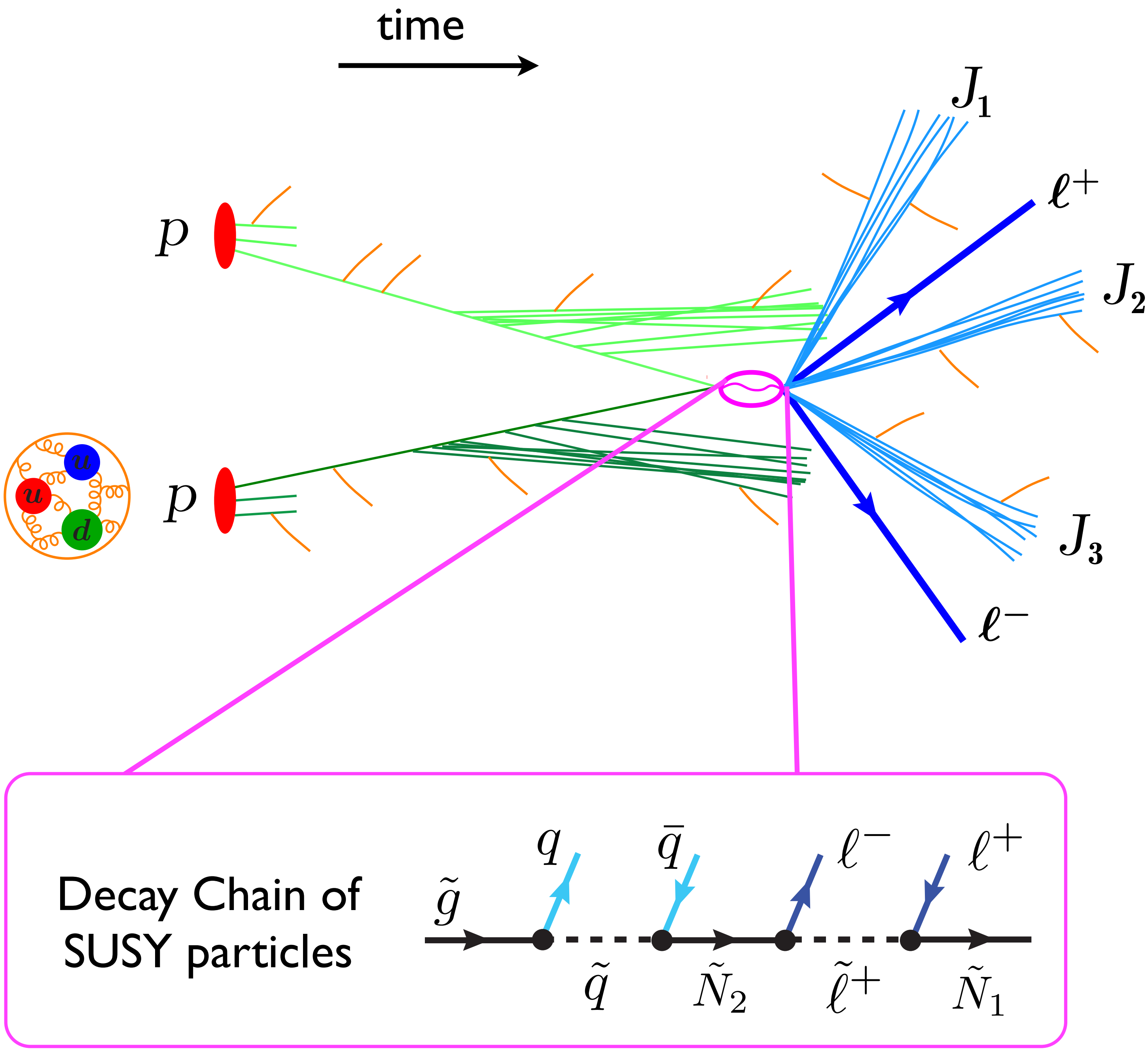
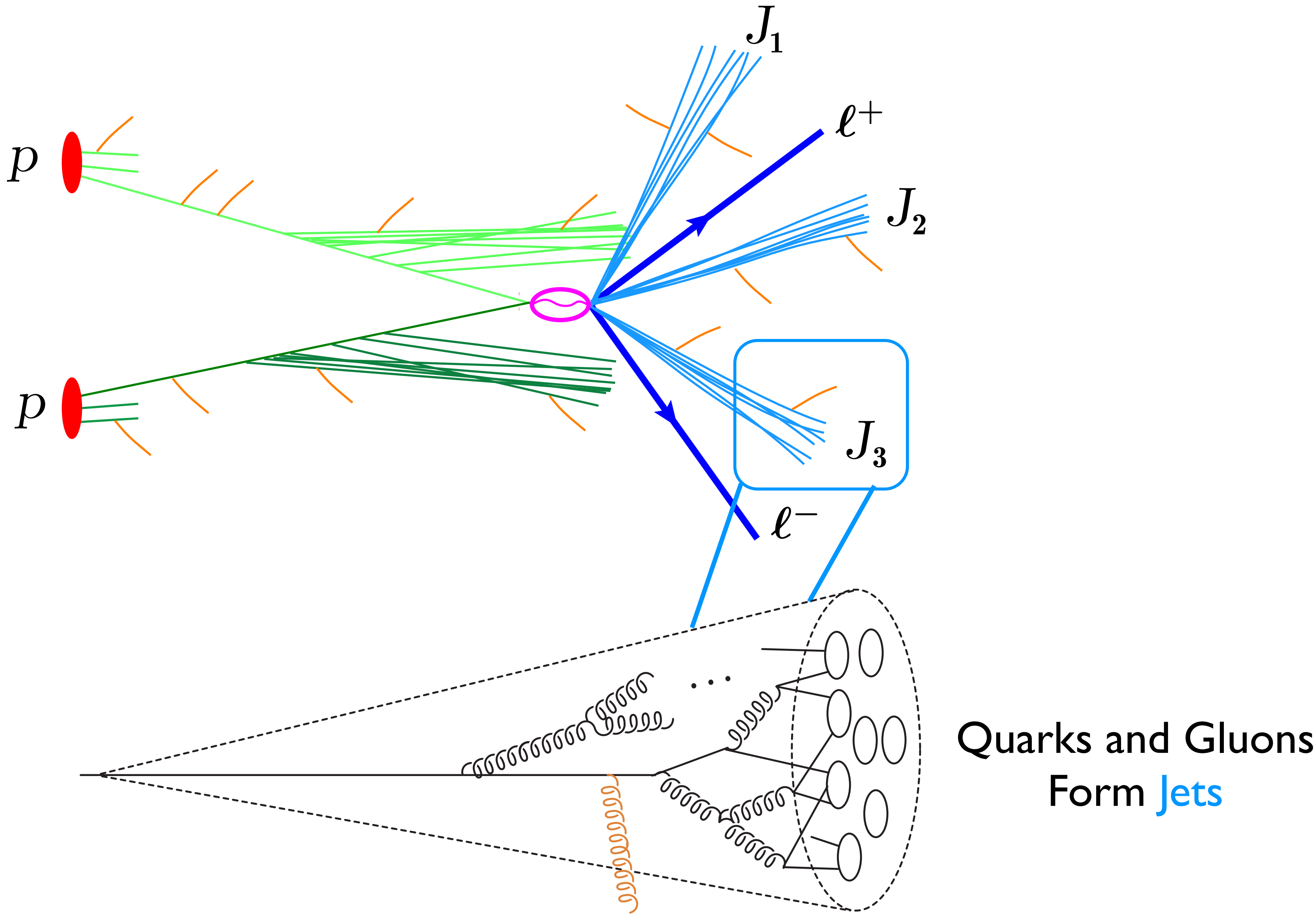
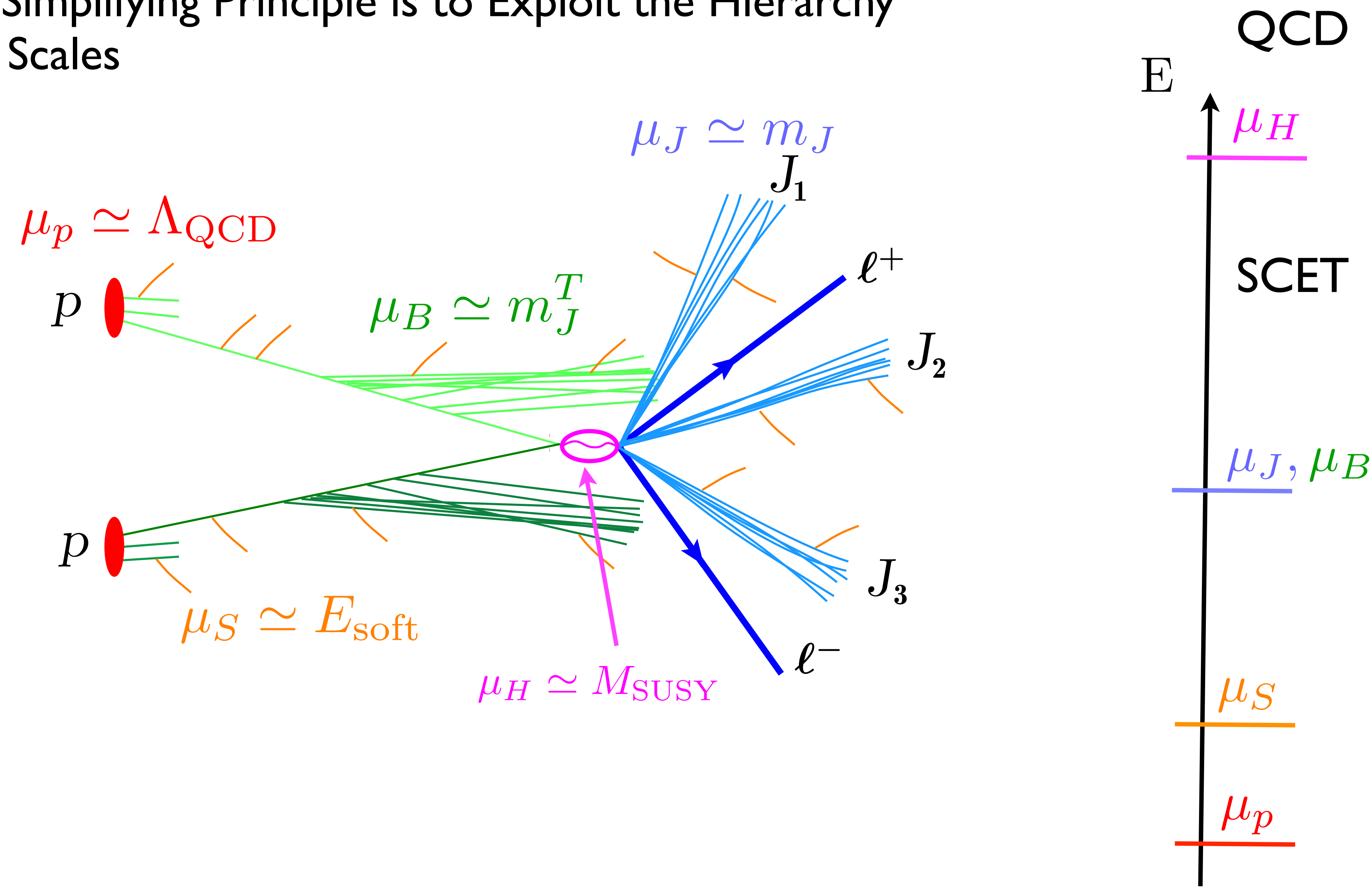


Search for New Heavy Particles at short distances

Exclusive Jet Production with a Hard Interaction:

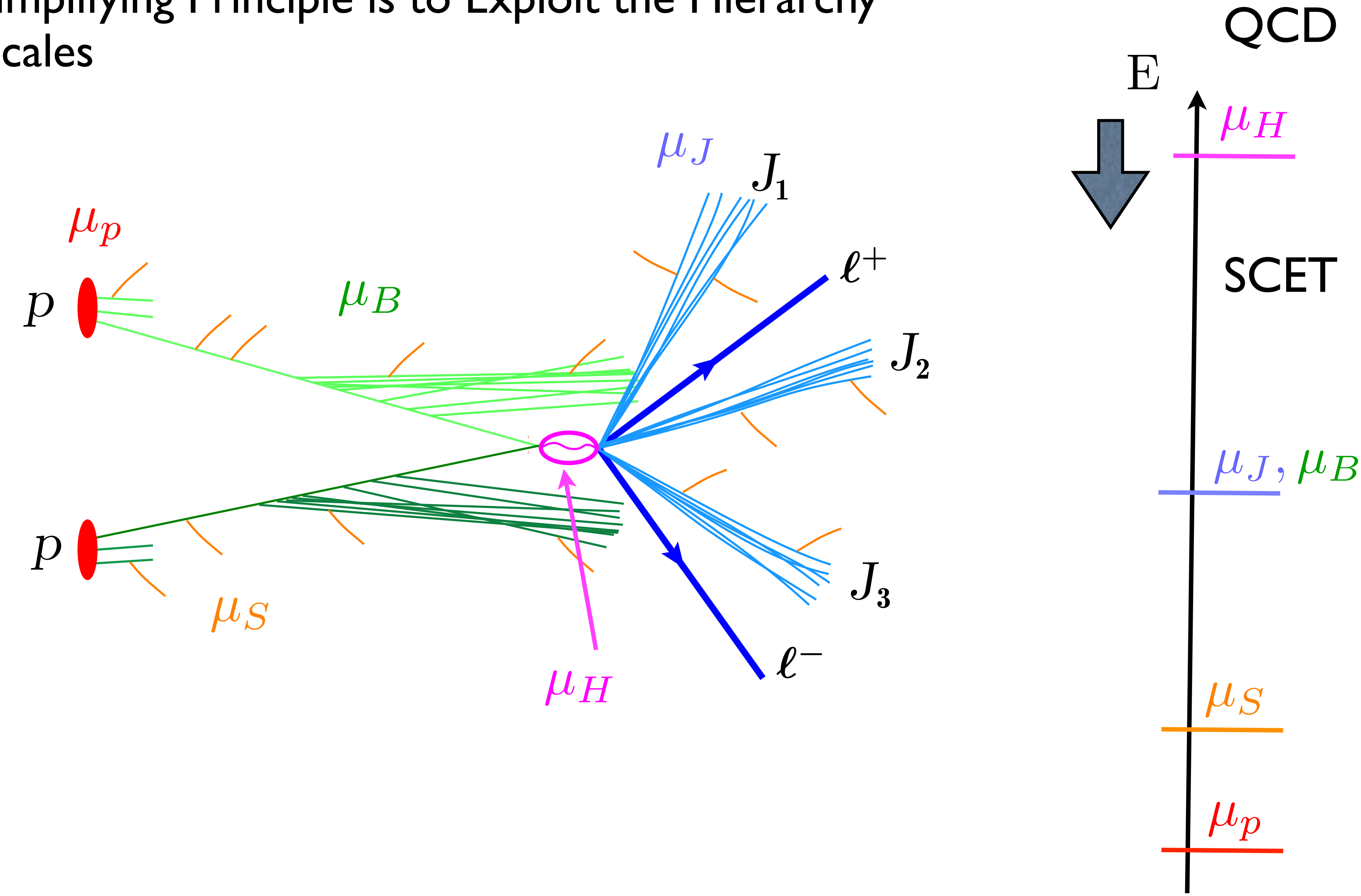


Key Simplifying Principle is to Exploit the Hierarchy of Scales



SCET = Soft-Collinear Effective Theory

Key Simplifying Principle is to Exploit the Hierarchy of Scales



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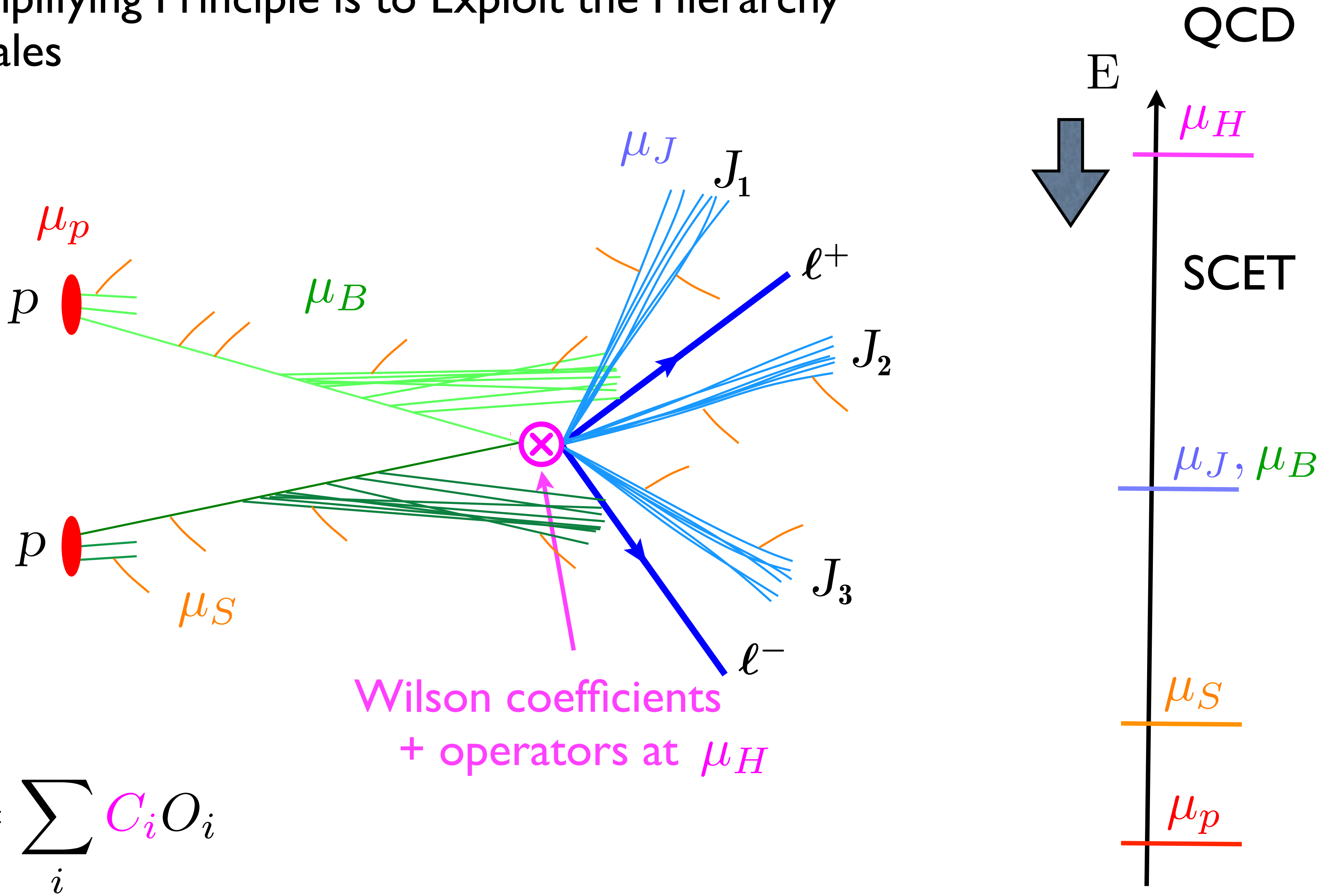
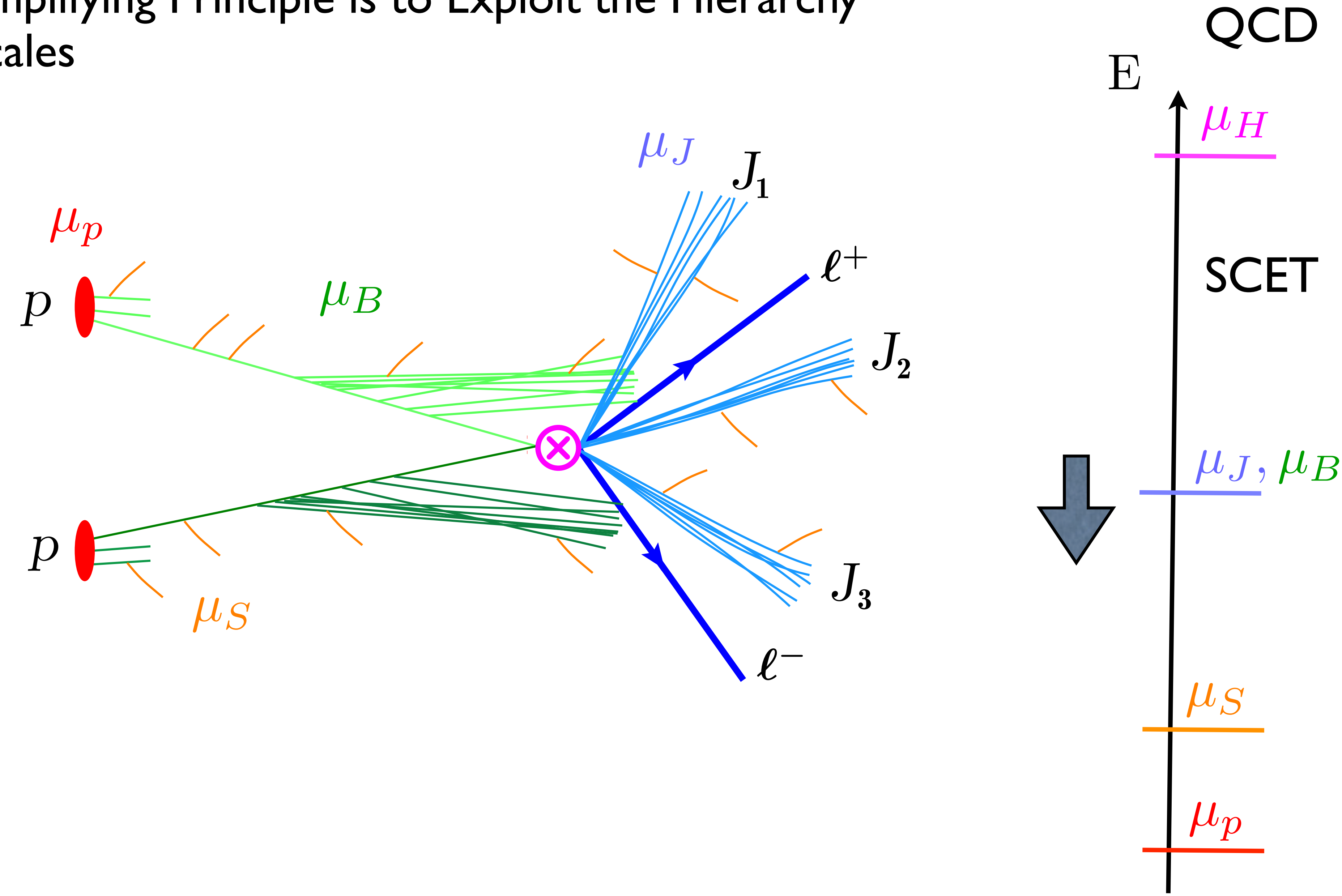
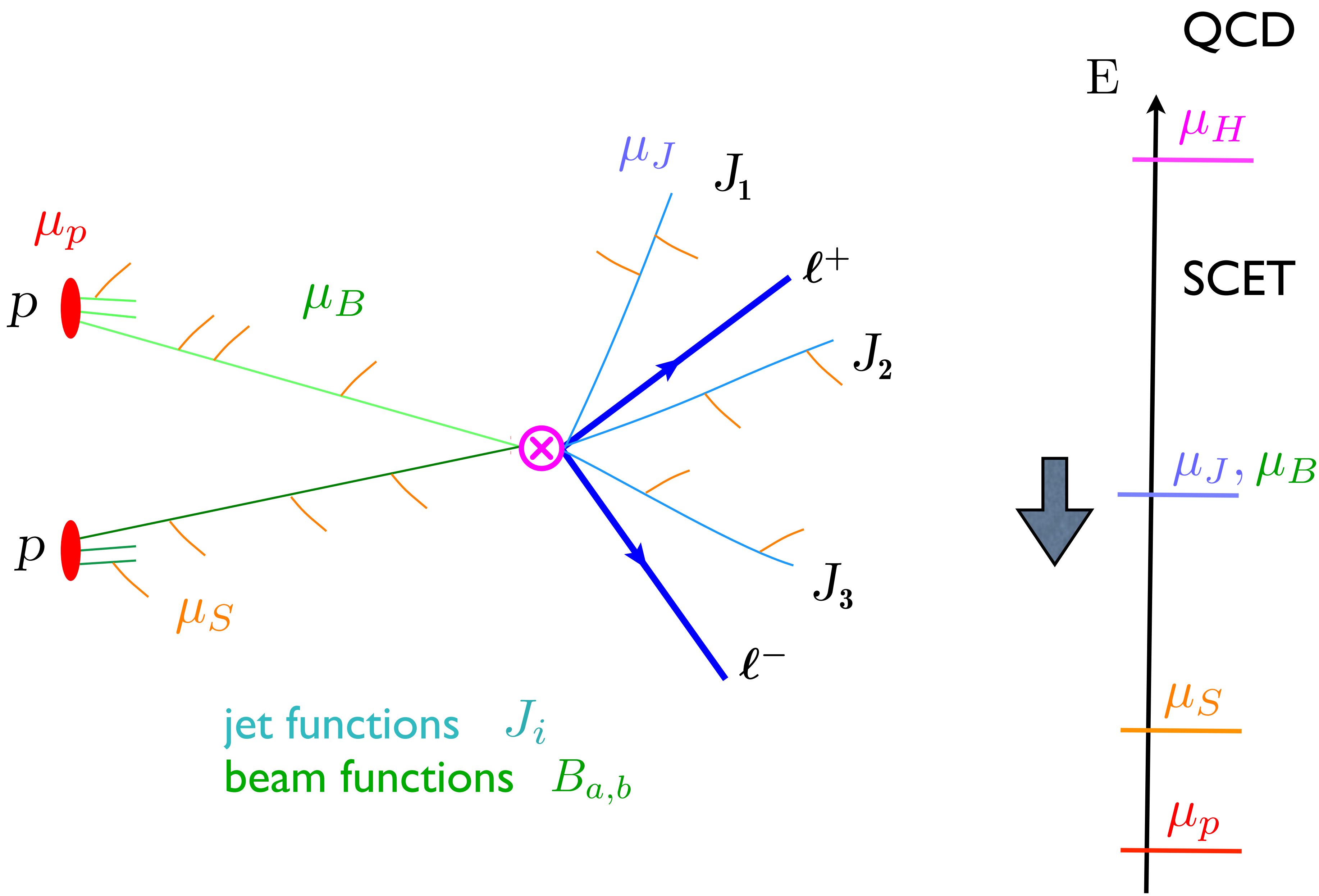


$$\mathcal{L} = \sum_i C_i O_i$$

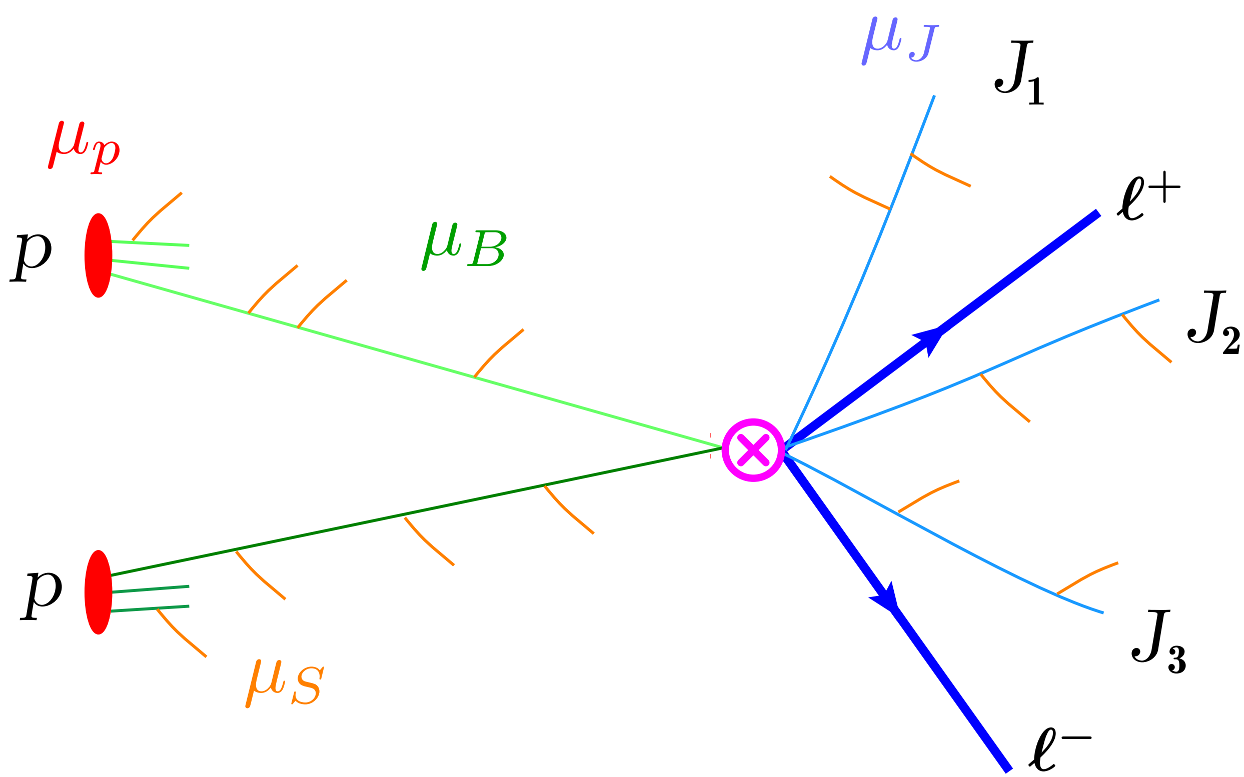
$$d\sigma = \int (\text{phase space}) \left| \sum_i C_i \langle O_i \rangle \right|^2 = \sum_j H_j \otimes (\text{longer distance dynamics})_j$$

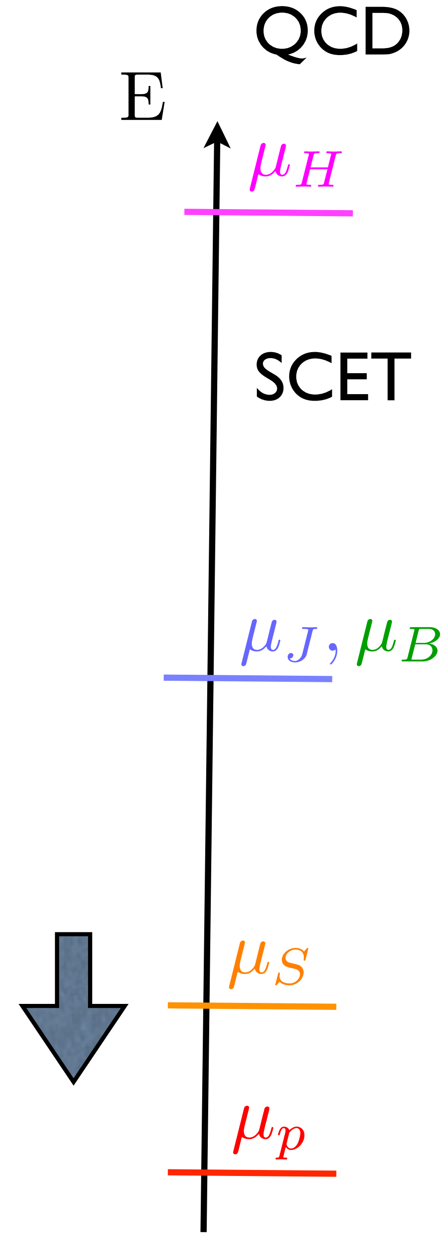
Key Simplifying Principle is to Exploit the Hierarchy of Scales





$$d\sigma = B_{a,b} \otimes H_j \otimes \prod_i J_i \otimes (\text{longer distance dynamics})$$

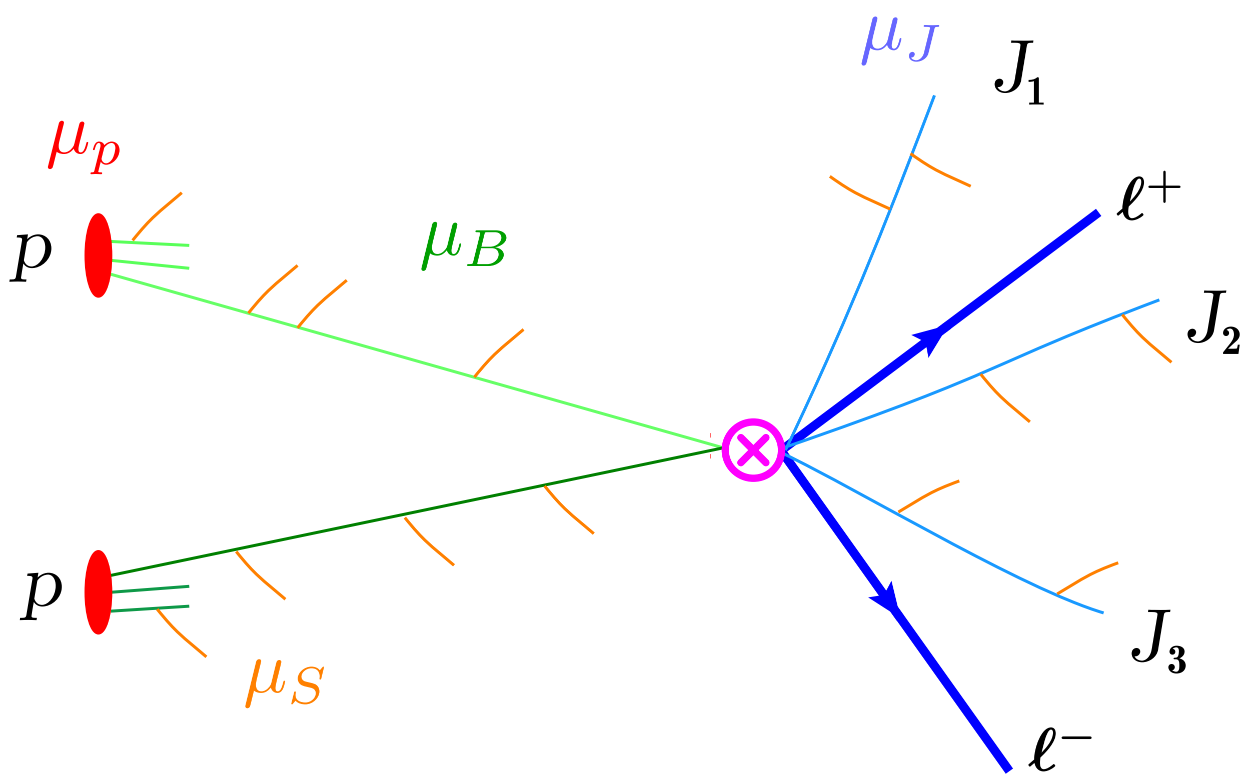


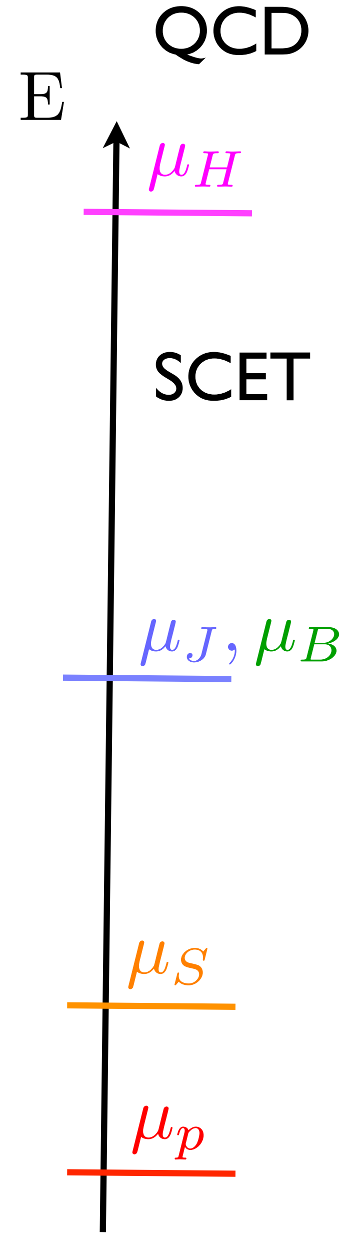


eikonal line matrix elt. for soft function S

PDFs $f_{a,b}$

Factorization: $$d\sigma = f_{a,b} \otimes \mathcal{I}_{a,b} \otimes H_j \otimes \prod_i J_i \otimes S$$





Exclusive N-jet Factorization:

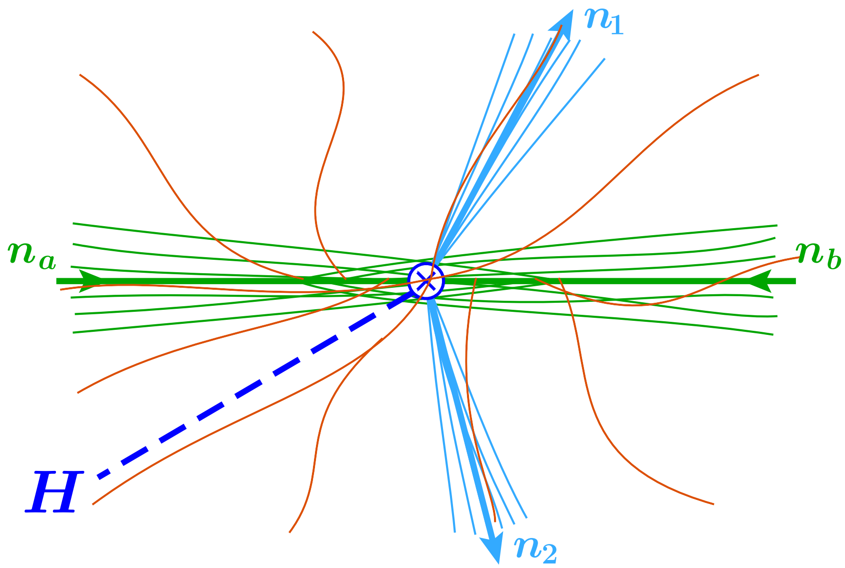
$$d\sigma = \text{PDFs} \otimes \text{ISR} \otimes \text{hard interactions} \otimes \text{FSR} \otimes \text{soft radiation}$$

$$d\sigma = f_{a,b} \otimes \mathcal{I}_{a,b} \otimes H_j \otimes \prod_i J_i \otimes S$$

Λ_{QCD} μ_B μ_H μ_J μ_S

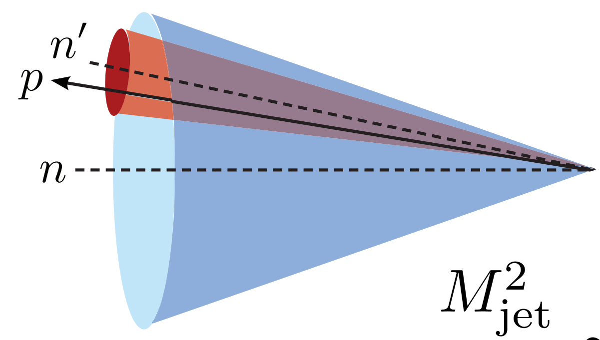
SCET energetic jets

eg. pp $\rightarrow$ H + 2 jets



Defining concepts:

- hard scale Q
- collinear sectors $\{[n_i]\}$
- power counting parameter λ



$$\frac{M_{\rm jet}^2}{E_{\rm jet}^2} \sim \lambda^2$$

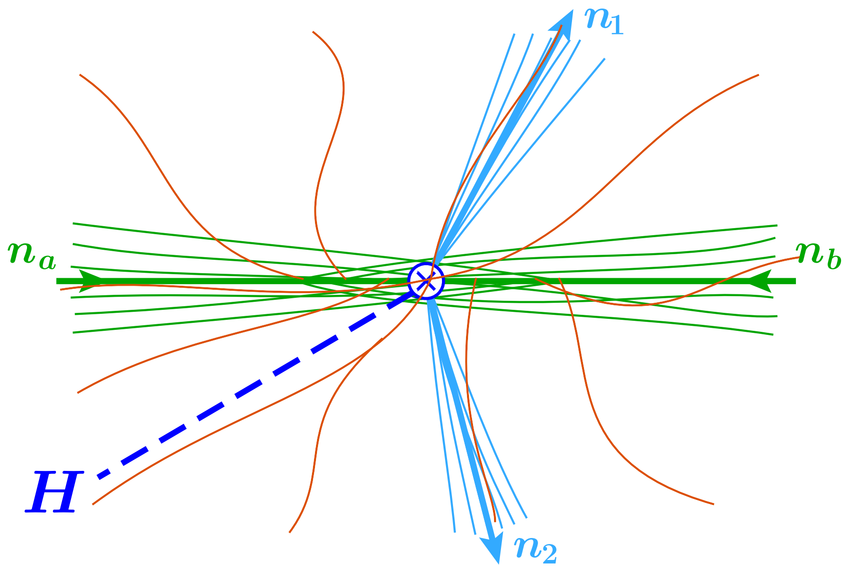
Start: determine relevant d.o.f.: collinear, soft, Coulomb?, Glauber?

(use known IR structure of QCD, test with matching calculations)

Then: derive factorization theorems without further assumptions, dominant terms require fixed order calculations for simpler objects, solve RGE to sum logs, etc

SCET   energetic jets

eg. pp $\to$ H + 2 jets



distinct collinear directions:

n_a, n_b, n_1, n_2

$n_i^\mu = (1, \hat{n}_i) \qquad n_i \cdot n_j \gg \lambda^2$

p is collinear to n_i **:**

$\bar{n}_i^\mu = (1, -\hat{n}_i)$

$$p^\mu = \frac{n_i^\mu}{2}(\bar{n}_i \cdot p) + \frac{\bar{n}_i^\mu}{2}(n_i \cdot p) + p_{\perp i}^\mu$$

$\mathcal{O}(1) \qquad \mathcal{O}(\lambda^2) \qquad \mathcal{O}(\lambda)$

Defining concepts:

- hard scale Q
- collinear sectors $\{[n_i]\}$
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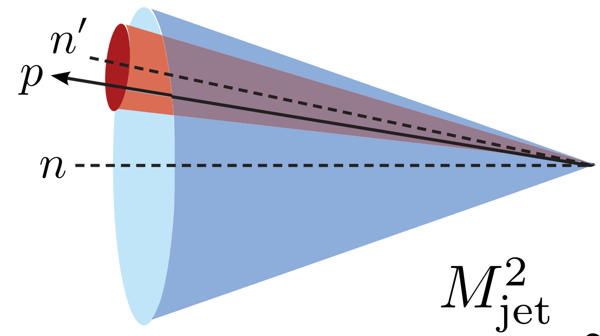


$\frac{M_{\rm jet}^2}{E_{\rm jet}^2} \sim \lambda^2$

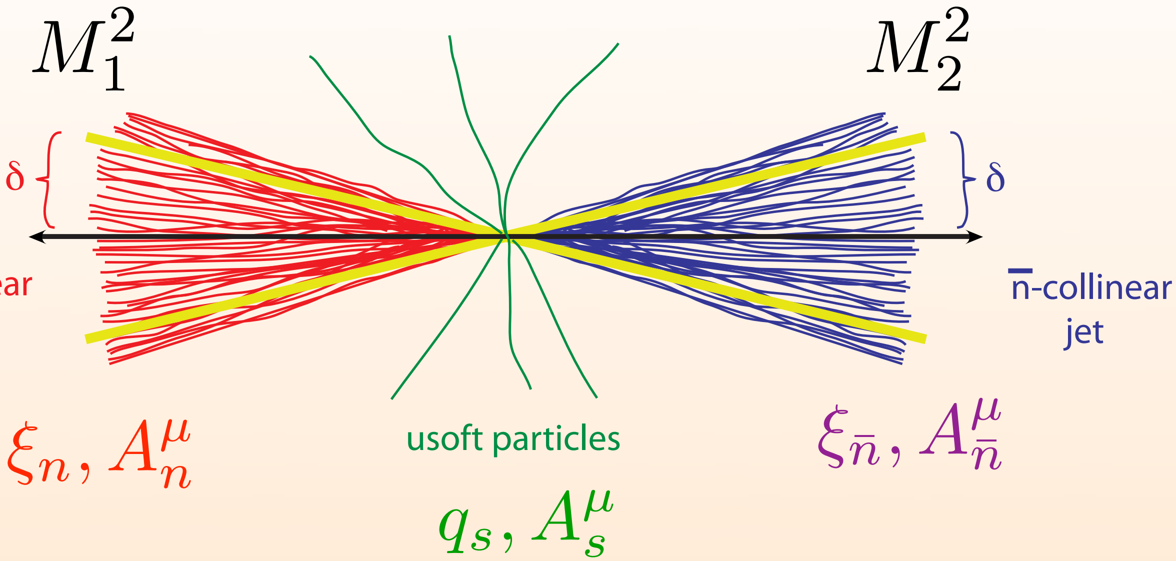
collinear fields: $\xi_{n_i}, A_{n_i}^\mu$

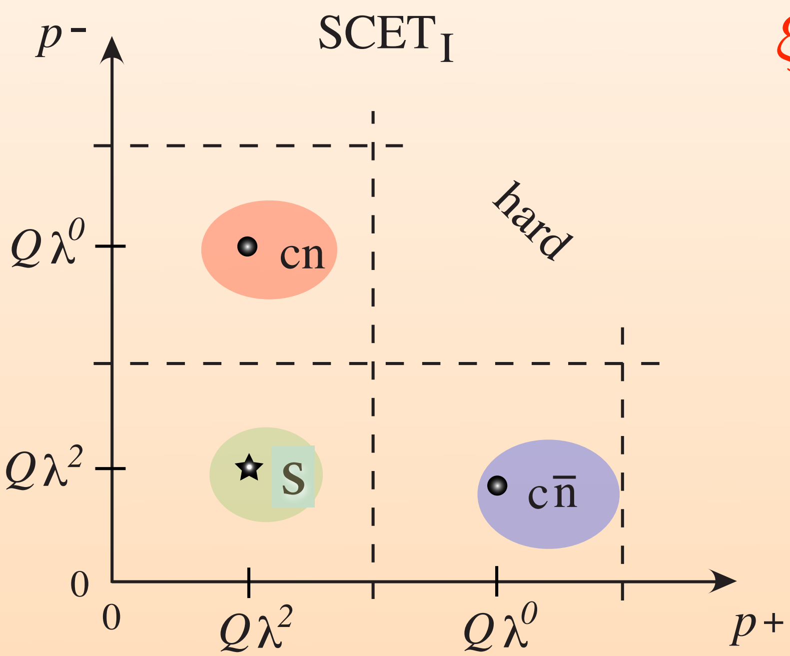
(u)soft fields: q_s, A_s^μ

$p^\mu = \mathcal{O}(\lambda^2)$ **or** $p^\mu = \mathcal{O}(\lambda)$

SCET energetic jets

eg. $e^+e^- \to 2$ jets





for thrust $\tau = 1 - T \ll 1$

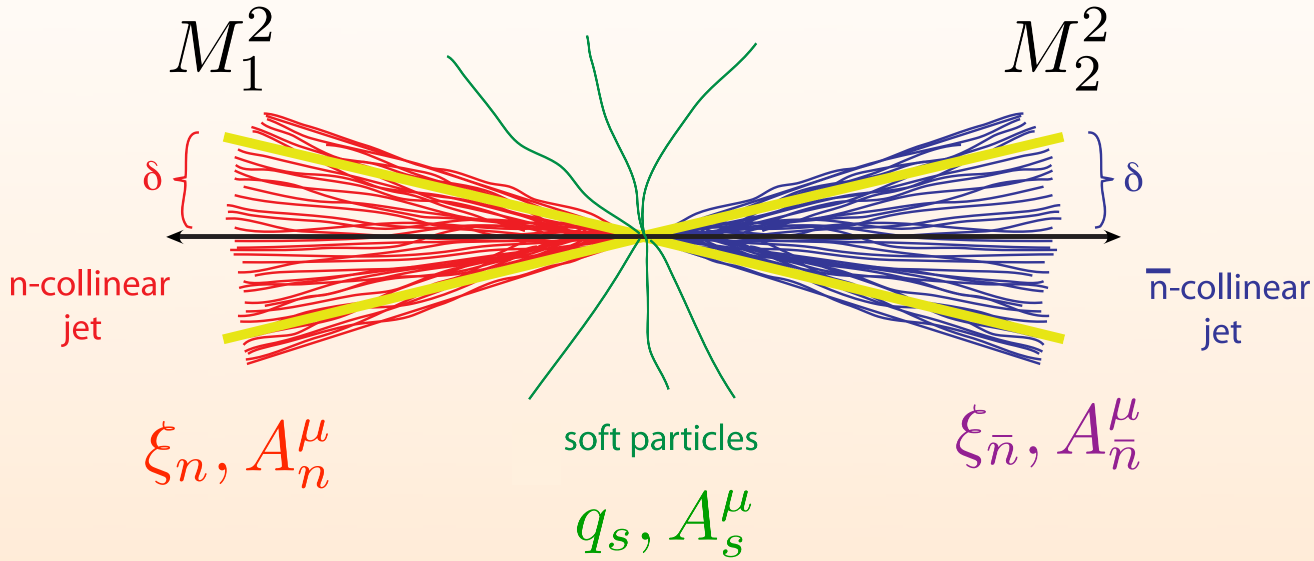
or

hemisphere jet masses $M_i^2 \ll Q^2$

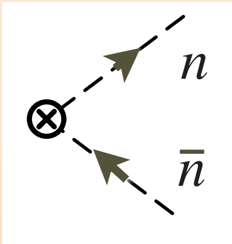
"event shapes"

SCET energetic jets

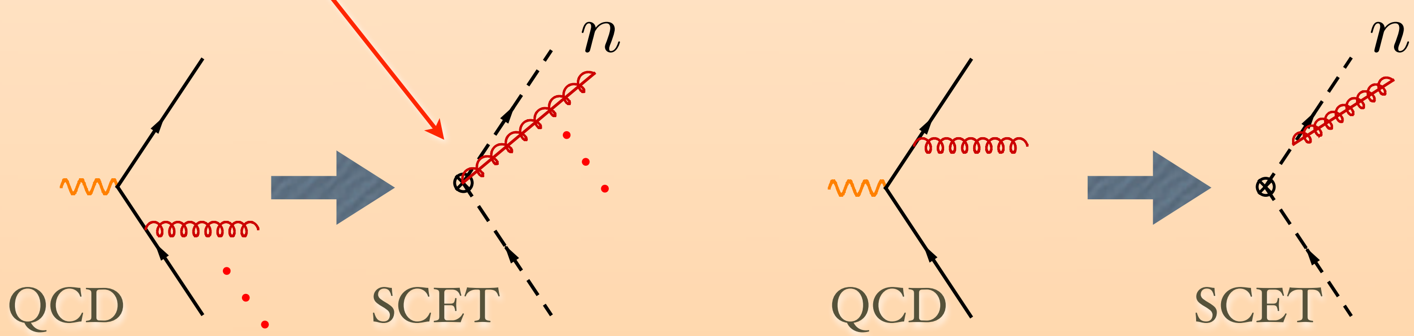
eg. $e^+e^- \to 2$ jets



Production Current:



$$\bar\psi\,\Gamma^\mu\psi \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu(W_{\bar n}^\dagger\xi_{\bar n})_{\bar\omega}$$



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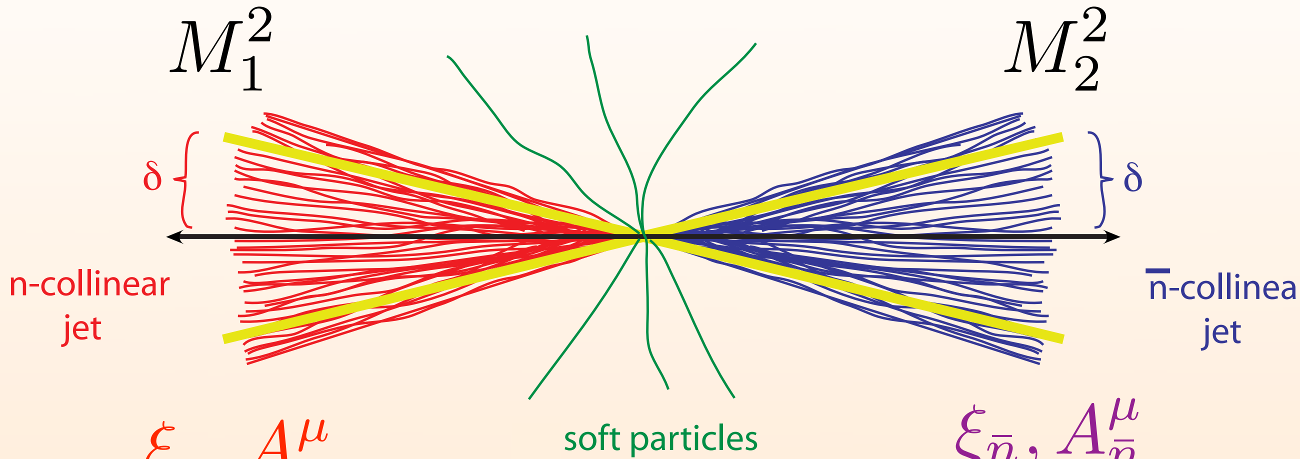
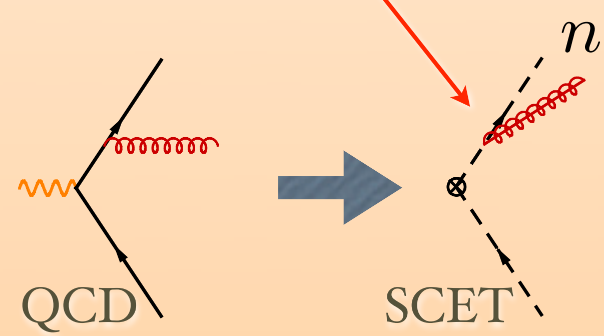


$\xi_n,\, A_n^\mu$ $\quad q_s,\, A_s^\mu$ $\quad \xi_{\bar n},\, A_{\bar n}^\mu$

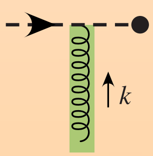
SCET Lagrangian:

$$\mathcal{L}_n^{(0)} = \bar\xi_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i\not{D}_\perp^n \frac{1}{i\bar n\cdot D_n} i\not{D}_\perp^n \Big\} \frac{\bar{\not{n}}}{2}\xi_n$$

propagator: $$\frac{i\not{n}}{2}\frac{\bar n\cdot p}{p^2+i\epsilon} = \frac{i\not{n}}{2}\frac{1}{n\cdot p - \frac{\vec p_\perp^{\,2}}{\bar n\cdot p} + i\epsilon\,\mathrm{sign}(\bar n\cdot p)}$$



eikonal softs:



$\frac{i}{n\cdot k + i\epsilon}$

$$\xi_n \to Y\xi_n$$
$$A_n \to Y A_n Y^\dagger$$
$$Y(x) = P\exp\Big(ig\int_{-\infty}^0 ds\, n\cdot A_{us}(x+ns)\Big)$$

SCET energetic jets

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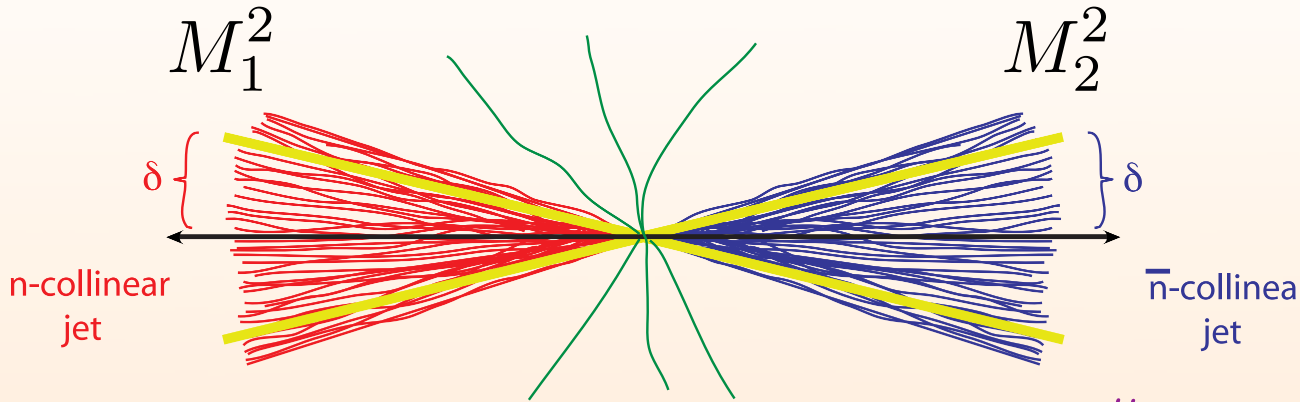
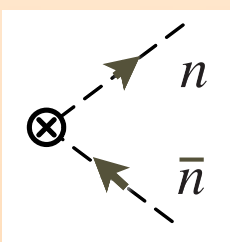
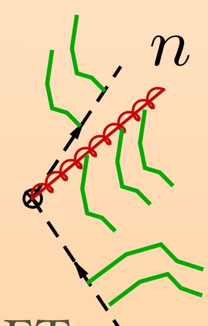


ξ_n, A_n^μ $\xi_{\bar n}, A_{\bar n}^\mu$ q_s, A_s^μ

Production Current: $Q \gg \Delta$



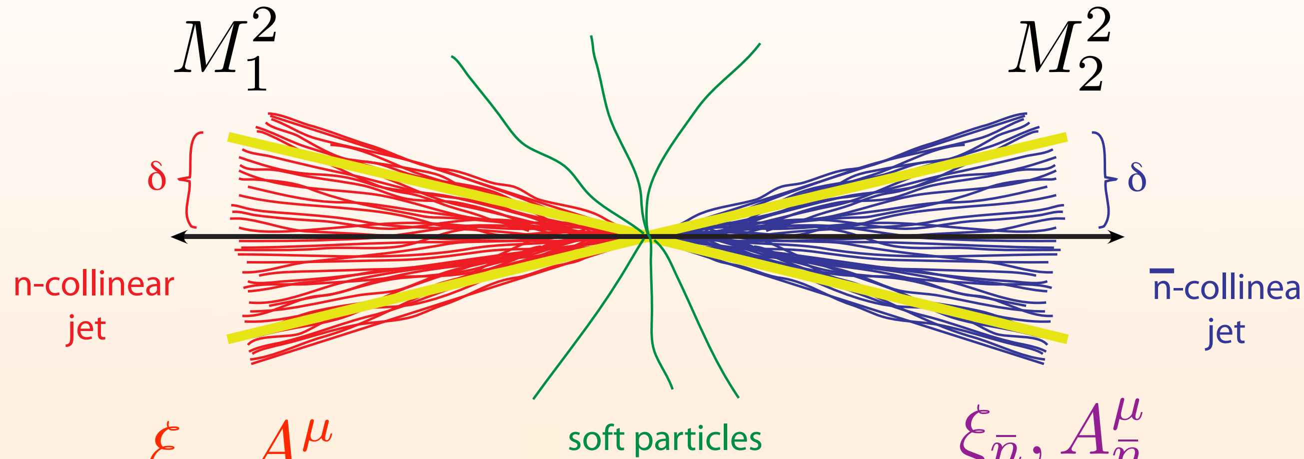
$$\bar\psi\,\Gamma^\mu\psi \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} \to (\bar\xi_n W_n)_\omega\, \underbrace{Y_n^\dagger \Gamma^\mu Y_{\bar n}}\, \underbrace{(W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}}_{\chi_{\bar n,\bar\omega}}$$



c. gauge invariant "parton" field

SCET energetic jets

eg. $e^+e^- \to 2$ jets



$\xi_n,\, A_n^\mu$ $\quad q_s,\, A_s^\mu$ $\quad \xi_{\bar n},\, A_{\bar n}^\mu$

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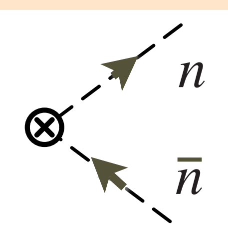
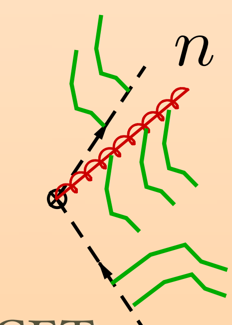


$\mathcal{L}_n^{(0)}$ $\quad \mathcal{L}_s^{(0)}$ $\quad \mathcal{L}_{\bar n}^{(0)}$

$$\bar\psi\, \Gamma^\mu \psi \to (\bar\xi_n W_n)_\omega\, \Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} \to (\bar\xi_n W_n)_\omega\; Y_n^\dagger \Gamma^\mu Y_{\bar n}\, (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$

$\chi_{\bar n, \bar\omega}$



SCET

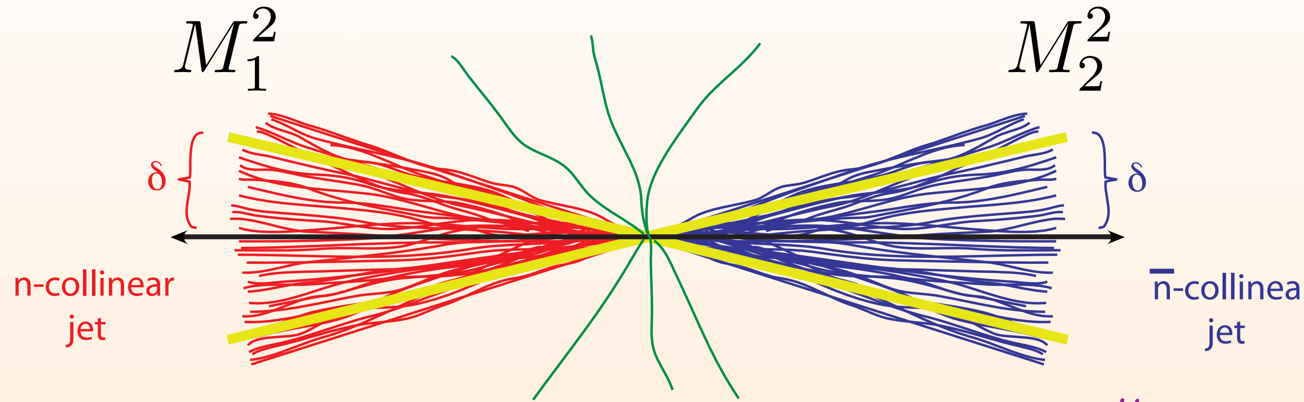
c. gauge invariant "parton" field

SCET energetic jets

eg. $e^+e^- \to 2$ jets

$$M_i^2 \sim \Delta^2$$

$$\Lambda^2 \ll \Delta^2 \ll Q^2$$



ξ_n, A_n^μ $\quad q_s, A_s^\mu$ $\quad \xi_{\bar n}, A_{\bar n}^\mu$

Factorization:

$$|X\rangle = |X_n X_{\bar n} X_s\rangle$$

$$\sigma = K_0 \sum_{\vec n} \sum^{res.}_{X_n X_{\bar n} X_s} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \langle 0|\overline{Y}_{\bar n}\, Y_n|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger|0\rangle$$

$$\times\ |C(Q,\mu)|^2 \langle 0|\hat{\bar{n}}\!\!\!/\,\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle \langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\hat{n}\!\!\!/\,\chi_{\bar n,\bar\omega}|0\rangle$$

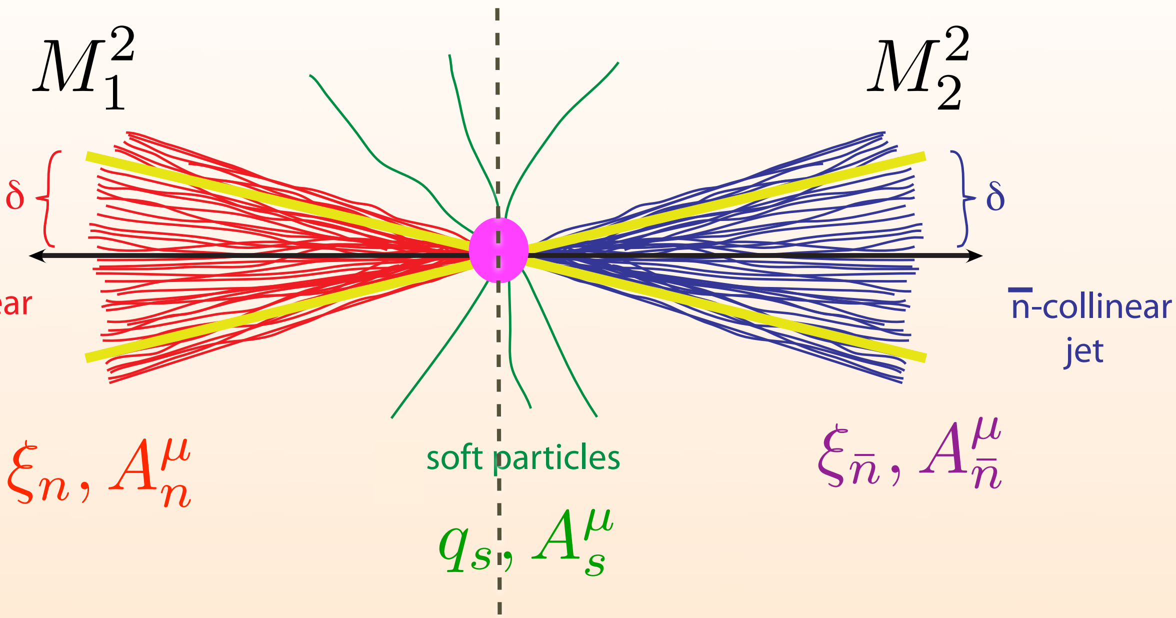
all-orders in α_s

SCET energetic jets

eg. $e^+e^- \to 2$ jets

$M_i^2 \sim \Delta^2$

$\Lambda^2 \ll \Delta^2 \ll Q^2$



Factorization:

$\mu_H \sim Q$ $\mu_J \sim M_i$ $\mu_S \sim \ell^\pm$

$$\frac{d^2\sigma}{dM_1^2 dM_2^2} = \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^- J_n\big(M_1^2 - Q\ell^+, \mu\big) J_{\bar n}\big(M_2^2 - Q\ell^-, \mu\big) S(\ell^+, \ell^-, \mu)$$

Hard Function Jet Functions Soft Function

SCET energetic jets

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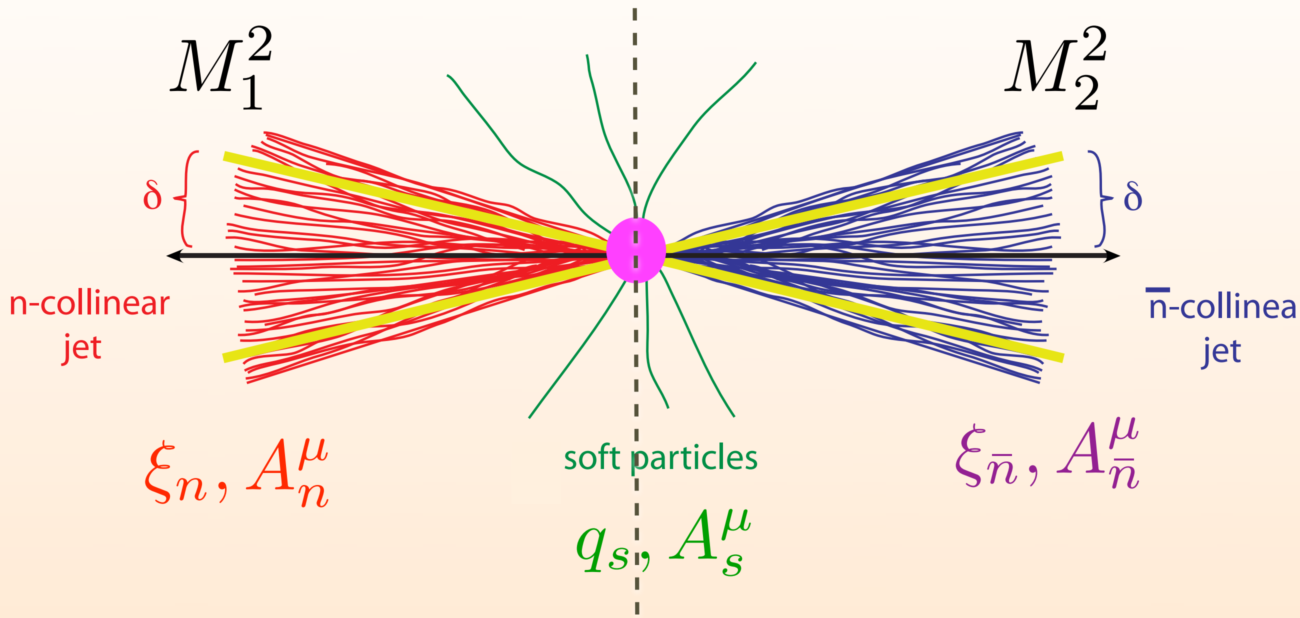
$M_i^2 \sim \Delta^2$

$\Lambda^2 \ll \Delta^2 \ll Q^2$



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state of art is

$N^3LL + \mathcal{O}(\alpha_s^3)$

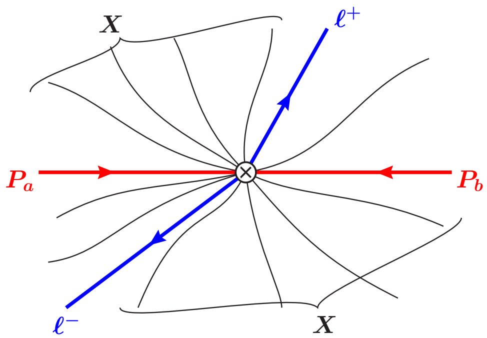
Becher, Schwartz
Chien, Schwartz
Abbate et al

using

Gehrmann et al
Weinzierl

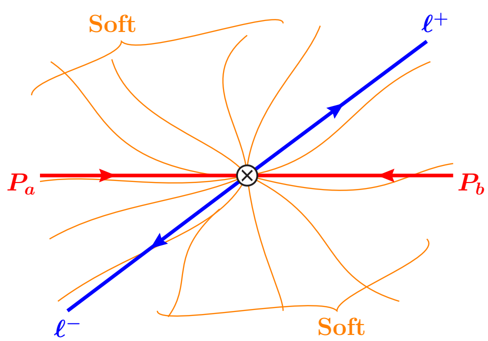
Factorization depends on choice of Measurement

eg. Drell-Yan $\quad pp \to X\ell^+\ell^-$ or $pp \to X(H \to W^+W^-)$



Inclusive: X=hard

$$\frac{d\sigma}{dq^2 dY} = \sum_{ij} H_{ij}^{\rm incl} \otimes f_i(\xi_a) f_j(\xi_b)$$

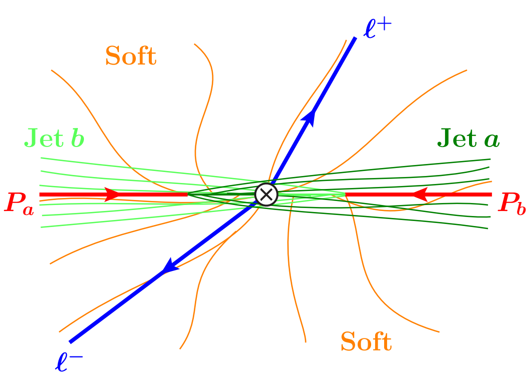


Threshold: X=soft

$$\frac{d\sigma}{dq^2} = \sum_{(ij)} H_{(ij)} S_{\rm thr} \otimes f_i(\xi_a) f_j(\xi_b)$$

partonic threshold

large double logs $\dfrac{\alpha_s^k \ln^{2k-1}(1-z)}{1-z}$ $\qquad z = \dfrac{q^2}{\xi_a \xi_b E_{\rm cm}^2} \to 1$



0-Jets: X= collinear & soft

large double logs $\dfrac{\alpha_s^k \ln^{2k-1}(t/Q^2)}{t}$

where $t \ll Q^2$ implements a Jet-Veto

IS, Tackmann, Waalewijn

a factorization friendly jet veto variable: Beam Thrust event shape

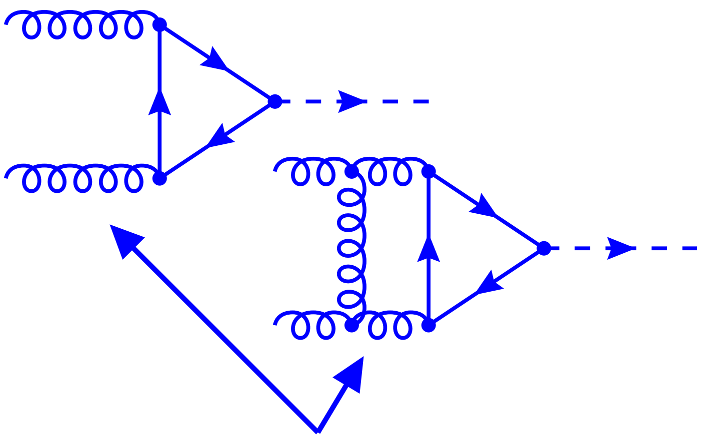
$$\mathcal{T}_{\rm cm} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|) \quad = \mathcal{T}_{\rm cm}^a + \mathcal{T}_{\rm cm}^b + \mathcal{T}_{\rm cm}^{\rm soft}$$

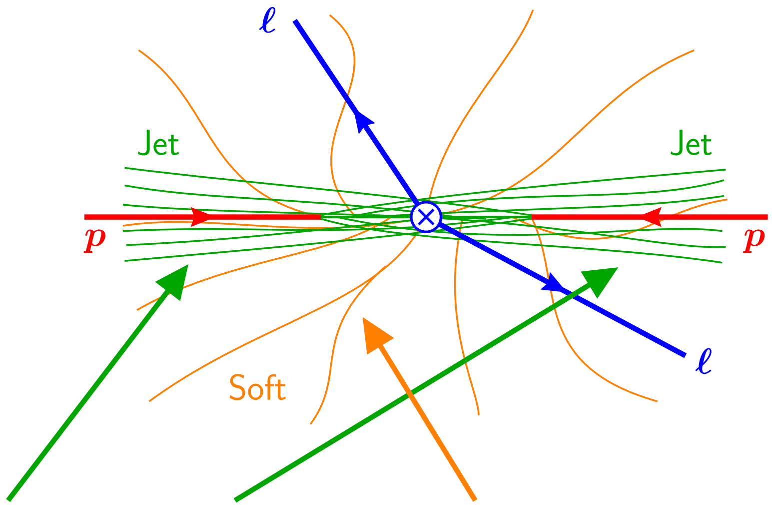
linear in momentum

$$\mathcal{T}_{\rm cm} \leq \mathcal{T}^{\rm cut}$$

implements a jet-veto

$pp \to H + 0\,\text{jets}$





Berger et al.

$$\frac{d\sigma^s}{d\mathcal{T}_{\rm cm}} = H_{gg}(\mu) \int dY\, dt_a\, dt_b\, B_g(t_a,\mu) B_g(t_b,\mu) S_B^{gg}\Big(\mathcal{T}_{\rm cm} - \frac{e^{-Y} t_a + e^{Y} t_b}{m_H}, \mu\Big)$$

inclusive gluon beam functions

$$B_g(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{gj}\Big(t, \frac{x}{\xi}, \mu\Big) f_j(\xi,\mu)$$

Use beam thrust to describe generic ingredients for cross section predictions

- resummation & evolution
- singular & nonsingular contributions
- profile functions: merging onto fixed order results
- perturbative uncertainties
- power corrections

Resummation

- evolution kernels U_X sum logs

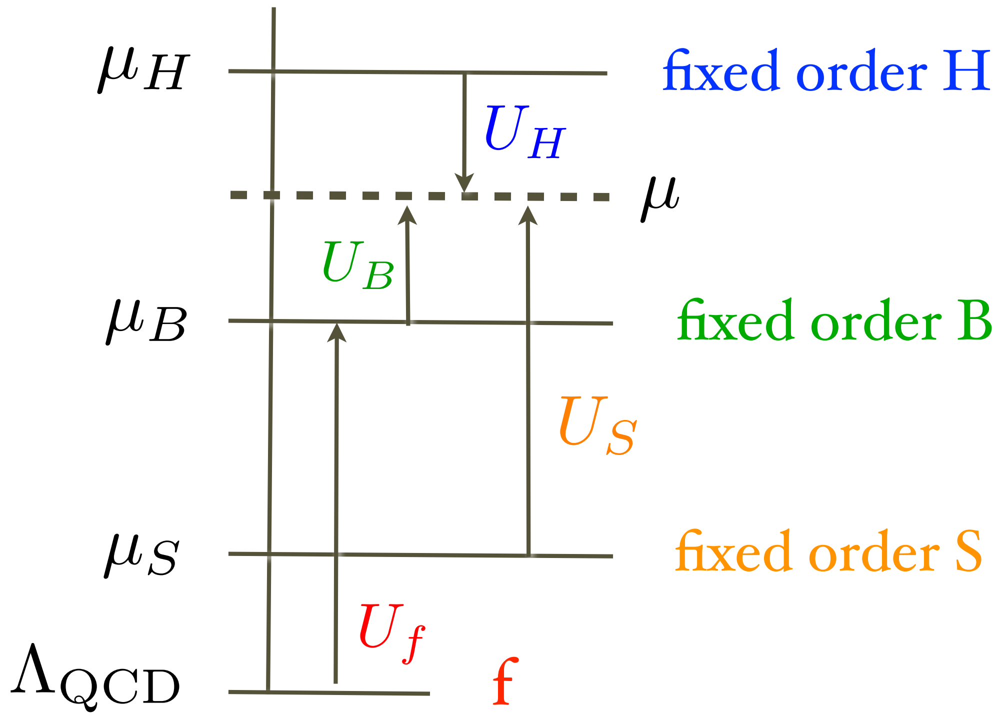
  $|\mu_H|^2 \simeq m_H^2 \quad \mu_B^2 \simeq m_H \mathcal{T}_{\rm cm}$

  $\mu_S^2 \simeq \mathcal{T}_{\rm cm}^2$

  determined after doing integrals*

- fixed order expansions at μ_H, μ_B, μ_S

- fixed order scale dependence cancels to the order one is working



$*$ ensures only perturbative anom.dim. are used & no Landau poles encountered

$$\frac{d\sigma^s}{d\mathcal{T}_{\rm cm}} = H_{gg}(\mu) \int dY\, dt_a\, dt_b\, B_g(t_a,\mu) B_g(t_b,\mu) S_B^{gg}\Big(\mathcal{T}_{\rm cm} - \frac{e^{-Y} t_a + e^{Y} t_b}{m_H}, \mu\Big)$$

$$\ln^2 \frac{\mathcal{T}_{\rm cm}}{m_H} = 2\ln^2 \frac{m_H}{\mu} \quad - \quad \ln^2 \frac{\mathcal{T}_{\rm cm} m_H}{\mu^2} \quad + \quad 2\ln^2 \frac{\mathcal{T}_{\rm cm}}{\mu}$$

Resummation

- evolution kernels U_X sum logs

  $|\mu_H|^2 \simeq m_H^2 \quad \mu_B^2 \simeq m_H \mathcal{T}_{\rm cm}$

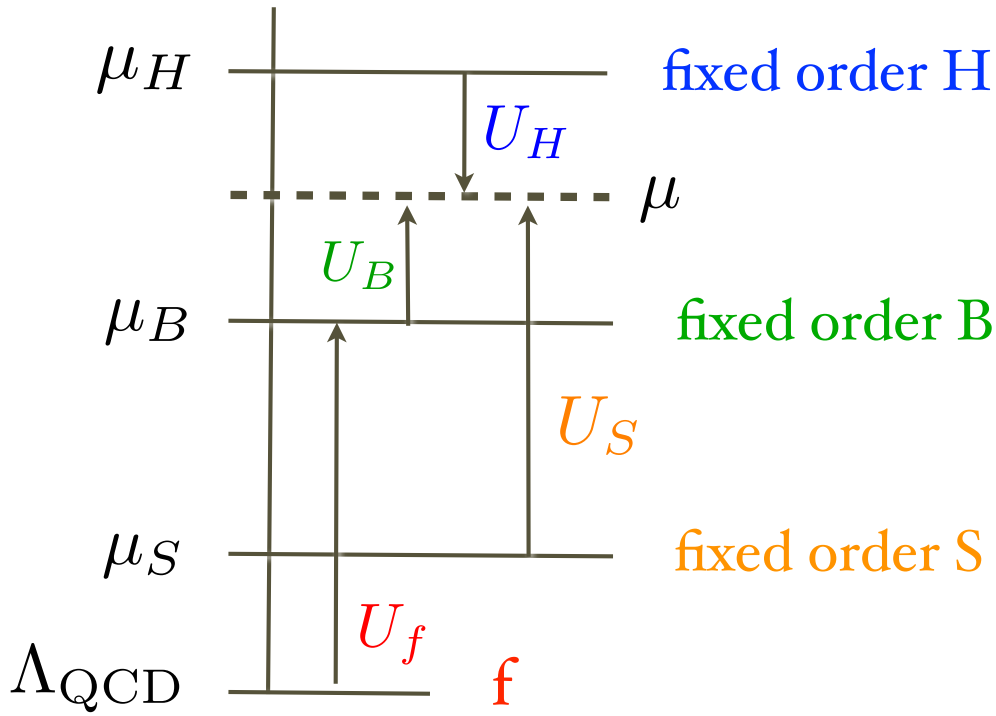
  $\mu_S^2 \simeq \mathcal{T}_{\rm cm}^2$

  determined after doing integrals*

- fixed order expansions

  at μ_H, μ_B, μ_S

- fixed order scale dependence cancels to the order one is working



* ensures only perturbative anom.dim. are used & no Landau poles encountered

$$\frac{d\sigma^s}{d\mathcal{T}_{\rm cm}} = \sigma_0 H_{gg}(m_t, m_H^2, \mu_H)\, U_H(m_H^2, \mu_H, \mu) \int dY \int dt_a\, dt_b$$

$$\times \int dt'_a\, B_g(t_a - t'_a, x_a, \mu_B)\, U_B^g(t'_a, \mu_B, \mu) \int dt'_b\, B_g(t_b - t'_b, x_b, \mu_B)\, U_B^g(t'_b, \mu_B, \mu)$$

$$\times \int dk\, S_B^{gg}\Big(\mathcal{T}_{\rm cm} - \frac{e^{-Y} t_a + e^{Y} t_b}{m_H} - k, \mu_S\Big)\, U_S(k, \mu_S, \mu)$$

$$B_g(t, x, \mu_B) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{gj}\Big(t, \frac{x}{\xi}, \mu_B\Big) \sum_{j'} \int d\xi'\, U_f^{jj'}(\xi, \xi', \mu_B, \mu_\Lambda) f_{j'}(\xi', \mu_\Lambda)$$

Resummation is in exponent:

counting is simplest in Fourier space $y = \mathrm{FT}[\mathcal{T}_{\mathrm{cm}}/m_H]$

$$\ln \frac{d\sigma}{dy} = \ln y(\alpha_s \ln y)^k + (\alpha_s \ln y)^k + \alpha_s(\alpha_s \ln y)^k + \alpha_s^2(\alpha_s \ln y)^k + \ldots$$

LL NLL NNLL $\mathrm{N^3LL}$

	matching (singular)	nonsingular	γ_x	Γ_{cusp}	β	PDF
LO	LO	LO	-	-	1-loop	LO
NLO	NLO	NLO	-	-	2-loop	NLO
NNLO	NNLO	NNLO	-	-	3-loop	NNLO
LL	LO	-	-	1-loop	1-loop	LO
NLL	LO	-	1-loop	2-loop	2-loop	LO
NNLL	NLO	-	2-loop	3-loop	3-loop	NLO
NLL′+NLO	NLO	NLO	1-loop	2-loop	2-loop	NLO
NNLL+NNLO	(N)NLO	NNLO	2-loop	3-loop	3-loop	NNLO
NNLL′+NNLO	NNLO	NNLO	2-loop	3-loop	3-loop	NNLO
$\mathrm{N^3LL}$+NNLO	NNLO	NNLO	3-loop	4-loop	4-loop	NNLO

Resummation is in exponent:

counting is simplest in Fourier space $\qquad y = \mathrm{FT}[\mathcal{T}_{\rm cm}/m_H]$

$$\ln \frac{d\sigma}{dy} = \ln y(\alpha_s \ln y)^k + (\alpha_s \ln y)^k + \alpha_s(\alpha_s \ln y)^k + \alpha_s^2(\alpha_s \ln y)^k + \ldots$$

	LL	NLL	NNLL	$\mathrm{N^3LL}$

	LO	NLO	NNLO	$\mathrm{N^3LO}$		
$\sigma(\mathcal{T}^{\rm cut}) =$	1	$+\alpha_s L^2$	$+\alpha_s^2 L^4$	$+\alpha_s^3 L^6$	$+\ldots$	LL
		$+\alpha_s L$	$+\alpha_s^2 L^3$	$+\alpha_s^3 L^5$	$+\ldots$	NLL
		$+\alpha_s$	$+\alpha_s^2 L^2$	$+\alpha_s^3 L^4$	$+\ldots$	NNLL
			$+\alpha_s^2 L$	$+\alpha_s^3 L^3$	$+\ldots$	NNLL
			$+\alpha_s^2$	$+\alpha_s^3 L^2$	$+\ldots$	$\mathrm{N^3LL}$
				$+\alpha_s^3 L$	$+\ldots$	$\mathrm{N^3LL}$
				$+\alpha_s^3$	$+\ldots$	

$$L = \ln(m_H^2/\mathcal{T}_{\rm cut}^2)$$

Singular and Nonsingular

$\mathrm{NNLL} + \mathrm{NNLO}$

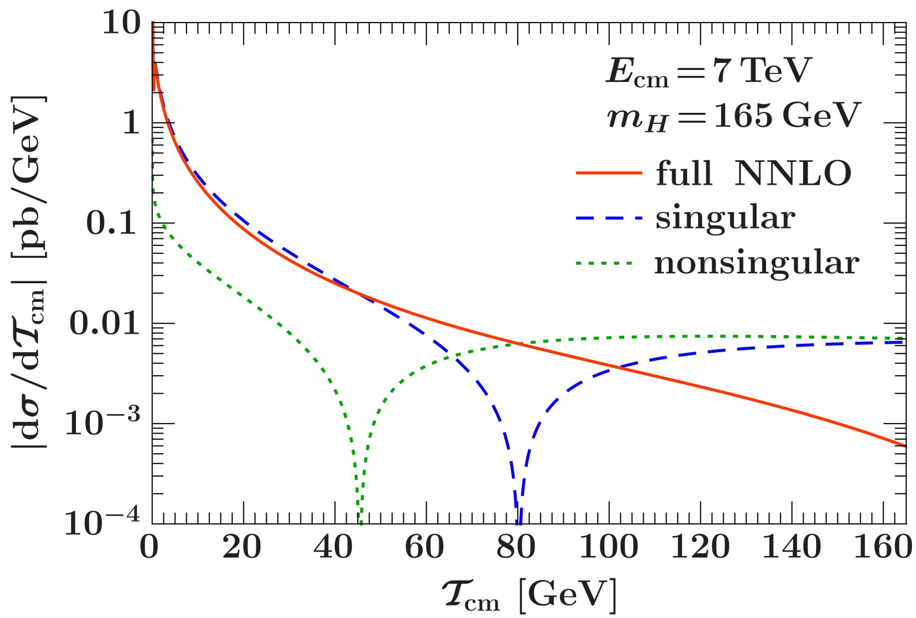
$$\underbrace{\frac{d\sigma}{d\tau} = C^{-1}\delta(\tau) + \sum_k C^k \left[\frac{\ln^k \tau}{\tau}\right]_+}_{} + \underbrace{\frac{d\sigma^{ns}}{d\tau}}_{} \qquad \text{with} \quad \tau = \frac{\mathcal{T}_{cm}}{m_H}$$

singular,
dominate for $\tau \ll 1$.
From factorization thm.

nonsingular,
suppressed by $\mathcal{O}(\tau)$
From fixed order matching

$$\sigma^{ns,NLO}(\tau^{cut}) = \sigma^{NLO}(\tau^{cut}) - \sigma^{s,NNLL}(\tau^{cut})\big|_{NLO}$$
$$\sigma^{res,NNLO}(\tau^{cut}) = \sigma^{NNLO}(\tau^{cut}) - \sigma^{s,NNLL}(\tau^{cut})\big|_{NNLO}$$

using FEHiP & MCFM



- fixed order singular and nonsingular terms are equally important for $\mathcal{T}_{cm} \gtrsim m_H/2$
- must turn off resummation to avoid spoiling cancellation between singular and nonsingular terms in this region

Profile Functions

$\mu_B(\mathcal{T}_{\rm cm}), \quad \mu_S(\mathcal{T}_{\rm cm})$

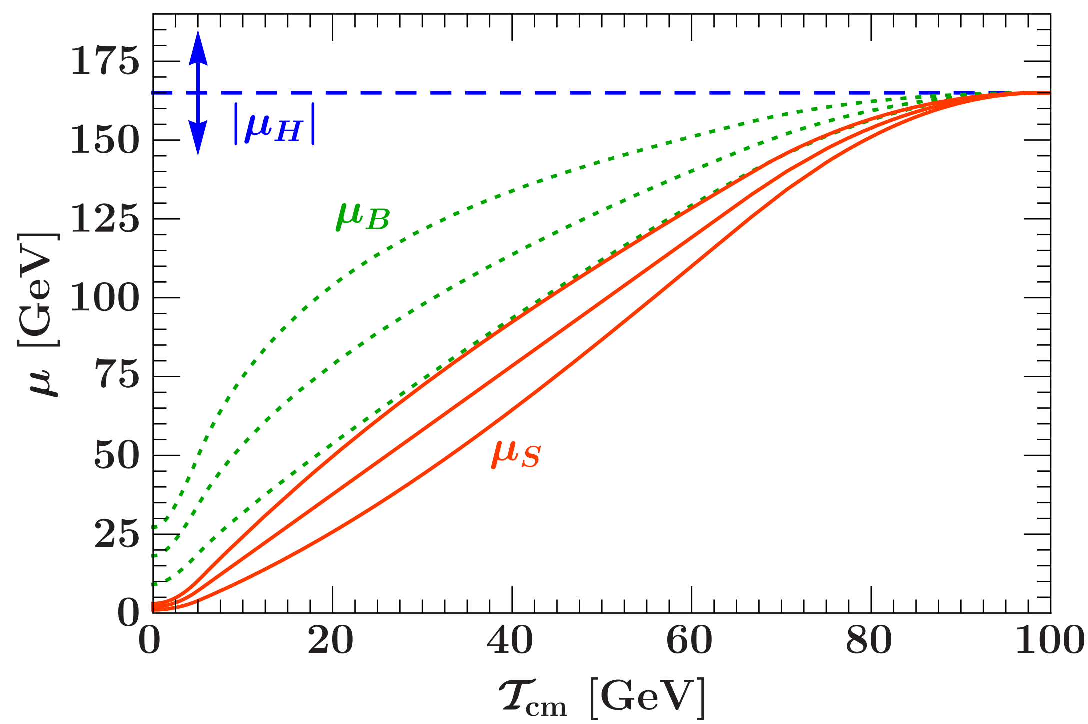
for:		must satisfy:	
• $\mathcal{T}_{\rm cm} \sim \Lambda_{\rm QCD}$	nonpert.	$\mu_S \gtrsim 1\,{\rm GeV}$	(only pert. anom. dimensions)
• $\Lambda_{\rm QCD} \ll \mathcal{T}_{\rm cm} \ll m_H$	pert. large logs	$\mu_S^2 \simeq \mathcal{T}_{\rm cm}^2$	$\mu_B^2 \simeq m_H \mathcal{T}_{\rm cm}$
• $\mathcal{T}_{\rm cm} \sim m_H/2$	no large logs	$\mu_S^2 \simeq \mu_B^2 \simeq \lvert\mu_H\rvert^2$	

$\mu \simeq -i m_H$ in gluon form factor

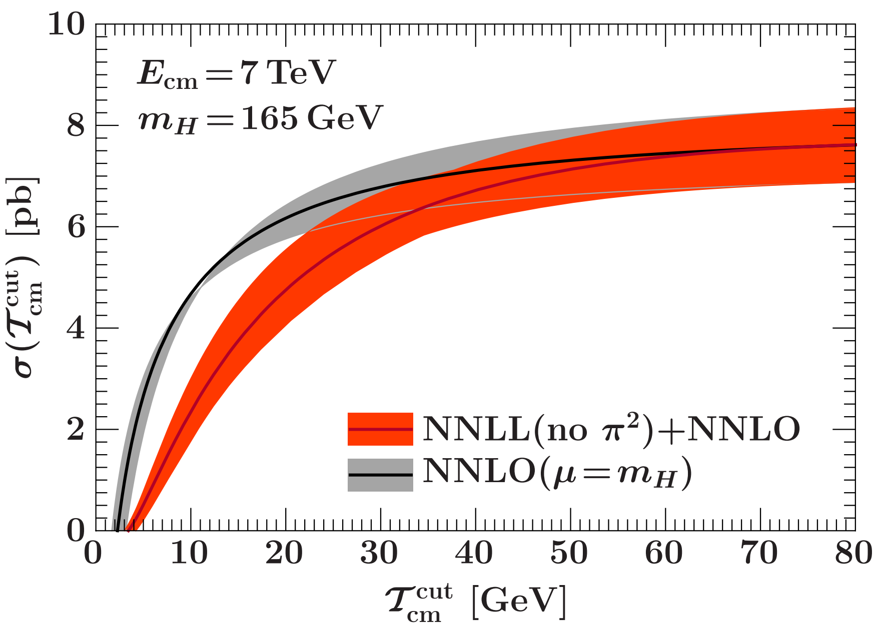
Scale variations

1. Overall scale by factors of 2
2. $\mu_B(\mathcal{T}_{\rm cm})$ profile
3. $\mu_S(\mathcal{T}_{\rm cm})$ profile

determines perturbative uncertainties



Reproducing Fixed-Order Result at Large $\mathcal{T}_{cm}$



$\mu \simeq m_H$ in gluon form factor

- Exactly reproduces fixed NNLO at $\mu = m_H$ for large $\mathcal{T}_{cm}$

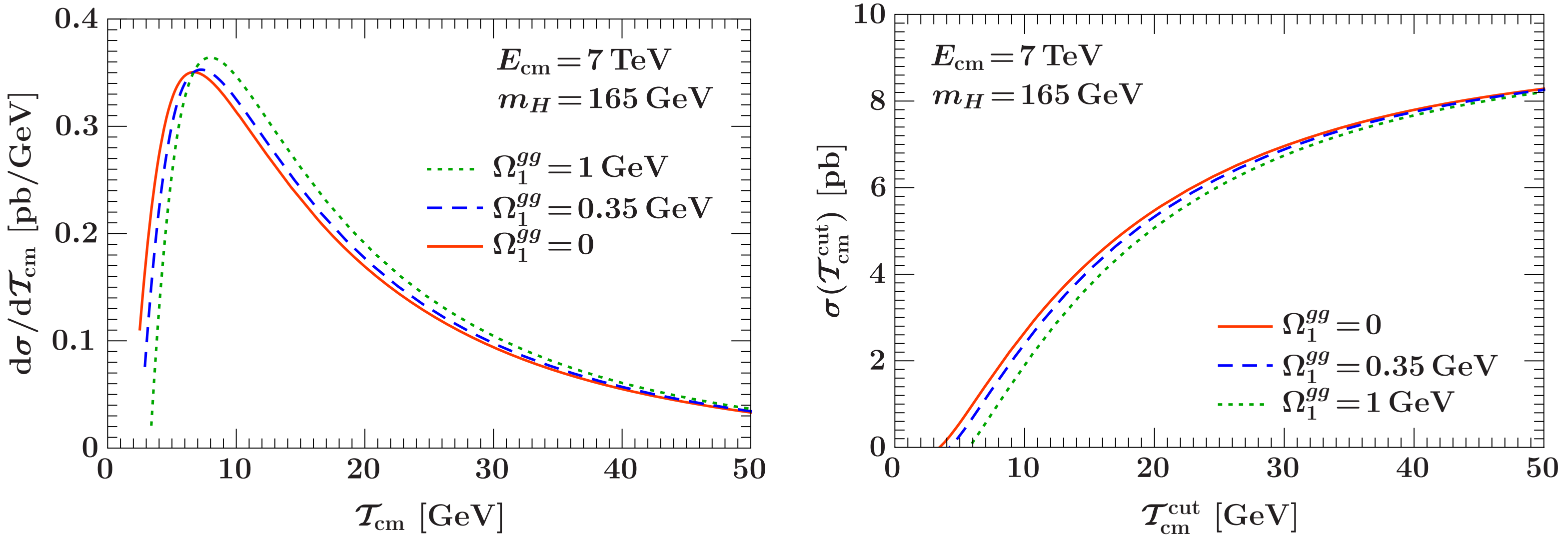
  (scale profiles are essential)

Nonperturbative Hadronization effects

- can be derived / parameterized with field theory matrix elements

For $\Lambda_{\rm QCD} \ll \mathcal{T}_{\rm cm} \ll m_H$ dominant correction is simply a shift:

$$S_B^{gg}(k,\mu_S) = S_{\rm pert}^{gg}(k,\mu_S) - 2\Omega_1^{gg}\,\frac{dS_{\rm pert}^{gg}(k,\mu_S)}{dk} + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^2}{k^3}\Big)$$



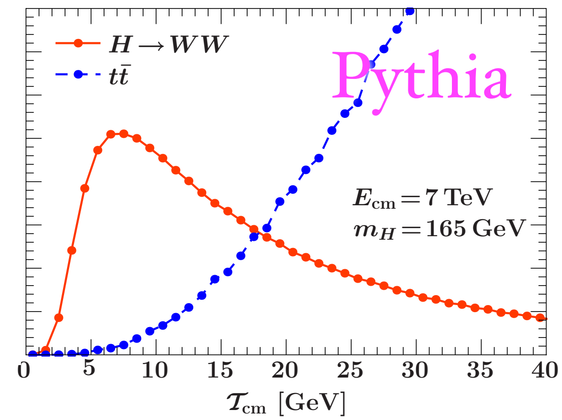
Pythia agrees with shift of $\Omega_1^{gg} = 1\,{\rm GeV}$

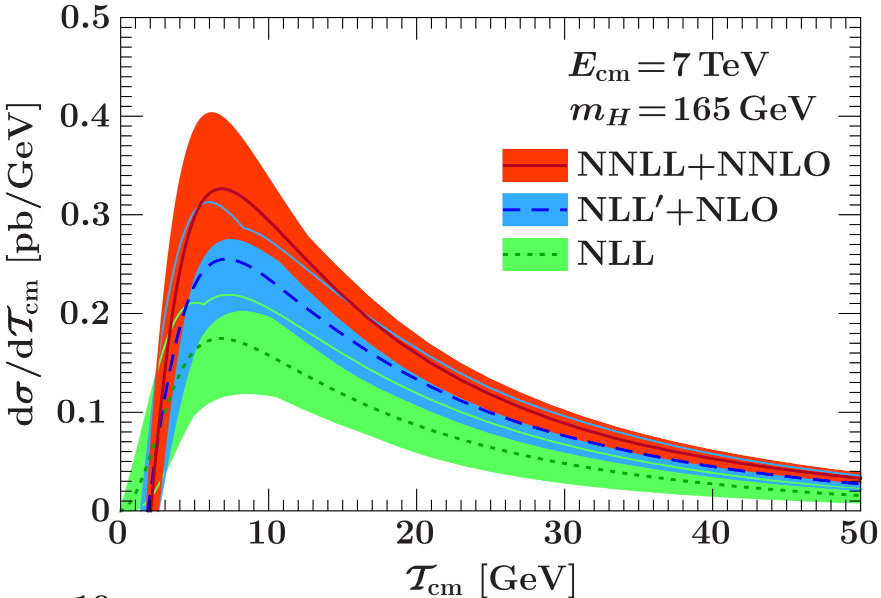
This is analog of the classic shift for e^+e^- event shapes — Dokshitzer, Webber

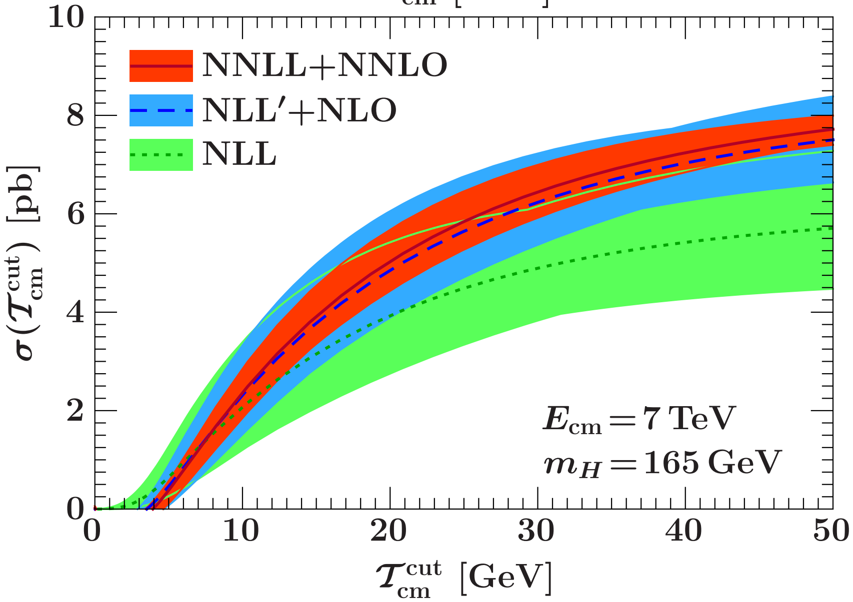
Can study universality of nonperturbative shifts with field theory methods — Lee, Sterman

$$\Omega_1^{gg} = \frac{1}{2N_A}\langle 0|{\rm tr}\; Y_{\bar n}(0) Y_n(0)\, i\widehat{\partial}_{\mathcal{T}}\; Y_n^\dagger(0) Y_{\bar n}^\dagger(0)|0\rangle$$

Beam Thrust Spectrum and Cumulant







$gg \to H$ production cross section for $m_H = 165\,\mathrm{GeV}$ at the LHC

Differential beam-thrust spectrum

- peaks at small $\mathcal{T}_{cm}$
- has rather large tail from ISR

Perturbative corrections are important

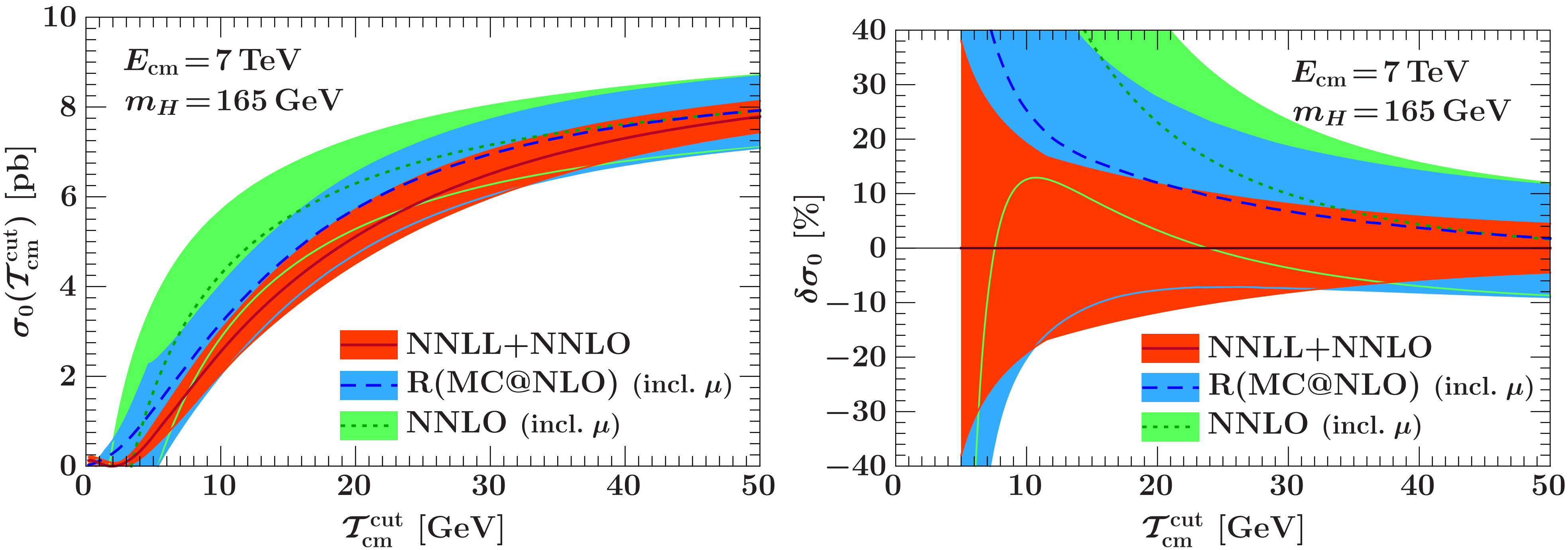
- Incoming gluons radiate a lot
- Very large at lower orders
- Good convergence at higher orders

Small $\mathcal{T}_{\rm cm}^{\rm cut}$

$$\sigma_0(\mathcal{T}^{\rm cut}) = \sigma_{\rm total} - \sigma_{\geq 1}(\mathcal{T}^{\rm cut})$$

combined inclusive scale variation shown for NNLO & MC@NLO

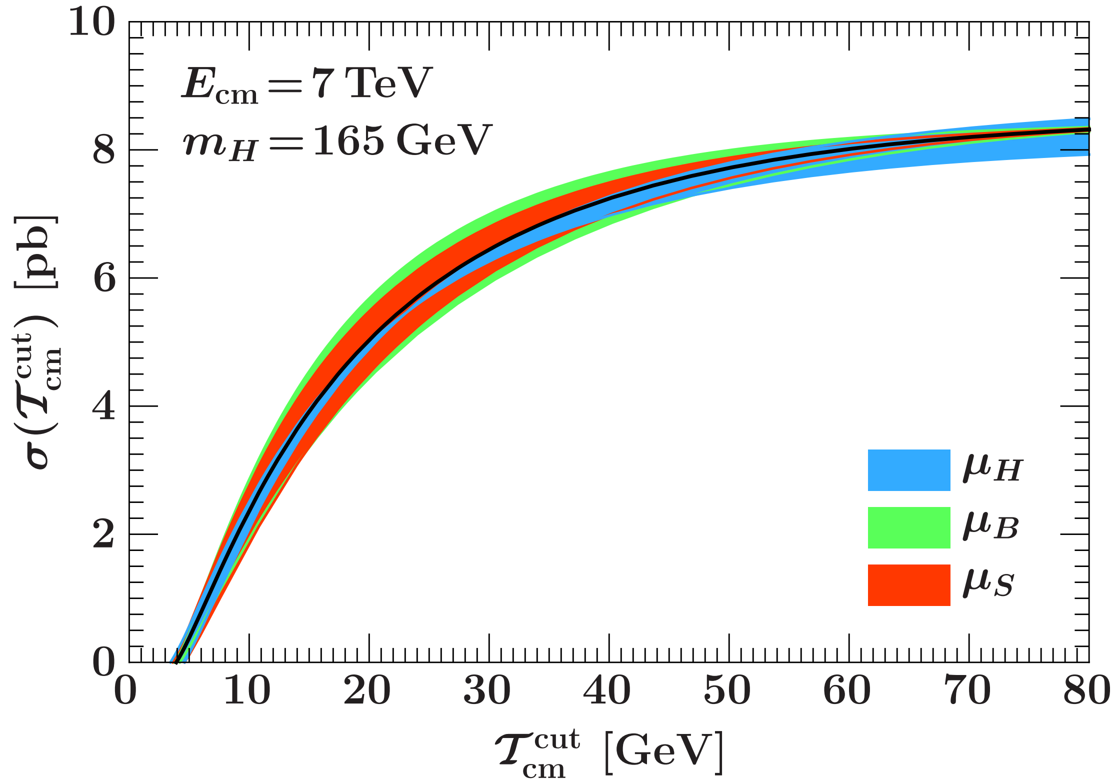
combined NNLL scale variations shown

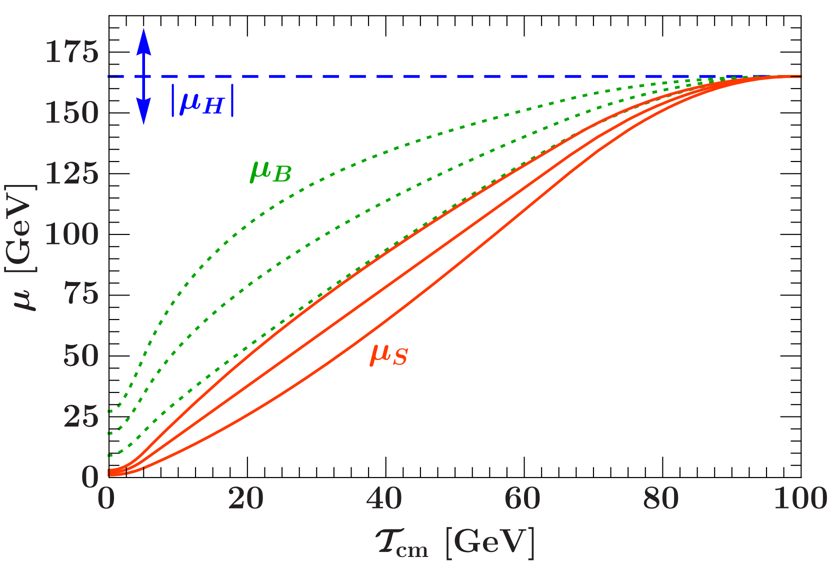


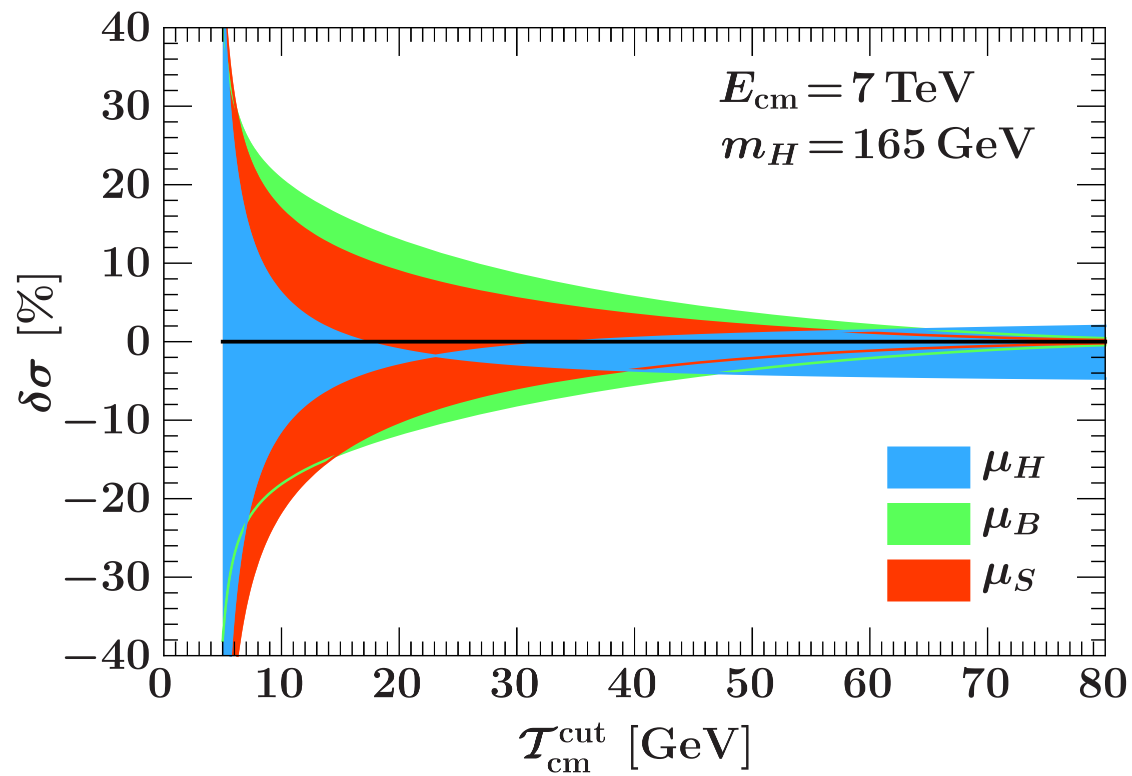
- NNLO band largely overlaps NNLL result
- reweigh MC@NLO to match NNLO incl. relative uncertainties (full spectrum). Overlaps nicely with NNLL.
- Factor of two improvement from resummation

Uncertainty Correlations

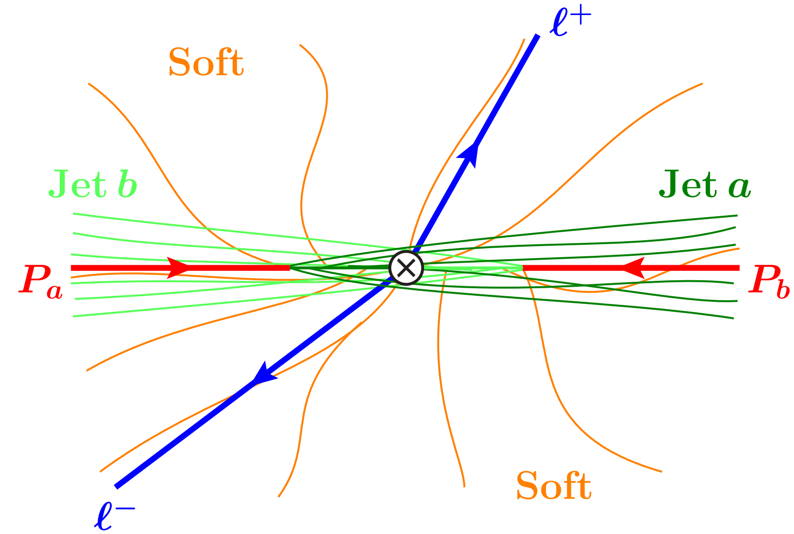
- three separate scale variations
- $\mu_H = \mu_{H0}$ 100% correlated with σ_{total}
- μ_B and μ_S give uncertainty from imposing jet-veto

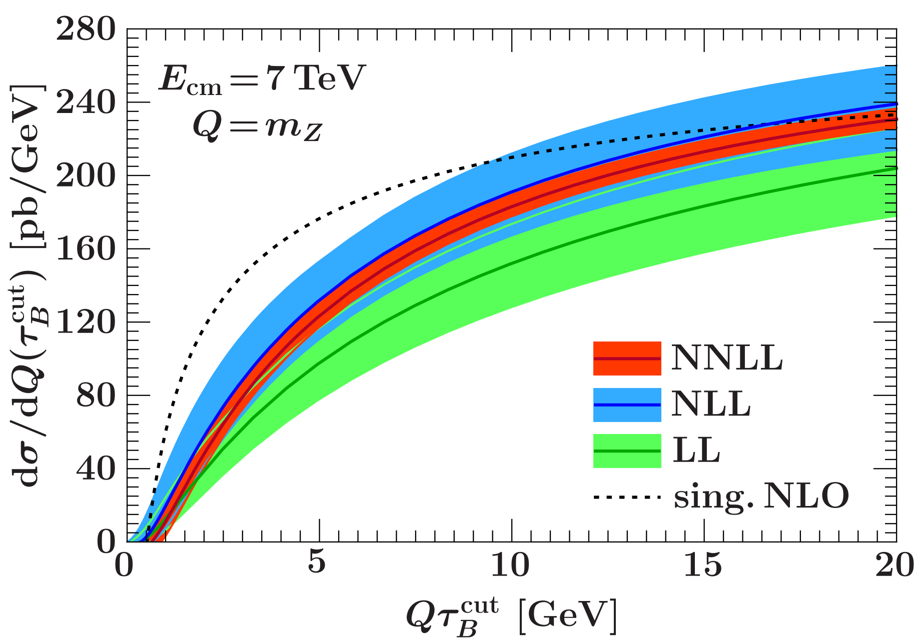


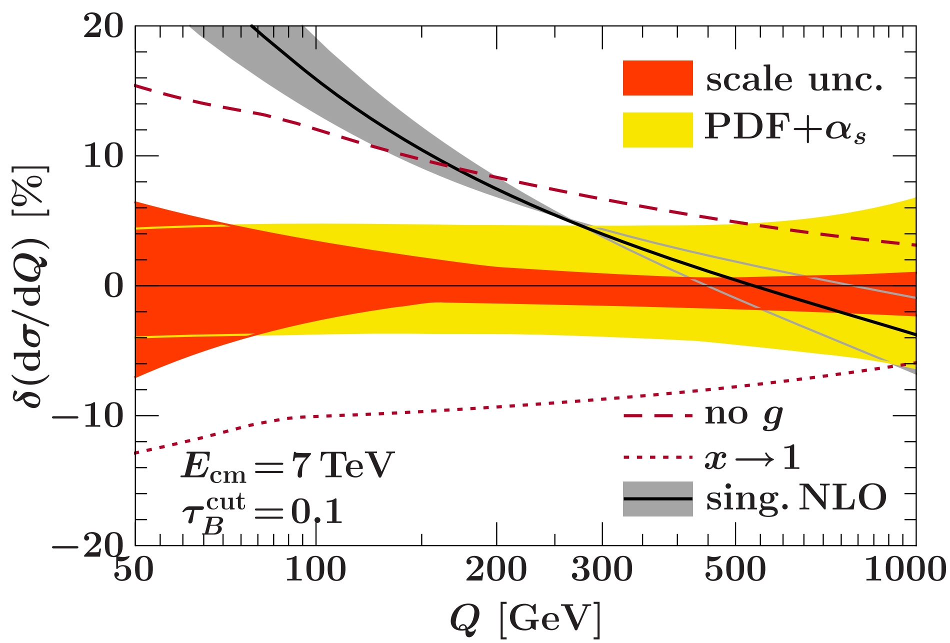




Analog for Drell-Yan pairs from γ^*, Z^* have smaller uncertainties





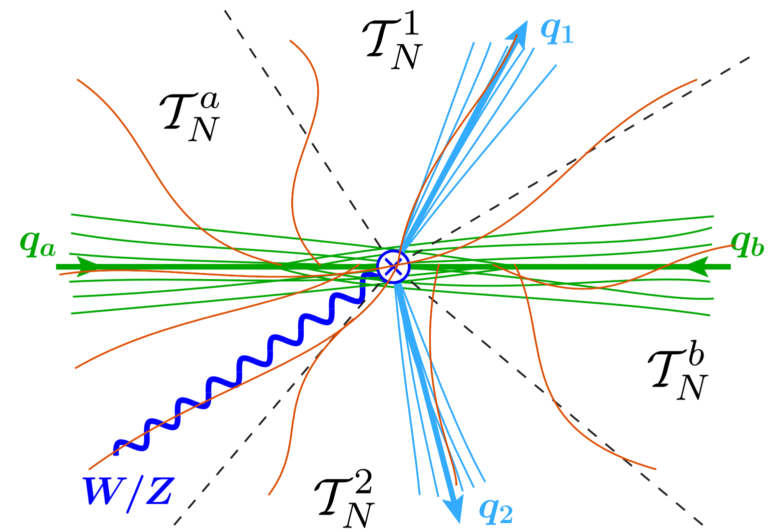


N-jets

IS, Tackmann, Waalewijn

N-Jettiness Event Shape

$$\mathcal{T}_N = \mathcal{T}_N(q_a, q_b, q_1, \ldots, q_N)$$



- $\mathcal{T}_N \to 0$ for N-jets

  Large $\mathcal{T}_N$ has >N jets

- Factorization Friendly

- Splits into a sum of observables for each jet-region

$$\mathcal{T}_N = \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \ldots + \mathcal{T}_N^N$$

- Can calculate N-jet exclusive cross-section differential in each region

$$\frac{d\sigma}{d\mathcal{T}_N^a \cdots d\mathcal{T}_N^N}$$

- Can be used for

$$pp \to \text{jets},\ pp \to H + \text{jets},\ \ldots$$

N-Jettiness $\mathcal{T}_N$

consider an inclusive N-jet sample with jet energies E_i & directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

light-like reference vectors

$$q_i^\mu = E_i(1, \hat{n}_i)$$

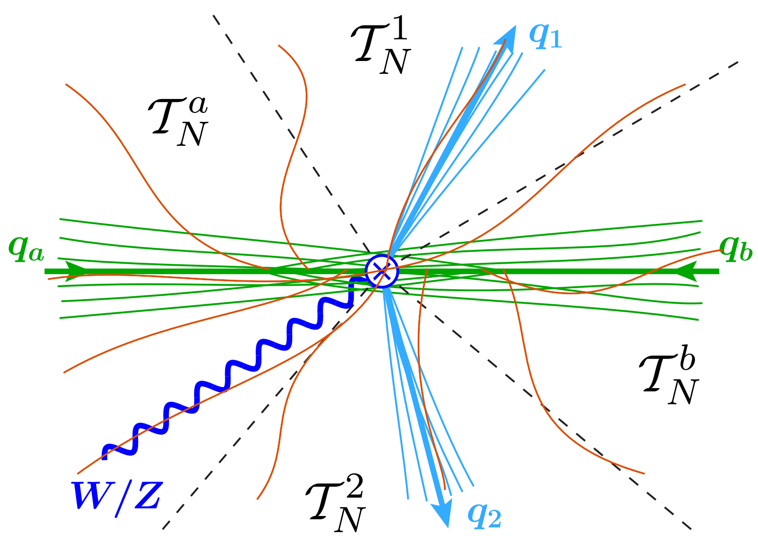
$$q_a^\mu = \frac{1}{2} x_a \, E_{\rm cm}(1, \hat{z}) \,, \quad q_b^\mu = \frac{1}{2} x_b \, E_{\rm cm}(1, -\hat{z})$$

$$x_a x_b = \frac{Q^2}{E_{\rm cm}^2} = \frac{(q_1 + \ldots + q_N + q)^2}{E_{\rm cm}^2}$$

$$2Y = \ln \frac{x_a}{x_b} = \ln \frac{(1, -\hat{z}) \cdot (q_1 + \cdots + q_N + q)}{(1, \hat{z}) \cdot (q_1 + \cdots + q_N + q)}$$

(set $x_a = x_b = 1$ for cases with MET)

measure $$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$



- Compares distance of particle k to beam and jet directions
- phase space divided into jet and beam regions $\mathcal{T}_N = \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$
- Q_j determines the jet measure

N-Jettiness $\mathcal{T}_N$

consider an inclusive N-jet sample with jet energies E_i & directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

light-like reference vectors

$$q_i^\mu = E_i(1, \hat{n}_i)$$

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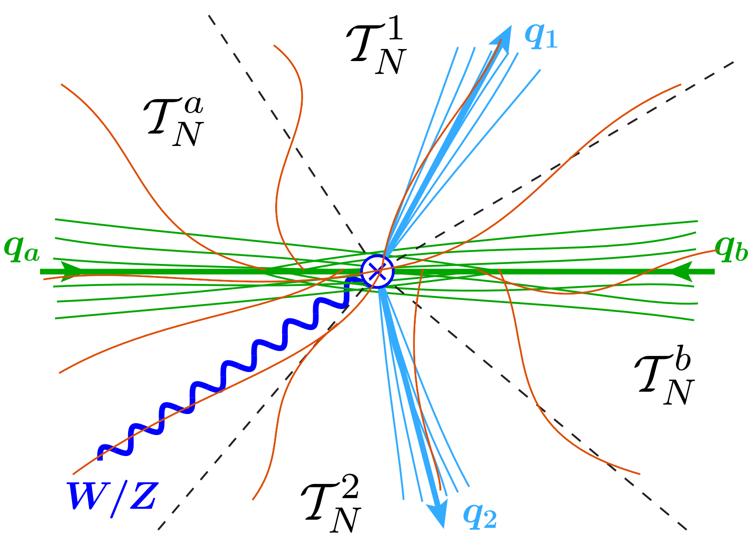
(set $x_a = x_b = 1$ for cases with MET)

measure $$\mathcal{T}_N = \sum_k \min\Big\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \Big\}$$



$$\mathcal{T}_N = \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$

- Related to Jet Masses:

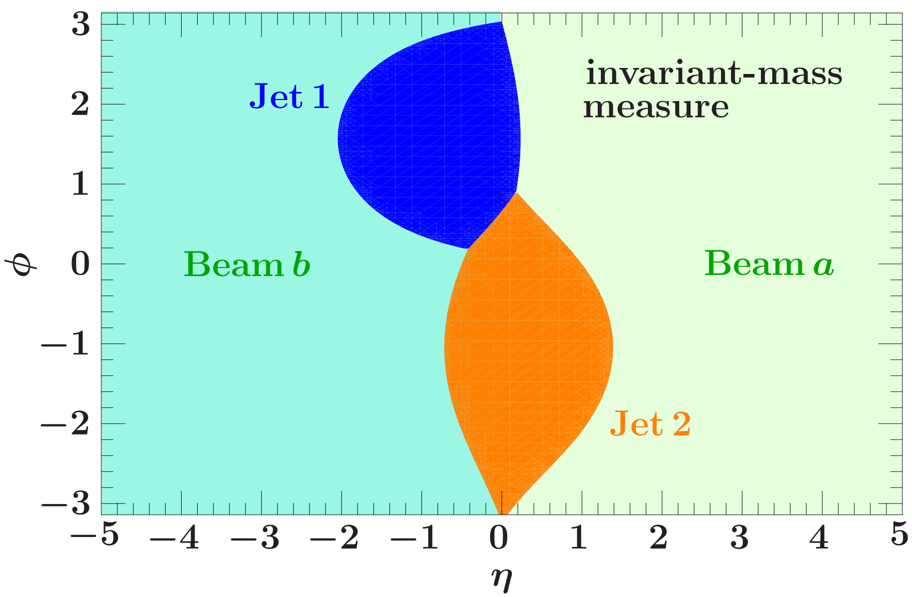
$$M_J^2 = P_J^2 = P_J^- P_J^+ = Q_i \mathcal{T}_N^i$$

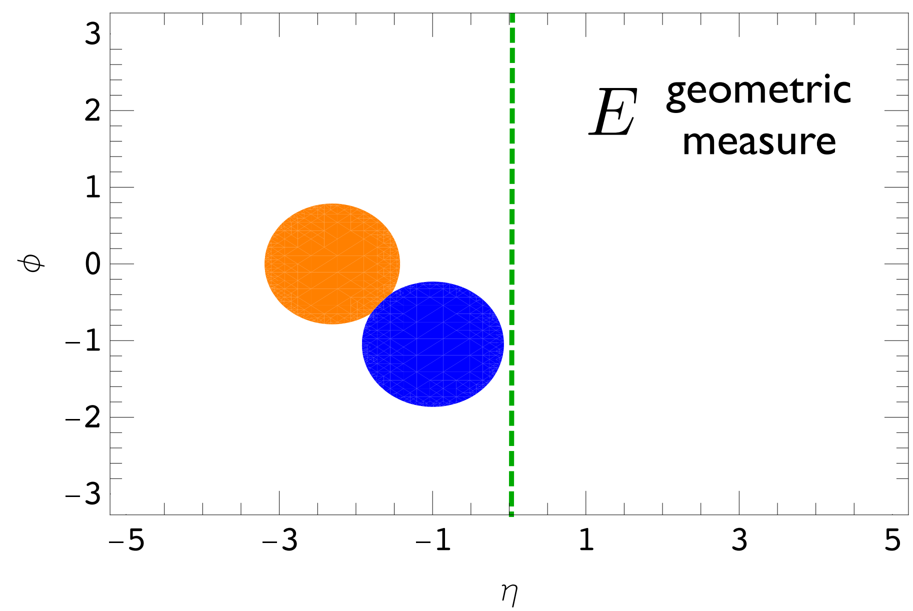
(with jet axes aligned)

example Jet definitions:

division into jet and beam regions fully specified by kinematics

$$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$





$$Q_{a,b} = Q_i = Q$$

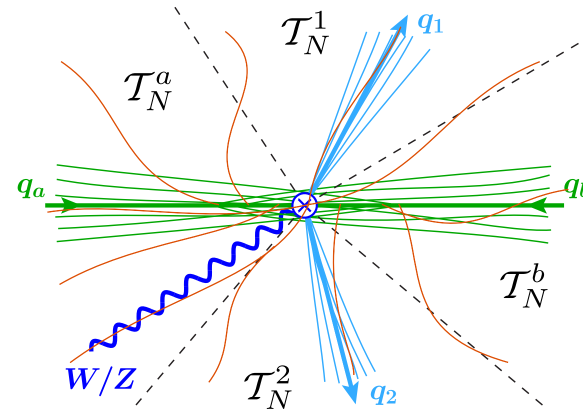
$$Q_a = x_a E_{\rm cm}$$

$$Q_b = x_b E_{\rm cm}$$

$$Q_i = E^i_{\rm jet}$$

circular jets like anti-kT

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times \operatorname{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

hard virtual corrections
$2 \to N + q$

$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$

obtain from NLO helicity amplitude calculations in QCD

beam function
$B_\kappa = \mathcal{I}_{\kappa\kappa'} \otimes f_{\kappa'}$
known to $\mathcal{O}(\alpha_s)$

IS, Tackmann, Waalewijn

Fleming, Leibovich, Mehen

N-jettiness soft function known to $\mathcal{O}(\alpha_s)$

Jouttenus, IS, Tackmann, Waalewijn

Bauer, Hornig, Dunn

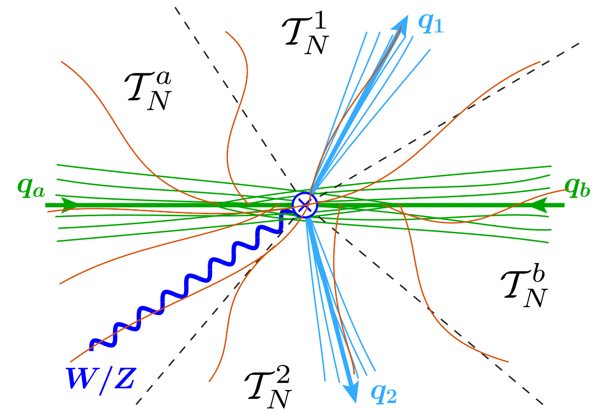
jet function known to $\mathcal{O}(\alpha_s^2)$

Becher & Neubert

Becher & Bell

All EFT ingredients exist for NNLL results

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times\, \mathrm{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$

$B_\kappa = \mathcal{I}_{\kappa\kappa'} \otimes f_{\kappa'}$

Assumptions used to sum logs with this formula:

1) $\mathcal{T}_i \sim \mathcal{T}_j$ ($\mathcal{T}_i \ll \mathcal{T}_j$ gives non-global logs of Dasgupta & Salam)

$$\alpha_s^2 \ln^2\left(\frac{\mathcal{T}_i}{\mathcal{T}_j}\right) + \ldots$$

2) $\hat{q}_i \cdot \hat{q}_j \gg \mathcal{T}_i / Q_i$

jets are well separated (avoid having jets merge, more later)

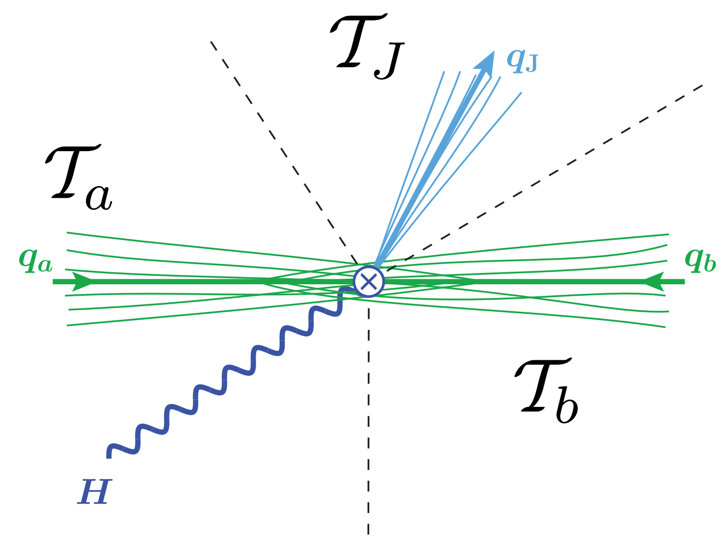
3) $Q_i \sim Q_j$

Jouttenus, IS, Tackmann, Waalewijn

$pp \to \text{Higgs} + \text{1-jet}$

- $ggg H,\ gq\bar{q}H$ channels

NLO Hard Fn's: C.Schmidt (2007)



- kinematic variables:

$m_J^2 = Q_1 \mathcal{T}_J = \text{jet-mass}$

$\mathcal{T}_{a,b} \le \mathcal{T}_B^{\rm cut}/2 = \text{restriction on beam radiation}$

$p_T^J = \text{jet } p_T$

$\eta^J = \text{jet rapidity}$

$Y = \text{event rapidity}$

these determine all others $\{x_a, x_b, Q^2, Q_{a,b,i}, \ldots\}$

- focus on region where

$p_T^J \sim m_H$ or $p_T^J > m_H$

(only have large logs from vetoing 2-jet events)

- NNLL results mostly analytic

- impose upper limit on beam radiation (cumulant)

$$\sigma(m_J, \mathcal{T}_B^{\rm cut}, p_T^J, \eta^J, Y) = \int_0^{\mathcal{T}_B^{\rm cut}/2} d\mathcal{T}_a \int_0^{\mathcal{T}_B^{\rm cut}/2} d\mathcal{T}_b \ \ \sigma(m_J, \mathcal{T}_a, \mathcal{T}_b, p_T^J, \eta^J, Y)$$

Normalizing the cross section makes it independent of $\mathcal{T}_B^{\rm cut}$

- $$\widehat{\sigma}(m_J, \not{\mathcal{T}}_B^{\rm cut}, p_T^J, y^J, Y) \equiv \frac{\sigma(m_J, \not{\mathcal{T}}_B^{\rm cut}, p_T^J, y^J, Y)}{\int_0^{m_J^{\rm max}} dm' \; \sigma(m', \not{\mathcal{T}}_B^{\rm cut}, p_T^J, y^J, Y)}$$

- When we integrate over phase space in numerator and denominator then the cancellation is approximate, but still very significant.

- if we look at gluon jets with fixed kinematics then Hard Function drops out so "Higgs" drops out.

(not true with color mixing or both quark and gluon channels)

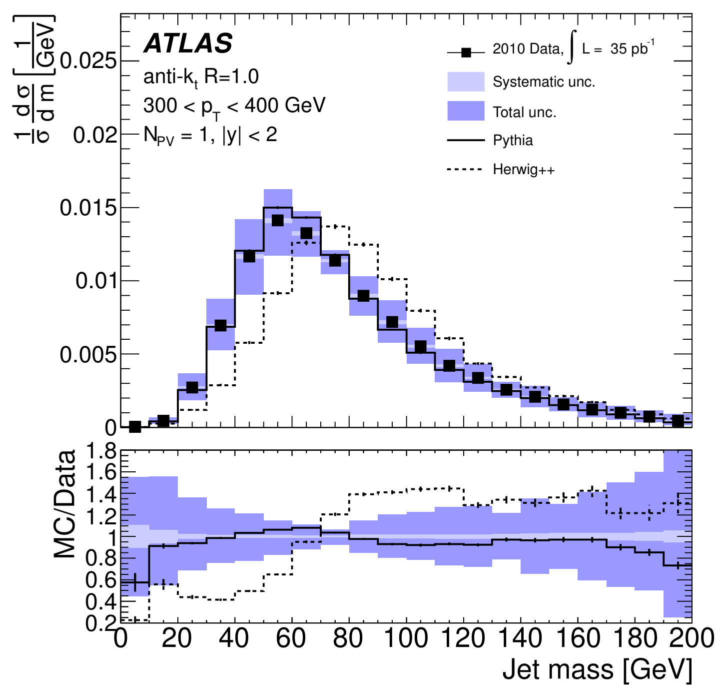
Status/Focus: gggH channel (code is fully cross-checked)

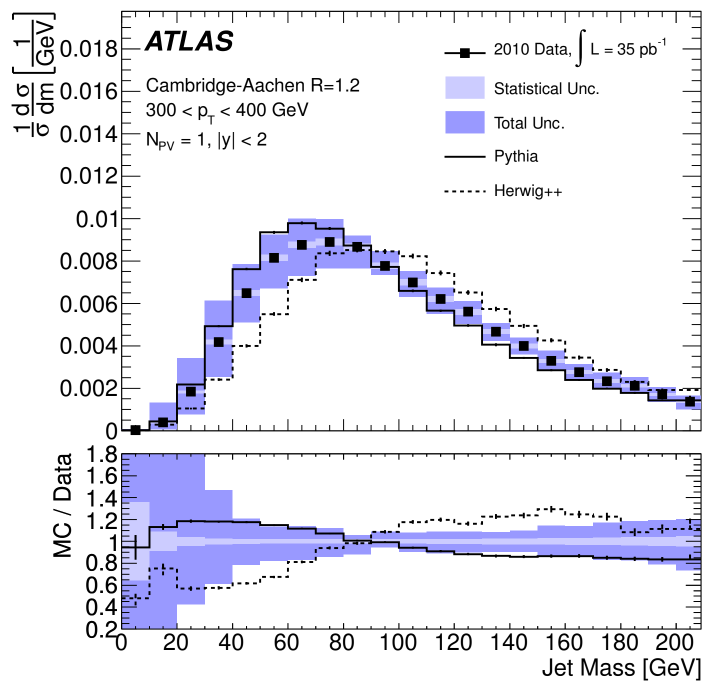
Description of Jet, in particular m_J

pick $m_H = 125\,\mathrm{GeV}$

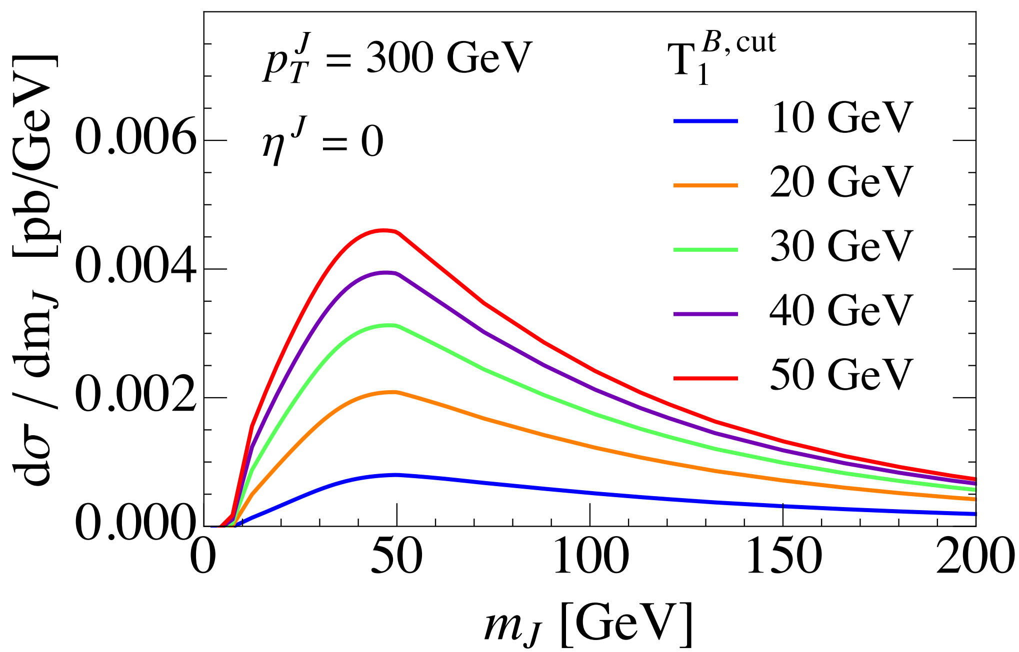
MSTW pdfs

- Gluon Jets tend to dominate the LHC jet masses



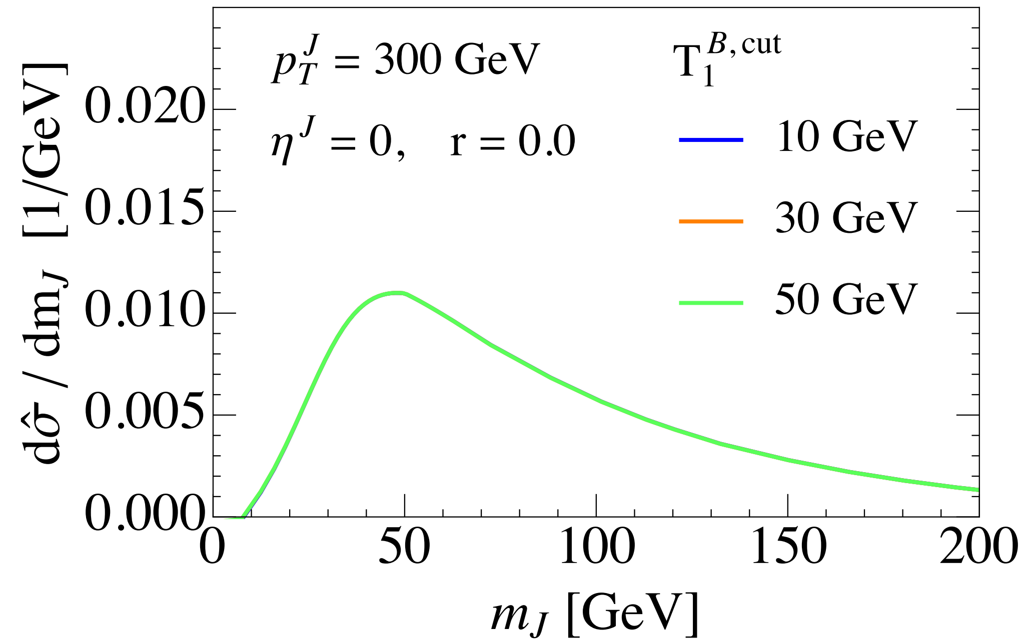


Not Normalized

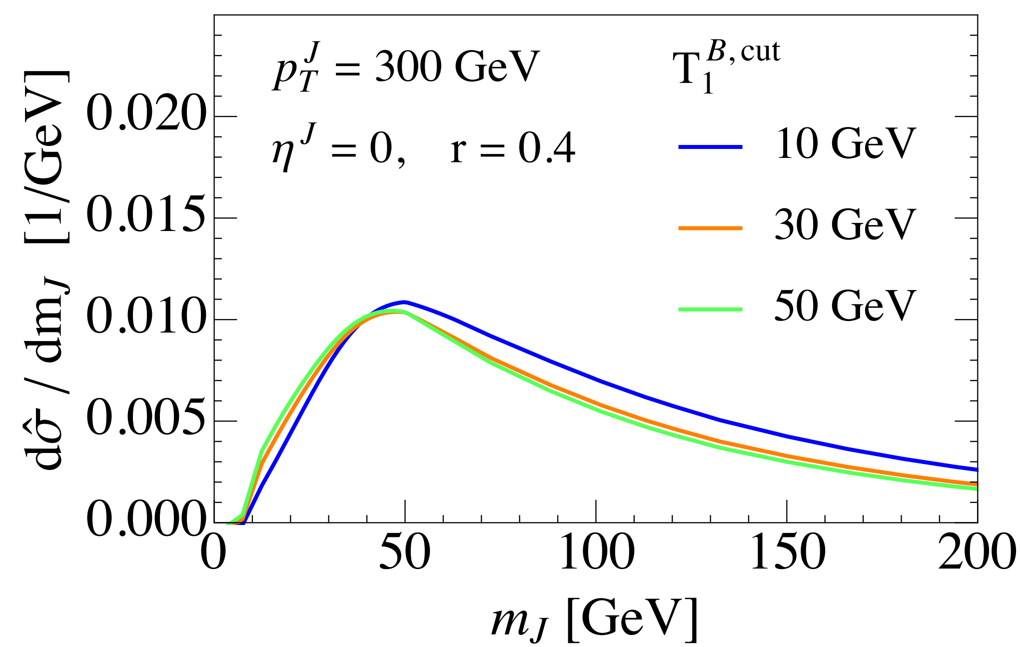


Normalized

$$\int dm_J \frac{d\widehat{\sigma}}{dm_J} = 1$$



here we allow some mixing in soft scales



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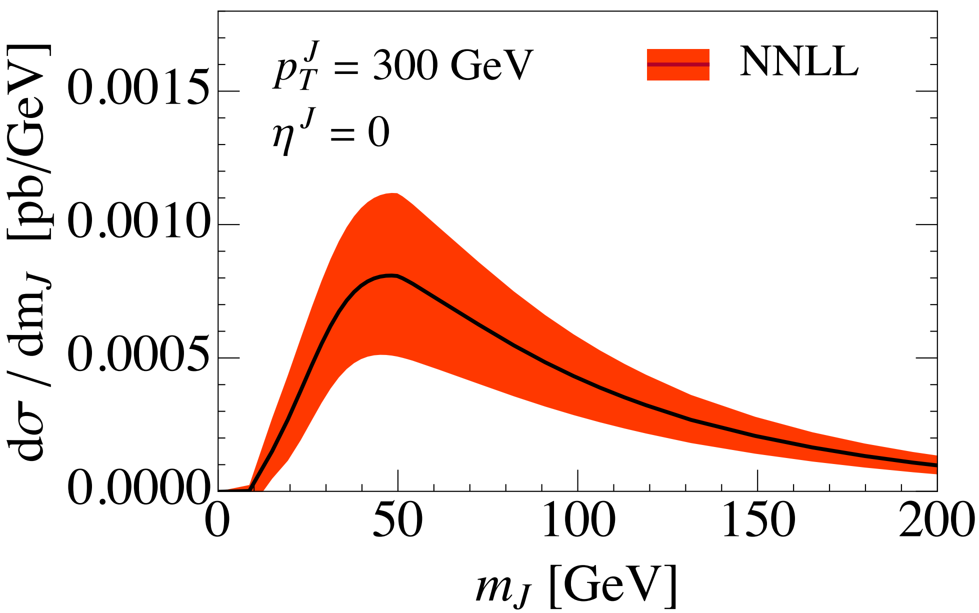

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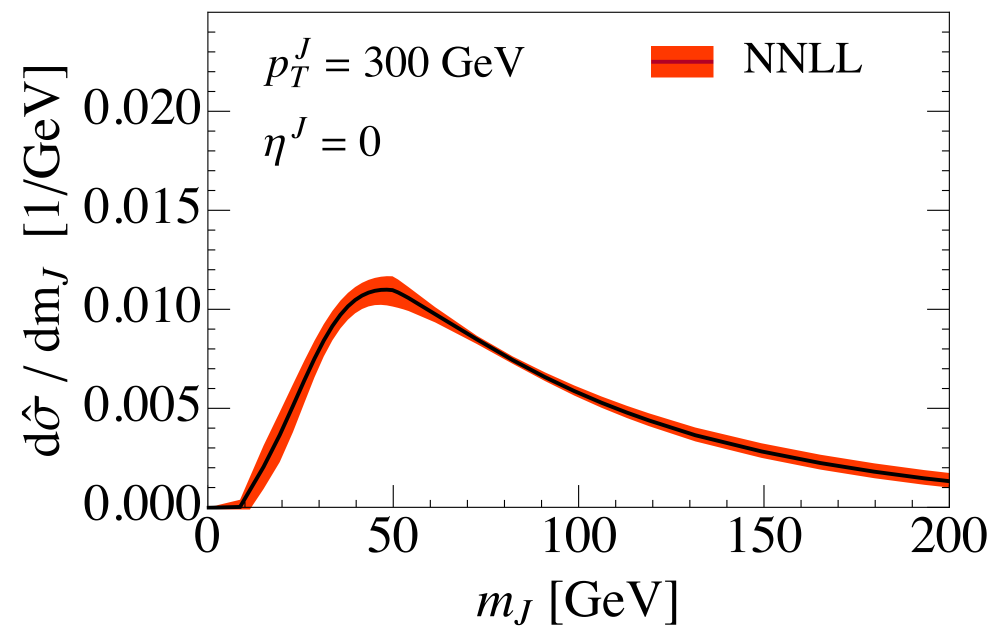
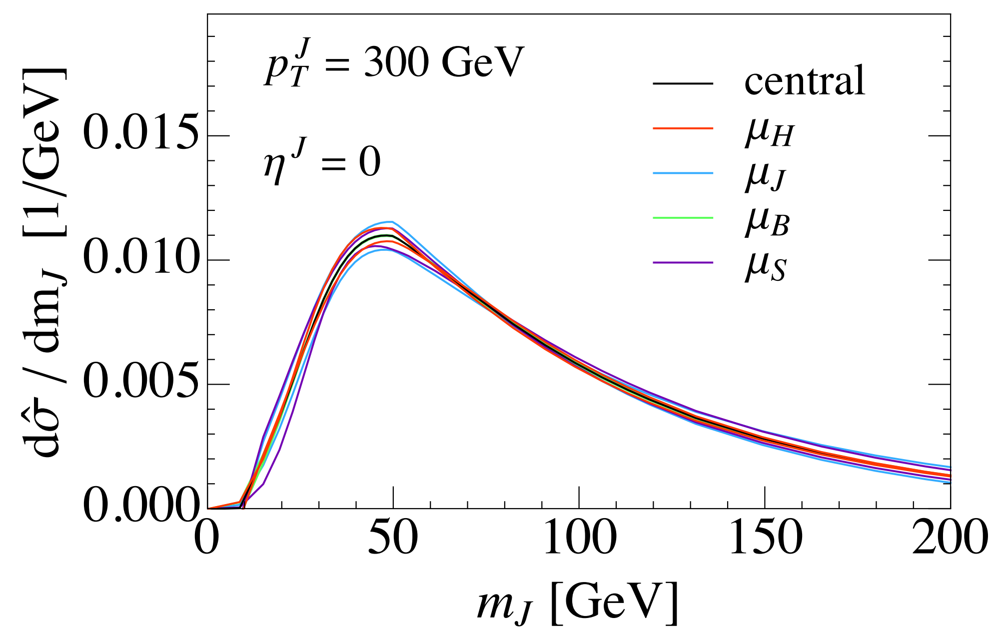
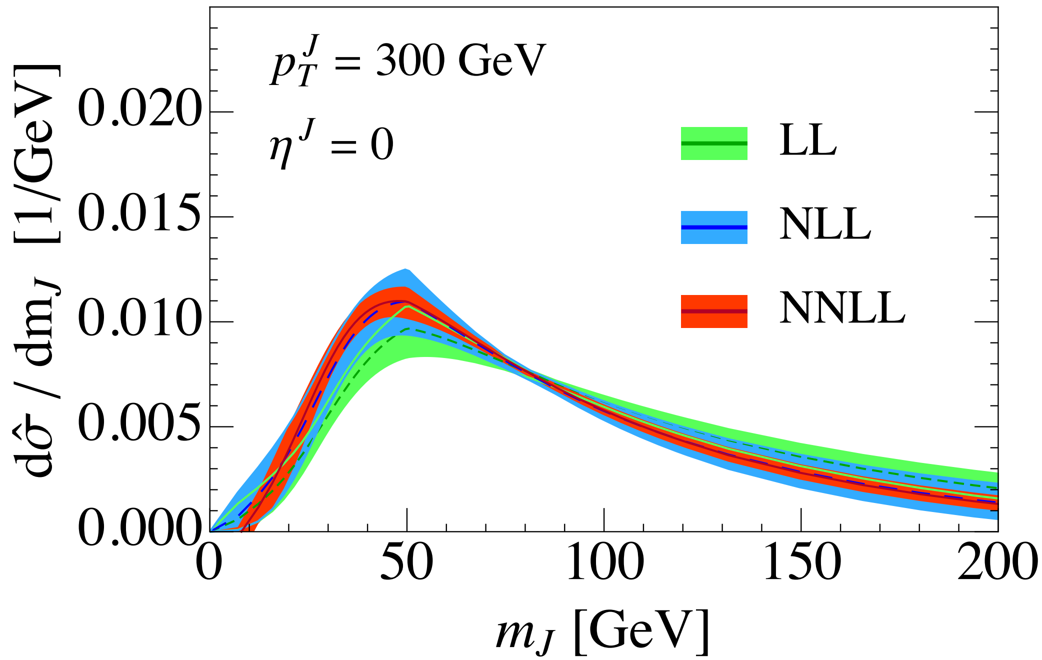




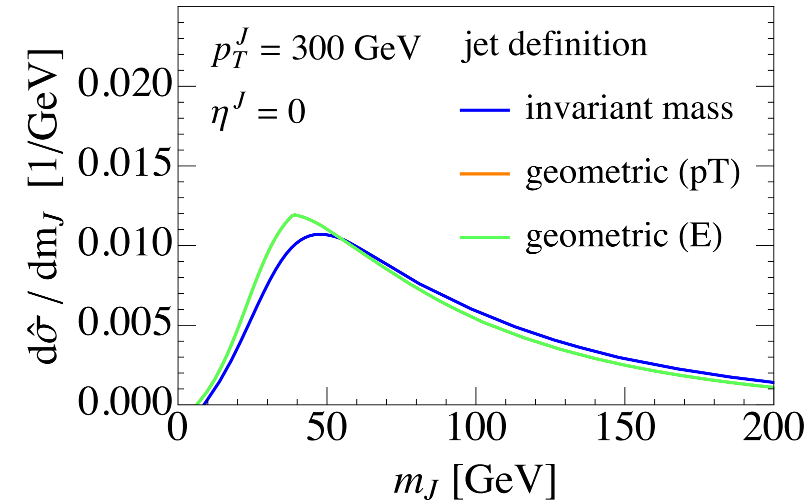
Normalized

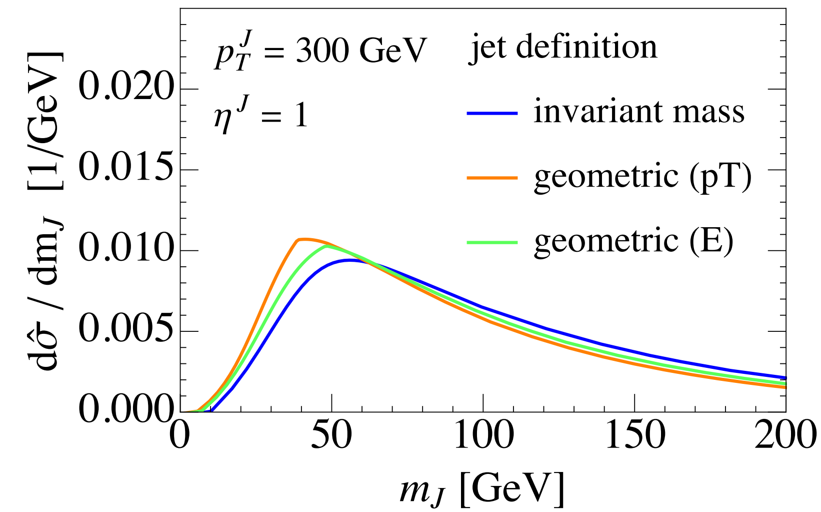
Order by Order Convergence

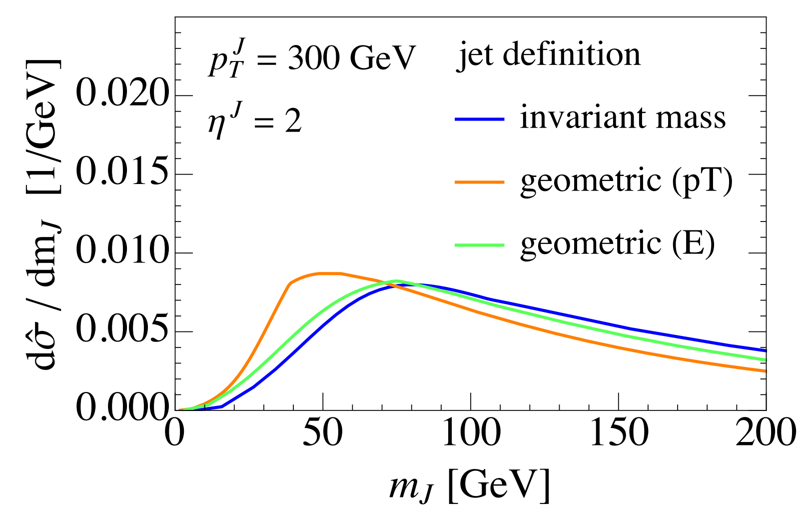


Dependence on the Jet Algorithm

invariant mass
vs
geometric pT
vs
geometric E

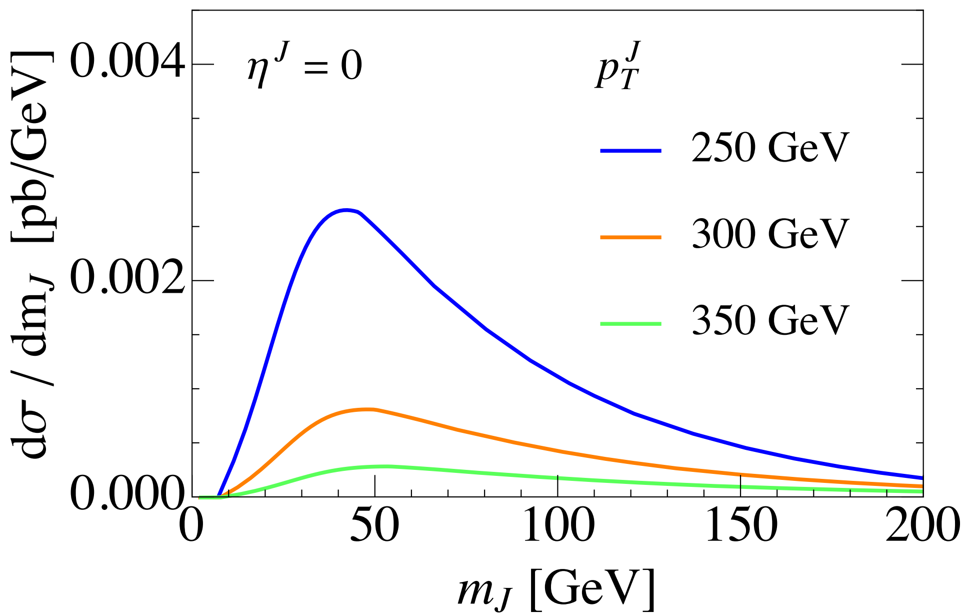






Jet Kinematic Variables: p_T^{jet}

Not Normalized



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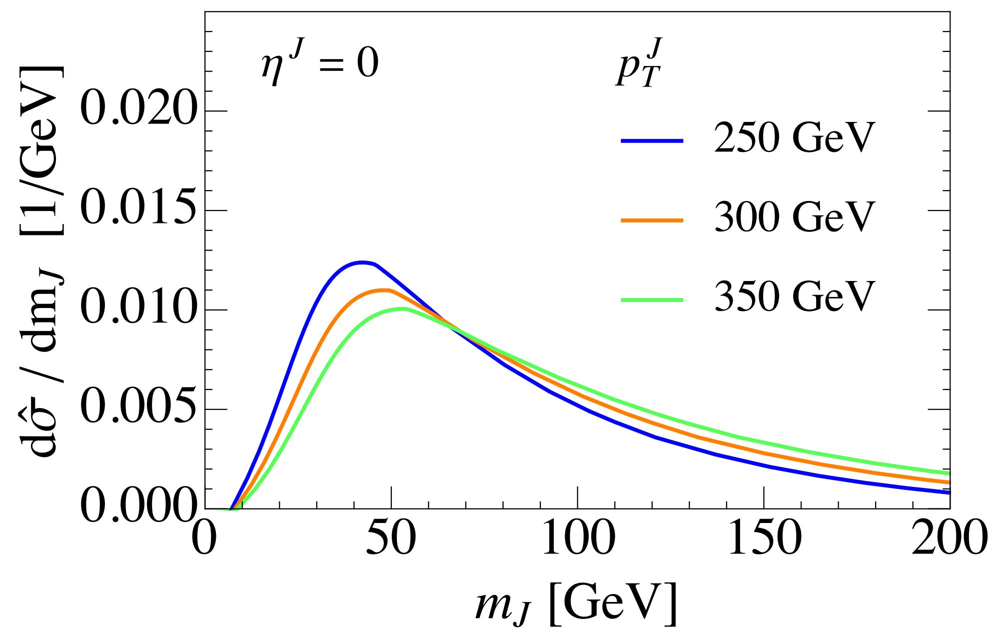
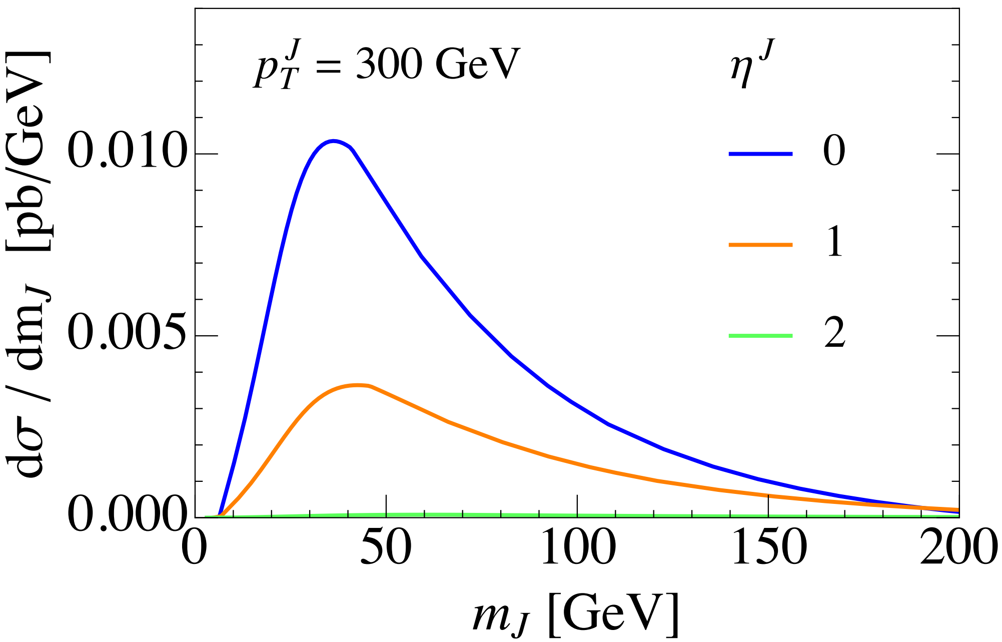


Jet Kinematic Variables: $\eta^{\rm jet}$

Not Normalized



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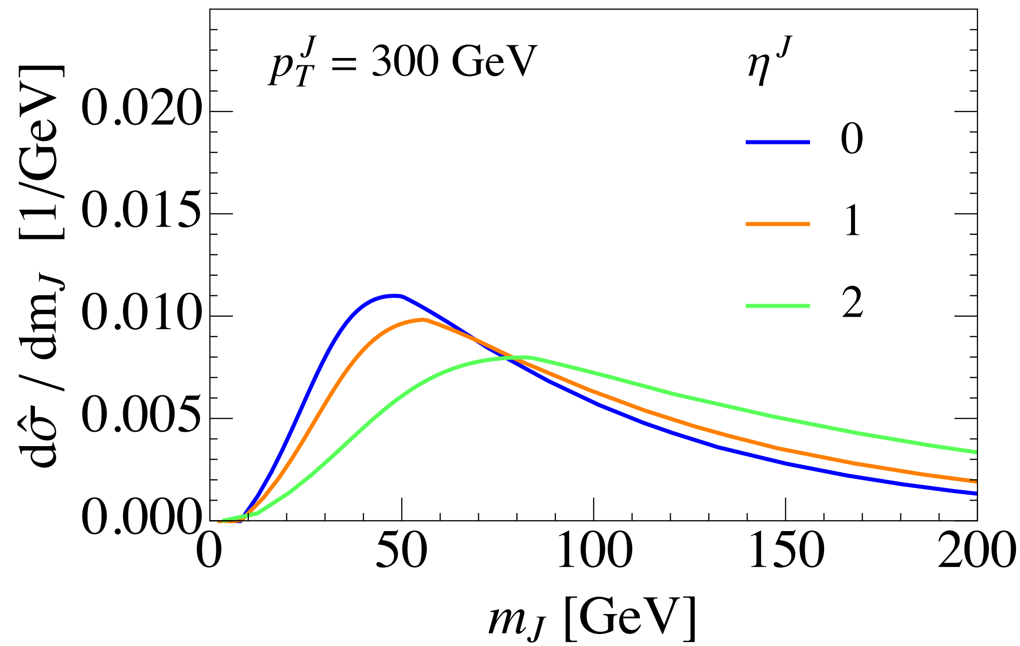
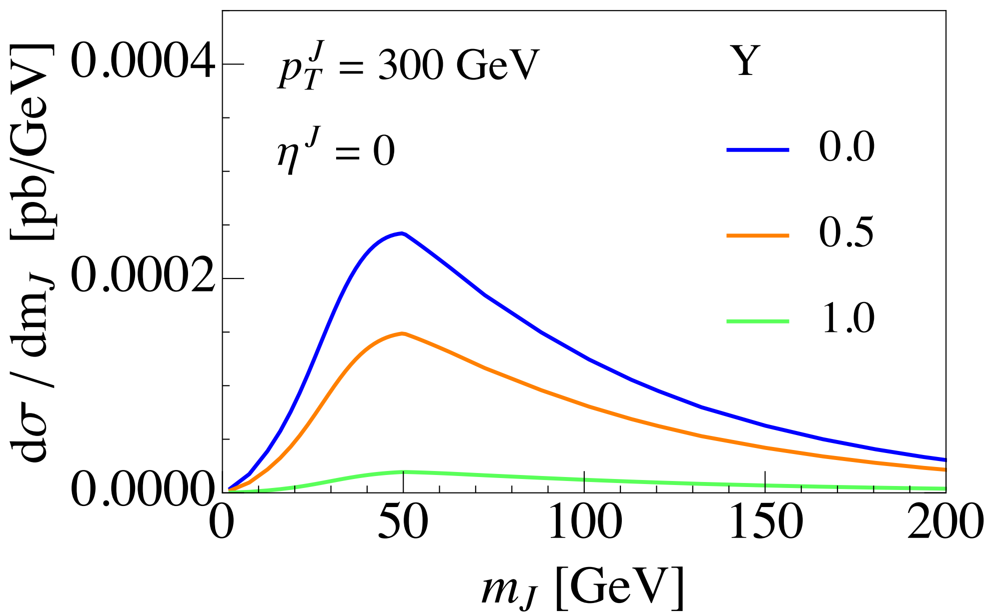


Kinematic Variables: Y (system rapidity)

Not Normalized



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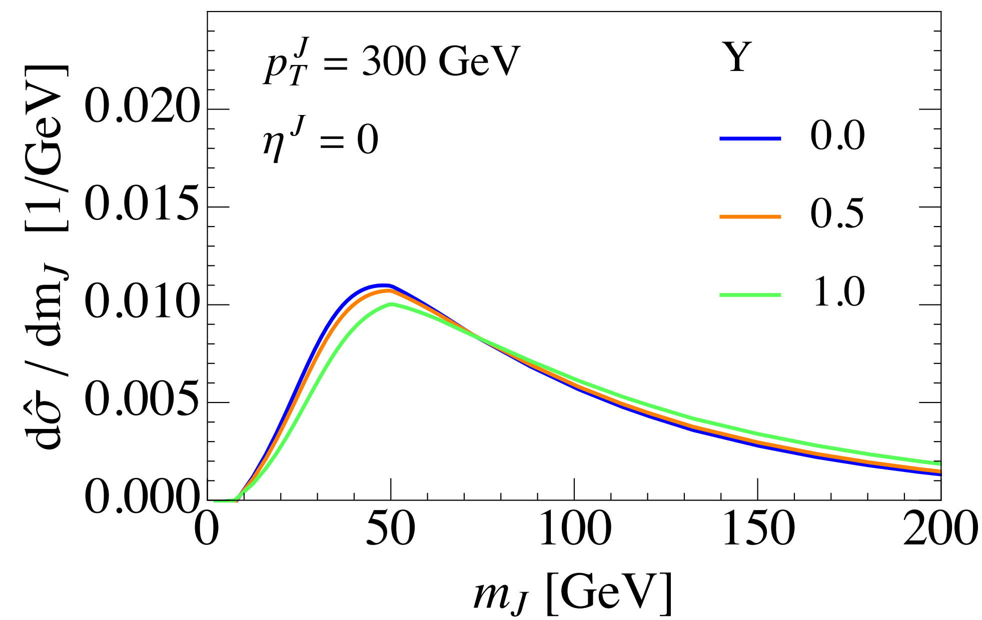


Applications:

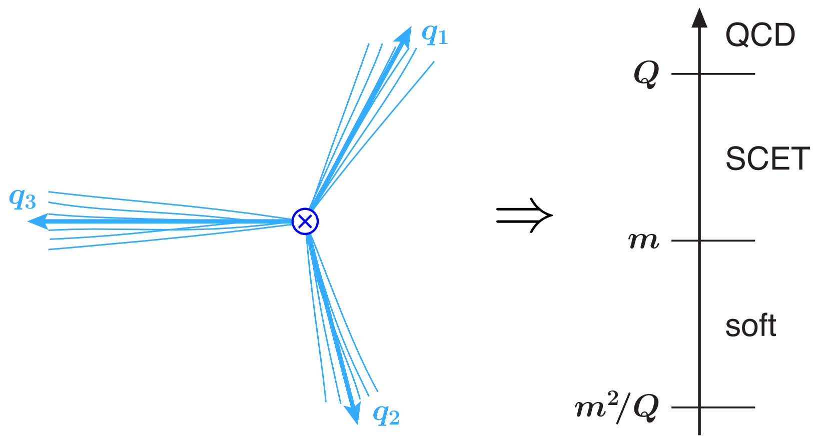
- can be used to study jet-veto uncertainties for Higgs 1-jet bin
- comparison to MC for exclusive 1-jet cross section

Nearby Jets

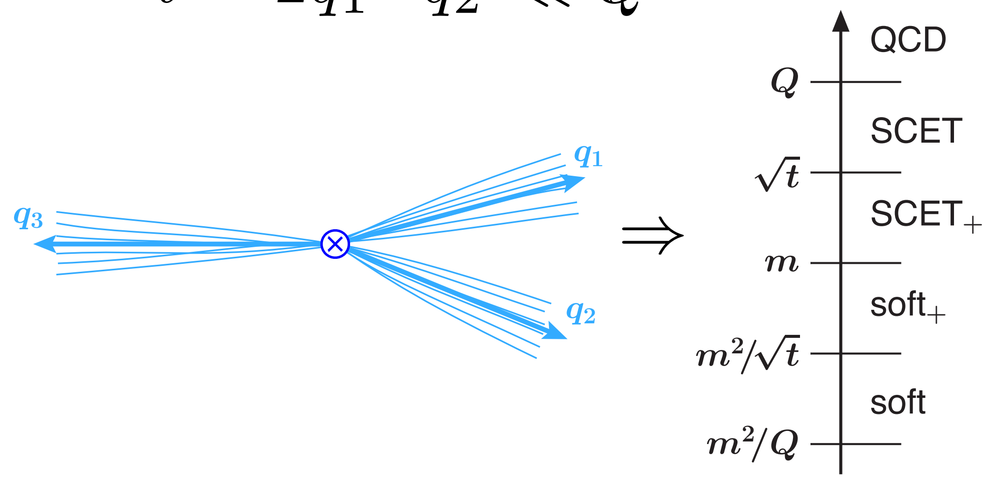
$n_1 \cdot n_2 \sim \lambda_t^2 \ll 1$

Bauer, Tackmann, Walsh, Zuberi

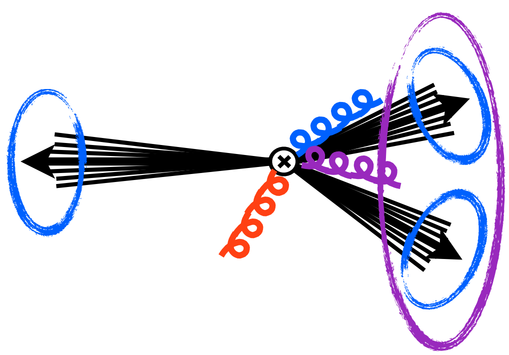
eg. $e^+e^- \to 3\text{-jets}$ $\qquad t = 2q_1 \cdot q_2 \ll Q^2$



(a) All jets equally separated.



(b) Two jets close to each other.



Need additional collinear-soft mode:

$$\lambda^2 = m^2/Q^2$$

$$\lambda_t^2 = t/Q^2$$

collinear: $p_c \sim E_J(1, \lambda^2, \lambda)$

csoft: $p_{cs} \sim E_J \dfrac{\lambda^2}{\lambda_t^2}(1, \lambda_t^2, \lambda_t)$

soft: $p_s \sim E_J(\lambda^2, \lambda^2, \lambda^2)$

$$H_3^\kappa(\{s_{ij}\}, \mu)\Big|_{s_{12} \ll s_{13} \sim s_{23}} = H_2(Q^2, \mu)\, H_+^\kappa(t, x, \mu)$$

$$S_3^\kappa(k_1, k_2, k_3, \mu)\Big|_{\hat{s}_t \ll \hat{s}_{13} = \hat{s}_{23}} = \int \mathrm{d}k_1'\, \mathrm{d}k_2'\, S_2(k_1 - k_1', k_2 - k_2', k_3, \mu)\, S_+^\kappa(k_1', k_2', \mu)$$

Sum logs of m_{jj}/Q

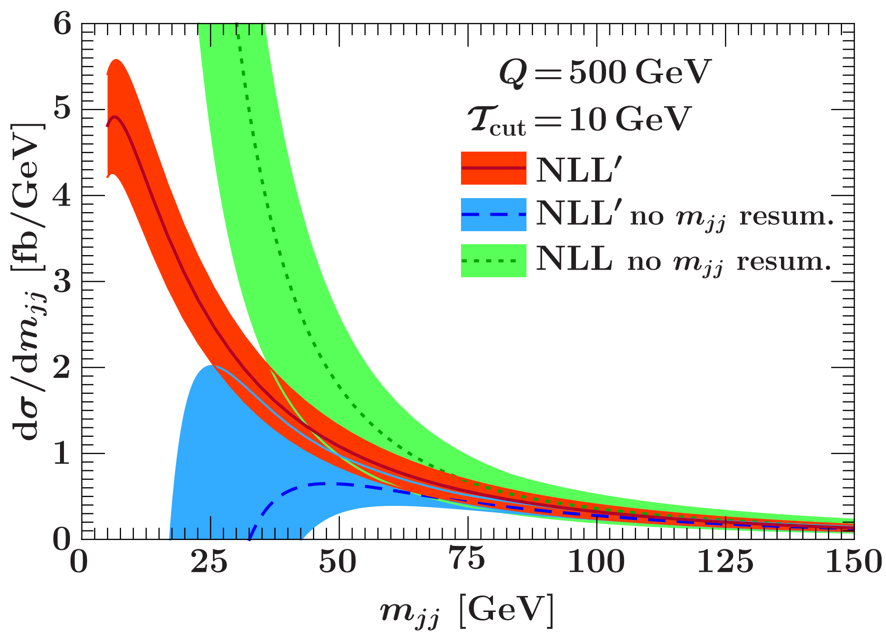
factorization (using N-jettiness to define jet masses):

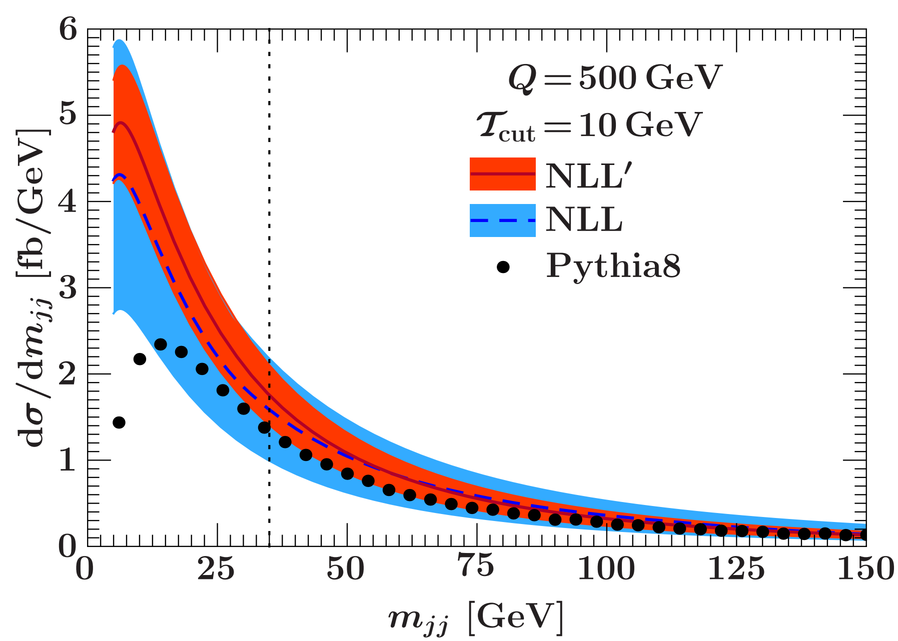
$$z = \frac{E_1}{E_1 + E_2}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_1\,\mathrm{d}\mathcal{T}_2\,\mathrm{d}\mathcal{T}_3\,\mathrm{d}t\,\mathrm{d}z} = \frac{\sigma_0}{Q^2}\sum_\kappa H_2(Q^2,\mu)\,H_+^\kappa(t,z,\mu)\prod_i \int \mathrm{d}s_i\,J_{\kappa_i}(s_i,\mu)$$
$$\times \int \mathrm{d}k_1\mathrm{d}k_2\,S_+^\kappa(k_1,k_2,\mu)\,S_2\Big(\mathcal{T}_1 - \frac{s_1}{Q_1} - k_1, \mathcal{T}_2 - \frac{s_2}{Q_2} - k_2, \mathcal{T}_3 - \frac{s_3}{Q_3}, \mu\Big)$$

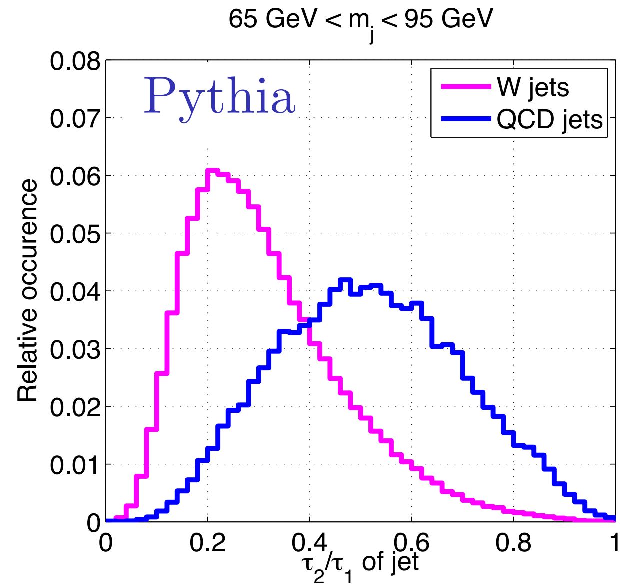
$$\frac{\mathrm{d}\sigma}{\mathrm{d}m_{jj}} = 2m_{jj}\int_{z_{\rm cut}}^{1-z_{\rm cut}} \mathrm{d}z \int_0^{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}\,\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}\,\mathrm{d}t\,\mathrm{d}z}$$

$$Q = 500\,\mathrm{GeV}\,,\quad \mathcal{T}_{\rm cut} = 10\,\mathrm{GeV}\,,\quad z_{\rm cut} = \frac{1}{3}$$





Jet Substructure



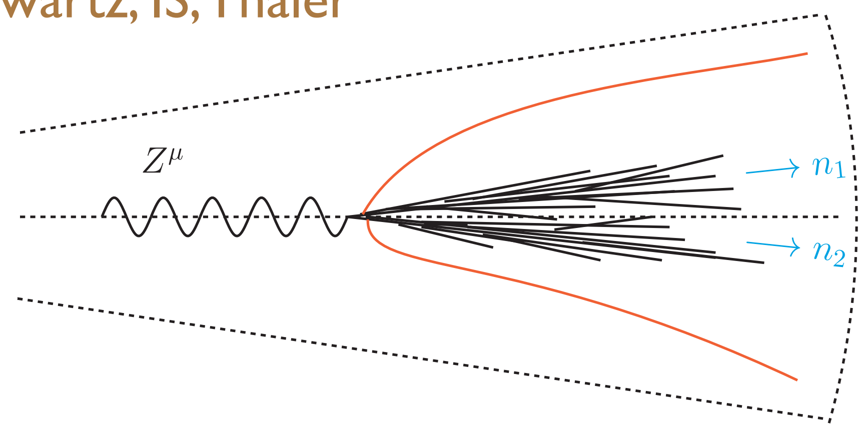
N-subjettiness

Thaler, Van Tilburg

$$\mathcal{T}_N \equiv \min_{n_1, n_2, \ldots, n_N} \sum_{j \in J} \min\{n_1 \cdot p_j, n_2 \cdot p_j, \ldots, n_N \cdot p_j\}$$

Calculation for the signal

Feige, Schwartz, IS, Thaler



$|\vec{p}_Z| = Q$ **large**

$$\widehat{\mathcal{T}}_1 \equiv P_Z^+ = \sqrt{Q^2 + m_Z^2} - Q$$

$\Delta\tau \equiv \mathcal{T}_1 - \widehat{\mathcal{T}}_1$ measures contamination from UE/ISR

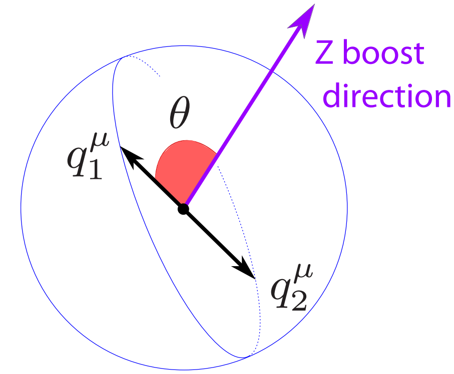
$$\tau_{21} \equiv \frac{\mathcal{T}_2 - \Delta\tau}{\mathcal{T}_1 - \Delta\tau}$$

has good large Q limit

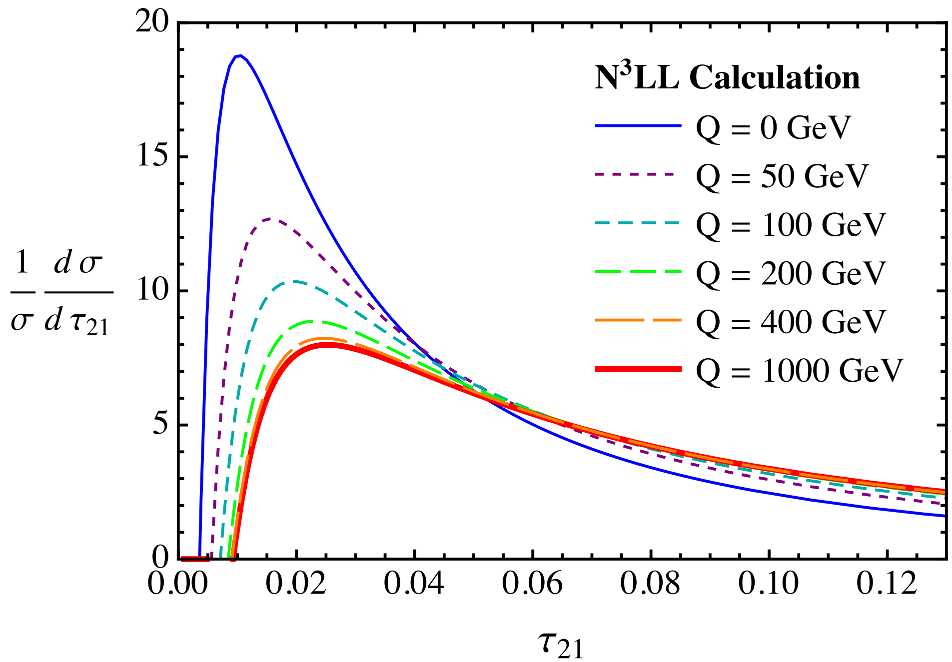
only has contamination at O(1/Q)

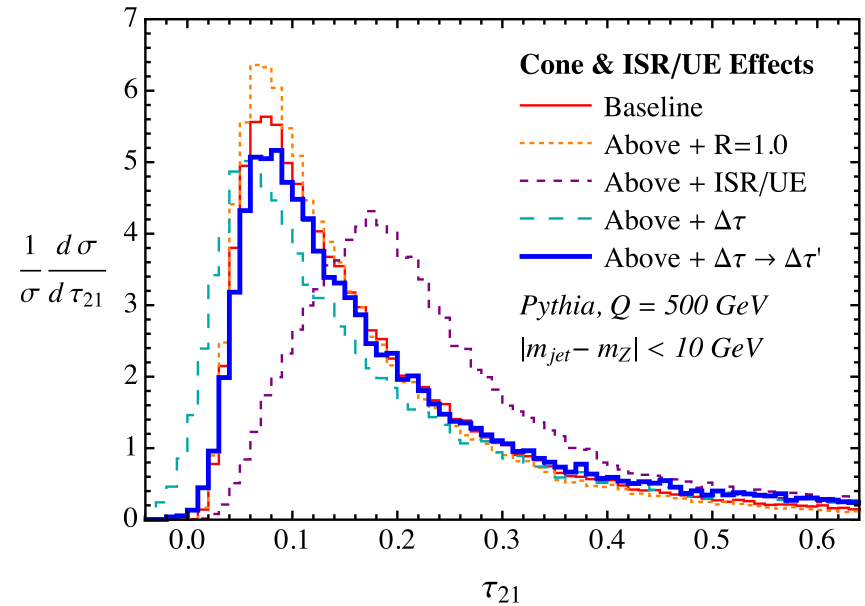
- boost event shape factorization theorem

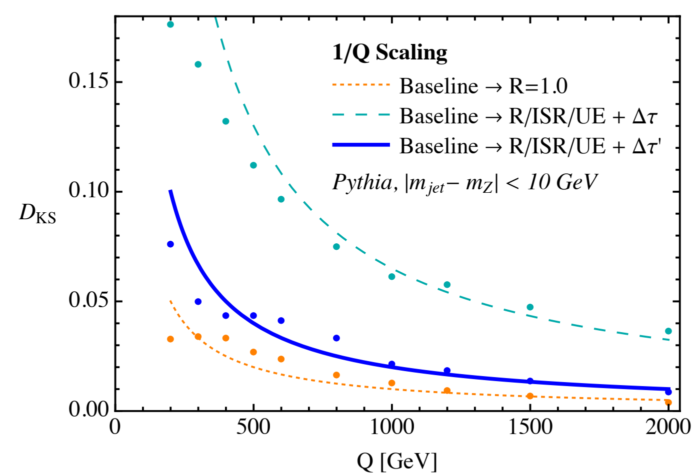
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_{21}} = H\int\frac{d\cos\theta}{2}\int ds_1 ds_2 dk_1 dk_2\, S(k_1,k_2,\{n_i\},\mu) \times J(s_1,\mu)J(s_2,\mu)\,\delta\Big(\tau_{21}-\frac{k_1+k_2}{\widehat{\mathcal{T}}_1}-\frac{s_1E_2+s_2E_1}{2E_1E_2\widehat{\mathcal{T}}_1}\Big)$$



Q=200 GeV already large



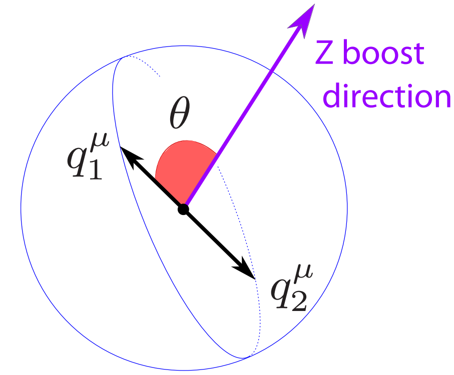




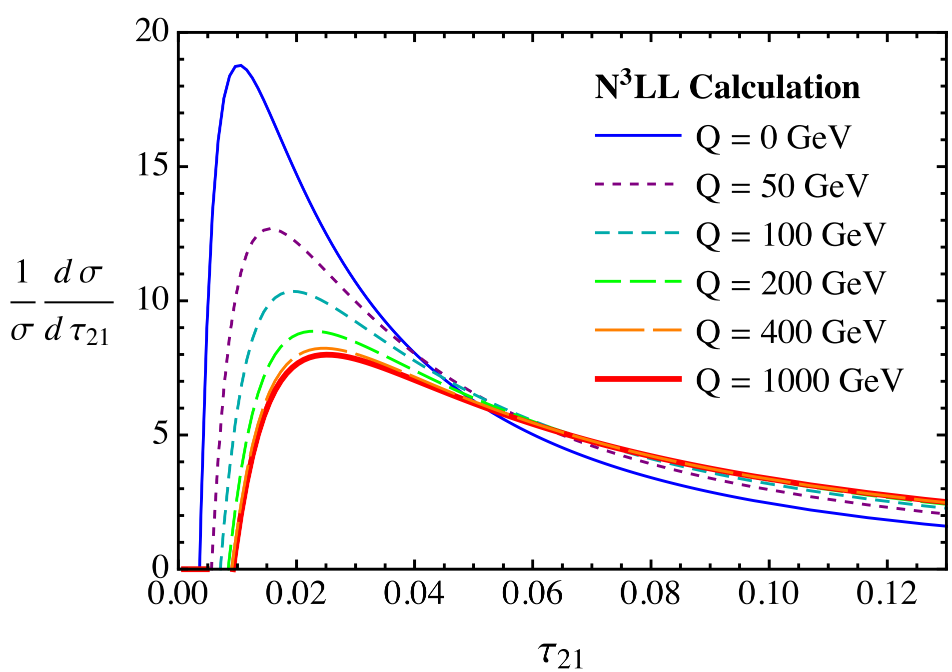
$$\Delta\tau \to \Delta\tau' = \Delta\tau\big(1-\frac{\pi m_Z}{2Q}\big)$$

- boost event shape factorization theorem

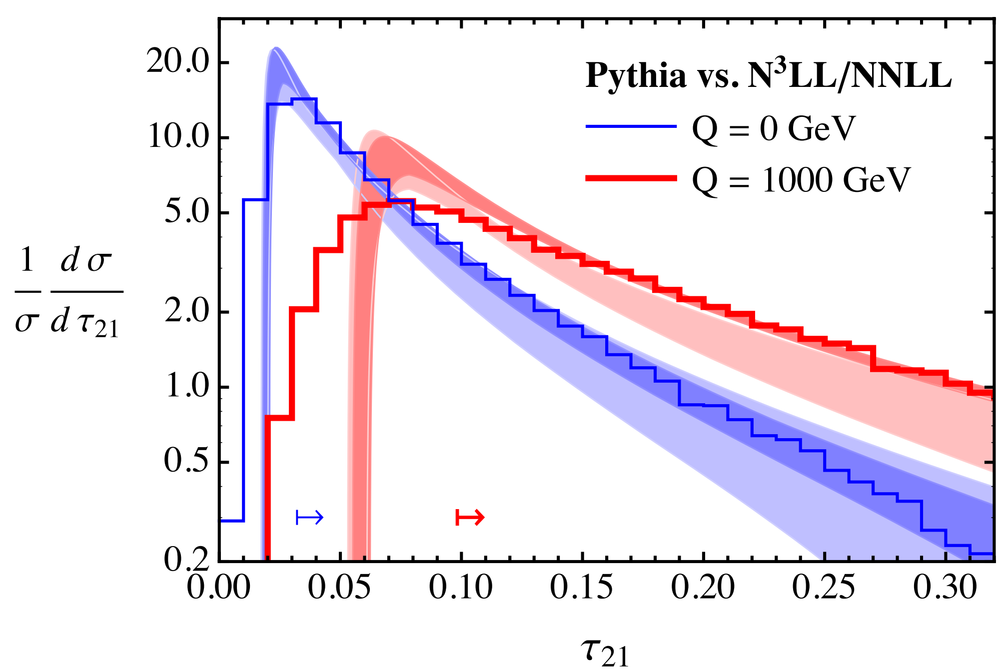
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_{21}} = H\int\frac{d\cos\theta}{2}\int ds_1 ds_2 dk_1 dk_2\, S(k_1,k_2,\{n_i\},\mu) \times \; J(s_1,\mu)J(s_2,\mu)\,\delta\Big(\tau_{21}-\frac{k_1+k_2}{\widehat{\mathcal{T}}_1}-\frac{s_1E_2+s_2E_1}{2E_1E_2\widehat{\mathcal{T}}_1}\Big)$$



Q=200 GeV already large



agrees well with (tuned) Pythia

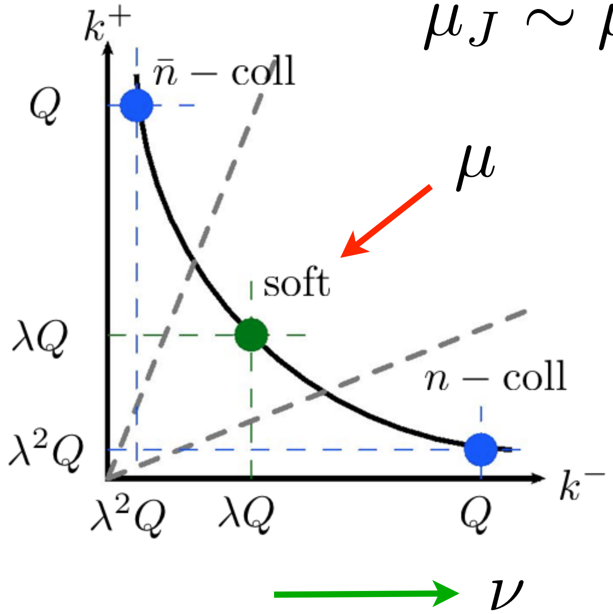


p_T resummation in SCET

many authors $\mathrm{SCET}_1 \rightarrow \mathrm{SCET}_2$

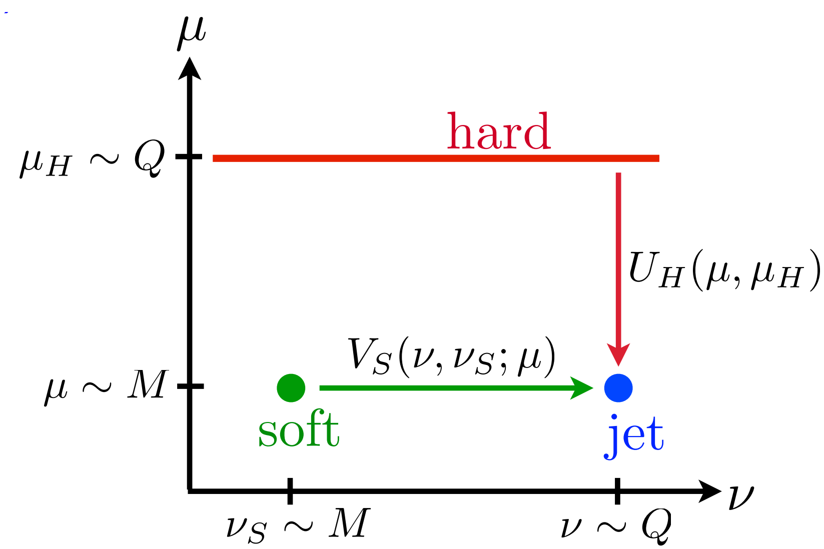
Chiu, Jain, Neill, Rothstein

$\mu_J \sim \mu_S$



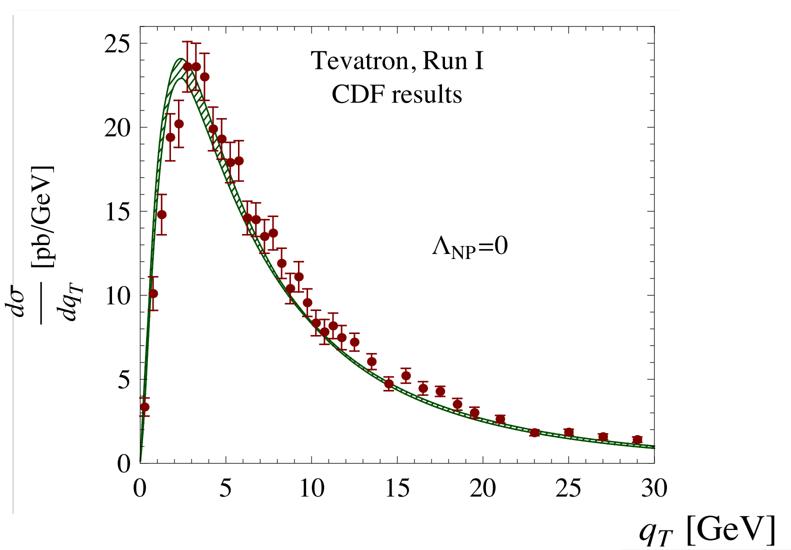
two parameter RGE

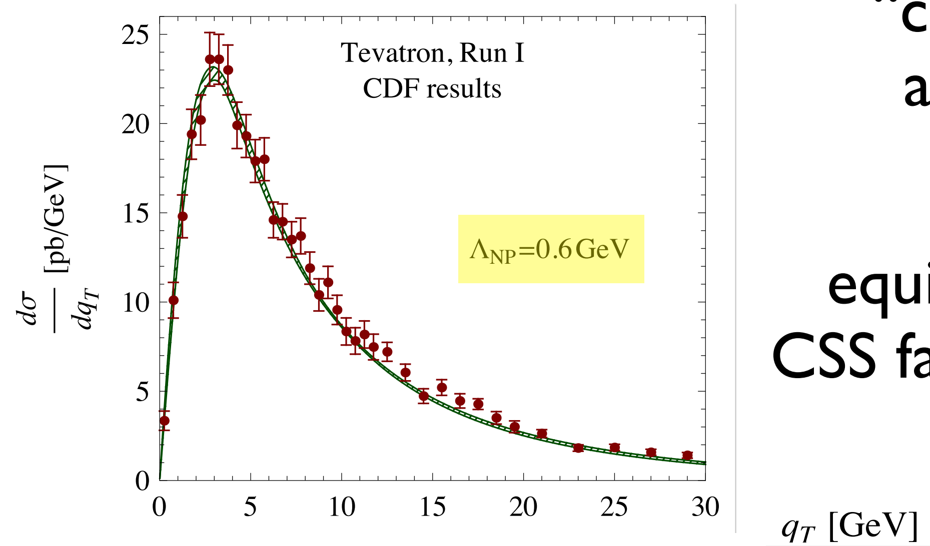
“rapidity RGE”



Z-boson production NNLL+NLO

Becher, Neubert, Wilhelm





“collinear anomaly”

equivalent to CSS factorization

Summary

Exclusive N-jet factorization

- factorization theorems with jet-vetoes, new ways to test MC

$$pp \to H + 0\,\text{jets} \qquad pp \to H + 1\,\text{jet}$$

N-jettiness

- simple yet powerful factorization friendly event shape

Beam Functions

- universal function that describes ISR for broad class of processes

Nearby Jets & Jet substructure

- Sensitive probe of events. Calculations tractable with SCET

Factorization with Jet Algorithms

- Not covered here. Must handle NGL's, clustering logs, ...