NLO parton shower for LHC physics – hard process and beyond

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NLO MC activity in Kraków IFJPAN

- **R&D on the NLO MC parton shower** pursued in Kraków since 2006. It features ladders of NLO kernels in the fully unintegrated/exclusive MC form. (arxiv.org/abs/1102.5083)
- **New method of NLO-correcting HARD process is part of the project (2011).** (arxiv.org/abs/1103.5015)
- **NLO-correcting LADDER parts "kT-ordering" within angular ordering reported here!**
- **Long term: NLO ladders + NNLO hard process:)**
- The whole project is still at the "feasibility study" stage:((

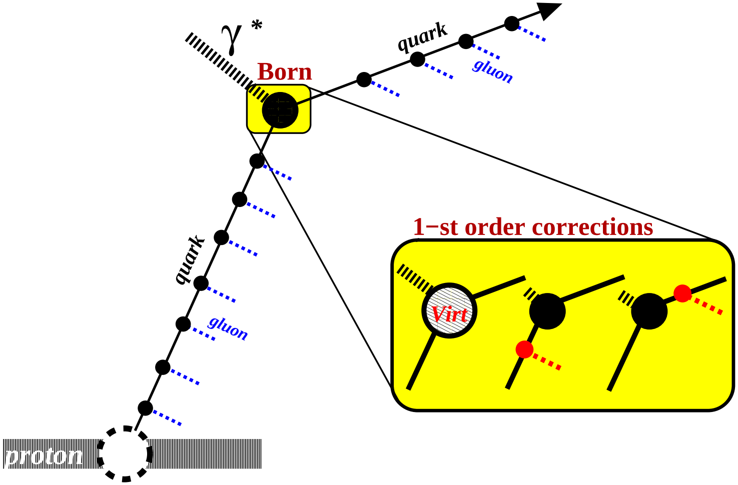
Relation to other works

- Departing from $\overline{MS}$ in order to simplify LO+NLO in the MC is independently advocated by Heidekazu Tanaka et.al. (2007). Who else???
- The basic 2009 method (sum over gluons in NLO wt) is reminiscent to YFS method (worked out in QED by S.J. with Bennie Ward and Zbyszek Wąs)
- Unintegrated PDFs in the context of small x resummation (CCFM) and unitarity saturation...

OVERVIEW

Overview of the new method of introducing NLO corrections in the hard process and the ladders

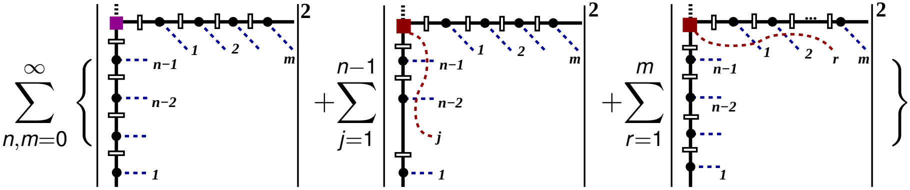
KRKMC method of NLO correcting HARD process



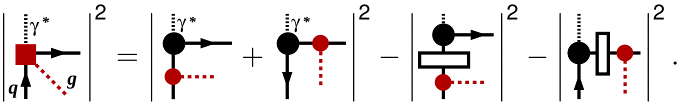
An alternative to MC@NLO and POWHEG

- All $\left(\frac{f(x)}{1-x}\right)_+$ in NLO corrections eliminated,
  annoying $\Sigma^{c\pm}$ of MC@NLO are gone.
  Virtual+soft (unresolved) corrs x-independent.
- Positive weight adds NLO corrections, as in POWHEG,
  but without generating separately hardest gluon,
  no need of vetoed and/or truncated LO showers.
- The price:
  (a) LO psMC must cover entire NLO phase space (retuning?)
  (b) the use of non-MS-bar coll. factoriz. scheme (theory)
  (c) multiple sums in NLO weight (CPU expensive?).

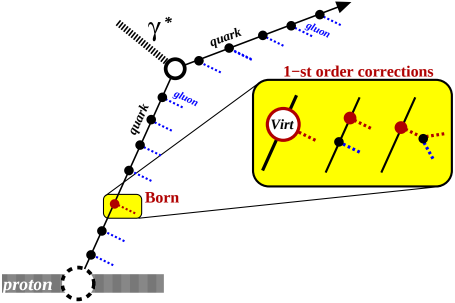
NLO correcting HARD process – details

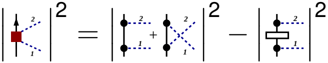


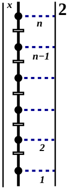
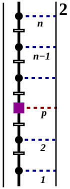
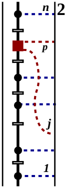
NLO real/virtual distributions (subtracted) from Feynman diags



free of any double/single collinear/soft singularities.





$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Big\{$  $+ \sum_{p=1}^{n}$  $+ \sum_{p=1}^{n}\sum_{j=1}^{p-1}$  $\Big\} = e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\}$$

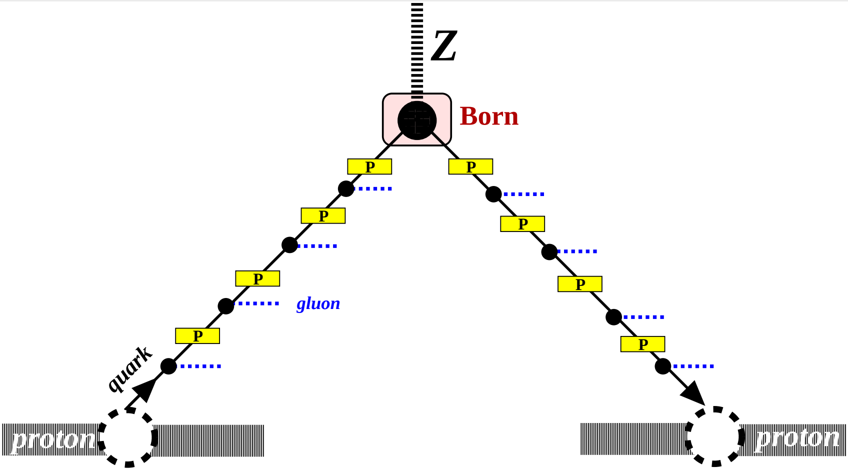
PART I: NLO Hard Process

Parton shower MC with two LO ladders and NLO-corrected hard process:

for W/Z production (Drell-Yan)

gluonstrahlung only –where the problems are

LO psMC is (re-)constructed from the scratch



$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\left\{\sigma\left[C_0^{(0)}(\mathbb{P}'K_{0F}^{(1)})^{n_1}(\mathbb{P}''K_{0B}^{(1)})^{n_2}\right]\right\}_{T.O.}$$

NLO-correcting MC weight

Once LO MC in the above FS scheme is in place, the NLO correction is introduced using simple positive MC weight:

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

where $\bar{P}(z) \equiv \frac{1+z^2}{2}$, the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F,\hat{p}_B;q_1,q_2,k) = \Big[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{B2})\Big]$$
$$-\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\,\frac{1}{2}$$

Terms like $\left(\frac{f(z)}{1-z}\right)_+$ completely absent! ($d\sigma_B$ cancels exactly with LO.)

EXACT analytical integration of the LO+NLO MC distributions over the multigluon phase space:

$$\sigma(C_0^{(1)}\Gamma_F\Gamma_B) = \int_0^1 d\hat{x}_F \, d\hat{x}_B \, dz \, D_F(t,\hat{x}_F) \, D_B(t,\hat{x}_B) \, \sigma_B(sz\hat{x}_F\hat{x}_B) \times \left\{ \delta_{z=1}(1+\Delta_{S+V}) + C_{2r}^{psMC}(z) \right\}$$

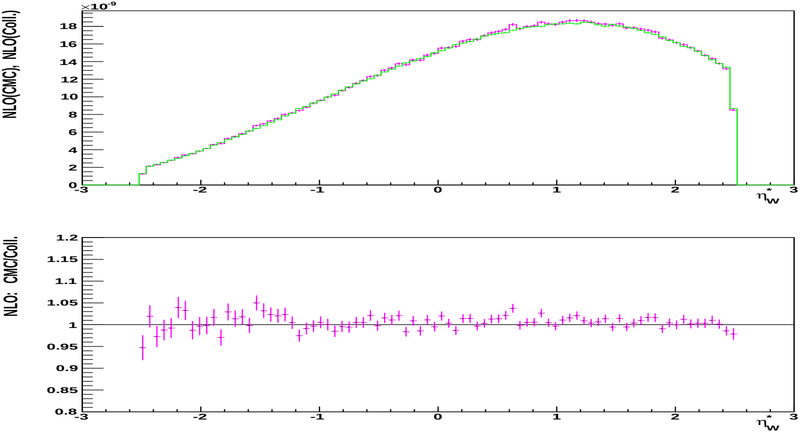
where t= rapidity diff. hadron – hard process,

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi} \left[-\frac{1}{2}(1-z) \right].$$

It differs from $\overline{MS}$ result (Altarelli-Ellis-Martinelli 1979)

$$C_{2r}^{\overline{MS}}(z) = \frac{C_F\alpha_s}{\pi} \left(\frac{1+z^2}{2(1-z)} [2\ln(1-z) - \ln z] \right)_+$$

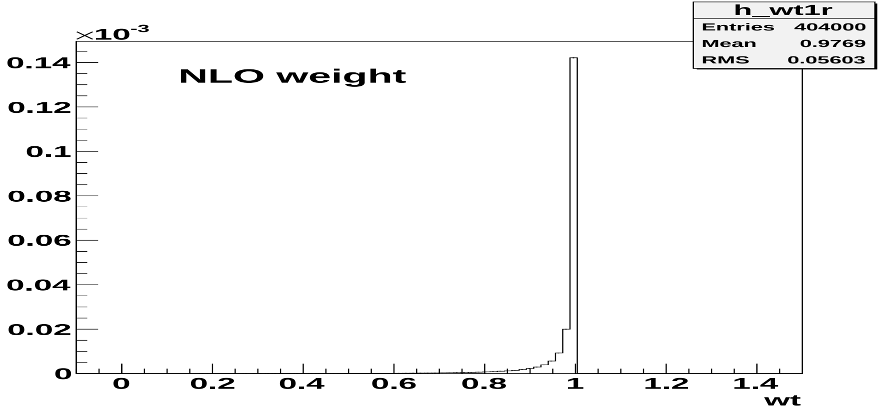
Precision numerical x-check of NLO corr. (Feb. 2012)



Pure NLO correction (real part) of the NLO weight $1 - W^{NLO}_{MC}$
compared with collinear formula $-C_{2r} \otimes D_{q\in p}(t) \otimes D_{\bar{q}\in p}(t)$
in the distribution of $\eta^*_W = \frac{1}{2}\ln(\hat{x}_F/\hat{x}_B)$.

Coll. LO PDFs $D(t,x)$ from special Markov MC run.

How good is NLO weight distribution?



The distribution of the MC weight W_{MC}^{NLO} implementing NLO corrections in gluonstrahlung FROM quark and antiquark annihilating into W boson is just perfect.

Coeff. functions in $\overline{MS}$ and MC factoriz. scheme

Our "coeff. function"

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi} \left[-\frac{1}{2}(1-z)\right]_+$$

differs from $\overline{MS}$ result (Altarelli-Ellis-Martinelli 1979)

$$C_{2r}^{\overline{MS}}(z) = \frac{C_F\alpha_s}{\pi} \left(\frac{1+z^2}{2(1-z)}[2\ln(1-z) - \ln z]\right)_+$$

by the difference of collinear counterterms in $\overline{MS}$ and MC schemes.

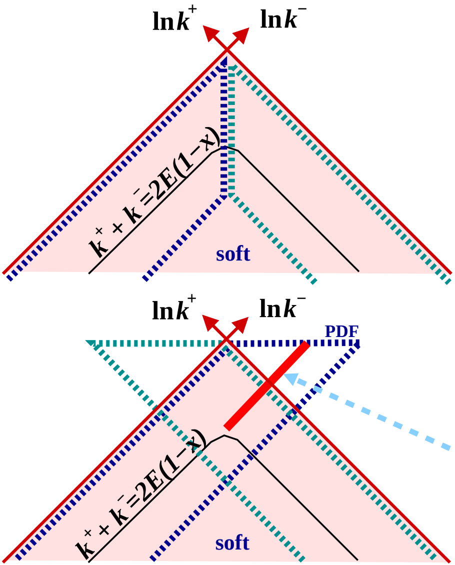
In MC@NLO and POWHEG this convolution is part of NLO corr. to hard proc. – absent in KRKMC NLO scheme.

In KRKMC PS the use of MC FS is a built-in feature anyway.

No problems with universality, see arxiv.org/abs/1103.5015.

Difference between $\overline{MS}$ and MC fact. schemes

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



psMC fact. scheme:

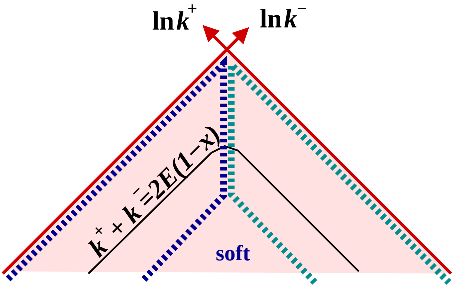
$$0\ \frac{|\ln(1-x)|}{1-x}$$

$\overline{MS}$ fact. scheme:

$$\frac{1}{1-x}\int_{1-x}^{1/(1-x)}\frac{d\beta}{\beta} = 2\frac{|\ln(1-x)|}{1-x}$$

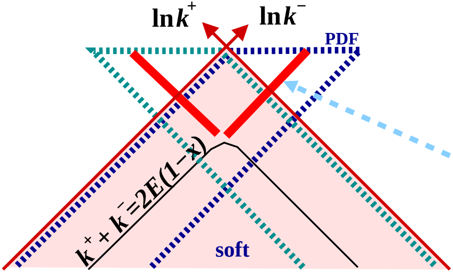
Difference between $\overline{MS}$ and MC fact. schemes

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



psMC fact. scheme:

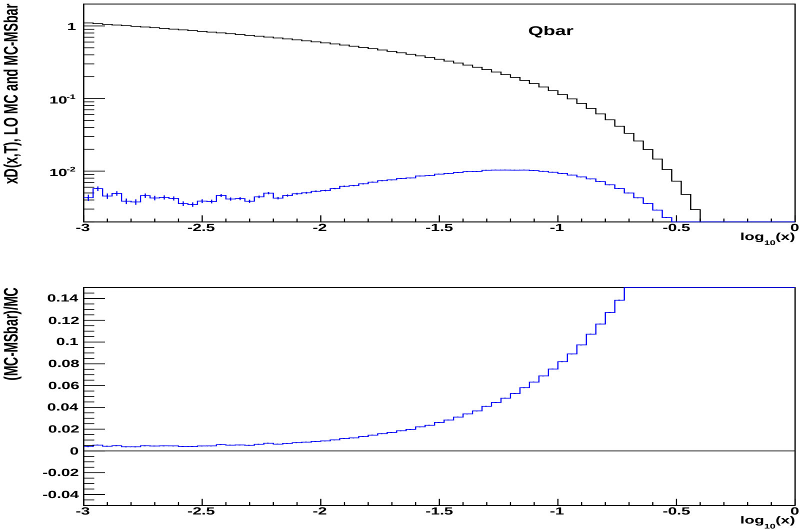
$$0\ \frac{|\ln(1-x)|}{1-x}$$



$\overline{MS}$ fact. scheme:

$$\frac{2}{1-x}\int_{1-x}^{1/(1-x)}\frac{d\beta}{\beta} = 4\,\frac{|\ln(1-x)|}{1-x}$$

PDF translation MC $\rightarrow$ $\overline{MS}$ fact. scheme



$xD_{\bar{q}\in p}(x)$ convoluted with $\Delta C^{\overline{MS}\rightarrow MC}(z) = C_{2r}^{\overline{MS}}(z) - C_{2r}^{psMC}(z)$.

Summation over spectator LO gluons in W_{MC}^{NLO}

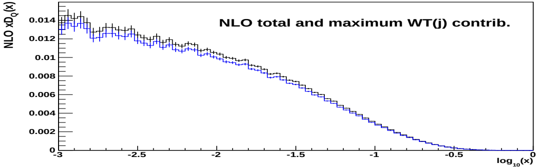
$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

The summation over all LO (spectator) gluons in W_{MC}^{NLO} is important feature of our NLO MC scheme,

Gluon entering NLO corr. to hard process in POWHEG is selected "by hand"; in MC@NLO exported to non-positive H-events. Hence negative WT events in MC@NLO and vetoed/truncated gluons in POWHEG (for angul.ord.).

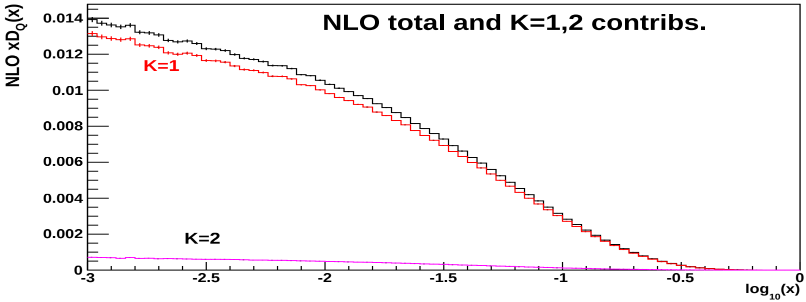
IS THIS SUM REALLY ESENTIAL?

It can be checked that *only one single term dominates the sum:*



which one?

Only one single term dominates the sum in W_{MC}^{NLO}



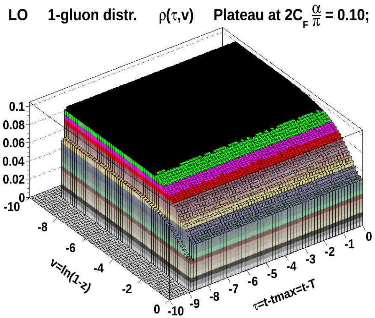
The (-)NLO contributions from $K = 1, 2$ single gluons, the one with maximum and another one with next-to-max. kT, in the x-distribution of quark entering W boson.

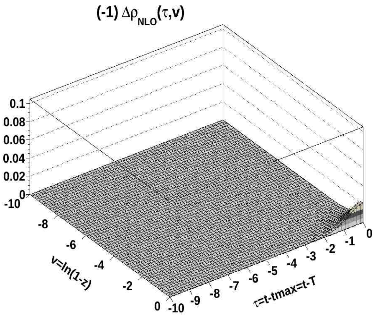
POWHEG exploits the above. We can do it differently, without vetoed/truncated gluons, see next slides...

The location and size of the (real) NLO correction on the Sudakov plane (rapidity, ln(1-z))

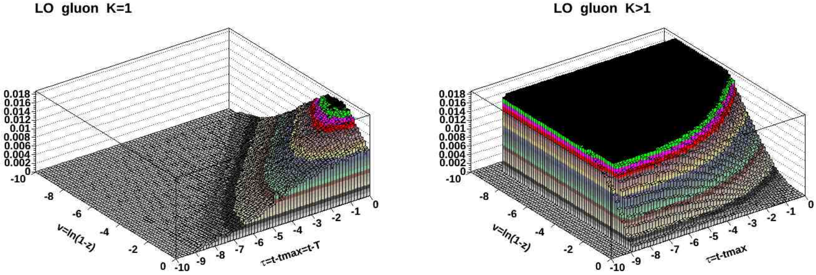
LO inclusive

pure NLO





Gluons generated in ang.ord. and re-ordered in kT



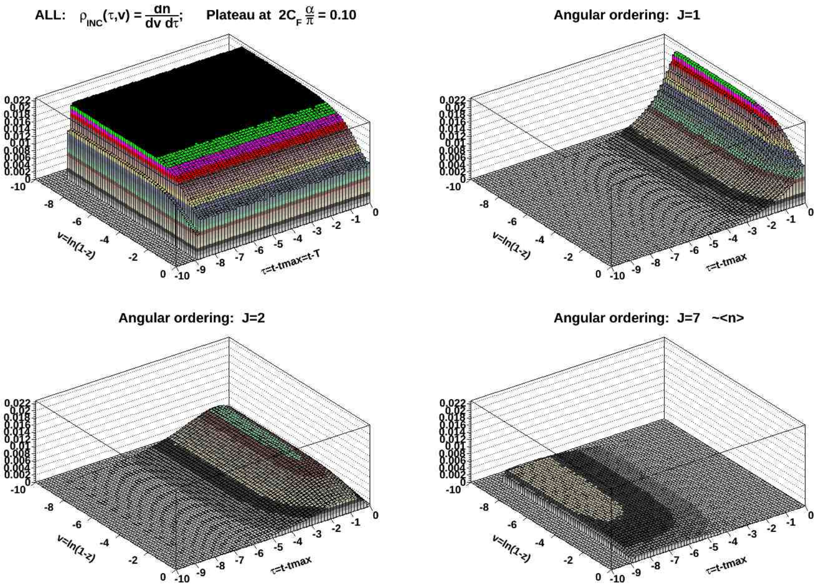
Sudakov suppression for the highest kT gluon (K=1)!

Our NLO weight with summation is ignorant about the above kT (re-)ordering.

The summation (or max. kT-selection) takes care of picking up correctly the hardest gluon.

No need of truncated/vetoed gluon showers.

Gluons ordered in rapidity (in our LO MC)



Rapidity *labeling* used in our LO MC (and in HERWIG).

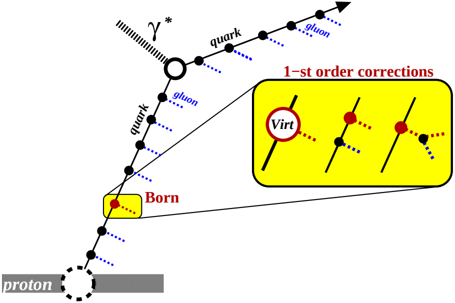
PART II: More on NLO in the ladder

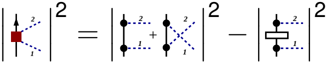
NEW!!!!

Sudakov suppression exploited to reduce multiple sums over spectator gluons,

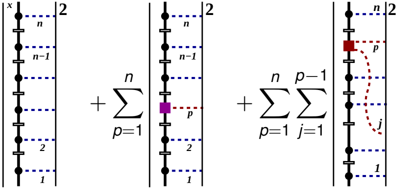
while introducing NLO corrections in the ladder!

NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$



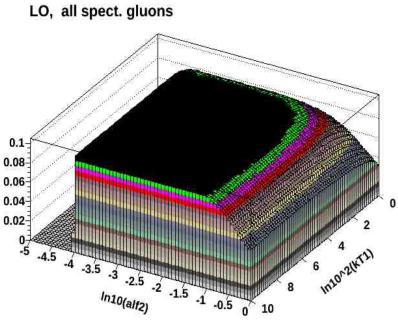


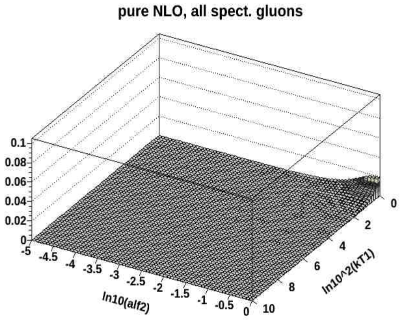
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{ \ldots + \sum_{p=1}^{n} \ldots + \sum_{p=1}^{n}\sum_{j=1}^{p-1} \ldots \Bigg\}$$



$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho^{(1)}_{1B}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n}\sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

Location and size of the (real) NLO correction in the ladder on the Sudakov log space

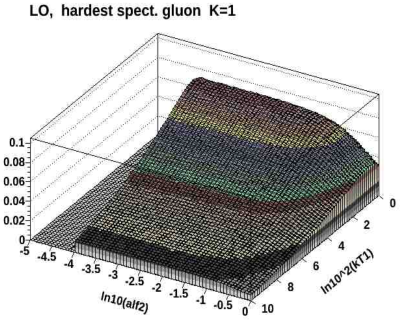


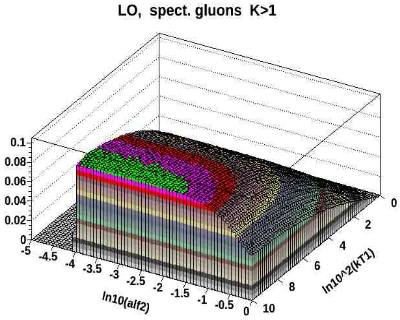


LO inclusive distribution features triple-log IR/coll. singularity, seen as a plateau in 2-dim. projection.

NLO correction IR/coll. finite, nonzero in the corner of the size ~ 1.

Split of inclusive LO distribution of gluons into contr. from the hardest one and the rest

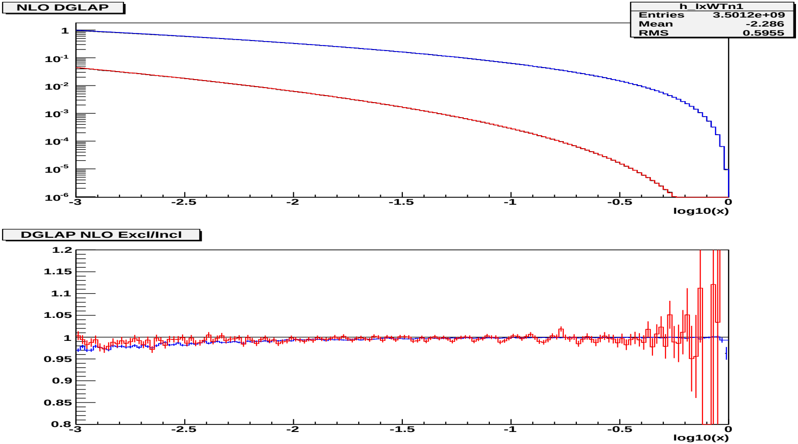




Distribution of the hardest LO spectator gluon approximates the total distribution in small corner where NLO is non-zero.

Repetition of 2009 test for NLO-corrected ladder

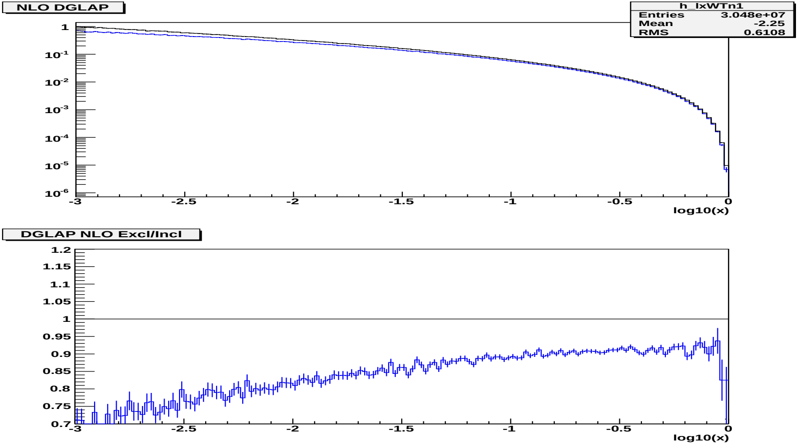
RADCOR 2009: NLO MC vs. analyt. NLO kernels. Perfect agreement



Single ladder, 1GeV-1TeV, 1 or 2 kernels NLO-corrected (3G ev.)

Repetition of 2009 test for NLO-corrected ladder

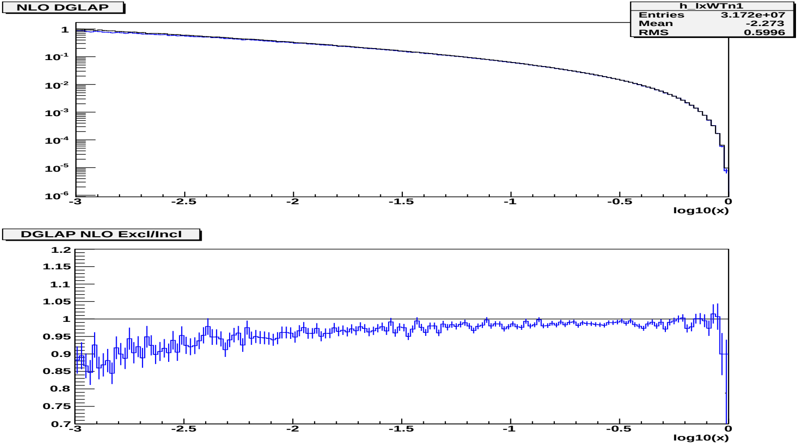
PRELIMINARY!!! May 2012: single contrib. from gluon with max. kT



This difference $\sim 30\%$ is formally the NNLO/NLO class. (30M evts)

Repetition of 2009 test for NLO-corrected ladder

PRELIMINARY!!! May 2012: contrib. from 2 gluon with max. kT



The difference is acceptable but still $\sim$ 10% (formally N3LO/NLO).

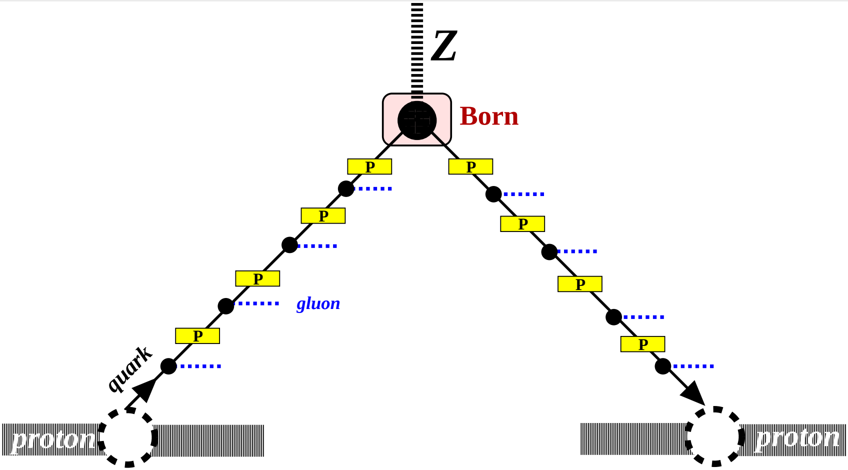
Summary on KRKMC project

- **Parton shower MC implementing complete NLO DGLAP in the ladders in exclusive way is feasible.**
- **Alternative (to MC@NLO or POWHEG) scenario for NLO-corrected hard proc. and LO PSMC is proposed.**
- Long term: NLO ladder + NNLO hard process, but (LO ladder + NLO hard proc. to be optimized first!!!)
- Most likely application: high quality QCD+EW+QED MC with hard process like $W/Z/H$ boson production.
- Potential gains from new QCD methods are:
  – reducing uncertainties due to distributions of partons in hadrons (PDFs, parton luminosities etc.)
  – easier implementation of NLO and NNLO corrections to hard process due to elimination of “trivial” (albeit numerically sizeable) soft gluon corrections
  – better environment for low x resummation (BFKL, CCFM) and heavy quark masses.

Description of LO parton shower Monte Carlo

for W/Z production (Drell-Yan)

LO psMC is (re-)constructed from the scratch



$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\left\{\sigma\left[C_0^{(0)}(\mathbb{P}'K_{0F}^{(1)})^{n_1}(\mathbb{P}''K_{0B}^{(1)})^{n_2}\right]\right\}_{T.O.}$$

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int d\hat{x}_F \, d\hat{x}_B$$

$$\times\, e^{-S_F} \int_{\Xi>\eta_{n_1}} \left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Fi}) \right) \delta_{\hat{x}_F=1-\sum_j \hat{\alpha}_j}$$

$$\times\, e^{-S_B} \int_{\Xi<\eta_{n_2}} \left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}} \frac{2C_F\alpha_s}{\pi^2} \bar{P}(z_{Bi}) \right) \delta_{\hat{x}_B=1-\sum_j \hat{\beta}_j}$$

$$\times\, d\tau_2(P - \sum_{j=1}^{n_1+n_2} k_j; q_1, q_2) \, \frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B, \hat{\theta}),$$

S_F and S_B = Sudakov formfactors, $\bar{P}(z) = \frac{1}{2}(1+z^2)$,
Ξ = Rapidity of the division plane between F and B hemispheres.
θ = angle of decay products (leptons) in Z rest frame.
$\hat{s} = s\hat{x}_F\hat{x}_B$ = effective mass of Z boson.

LO psMC for W/Z production: Details

$$
\begin{aligned}
\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = & \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B \\
& \times e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j} \\
& \times e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j} \\
& \times d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),
\end{aligned}
$$

Eikonal phase space for real gluon:
$d^3\mathcal{E}(k) = \frac{d^3k}{2k^0}\,\frac{1}{\mathbf{k}^2} = \pi\frac{d\phi}{2\pi}\frac{d\alpha}{\alpha}d\eta = \pi\frac{d\phi}{2\pi}\frac{d\beta}{\beta}d\eta,$
Lightcone variables: $\alpha = \frac{k^+}{2E},\ \beta = \frac{k^-}{2E}$; rapidity: $\eta = \frac{1}{2}\ln\frac{k^+}{k^-}$,
$d\tau_2(Q;q_1,q_2)$ = two-body phase space element.

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B$$

$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$

$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$

$$\times\, d\tau_2(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

Variables in LO evolution kernels:

$z_{Fi}=\frac{\hat{x}_{Fi}}{\hat{x}_{F(i-1)}},\ \hat{x}_{Fi}=1-\sum_{j=1}^{i}\hat{\alpha}_i=\prod_{j=1}^{i}z_{Fj},$

$z_{Bi}=\frac{\hat{x}_{Bi}}{\hat{x}_{B(i-1)}}\ \hat{x}_{Bi}=1-\sum_{j=1}^{i}\hat{\beta}_i=\prod_{j=1}^{i}z_{Bj},$

Lightcone variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ in the tangent space, see next slide.

Define hat-variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ of tangent space

Mappings entering definition of $\mathbb{P}'$ and $\mathbb{P}''$ proj. operators of CFT

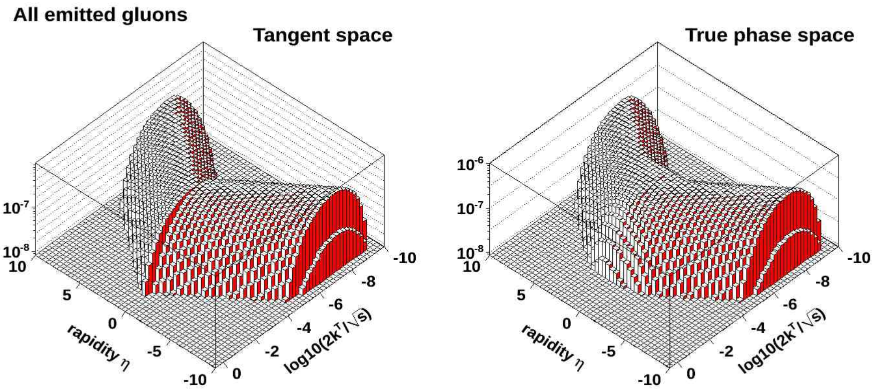
Order ALL gluons according to rapidity distance from Ξ, rapidity position of the hard process (W/Z+G):
Permutation $\pi = \{\pi_1, \pi_2, ..., \pi_{n_1+n_2}\}$ defined such that
$|\eta_{\pi_i} - \Xi| > |\eta_{\pi_{i-1}} - \Xi|, \quad i = 1, ..., n_1 + n_2$

Recursively defined dilatations transform from tangent space to true phsp:
$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}, \quad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2(P - \sum_{j=1}^{i-1} k_{\pi_j}) \cdot \bar{k}_{\pi_i}}, \quad i = 1, 2, ..., n_1 + n_2.$

Rescaling factor λ_i obey:
$\bar{s}_i = s\bar{x}_i = s \prod_{j=1}^{i} \hat{z}_{(F,B)\pi_j} = (P - \sum_{j=1}^{i} k_{\pi_j})^2 = (P - \sum_{j=1}^{i} \lambda_j \bar{k}_{\pi_j})^2.$

In F hemisphere $\hat{\alpha}_i = \lambda_i \alpha_i$ and in B hemisphere $\hat{\beta}_i = \lambda_i \beta_i$.
(An improvement over 1st such scenario publ. in 2007 in APP, Stephes at.al.)

Transformation from the tangent space to real phase space. W production at $\sqrt{s} = 7$TeV



The inclusive distribution of gluons emitted from quark and antiquark in the "tangent space" and in the true phase space (after rescaling) on the Sudakov plane of rapidity and $ln(k^T)$.

Overview of the LO Monte Carlo algorithm:

- Variables $\hat{z}_F$ and $\hat{z}_B$ are generated by FOAM. $\Xi = 0$ is used.
- Four momenta $\bar{k}_i^\mu$ are generated separately in F and B tangets spaces using CMC module, with the constraints $\sum_{j\in F} \hat{\alpha}_j = 1 - \hat{z}_F$ and $\sum_{j\in B} \hat{\beta}_j = 1 - \hat{z}_B$.
- Double ordering permutation π is established.
- Using P and $\bar{k}_{\pi_1}$ rescaling parameter λ_1 is calculated, $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set. At this stage $(P - k_{\pi_1})^2 = sx_1$, where $x_1 = z_{\pi_1} = 1 - \hat{\alpha}_{\pi_1}$ or $x_1 = z_{\pi_1} = 1 - \hat{\beta}_{\pi_1}$, depending whether k_{π_1} was in F or B part of LIPS.
- Using $P - k_{\pi_1}$ and $\bar{k}_{\pi_2}$ parameter λ_2 is found and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set. At this stage we enforce $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$. This recursive procedure continues until the last gluon.
- In the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ 4-momenta of q_1^μ and q_2^μ are generated according to Born angular distribution.
- Get rapidity η_h of the hard process (W/Z + hard G), take $\Xi = \eta_h$ and repeat mapping from tangent space to true phsp.