Parton Showers

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Event Generators and Resummation, 29 May – 1 Jun 2012

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Quarks and gluons in final state, pointlike.

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Generated from emissions *ordered* in Q.
Soft and/or collinear emissions.

Outline

Review

- Building a parton shower, what's inside?
- Splitting functions, Sudakov FF, Angular ordering…
- Jet rates and resummations.
- Some data comparison.
- Some non–perturbative issues.

Covered in other talks:

- Matching to NLO.
- Merging higher multiplicity MEs with PS.
- Subleading Colour.
- MPI.
- Alternatve formulations.

Good starting point: $e^+e^- \to q\bar{q}g$:

Final state momenta in one plane (orientation usually averaged).
Write momenta in terms of

$$x_i = \frac{2p_i \cdot q}{Q^2} \quad (i = 1,2,3) \, ,$$

$$0 \le x_i \le 1 \, , x_1 + x_2 + x_3 = 2 \, ,$$

$$q = (Q, 0, 0, 0) \, ,$$

$$Q \equiv E_{cm} \, .$$

$(x_1, x_2) = (x_q, x_{\bar{q}})$ –plane:

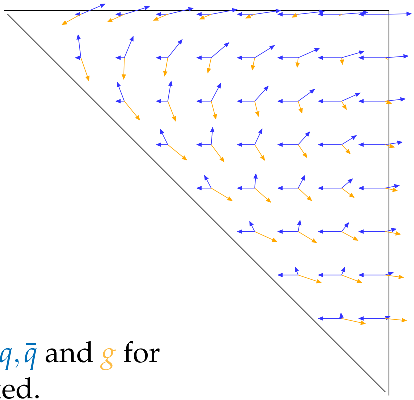
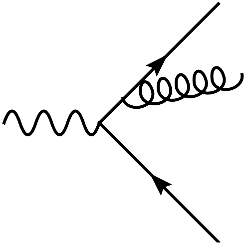


Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1, x_2), $\bar{q}$ direction fixed.

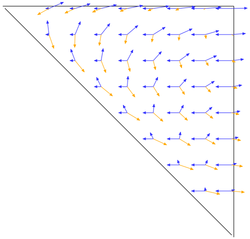
e^+e^- annihilation

Differential cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \frac{x_1 + x_2}{(1-x_1)(1-x_2)}$$



Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$.
Soft singularity: $x_1, x_2 \to 1$.



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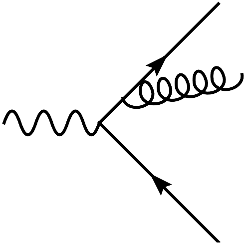

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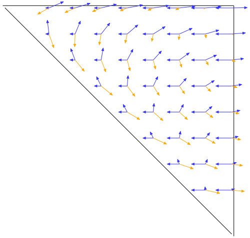
Rewrite in terms of x_3 and $\theta = \angle(q,g)$:

$$\frac{d\sigma}{d\cos\theta\, dx_3} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \left[\frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right]$$



Singular as $\theta \to 0$ and $x_3 \to 0$.

Can separate into two jets as

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta}$$
$$= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}$$
$$\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

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So, we rewrite $\mathrm{d}\sigma$ in collinear limit as

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z^2} \mathrm{d}z$$

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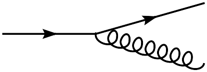
$$= \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

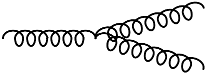
with DGLAP splitting function $P(z)$.

Universal DGLAP splitting kernels for collinear limit:

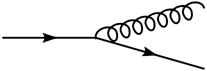
$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z)\mathrm{d}z$$



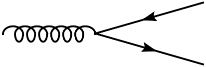
$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{g\to gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{q\to gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g\to qq}(z) = T_R(1-2z(1-z))$$

Collinear limit

Universal DGLAP splitting kernels for collinear limit:

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

Note: Other variables may equally well characterize the collinear limit:

$$\frac{\mathrm{d}\theta^2}{\theta^2} \sim \frac{\mathrm{d}Q^2}{Q^2} \sim \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \sim \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \sim \frac{\mathrm{d}t}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means “collinear”.

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- θ: HERWIG
- Q^2: PYTHIA ≤ 6.3, SHERPA.
- $p_\perp$: PYTHIA ≥ 6.4, ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Resolution

Need to introduce resolution t_0, e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \to 0$.

Emissions below t_0 are unresolvable.

Finite result due to virtual corrections:

= finite.

unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0)\int_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\int_{z_-}^{z_+}\mathrm{d}z\,\frac{\alpha_S}{2\pi}\hat{P}(z) \;= \sigma_2(t_0)\int_{t_0}^{t}\mathrm{d}t\,W(t)\ .$$

Multiple emissions (probabilistic)

$$\sigma_{>2}(t_0) = \sigma_2(t_0)\sum_{k=1}^{\infty}\frac{2^k}{k!}\left(\int_{t_0}^{t}\mathrm{d}t\,W(t)\right)^k = \sigma_2(t_0)\left(\mathrm{e}^{2\int_{t_0}^{t}\mathrm{d}t\,W(t)}-1\right)$$

$$= \sigma_2(t_0)\left(\frac{1}{\Delta^2(t_0,t)}-1\right)$$

Sudakov Form Factor

$$\Delta(t_0,t) = \exp\left[-\int_{t_0}^{t}\mathrm{d}t\,W(t)\right]$$

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$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor in QCD

$$\Delta(t_0, t) = \exp \left[- \int_{t_0}^{t} \mathrm{d}t \, W(t) \right] = \exp \left[- \int_{t_0}^{t} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z,t)}{2\pi} \hat{P}(z,t) \mathrm{d}z \right]$$

Note that

$$\sigma_{\text{all}} = \sigma_2 + \sigma_{>2} = \sigma_2 + \sigma_2 \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right) ,$$
$$\Rightarrow \Delta^2(t_0, t) = \frac{\sigma_2}{\sigma_{\text{all}}} .$$

$$\text{Two jet rate} = \Delta^2 = P^2(\text{No emission in the range } t \to t_0) .$$

$$\text{Sudakov form factor} = \text{No emission probability} .$$

Often $\Delta(t_0, t) \equiv \Delta(t)$.

- Hard scale t, typically CM energy or $p_\perp$ of hard process.
- Resolution t_0, two partons are resolved as two entities if inv mass or relative $p_\perp$ above t_0.
- P^2 (not P), as we have two legs that evolve independently.

Sudakov form factor from Markov property

Unitarity

$$P(\text{“some emission”}) + P(\text{“no emission”}) \\ = P(0 < t \le T) + \bar{P}(0 < t \le T) = 1 \; .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \le T) = \bar{P}(0 < t \le t_1)\bar{P}(t_1 < t \le T)$$

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$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \leq i \leq n$.

$$\bar{P}(0 < t \leq T) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\bar{P}(t_i < t \leq t_{i+1}) = \lim_{n\to\infty}\prod_{i=0}^{n-1}\big(1 - P(t_i < t \leq t_{i+1})\big) \\ = \exp\left(-\lim_{n\to\infty}\sum_{i=0}^{n-1}P(t_i < t \leq t_{i+1})\right) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\ .$$

Again, no–emission probability!

$$\bar{P}(0 < t \leq T) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

So,

$$\begin{aligned}\mathrm{d}P(\text{first emission at } T) &= \mathrm{d}P(T)\bar{P}(0 < t \leq T) \\ &= \mathrm{d}P(T)\exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

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Conveniently, the probability distribution is $\Delta(t)$ itself.

Parton shower Monte Carlo

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Hence, parton shower very roughly from (HERWIG):

1. Choose flat random number $0 \le \rho \le 1$.
2. If $\rho < \Delta(t_{\max})$: no resolvable emission, stop this branch.
3. Else solve $\rho = \Delta(t_{\max})/\Delta(t)$
   (= no emission between $t_{\max}$ and t) for t.
   Reset $t_{\max} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

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Conveniently, the probability distribution is $\Delta(t)$ itself.

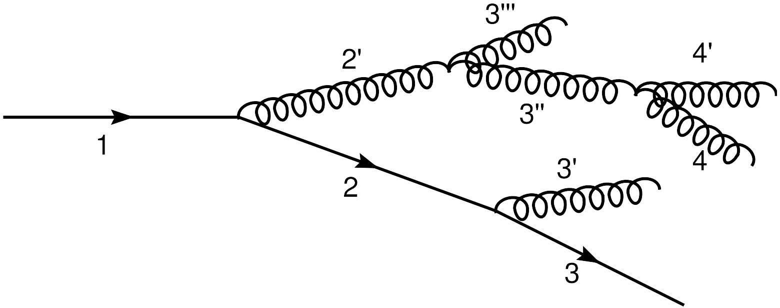
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the Veto Algorithm.
- Method to sample x from distribution of the type

$$\mathrm{d}P = F(x)\exp\left[-\int^{x}\mathrm{d}x'\,F(x')\right]\mathrm{d}x \, .$$

Simpler, more flexible, but slightly slower.

Parton cascade

Get tree structure, ordered in evolution variable t:



Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

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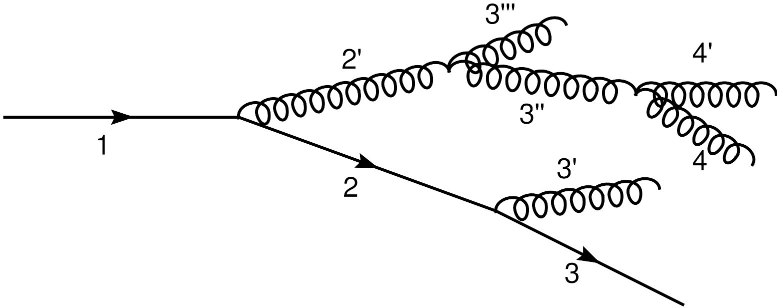

Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
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Not at all unique!
Many (more or less clever) choices still to be made.

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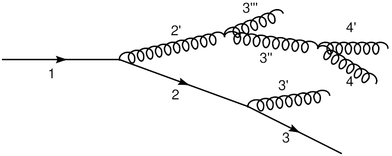


- t can be θ, Q^2, $p_\perp$, ...
- Choice of hard scale $t_{\max}$ not fixed. “Some hard scale”.
- z can be light cone momentum fraction, energy fraction, ...
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- ...

Good choices needed here to describe wealth of data!

Parton shower kinematics (Herwig++)

Sudakov-basis p, n with $p^2 = M^2$ ('forward'), $n^2 = 0$ ('backward'),

$$
\begin{aligned}
p_q &= zp + \beta_q n - q_\perp \\
p_g &= (1-z)p + \beta_g n + q_\perp
\end{aligned}
$$

Collinear limit for radiation off heavy quark,

$$
\begin{aligned}
P_{gq}(z, \mathbf{q}^2, m^2) &= C_F \left[\frac{1+z^2}{1-z} - \frac{2z(1-z)m^2}{\mathbf{q}^2 + (1-z)^2 m^2} \right] \\
&= \frac{C_F}{1-z} \left[1 + z^2 - \frac{2m^2}{z\tilde{q}^2} \right]
\end{aligned}
$$

$\longrightarrow$ $\tilde{q}^2 \sim \mathbf{q}^2$ may be used as evolution variable.

Parton shower kinematics (Herwig++)

- Introduce a kinematical cutoff Q_g in order to avoid the soft gluon singularity (usually $z \to 1$).
- Reconstruct transverse momentum $\mathbf{q}$ from our *definition* of $\tilde{q}$ for $q \to qg$ splitting [$\mu = \max(m_q, Q_g)$],

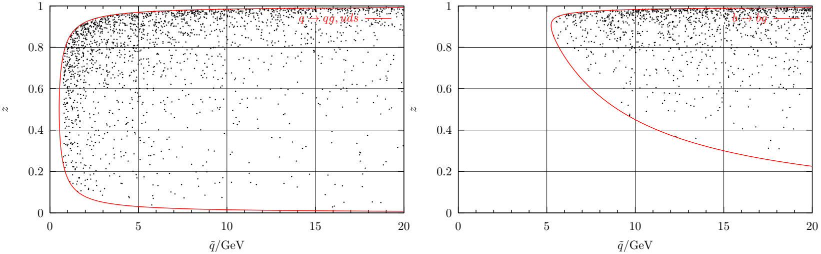
$$z(1-z)\tilde{q}^2 = \frac{\mathbf{q}^2}{z(1-z)} + \frac{Q_g^2}{1-z} + \frac{1-z}{z}\mu^2$$

- Allowed phase space, where $\mathbf{q}$ real. For large $\tilde{q}$

$$\frac{\mu}{\tilde{q}} < z < 1 - \frac{Q_g}{\tilde{q}}.$$

Parton shower kinematics (Herwig++)

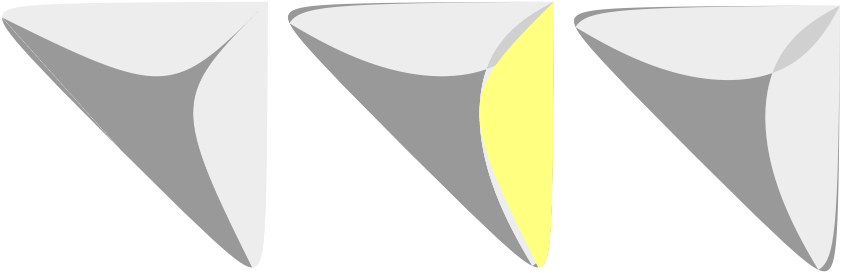
For given $\tilde{q}$ get phase space limits $z_{\pm}(\tilde{q})$.



$(q \to qg/b \to bg)$.

Parton shower kinematics (Herwig++)

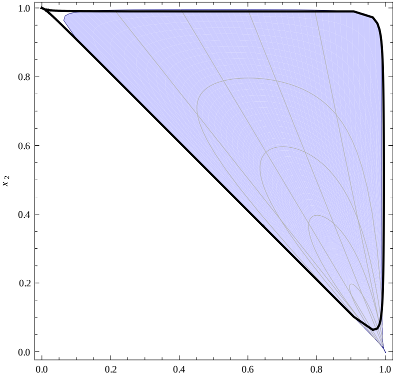
Evolution variables for parton shower evolution in Herwig++.
Consider $(x, \bar{x})$ phase space for $e^+e^- \to q\bar{q}g$



Herwig++ Comparison HERWIG

Possible pitfalls: dead regions, overlap.

Alternative picture with different evolution variables (Catani–Seymour dipole shower).



Lines of const z straight, const $|\mathbf{q}|$. Soft cutoff blown up.

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

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Soft emission: consider *eikonal factors*,
here for $q(p+q) \to q(p)g(q)$, soft g:

$$u(p)\ \not{\varepsilon}\frac{\not{p}+\not{q}+m}{(p+q)^2-m^2} \longrightarrow u(p)\frac{p\cdot\varepsilon}{p\cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.
In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_S}{2\pi}\sum_{ij} C_{ij}W_{ij} \quad (\text{"QCD–Antenna"})$$

with

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})}\ .$$

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .

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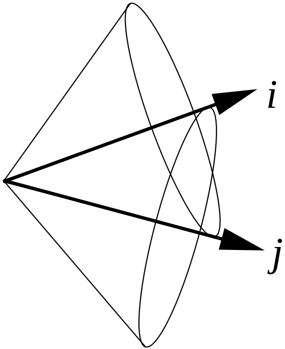
$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .
After integrating out the azimuthal angles, we find

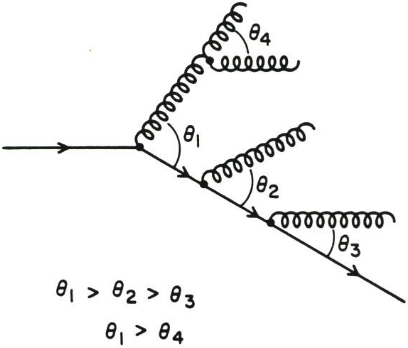
$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \frac{1}{1-\cos\theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

That's angular ordering.

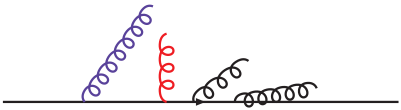
Radiation from parton *i* is bound to a cone, given by the colour partner parton *j*.



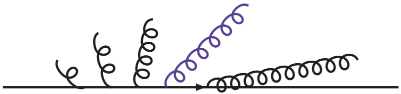
Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.



Angular ordered parton showers



$p_\perp$ ordered shower. Angular ordering from additional vetos.



Angular ordered shower. Some softer emissions before hardest one.

Colour coherence from CDF

Events with 2 hard ($> 100\,\text{GeV}$) jets and a soft 3rd jet ($\sim 10\,\text{GeV}$)

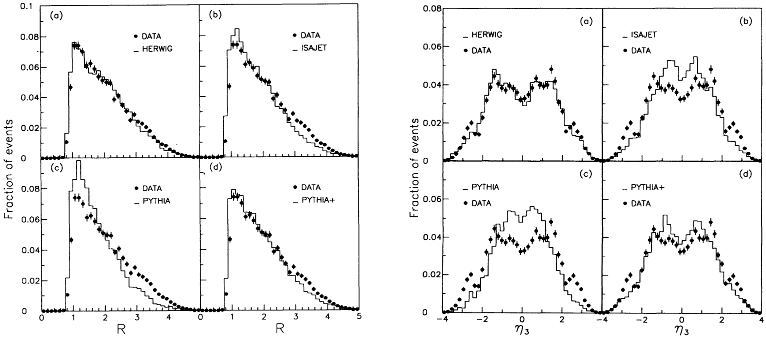


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

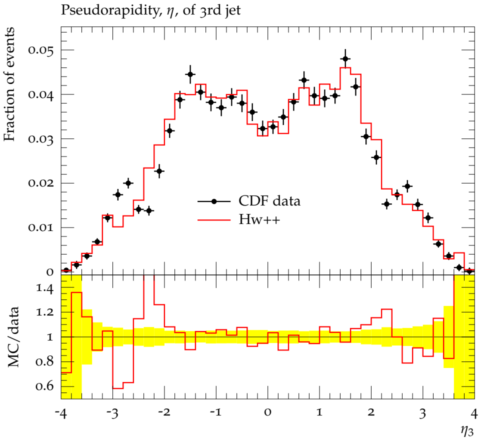
FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Colour coherence from CDF

Events with 2 hard ($> 100\,\text{GeV}$) jets and a soft 3rd jet ($\sim 10\,\text{GeV}$)



F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Parton showers

Herwig++

- New parton shower variables introduced for Herwig++

  [SG, P. Stephens and B. Webber, JHEP **0312** (2003) 045 [hep-ph/0310083]]

- More under development → dipole shower, based upon Catani–Seymour dipoles.

  [SG, S. Plätzer, 0909.5593]]

Sherpa

- CS-Shower default by now, always matched via CKKW (see later).

Pythia

- p_T ordered shower (simple matching).
- Interleaved with Multiple partonic interactions.

Resummation

- Sudakov FF basis for resummation.
- Radiator functions $\Gamma_i(t,t')$ only retaining the leading log terms of the splitting function.
- Formulate parton shower in terms of generating functionals and obtain jet rates (in e^+e^-, at k_T resulution y).

$$f_2(s,y) = \left[\Delta_q(s)\right]^2 ,$$

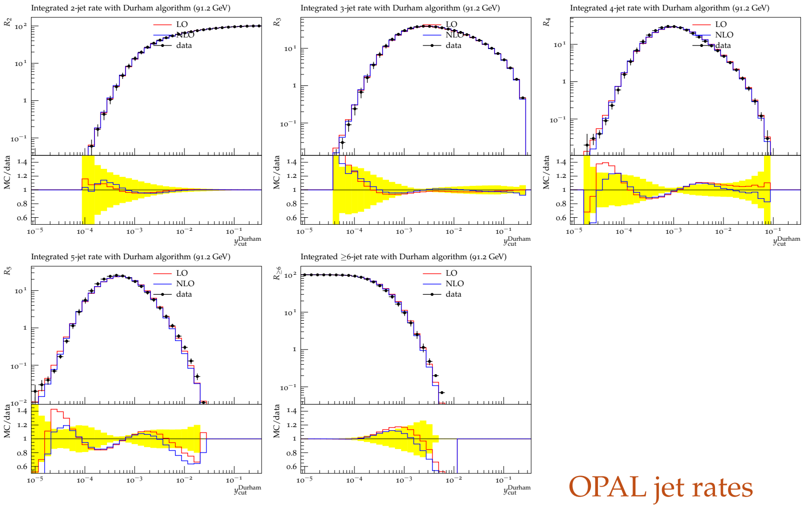
$$f_3(s,y) = 2\left[\Delta_q(s)\right]^2 \int_{t_0}^{s} \mathrm{d}t\, \Gamma_q(s,t)\Delta_g(t) ,$$

$$f_4(s,y) = 2\left[\Delta_q(s)\right]^2 \left\{ \left[\int_{t_0}^{s} \mathrm{d}t\, \Gamma_q(s,t)\Delta_g(t)\right]^2 + \int_{t_0}^{s} \mathrm{d}t\, \Gamma_q(s,t)\Delta_g(t) \int_{t_0}^{s} \mathrm{d}t' \left[\Gamma_g(t,t')\Delta_g(t') + \Gamma_f(t,t')\Delta_f(t')\right] \right\} .$$

Resummation in the parton shower

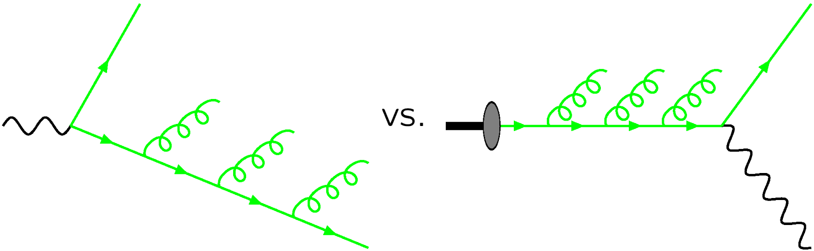
Herwig++ dipole shower.

LO and NLO matched (POWHEG like).



OPAL jet rates

Initial state radiation



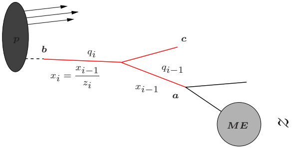
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

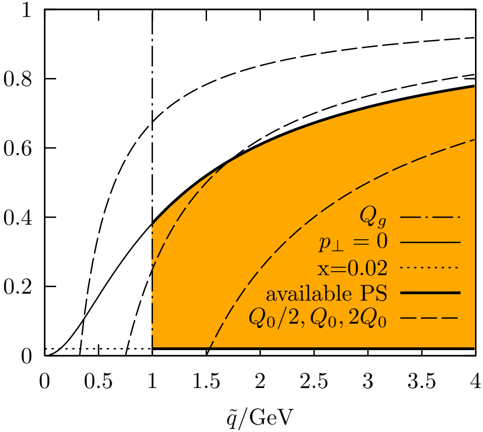
$$\Delta(t, t_{\max}) = \exp\left[-\sum_b \int_t^{t_{\max}} \frac{dt}{t} \int_{z_-}^{z_+} dz \frac{\alpha_S(z,t)}{2\pi} \frac{x' f_b(x', t)}{x f_a(x, t)} \hat{P}_{ba}(z, t)\right]$$

Have to divide out the pdfs.

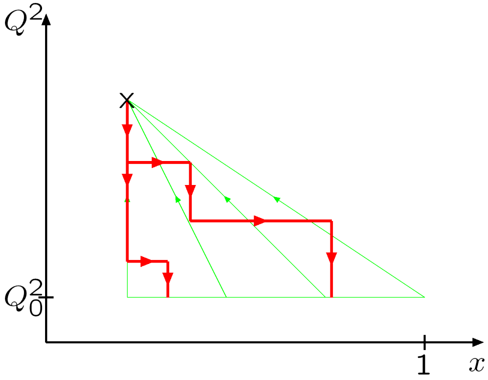
Available phase space limited by several factors:

- Real $\mathbf{q} \Rightarrow x < z < z_{\max}(\tilde{q}; Q_g)$.
- $\tilde{q} > Q_g$
- Maybe $\alpha_S(Q) =$ frozen at low $\tilde{q}$.





Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2. Thereby increase x.



With parton shower we *undo* the DGLAP evolution of the pdfs.

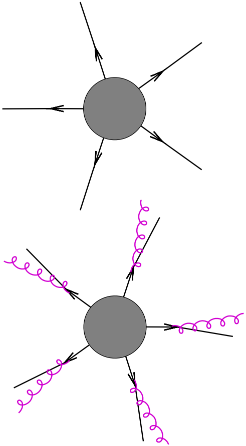
After shower: original partons acquire virtualities q_i^2
→ boost/rescale jets:
Started with

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{m_i^2 + \vec{p}_i^2}$$

we *rescale* momenta with common factor k,

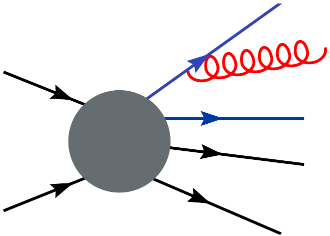
$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{q_i^2 + k\vec{p}_i^2}$$

to preserve overall energy/momentum.
→ resulting jets are boosted accordingly.



Exact kinematics when recoil is taken by spectator(s).

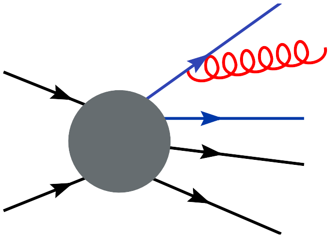
- Dipole showers.
- Ariadne.
- Recoils in Pythia.



Dipoles

Exact kinematics when recoil is taken by spectator(s).

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
  - Catani Seymour dipoles.
  - QCD Antennae.
  - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.



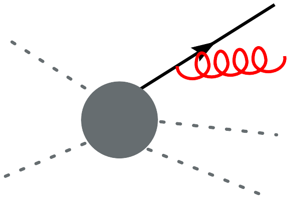
Dipole showers

- (ARIADNE, of course. Not invented for NLO matching.)
- Idea to use CS subtraction terms for parton shower immediately clear from MC@NLO. Written up in

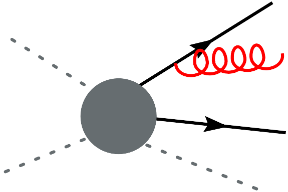
  [Nagy, Soper, JHEP 0510:024 (2005)]
- Catani–Seymour Dipole cascade codes recently implemented by Schumann, Krauss [0709.1027] (with CKKW like matching) and (independently) Dinsdale, Ternick, Weinzierl [0709.1026]
- Lund Dipoles revisited [J. C. Winter and F. Krauss, JHEP **0807**, 040 (2008)]
- Similar approach (VINCIA), based on Antenna subtraction by Giele, Kosower, Skands (toy process) [0707.3652]
- Dipole shower in Herwig++, Coherence/angular ordering clearified see below. [Simon Plätzer, SG, 0909.5593]

Why all those dipoles?

Momentum conservation in shower emissions.



$1 \to 2$ emissions. No momentum conservation. Kinematic reshuffling needed afterwards. Difficult for matching with NLO.



$2 \to 3$ emissions. Momentum conservation simple due to recoil against 3rd parton.

Coherent dipole shower

What about angular ordering/soft coherence?

- Investigate parton showers with local recoils (that's what a CS parton shower is like).
- Angular ordering/soft coherence manifest in Sudakov anomalous dimension

$$\Gamma_q(p_\perp^2, Q^2) = C_F \left(\ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} \right) ,$$

$$\Gamma_g(p_\perp^2, Q^2) = C_A \left(\ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} \right) .$$

[A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. 100 (1983) 201]

[G. Marchesini, B.R. Webber, NPB 238 (1984) 1]

Coherent dipole shower

What about angular ordering/soft coherence?

- Investigate parton showers with local recoils (that's what a CS parton shower is like).
- Angular ordering/soft coherence manifest in Sudakov anomalous dimension

$$\Gamma_q(p_\perp^2, Q^2) = C_F \left(2 \ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} \right) ,$$

$$\Gamma_g(p_\perp^2, Q^2) = C_A \left(2 \ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} \right) .$$

[A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. 100 (1983) 201]

[G. Marchesini, B.R. Webber, NPB 238 (1984) 1]

- Not correct for virtuality ordered showers.

Result for Herwig++ like shower (ordering in $\tilde{q}$) with recoils

$$\Gamma_q^{AO}(p_\perp^2, Q^2) = C_F \left(\ln \frac{Q^2}{p_\perp^2} - \frac{3}{2} \right) + C_F \frac{p_\perp}{Q} \left(1 - 2\lambda \frac{Q^2}{s_{ik}} \right) + \mathscr{O}\left(\frac{p_\perp^2}{Q^2} \right) ,$$

$$\Gamma_g^{AO}(p_\perp^2, Q^2) = C_A \left(\ln \frac{Q^2}{p_\perp^2} - \frac{11}{6} \right) + 2C_A \frac{p_\perp}{Q} \left(1 - \lambda \frac{Q^2}{s_{ik}} \right) + \mathscr{O}\left(\frac{p_\perp^2}{Q^2} \right) .$$

Recoil $\rightarrow$ power correction, beyond NLL.
(Only enters in phase space ($\rightarrow \lambda$) as spectator is only rescaled.)

[Simon Plätzer, SG, 0909.5593]

Coherent dipole showers

Result for CS dipole shower with $p_\perp$ ordering and right choice of phase space boundaries.

$$\Gamma_q^{CS}(p_\perp^2,\cdot) = C_F\left(\ln\frac{s_{ik}}{p_\perp^2} - \frac{3}{2}\right) - C_F\pi\lambda\frac{p_\perp}{\sqrt{s_{ik}}} + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right),$$

$$\Gamma_g^{CS}(p_\perp^2,\cdot) = C_A\left(\ln\frac{s_{ik}}{p_\perp^2} - \frac{11}{6}\right) - C_A\pi\lambda\frac{p_\perp}{\sqrt{s_{ik}}} + \mathcal{O}\left(\frac{p_\perp^2}{Q^2}\right).$$

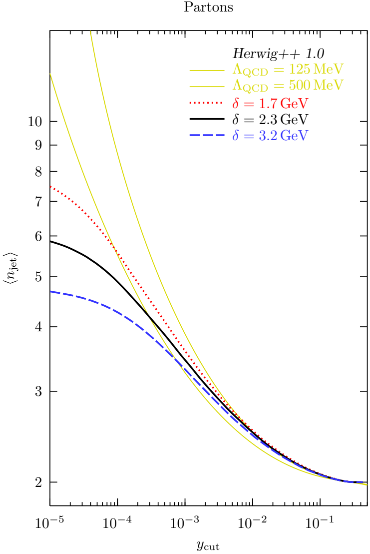
Hard scale $Q^2 = s_{ik}$ = dipole inv mass.
Recoil → power correction, beyond NLL.
$p_\perp$ ordering → simple POWHEG matching.

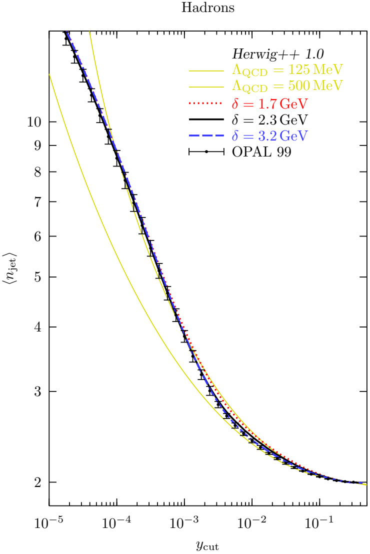
[Simon Plätzer, SG, 0909.5593]

How well do parton showers work?

- Jet shapes, jet rates, event shapes, identified particles…
- 'Tuning' of parameters ($e^+e^- \to$ hadrons, mostly at LEP).
- Want to get *everything* right with *one* parameter set.
- Compare to literally 100s of plots.

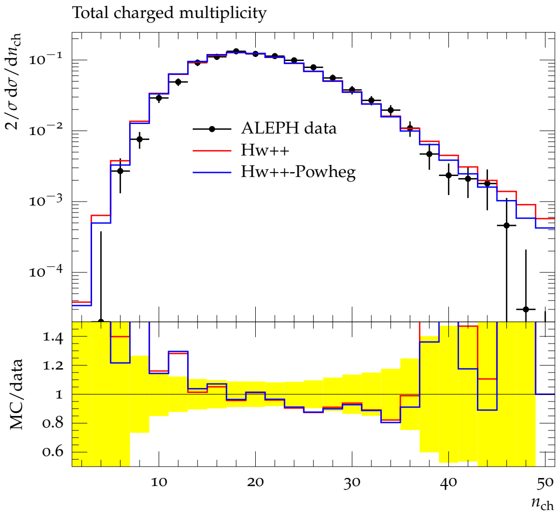
Smooth interplay between shower and hadronization.





How well does it work?

N_{ch} at LEP. Crucial for t_0 (Herwig++ 2.5.2)

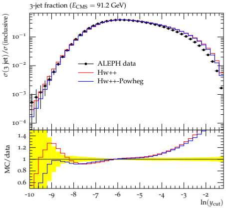


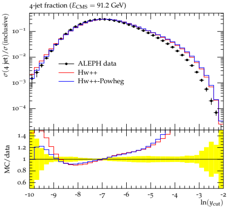
How well does it work?

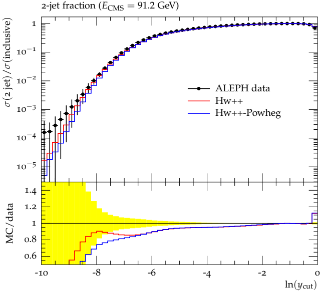
Jet rates at LEP.

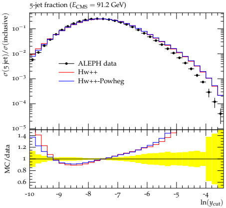
$$R_n = \sigma(n\text{–jets})/\sigma(\text{jets})$$
$$R_6 = \sigma(>5\text{–jets})/\sigma(\text{jets})$$

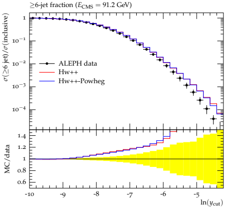
(Herwig++ 2.5.2)





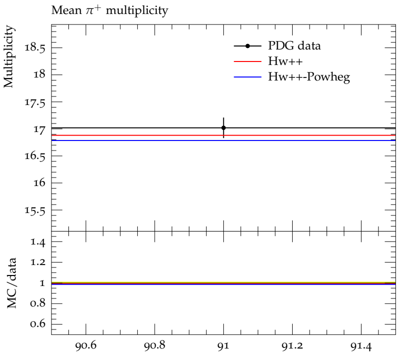


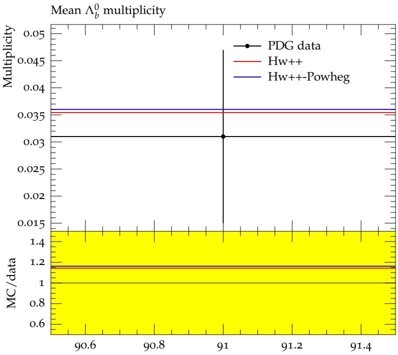




How well does it work?

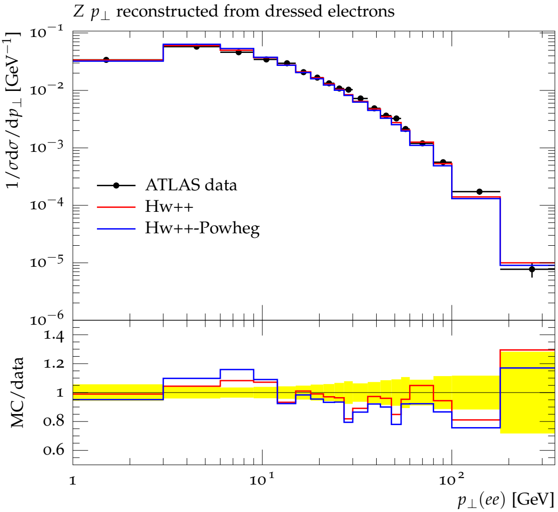
Hadron Multiplicities at LEP (e.g. π^+, Λ_b^0).





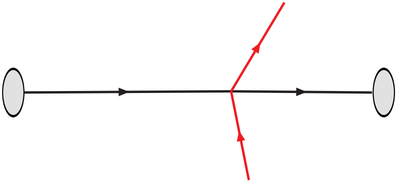
How well does it work?

$p_\perp(Z^0) \to$ intrinsic $k_\perp$ (LHC 7 TeV).



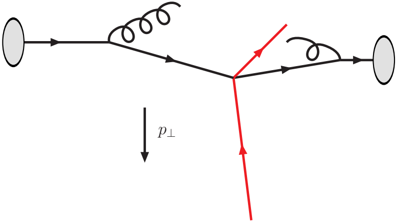
Nonperturbative issues (Drell Yan)

1)



Born level process.
Net $p_\perp$ vanishes.

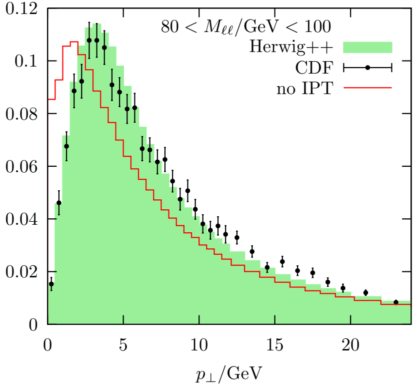
2)



$p_\perp$ from recoil against gluon emissions.

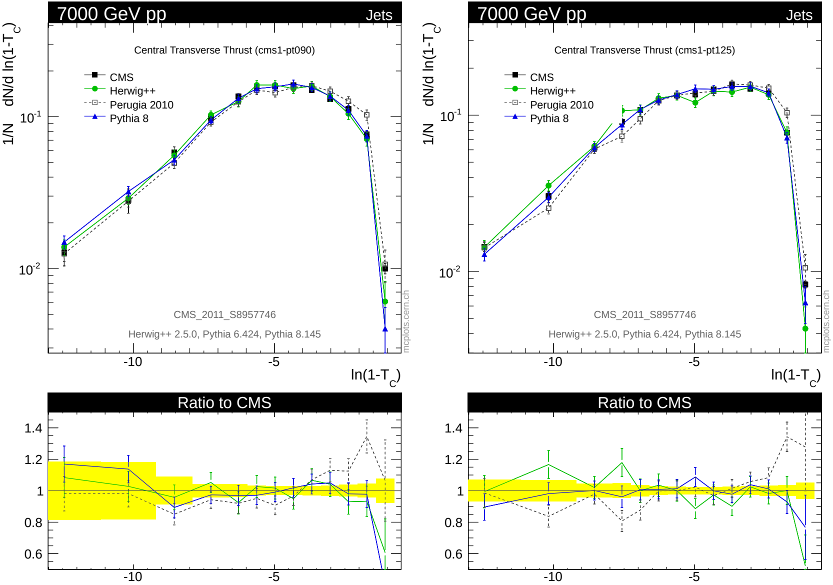
Nonperturbative issues (Drell Yan)

- Primordial $k_\perp$ from soft, non–perturbative(?) gluons.
- Gaussian smearing. Default: $\langle p_\perp \rangle = 2.1\,\mathrm{GeV}$!
- Modeled by non–perturbative gluon emissions.



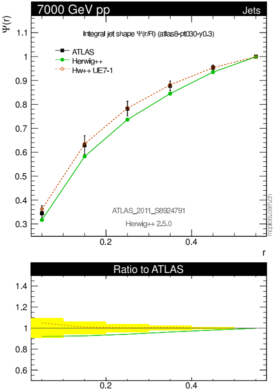
[SG, M. Seymour, A. Siodmok, JHEP 0806 (2008) 001]

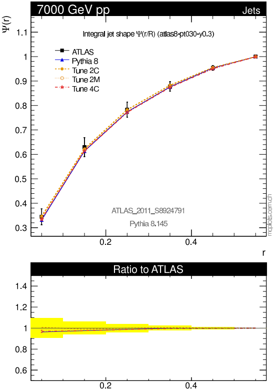
Transverse thrust

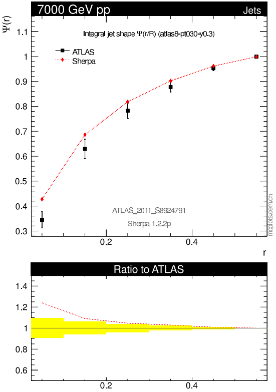


Integral jet shapes

not too hard, central ($30 < p_T/\mathrm{GeV} < 40; 0 < |y| < 0.3$)

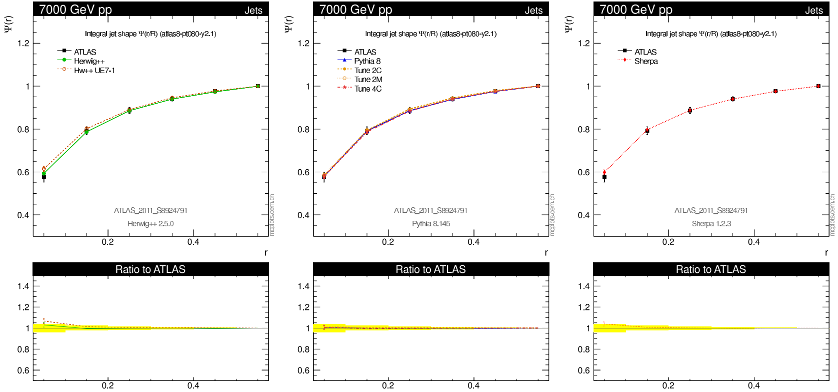




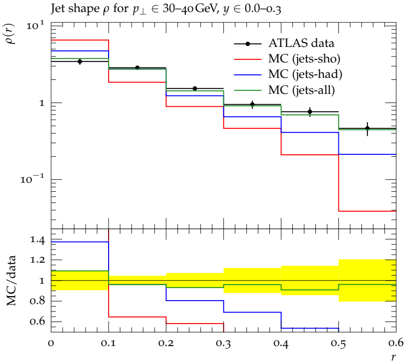


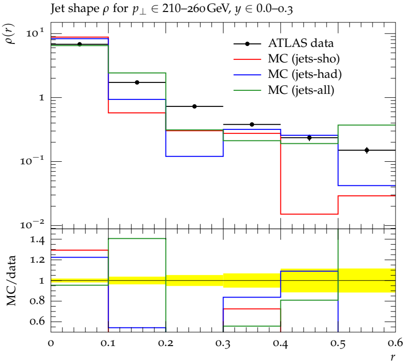
Integral jet shapes

harder, more forward ($80 < p_T/\mathrm{GeV} < 110; 1.2 < |y| < 2.1$)



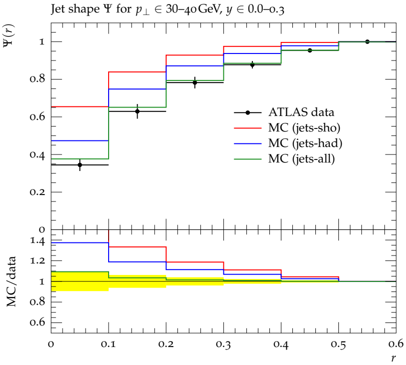
Hadronization and UE effects both present in jet substructure!

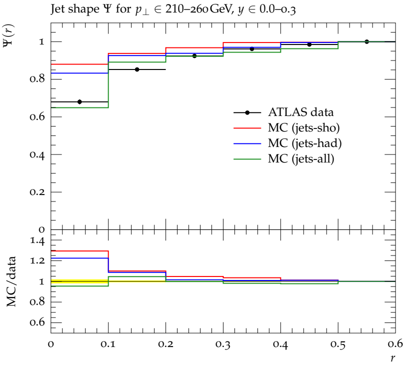




$\rho(r)$, energy flow at $r < R$ within jet of radius R.
(Herwig++ 2.5.2 shower/+hadronization/+MPI)

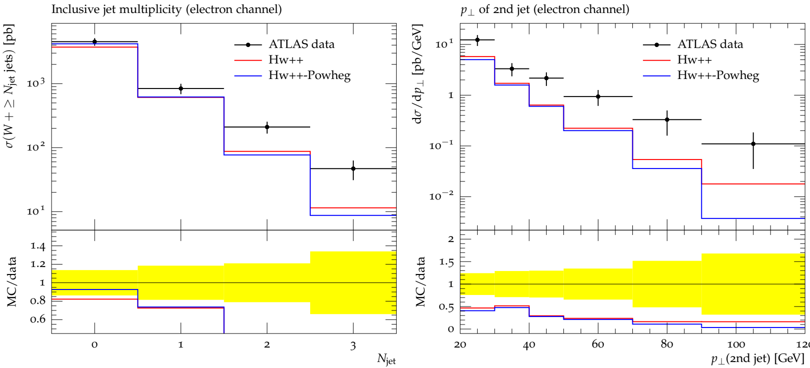
Hadronization and UE effects both present in jet substructure!





$\Psi(r)$, integrated energy flow.
(Herwig++ 2.5.2 shower/+hadronization/+MPI)

$W + \text{jets}$, LHC 7 TeV.



Higher jets not covered by parton shower only → matching.

Conclusion

- Parton shower MCs amongst core tools for LHC.
- Devil's in the details.
- Strong ties between perturbative/non–perturbative physics.
- Use resummation to clearify entanglement of perturbative/non–perturbative interplay?