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Matching @LO&NLO

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Introduction

- ▶ Parton Showers
- ▶ Tree-level matching
- ▶ NLO matching



No Introduction

- ▶ Why matching and merging
- ▶ Tree-level matrix element generation
- ▶ NLO generation



Parton Showers

- ▶ Exclusive final states
- ▶ (N)LL Resummation to all orders
- ▶ Soft and collinear approximation
- ▶ Crappy hard wide-angle emissions
- ▶ Unitary procedure
- ▶ Crappy total cross section



Exclusive n -jet cross section, Parton Shower style

$$d\sigma_n^{\text{ex}} = F_0 |\mathcal{M}_0|^2 d\phi_0 \times \left[\prod_{i=1}^n \alpha_S \frac{F_i}{F_{i-1}} P_i d\rho_i dz_i \Pi_{i-1} \right] \Pi_n(\rho_n, \rho_{\text{MS}})$$

- ▶ $|\mathcal{M}_0|^2 d\phi_0$: Born-level ME and phase space.
- ▶ F_i : PDF's from both sides for the i .
- ▶ $P_i(\rho, z) d\rho dz \approx \frac{|\mathcal{M}_i|^2 d\phi_i}{|\mathcal{M}_{i-1}|^2 d\phi_{i-1}} \equiv P_i^{\text{ME}}(z) d\rho dz$
- ▶ ρ, z : Splitting variables. Assume ρ is a suitable jet scale.
- ▶ ρ_{MS} : jet resolution scale.
- ▶ $\Pi_i(\rho_{i-1}, \rho_i)$: No-emission probabilities.
- ▶ Ignore running of α_S and PDF's for now.



No-emission probabilities

$$\Pi_i(\rho_i, \rho_{i+1}) = \exp \left(- \int_{\rho_{i+1}}^{\rho_i} d\rho dz \alpha_S \frac{F_{i+1}}{F_i} P_{i+1} \right)$$

The probability of not having any splittings above the scale ρ_{i+1} starting the shower from the state i at scale ρ_i .

For initial-state radiation, distinguish from Sudakov factor

$$\Delta_i(\rho_i, \rho_{i+1}) = \frac{F_i(\rho_i)}{F_i(\rho_{i+1})} \Pi_i(\rho_i, \rho_{i+1})$$

(Webber's trick relating forward and backward evolution)



With the veto-algorithm it is easy to generate a hardest emission using the no-emission probabilities, even if the integrand is complicated and we have several possible processes.

Also, if we generate one emission (ρ, z) from a given state, i , starting from a maximum scale ρ_i

$$P(\rho < \rho_{i+1}) = \Pi_i(\rho_i, \rho_{i+1})$$



In addition, if we get a $\rho > \rho_{i+1}$, we can continue generating another emission from state i below ρ (discarding the first one), and then another one, etc. The average number of emissions above ρ_{i+1} is then given by the exponent

$$\langle n \rangle = \int_{\rho_{i+1}}^{\rho_i} d\rho dz \alpha_S \frac{F_{i+1}}{F_i} P_{i+1} = -\log \Pi_i(\rho_i, \rho_{i+1})$$

and

$$\langle n(n-1) \rangle = \left(\int_{\rho_{i+1}}^{\rho_i} d\rho dz \alpha_S \frac{F_{i+1}}{F_i} P_{i+1} \right)^2$$



Fixed-order expansion of a parton shower

(using $\mathcal{P}_i = \frac{F_i}{F_{i-1}} \mathcal{P}_i$)

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1 + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1 \right)^2 \right]$$

$$\begin{aligned} \frac{d\sigma_1^{\text{ex}}}{d\phi_0} &= F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1 d\rho_1 dz_1 \\ &\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right] \end{aligned}$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1 d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

Unitary to all orders in α_S .



Tree-level matching

We now want to improve our parton shower.

The easiest thing is

$$P_i \rightarrow P_i^{ME} \equiv \frac{|\mathcal{M}_i|^2 d\phi_i}{|\mathcal{M}_{i-1}|^2 d\phi_{i-1} d\rho dz}$$

This has been around quite a while in PYTHIA for the first splitting in some processes.



Leading-order tree-level matching

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} \right)^2 \right]$$

$$\begin{aligned} \frac{d\sigma_1^{\text{ex}}}{d\phi_0} &= F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \\ &\quad \times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right] \end{aligned}$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

Still unitary to all orders of α_S . We can decrease ρ_{MS} to the non-perturbative boundary ρ_{cut} .

CKKW(-L)

For higher jet-multiplicities things get a bit more complicated.

It is solved in CKKW(-L) by generating inclusive few-jet samples according to exact tree-level $|\mathcal{M}_n|^2$ using some merging scale ρ_{MS} .

These are then made exclusive by reweighting no-emission probabilities (in CKKW-L generated by the shower itself)

Also fix the running of α_S and PDF's.

Add normal shower emissions below ρ_{MS} .

Add all samples together.



- ▶ Dependence on the merging scale cancels to the precision of the shower.
- ▶ If the merging scale is not defined in terms of the shower ordering variable, we need vetoed and truncated showers.
- ▶ Breaks the unitarity of the shower.



Higher-order tree-level matching

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} \right)^2 \right]$$

$$\frac{d\sigma_1^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1$$
$$\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2^{\text{ME}} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

NOT unitary. Gives artificial dependence of ρ_{MS} .

Can be alleviated for some processes in PYTHIA



Higher-order tree-level matching

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} \right)^2 \right]$$

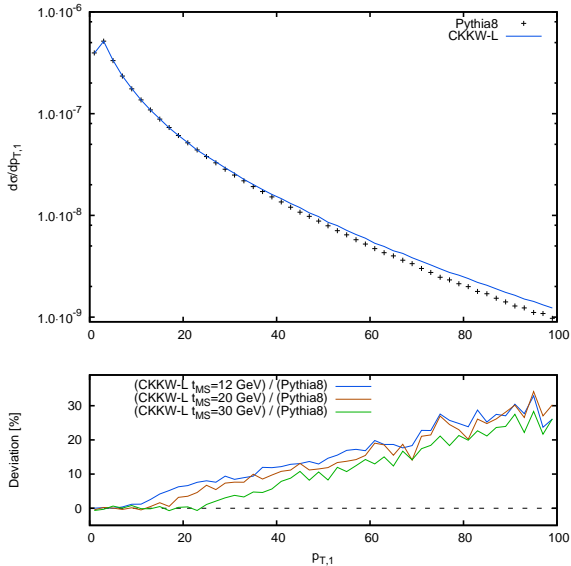
$$\frac{d\sigma_1^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1$$
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$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2^{\text{ME}} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

NOT unitary. Gives artificial dependence of ρ_{MS} .

Can be alleviated for some processes in PYTHIA





Mature procedure. Available in

- ▶ (HERWIG++)
- ▶ SHERPA
- ▶ PYTHIA8
- ▶ Also MLM-procedure, ALPGEN + HERWIG/PYTHIA



Vincia

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} \right)^2 \right]$$

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$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2^{\text{ME}} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

Unitarity restored $\rho_{\text{MS}} \rightarrow \rho_{\text{cut}}$.

See next talk by Juan.



Why worry about unitarity when cross section anyway is only leading order?

Parton Showers get many shapes of observables right. Maybe it is enough to just calculate a NLO K -factor

$$K_0 = \frac{\int d\sigma_0^{NLO}}{\int d\sigma_0^{LO}}$$

But we want to do better than that.



NLO matching: POWHEG

Let's calculate a phase-space dependent K -factor:

$$K(\phi_0) = \frac{d\sigma_0^{NLO}}{d\sigma_0^{LO}}$$

and then use the ME-corrected splitting function in our parton shower, $P_i \rightarrow P_i^{ME}$.



POWHEG

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = K(\phi_0) F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} \right)^2 \right]$$

$$\frac{d\sigma_1^{\text{ex}}}{d\phi_0} = K(\phi_0) F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1$$

$$\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right]$$

$$\frac{d\sigma_2}{d\phi_0} = K(\phi_0) F_0 |\mathcal{M}_0|^2 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

$$\rho_{\text{MS}} \rightarrow \rho_{\text{cut}}$$



It is not always possible to project the real emission ME down to a Born-level phase space point. These are instead added as a separate contribution to the one-jet cross section, which is not exponentiated in the no-emission probability.

In general one can shuffle non-singular pieces of the real emission ME to the non-exponentiated contribution to the one-jet contribution. This will modify the K -factor, but the end result will still be valid to NLO.



NLO matching: MC@NLO

Here we shuffle everything except the Parton Shower splitting function to the non-exponentiated one-jet contribution.

In my Parton Shower language this corresponds to calculating a phase space dependent K -factor using only the Parton Shower splitting, rather than the whole full real-emission ME.



MC@NLO

$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = K_S(\phi_0) F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1 + \frac{\alpha_S^2}{2} \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1 \right)^2 \right]$$

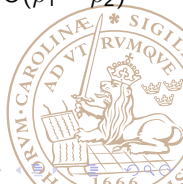
$$\frac{d\sigma_1^{\text{ex}}}{d\phi_0} = K_S(\phi_0) F_0 |\mathcal{M}_0|^2 \alpha_S \mathcal{P}_1 d\rho_1 dz_1 \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right]$$

$$+ \alpha_S (\mathcal{P}_1^{\text{ME}} - \mathcal{P}_1) d\rho_1 dz_1 \left[1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \alpha_S^2 (K_S(\phi_0) \mathcal{P}_1 + \mathcal{P}_1^{\text{ME}} - \mathcal{P}_1) d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

$\rho_{\text{MS}} \rightarrow \rho_{\text{cut}}$

Note that $\mathcal{P}_1^{\text{ME}} - \mathcal{P}_1$ may become negative.



MC@NLO and POWHEG are equivalent to NLO.

They both resum whatever the Parton Shower resums (to LL or NLL).

But they differ in finite terms. In particular POWHEG gets an extra boost in the two-jet cross section as the K -factor multiplies everything.

Also differences depending which shower is added. (PYTHIA, HERWIG, SHERPA)

Can be combined with CKKW (MENLOPS)



Let's try to also correct the one-jet cross section to NLO.

For this we go back to the CKKW(-L) matching using a merging scale ρ_{MS} to separate the ME and PS region.

Let's first look at the 0-jet exclusive cross section in more detail.

$$\begin{aligned}\frac{d\sigma_0^{\text{ex}}}{d\phi_0} &= K_0 F_0 |\mathcal{M}_0|^2 \Pi_0(\rho_0, \rho_{\text{MS}}) \\ &= K_0 F_0 |\mathcal{M}_0|^2 \left[1 - \alpha_S(\rho) \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \frac{F_1(\rho)}{F_0(\rho)} P_1 + \dots \right]\end{aligned}$$



How do we get a NLO exclusive cross section?

Generate a Born-level sample point using POWHEG
(switching off the first emission).

$$\frac{d\sigma_0^{NLO}}{d\phi_0} = K(\phi_0) F_0(\mu_F) |\mathcal{M}_0|^2$$

Generate a one-jet sample using the tree-level ME

$$\frac{d\sigma_1}{d\phi_0} = F_0(\mu_F) |\mathcal{M}_0|^2 \alpha_S(\mu_R) \int_{\rho_{MS}}^{\rho_0} \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1^{ME}$$

But recluster it to a 0-jet state.

This corresponds directly to the α_S term in the
PS no-emission probability.

How do we get a NLO exclusive cross section?

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But recluster it to a 0-jet state.

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If we now take the CKKW-L sample but reweight it by subtracting

$$F_0 |\mathcal{M}_0|^2 \left[K_0 \Pi_0(\rho_0, \rho_{\text{MS}}) - K_0 + \alpha_S(\mu_R) \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1 \right]$$

Then we add the POWHEG sample and the reclustered 1-jet sample Only adding a parton shower below ρ_{MS} .

The 1-jet and higher order exclusive CKKW-samples stays the same.

We then get something which is correct to NLO but with all higher order terms taken from the Parton Shower.

NL_{SP}³

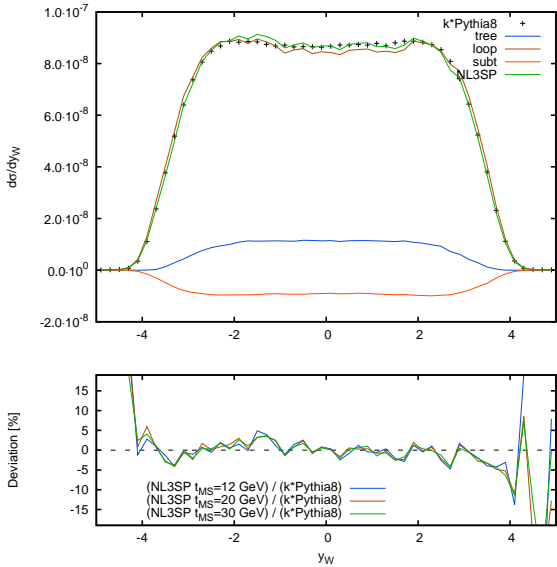
$$\frac{d\sigma_0^{\text{ex}}}{d\phi_0} = F_0 |\mathcal{M}_0|^2 \left[K(\phi_0) - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1^{\text{ME}} + \frac{\alpha_S^2}{2} K_0 \left(\int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \mathcal{P}_1 \right)^2 \right]$$

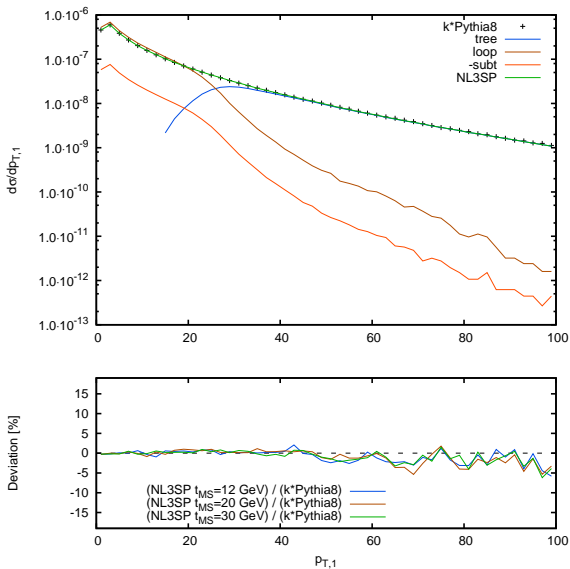
$$\begin{aligned} \frac{d\sigma_1^{\text{ex}}}{d\phi_0} &= F_0 |\mathcal{M}_0|^2 K_0 \alpha_S \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \\ &\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \mathcal{P}_1 - \alpha_S \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \mathcal{P}_2 \right] \end{aligned}$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 |\mathcal{M}_0|^2 K_0 \alpha_S^2 \mathcal{P}_1^{\text{ME}} d\rho_1 dz_1 \mathcal{P}_2^{\text{ME}} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$









Now let's go to the 1-jet exclusive cross section in CKKW-L

$$\begin{aligned}
 \frac{d\sigma_1^{\text{ex}}}{d\phi_0} &= F_1(\mu_F) |\mathcal{M}_0|^2 P_1^{\text{ME}} \alpha_S(\mu_R) d\rho_1 dz_1 \\
 &\quad \times K \frac{F_0(\mu_F) F_1(\rho_1) \alpha_S(\rho_1)}{F_1(\mu_F) F_0(\rho_1) \alpha_S(\mu_R)} \Pi_0(\rho_0, \rho_1) \Pi_1(\rho_1, \rho_{\text{MS}}) \\
 &= F_1(\mu_F) |\mathcal{M}_0|^2 P_1^{\text{ME}} \alpha_S(\mu_R) d\rho_1 dz_1 \\
 &\quad \times \left[K + A\alpha_S(\mu_R) + B\alpha_S(\mu_R) - \alpha_S(\mu_R) \int_{\rho_1}^{\rho_0} d\rho dz \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1 \right. \\
 &\quad \left. - \alpha_S(\mu_R) \int_{\rho_{\text{MS}}}^{\rho_1} d\rho dz \frac{F_2(\mu_F)}{F_1(\mu_F)} P_2 \right] + \mathcal{O}(\alpha_S^2(\mu_R))
 \end{aligned}$$

A and B can be calculated from the leading order running of α_S and the PDF's.



Again we get the 1-jet NLO inclusive sample from POWHEG (using ρ_{MS} as cutoff) and make it exclusive by subtracting the reclustered 2-jet tree-level ME

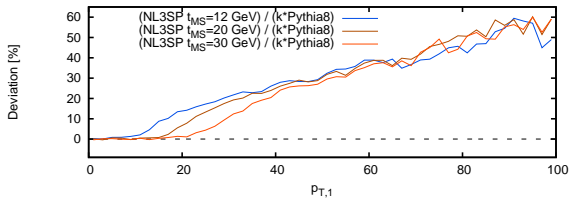
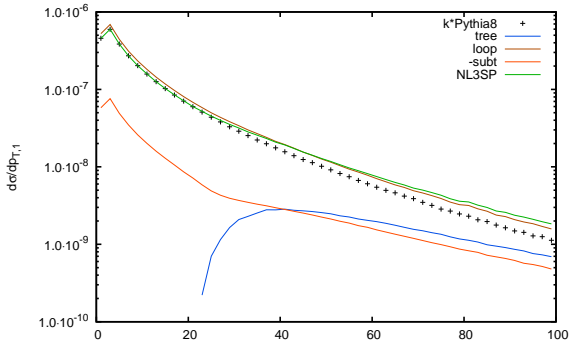
$$\frac{d\sigma_2^{\text{rec}}}{d\phi_0} = F_0(\mu_F) |\mathcal{M}_0|^2 \alpha_S^2(\mu_R) \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1^{\text{ME}} d\rho_1 dz_1 \int_{\rho_{\text{MS}}}^{\rho_0} \frac{F_2(\mu_F)}{F_1(\mu_F)} P_2^{\text{ME}}$$

(Note that this gives both ordered and un-ordered emissions)



- ▶ 0-jet LO sample, reweighted with subtracted CKKW
- + 0-jet NLO sample.
 - 1-jet LO sample, reclustered to 0-jet
- ▶ 1-jet LO sample, reweighted with subtracted CKKW
- + 1-jet NLO sample.
 - 2-jet LO sample, reclustered to 1-jet
- ▶ 2-jet LO sample, reweighted with CKKW
- ▶ ...





NL_{SP}³ summary

- ▶ Has been implemented in PYTHIA8, but a lot more testing to be done.
- ▶ A bit complicated with many samples to be combined.
- ▶ Lots of technical details omitted here.
- ▶ There are some non-trivial ρ_{MS} dependencies left which may be large (α_S^2 term in 0-jet contribution).
- ▶ Can be generalized to higher jet multiplicities.
- ▶ In principle we can go to NNLO 0-jet (need to recluster 2-jet ME twice).



Summary

- ▶ We need to improve the precision of our Parton Showers
- ▶ The distribution of multiple hard jets needs ME corrections
- ▶ The total cross sections needs to be NLO
- ▶ Multi-jet cross sections should also be NLO
- ▶ State-of-the-art (MLM, CKKW(-L), MC@NLO, POWHEG) works OK.
- ▶ Soon we will have multi-jet matching to NLO. (see also arXiv:1108.0909)



