Saturation, dijets and resummation in nonlinear evolution equations

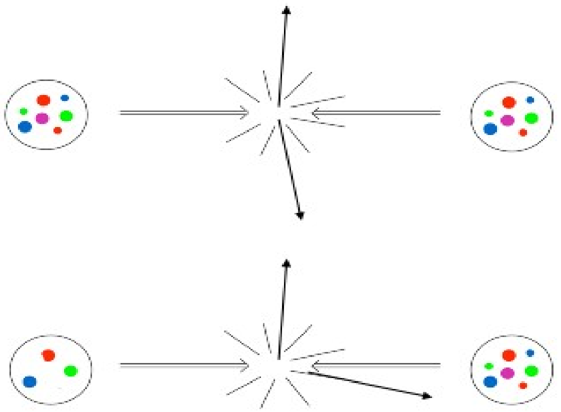
Krzysztof Kutak

Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

LIDER

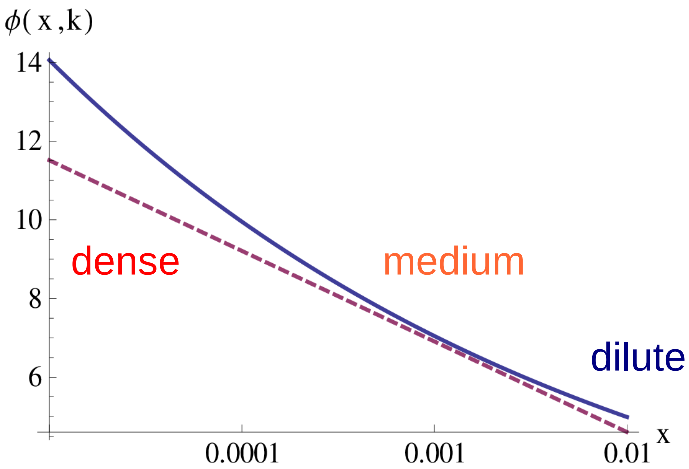
Narodowe Centrum Badań i Rozwoju

LHC as a scaner of gluon



central-central i.e.
medium-medium

forward-central i.e.
dilute – dense



From C. Marquet

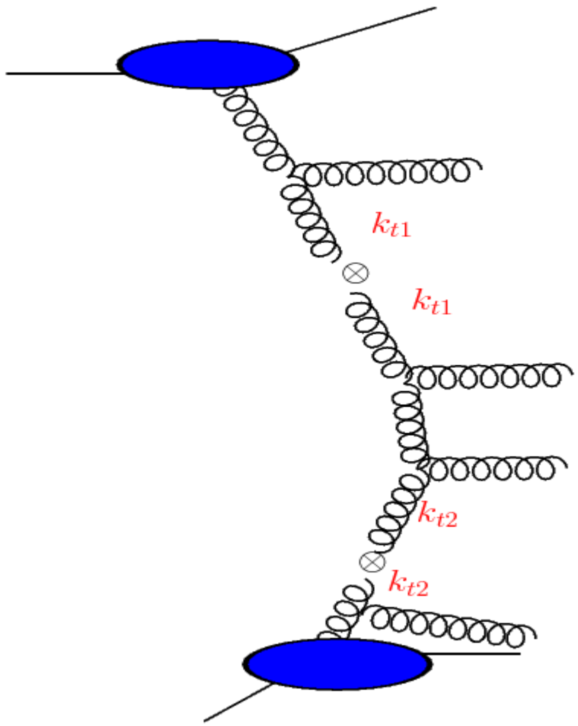
High energy limit of QCD

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 S)^2} \overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t})$$

$$\times\, \phi_{a/A}(x_1, k_{1t}^2, \mu^2)\, \phi_{b/B}(x_2, k_{2t}^2, \mu^2)\, \frac{1}{1+\delta_{cd}}$$

Ciafaloni, Catani, Hautman '93

Implemented in Monte Carlo generator CASCADE
H. Jung

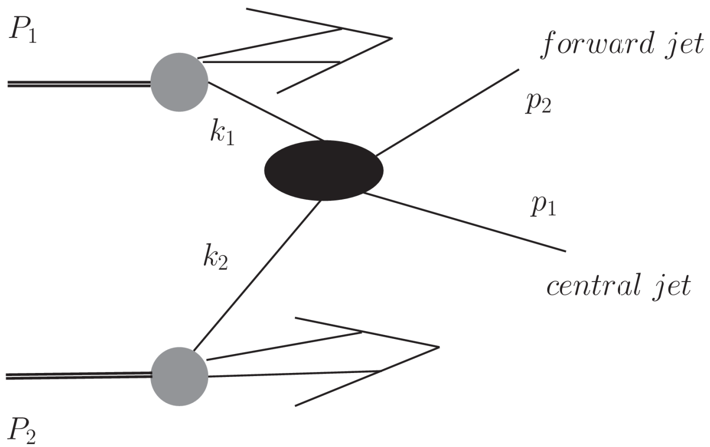


- *Gluon density depends on k_t*
  - *Off shell initial state partons with shellness ~k_t*

High energy prescription and forward-central dijets

Deak, Jung, Hautmann Kutak
JHEP 0909:121,2009

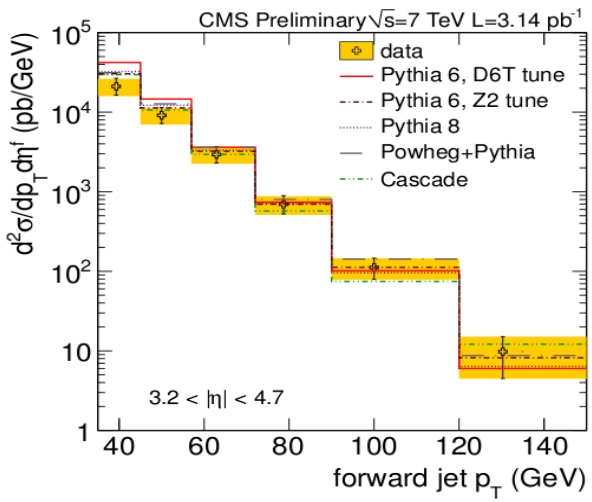
$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} \overline{|\mathcal{M}_{ag\to cd}|}^2 x_1 f_{a/A}(x_1, \mu^2)\, \phi_{g/B}(x_2, k_t^2, \mu^2) \frac{1}{1+\delta_{cd}}$$

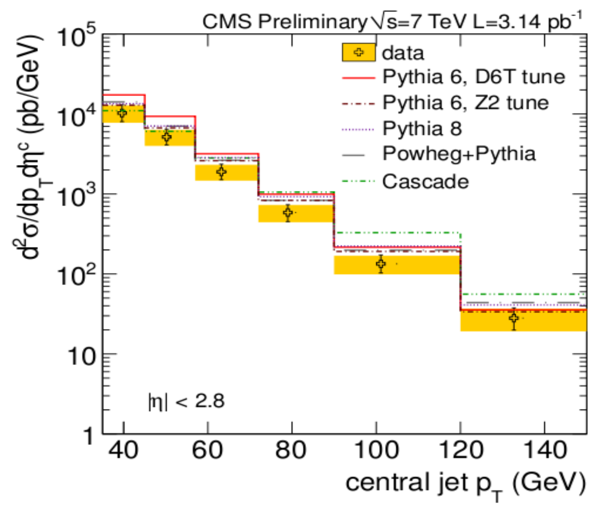


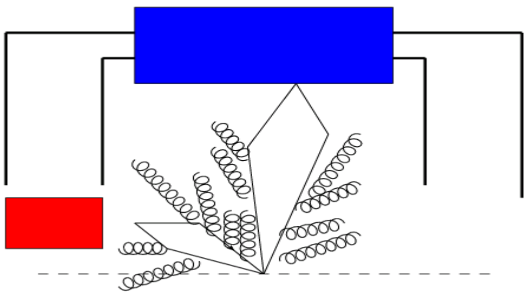
$$\begin{aligned} k_1^\mu &= x_1 P_1^\mu \\ k_2^\mu &= x_2 P_2^\mu + k_t^\mu \end{aligned}$$

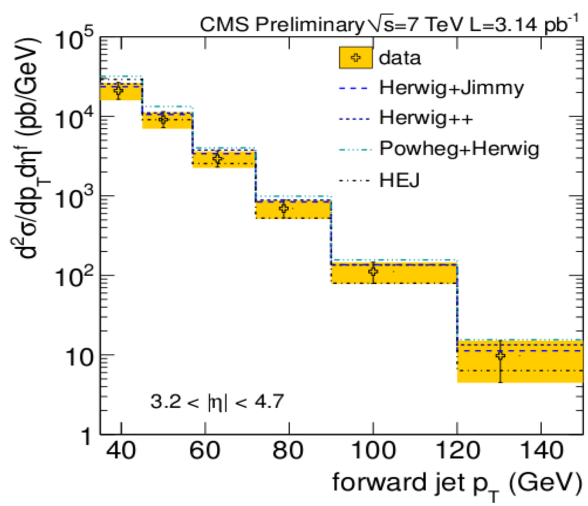
- Resummation of logs of x and logs of hard scale
- Knowing well parton densities at largr x one can get information about low x physics

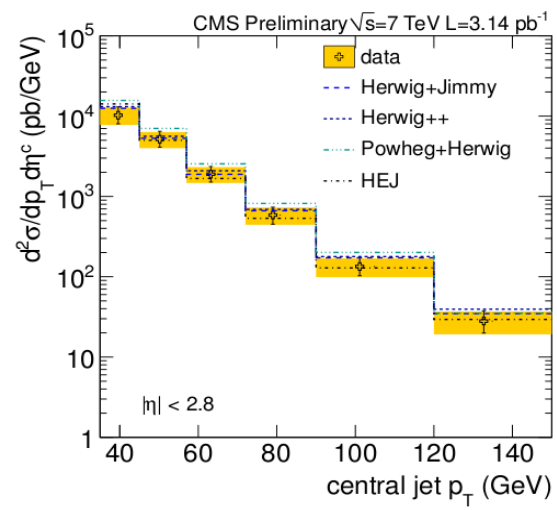
Forward central – jet production









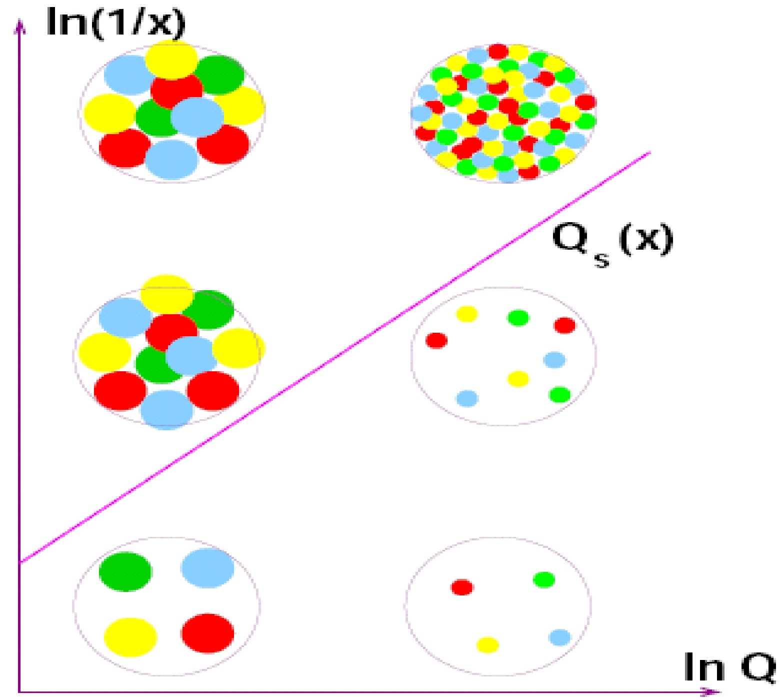


- HEJ and Cascade based on unordered in k_t emissions but use different parton densities
- Herwig and PYTHIA use k_t odered shower but differ in approximations in ME and ordering conditions in shower

Deak, Jung, Hautmann, Kutak,' 10

High energy factorization and saturation

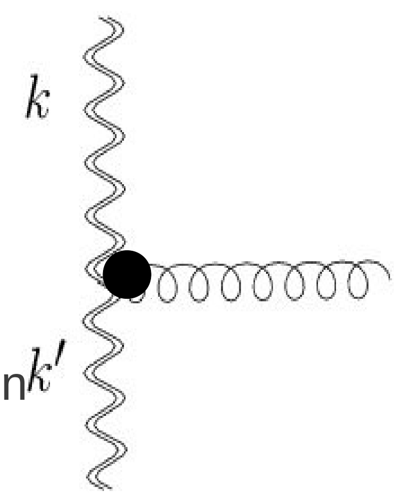
Saturation – state where number of gluons stops growing due to high occupation number.
More generally saturation is an example of percolation which has to happen since partons have size $1/k_t$ and hadron has finite size



Cross sections change their behavior from power like to logarithmic like.

On microscopic level it means that gluon apart splitting recombine

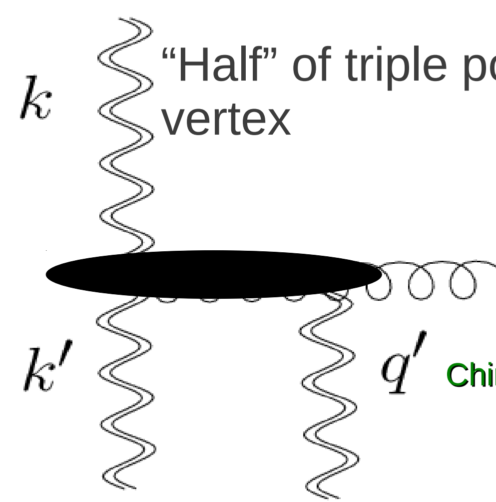
splitting



Linear evolution equation

recombination

Nonlinear evolution
equations
BK, JIMWLK
CGC framework
DIPSY



"Half" of triple pomeron vertex

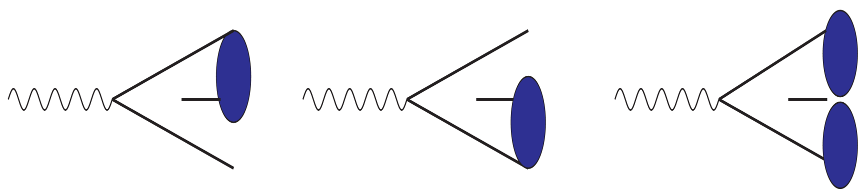
Bartels,Wusthoff
Z.Phys. C66 (1995) 157-180

Chirilli,Szymanowski,Wallon '10

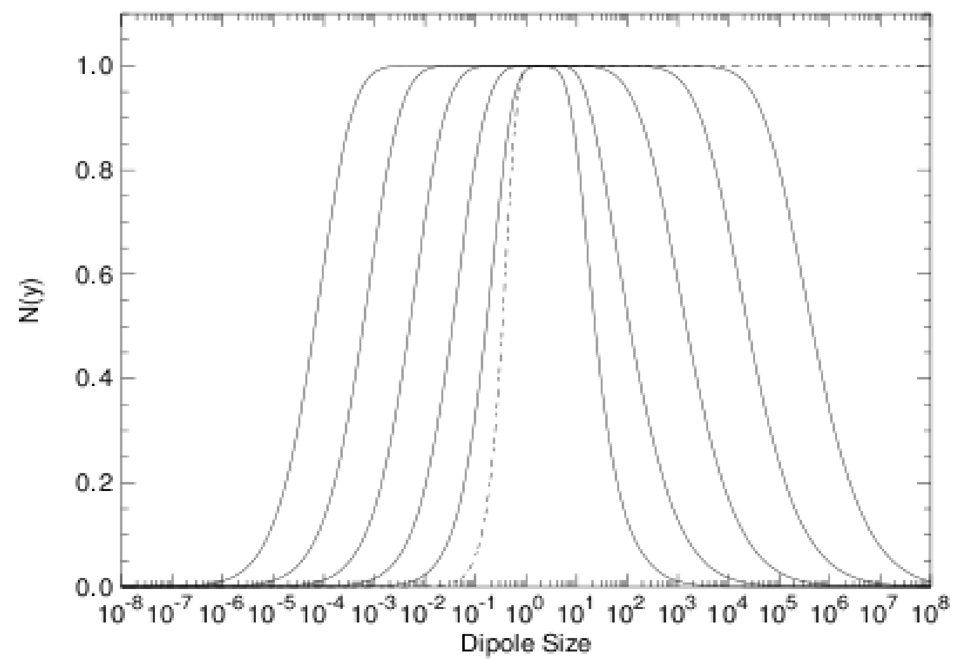
Simple evolution equation
with nonlinearities

Kovchegov '99

$$\frac{\partial N_{\mathbf{x}_0\mathbf{x}_1}}{\partial Y} = \overline{\alpha}_s \int \frac{d^2\mathbf{x}_2}{2\pi} \frac{(\mathbf{x}_0 - \mathbf{x}_1)^2}{(\mathbf{x}_0 - \mathbf{x}_2)^2(\mathbf{x}_1 - \mathbf{x}_2)^2} \left[N_{\mathbf{x}_0\mathbf{x}_2} + N_{\mathbf{x}_1\mathbf{x}_2} - N_{\mathbf{x}_0\mathbf{x}_1} - N_{\mathbf{x}_0\mathbf{x}_2} N_{\mathbf{x}_1\mathbf{x}_2}\right]$$



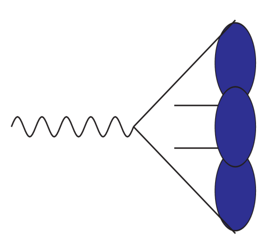
Nonlinear term allows for saturation



Recently solved with full impact parameter dependence

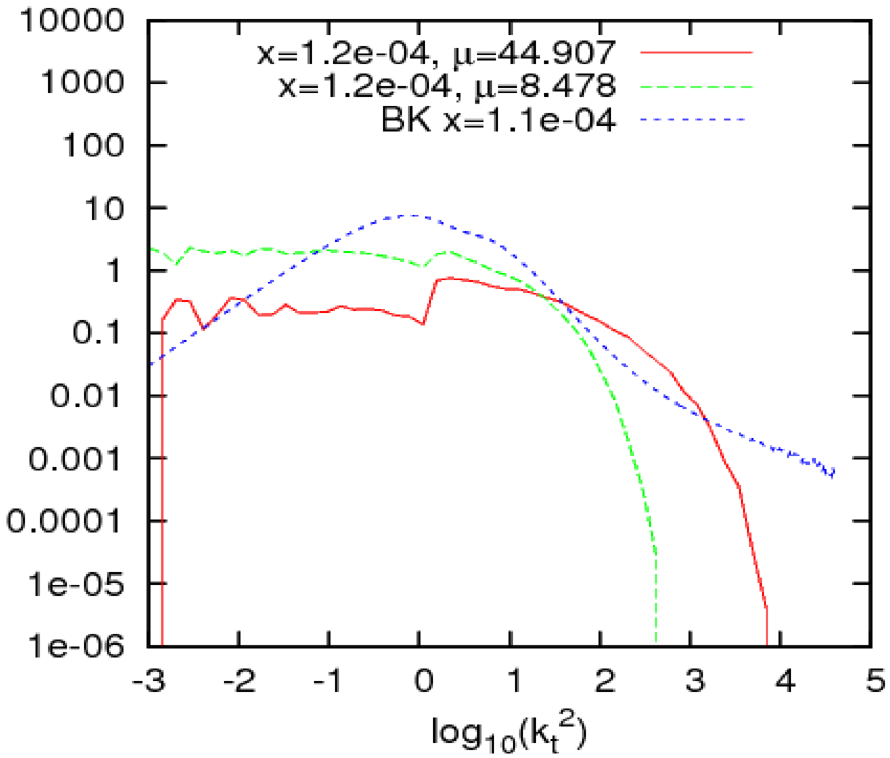
Berger, Stasto '11

BK is at present known up to NLO where such transitions are possible



Forward physics as the way to constrain gluon both at large and small pt

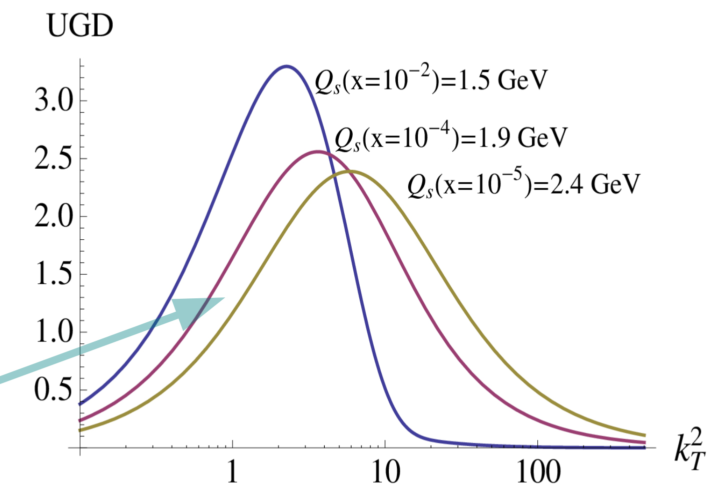
- Too flat behaviour of BK at large kt
- Lack of saturation in CCFM small k_t



Needed framework which unifies both correct behaviors

The BK equation in the momentum space

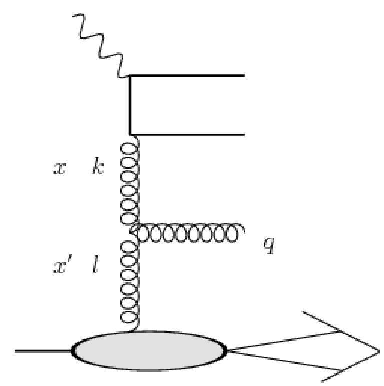
Go to momentum space



dipole density

$$\Phi(x,k^2) = \int \frac{d^2\mathbf{b}d^2\mathbf{r}}{2\pi} \exp(-i\mathbf{k}\cdot\mathbf{r}) \frac{N(x,r,b)}{r^2}$$

$$\frac{\partial\phi(x,k^2)}{\partial\ln 1/x} = \overline{\alpha}_s \int_0^\infty dl^2 \left[\frac{l^2\phi(x,l^2)-k^2\phi(x,k^2)}{|l^2-k^2|} + \frac{k^2\phi(x,k^2)}{\sqrt{4l^2+k^2}}\right] - \frac{\phi^2(x,k^2)}{R^2}$$



gluon density

Solved by: Enberg; Golec-Biernat; Lublinsky; Soyez,...

$$\mathcal{F}_{BK}(x,k^2) = \frac{N_c}{\alpha_s\pi^2} k^2 \nabla_k^2 \Phi(x,k^2)$$

$$\frac{\partial\mathcal{F}_{BK}(x,k^2)}{\partial\ln 1/x} = \frac{N_c\alpha_s}{\pi}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}_{BK}(x,l^2)-k^2\mathcal{F}_{BK}(x,k^2)}{|k^2-l^2|} + \frac{k^2\mathcal{F}_{BK}(x,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{\alpha_s^2}{R^2}\left\{\left[\int_{k^2}^\infty \frac{dl^2}{l^2}\mathcal{F}_{BK}(x,l^2)\right]^2 + \mathcal{F}_{BK}(x,k^2)\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}_{BK}(x,l^2)\right\}$$

Leading order equation
not applicable at large k_t
Not obvious probabilistic interpretation
Might be difficult for MC

Kutak, Kwiecinski '03; Nikolaev, Scheffer '04

Resummed form of the BK

The strategy:

- Use the equation for dipole density. Simple nonlinear term
- Split linear kernel into resolved and unresolved parts
- Resumm the virtual contribution and unresolved ones in the linear part
- Use analogy to postulate nonlinear CCFM

The starting point:

$$\frac{\partial \phi(x,k^2)}{\partial \ln 1/x} = \overline{\alpha}_s \int_0^\infty dl^2 \left[\frac{l^2 \phi(x,l^2) - k^2 \phi(x,k^2)}{|l^2 - k^2|} + \frac{k^2 \phi(x,k^2)}{\sqrt{4l^2 + k^2}} \right] - \frac{\phi^2(x,k^2)}{R^2} \, \overline{\alpha}_s$$

Resummed form of the BK

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek
JHEP 1202 (2012) 117

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z,|\mathbf{k}+\mathbf{q}|^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)$$

Write in exclusive form

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \Phi(x/z,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z,|\mathbf{k}+\mathbf{q}|^2)\theta(\mu^2-q^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)\,.$$

$$R = 1GeV^{-1}$$

Resolution scale introduced

Perform Mellin transform w.r.t x to get rid of "z" integral

$$\overline{\Phi}(\omega,k^2) = \int_0^1 dx x^{\omega-1}\Phi(x,k^2)$$

$$\Phi(x,k^2) = \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega}\overline{\Phi}(\omega,k^2)$$

Resummed form of the BK

Using in unresolved real part $|\mathbf{k}+\mathbf{q}|^2 \approx \mathbf{k}^2 \longleftarrow q^2 < \mu^2$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}[\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2)] + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{q^2}\overline{\Phi}(\omega,k^2)[\theta(\mu^2-q^2)-\theta(k^2-q^2)] - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)$$

$$\overline{\Phi}(\omega,k^2) = \overline{\Phi}^0(\omega,k^2) + \frac{\overline{\alpha}_s}{\omega}\int \frac{d^2\mathbf{q}}{\pi q^2}\overline{\Phi}(\omega,|\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) - \frac{\overline{\alpha}_s}{\omega}\overline{\Phi}(\omega,k^2)\ln\frac{k^2}{\mu^2} - \frac{\overline{\alpha}_s}{\omega}\int_0^1 dy y^{\omega-1}\Phi^2(y,k^2)\,.$$

BK equation in the resummed exclusive form

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek

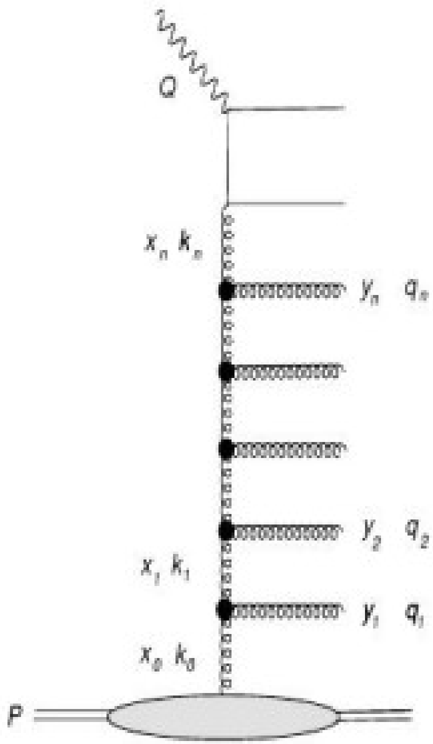
JHEP 1202 (2012) 117

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\, \theta(q^2-\mu^2)\, \frac{\Delta_R(z,k,\mu)}{z} \left[\Phi\left(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2\right) - q^2\delta(q^2-k^2)\,\Phi^2\left(\frac{x}{z},q^2\right)\right]$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z}\ln\frac{k^2}{\mu^2}\right)$$

- The same resumed piece for linear and nonlinear
- Initial distribution also gets multiplied by the Regge form factor
- New scale introduced to equation. One has to check dependence of the solution on it
- Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states

CCFM evolution equation - evolution with observer



- p - incoming proton, $p = (1,0,0,1)P$
- q_i - emitted gluons, $q_i = y_i p + \bar{y}_i \bar{p} + q_{i\perp}$
- axial gauge with the gauge vector $\bar{p} = (1,0,0,-1)P$
- gluon polarization vector purely transverse $\varepsilon_\mu^{(\lambda)}(q) = g_\mu^{(\lambda)} - \frac{q_\mu \bar{p}^{(\lambda)}}{q\bar{p}}$

CCFM evolution equation - evolution with observer

$$\mathcal{A}(x,k,p) = \bar{\alpha}_S \int_x^1 dz \int \frac{d^2\bar{q}}{\pi\bar{q}^2}\theta(p - z\bar{q})P(z,k^2,p)\mathcal{A}(\frac{x}{z},k',\bar{q}) + \mathcal{A}_0(x,k,p)$$

- angular variable used $\xi_i = \frac{\bar{y}_i}{y_i}$
- $\eta_i = \frac{1}{2}\ln\xi_i = \ln\left(\frac{|\mathbf{q_i}|}{\sqrt{s}y_i}\right) \quad \tan\frac{\theta_i}{2} = \frac{|\mathbf{q_i}|}{\sqrt{s}y_i}$
- $\bar{q} \equiv \frac{q_T}{1-z} \approx \theta E$
- the scale p is defined via maximal angle $\bar{\xi}$: $\bar{\xi} = p^2/(x_n^2 s)$
- $k' = |\mathbf{k} + (1-z)\bar{\mathbf{q}}|$, $Q_T = k_{n_\perp}$
- $\overline{\alpha} = \alpha_S \frac{N_C}{\pi}$

CCFM evolution equation - evolution with observer

$$P(z,k^2,p) = \frac{\alpha_S}{2\pi} 2C_A \; \Delta_S(zq,p) \left(\frac{\Delta_{NS}(z,q,k^2)}{z} + \frac{1}{1-z} \right),$$

regulates 1/(1-z)

regulates 1/z

emissions k

Compare with DGLAP LO evolution kernel

$$P_{gg}^{\mathrm{DGLAP}} = \frac{\alpha_S}{2\pi} 2C_A \; \Delta_S(z) \left(\frac{1}{z} + \frac{1}{1-z} - 2 + z(1-z) \right)$$

$$\Delta_{ns}(z,k,q) = \exp\left(-\alpha_s \ln\frac{1}{z} \ln\frac{k^2}{zq^2} \right)$$

Extension of CCFM to non linear equation

- *The second argument should be kt motivated by analogy to BK*
- *The third argument should reflect locally the angular ordering*

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 dz \int \frac{d^2\mathbf{q}}{\pi q^2}\, \theta(q^2-\mu^2) \frac{\Delta_R(z,k,\mu)}{z} \left[\Phi(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2}\, \theta(p - z\bar{q})\Delta_s(p,z\bar{q}) \left(\frac{\Delta_{ns}(z,k,q)}{z} + \frac{1}{1-z}\right) \left[\mathcal{E}\left(\frac{x}{z}, k'^2, \bar{q}\right) - \bar{q}^2\delta(\bar{q}^2-k^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\right].$$

Extension of CCFM to nonlinear equation

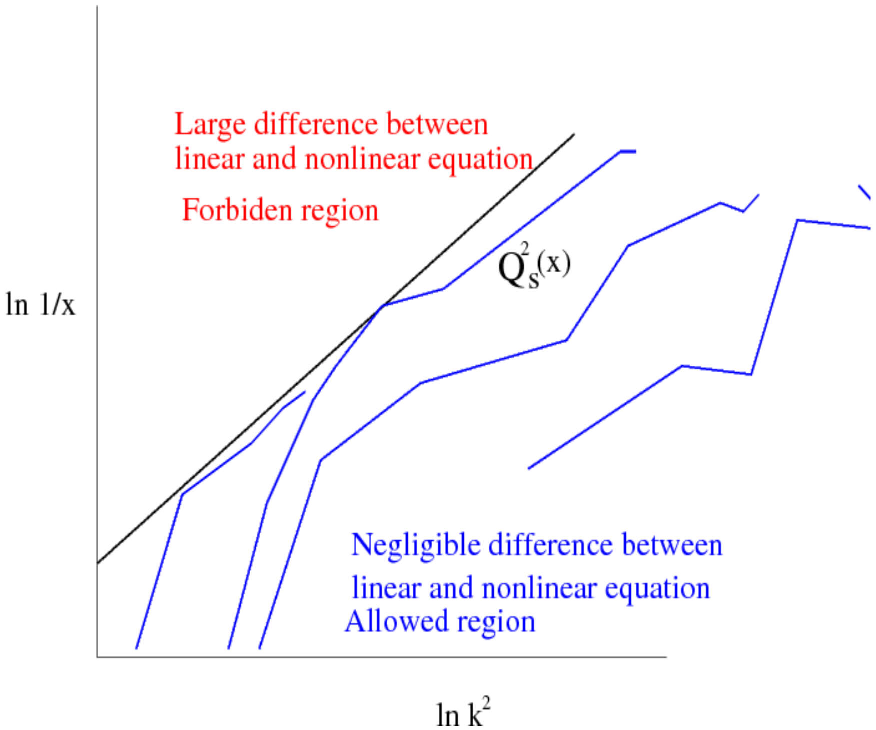
The unintegrated gluon density is obtained from

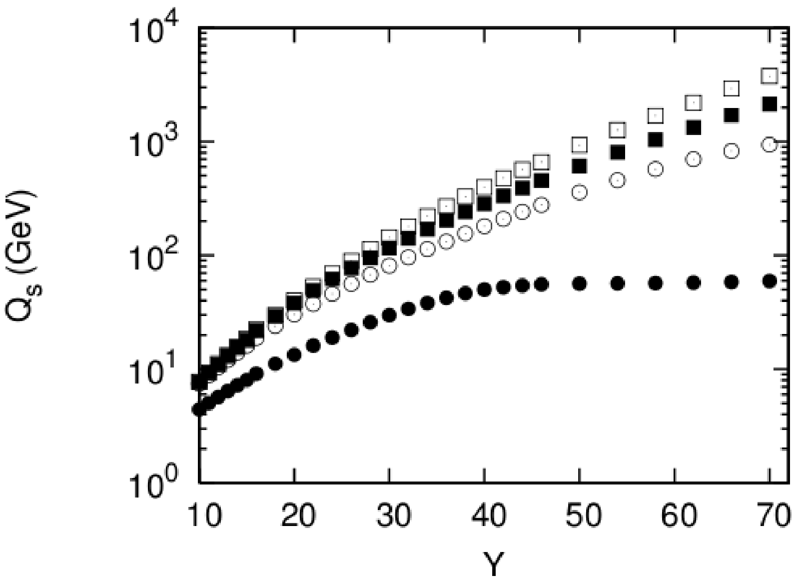
$$\mathcal{A}_{non-linear}(x,k^2,p) = \frac{N_c}{\alpha_s \pi^2} k^2 \, \nabla_k^2 \, \mathcal{E}(x,k^2,p)$$

The nonlinear term can be understood as a way to introduce the decoherence in emission of gluons which build unintegrated gluon density.

CCFM with saturation – consequences

Jung, Kutak '09
Avsar, Iancu '09





Avsar, Stasto '10

introduce line which will introduce effectively saturation effects in evolution. trajectories which enter the saturation region are rejected.

saturation scale saturates itself *because of limited phase space due to existence of hard scale*

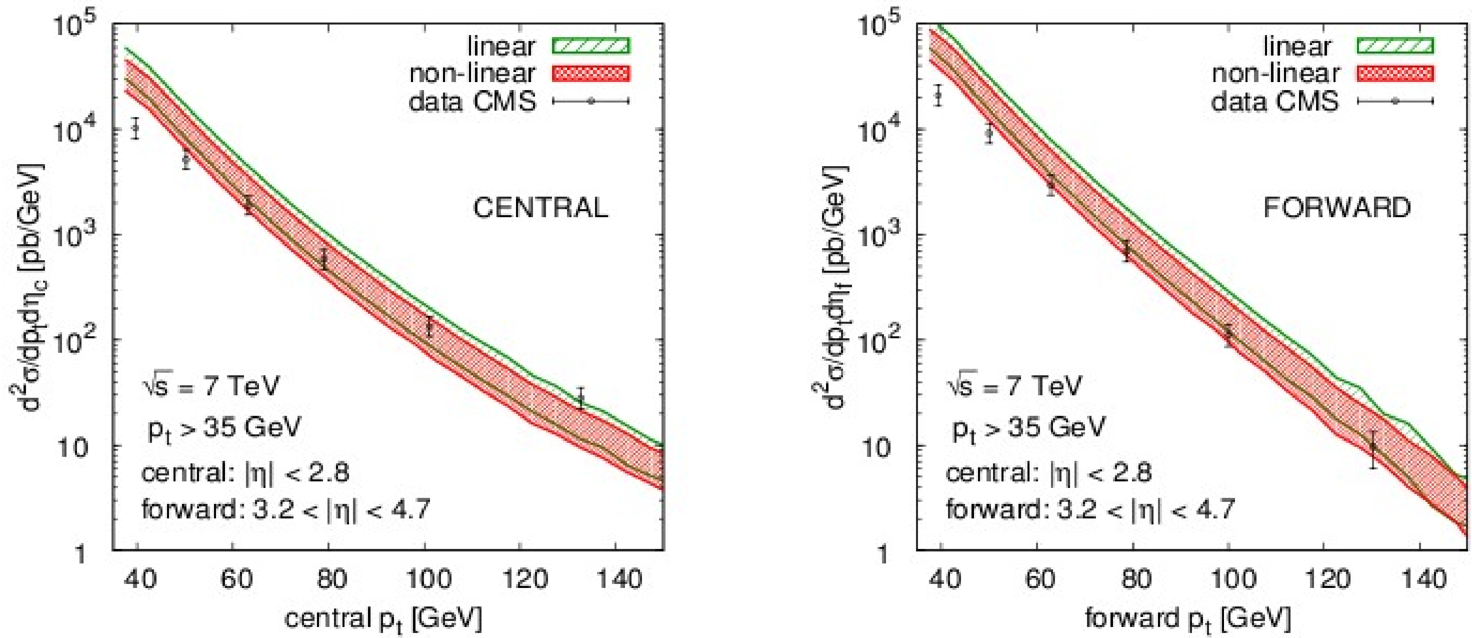
Consequences for entropy production

K.Kutak '11

$$S = \frac{6C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

Kiritsis, Tsalios '11

Jets and saturation



Kutak, Sapeta
arxiv:1205.5035

$$\mathcal{F}_p(x,k^2) \; = \; \mathcal{F}_p^{(0)}(x,k^2)$$

$$+ \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2) - k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\}$$

$$+ \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z) - \frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\mathcal{F}_p(\frac{x}{z},l^2)$$

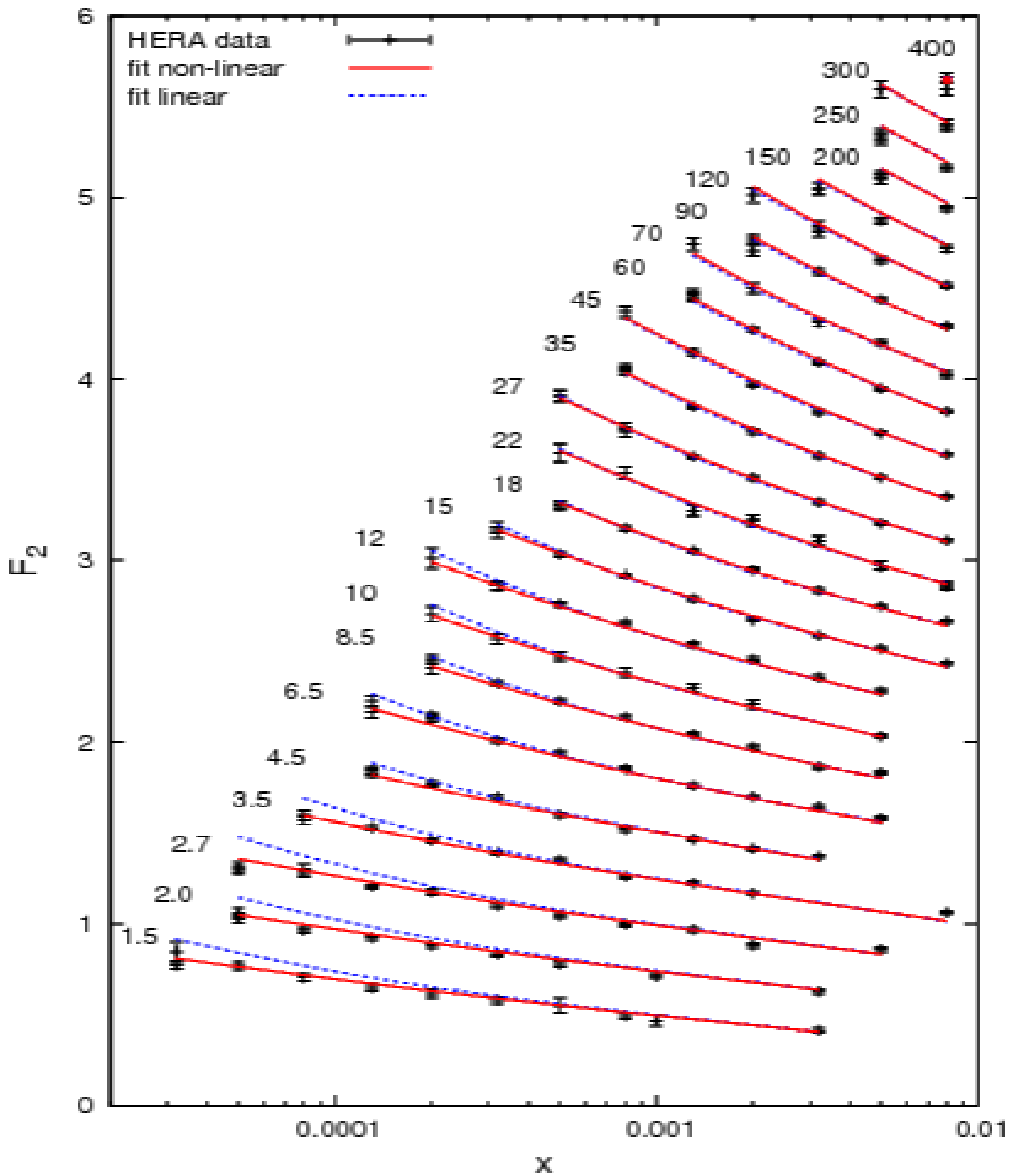
$$- \frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\mathcal{F}_p(x,l^2)\right)^2 + \mathcal{F}_p(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}_p(x,l^2)\right]$$

Kwiecinski,Kutak '03

Corrections of higher orders Included. Kin. Constr DGLAP spf

Andersson, Gustafson,Sammuelsson '96
Kwiecinski, Martin, Sutton '96

Further hints for saturation in F_2 data



Kutak, Sapeta
arxiv:1205.5035

Fit of BK-DGLAP and BFKL-DGLAP to combined H1-ZEUS data

Very good description with BK-DGLAP in range $Q^2 > 4.5$ GeV^2

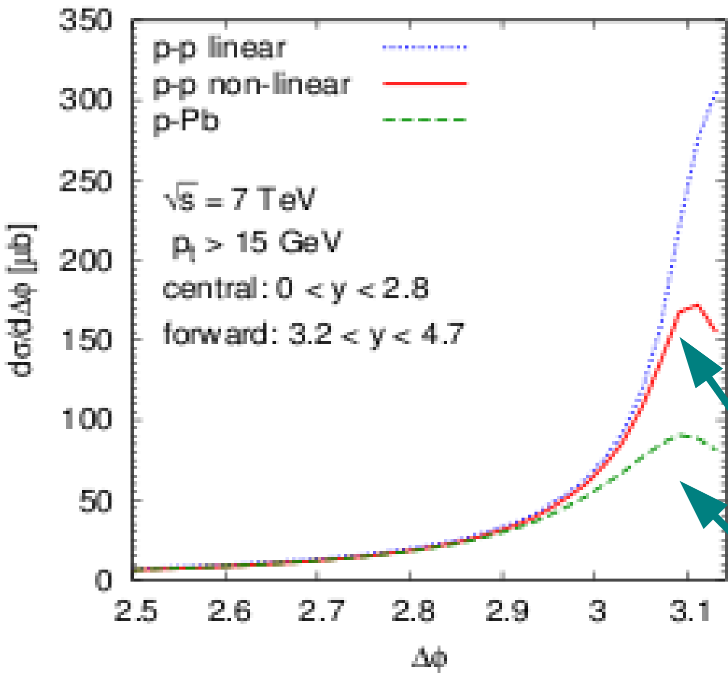
$$\chi^2 = 1.73$$

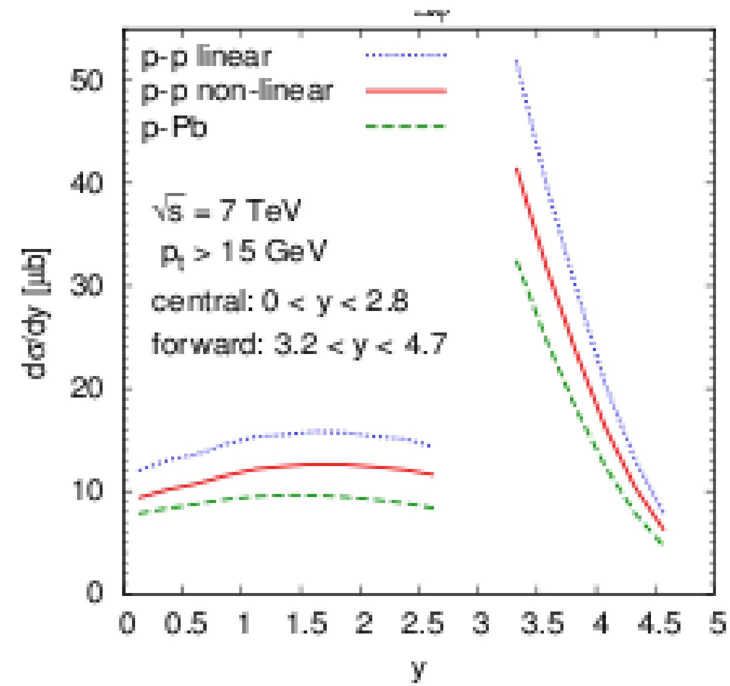
Very good description with BFKL-DGLAP in range $Q^2 > 4.5$ GeV^2

$$\chi^2 = 1.5$$

Signatures of saturation in p-p and p-Pb

Kutak, Sapeta
arxiv:1205.5035



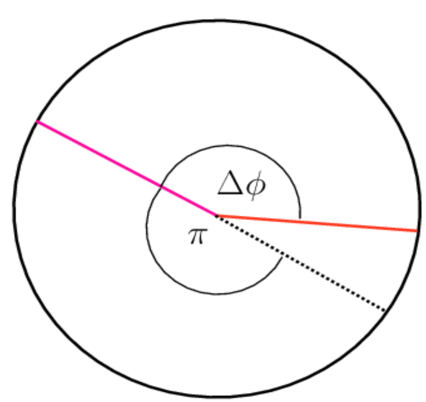


Reflects~ k^2

behavior of gluon at small k^2

Observable suggested to study BFKL effects

Sabio-Vera, Schwensen '06



Conclusions and outlook

- There comes oportunity to test parton densities both when the parton density is probed at low x and at high kt.

- Used so far equations did not allow for this

- New representation for BK equation allowed for ansatz for well motivated equation which incorporates both saturation effects and coherence

- In the future it will be interesting to check whether this equation predicts saturation of the saturation scale as in other frameworks

- Results based on BK/DGLAP approach support hints for saturation in F_2

- Results based on BK/DGLAP approach predict saturation in p-Pb and suggest its presence in p-p