Understanding parton showers

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Topics interleaved in this talk

- Some key ideas built into parton shower event generators.
- How some of these ideas germinated, influenced by DESY results.
- Formulas that sum series with large logarithms and their relation to parton showers.
- Prospects for improving the parton shower event generators.

Parton showers describe jets

Why are there jets?

- Bjorken, Berman and Kogut (1971) had it figured out before jets were seen and before QCD.
- "... the isolated high P_T partons will communicate with the 'wee' partons by cascade emission of partons."

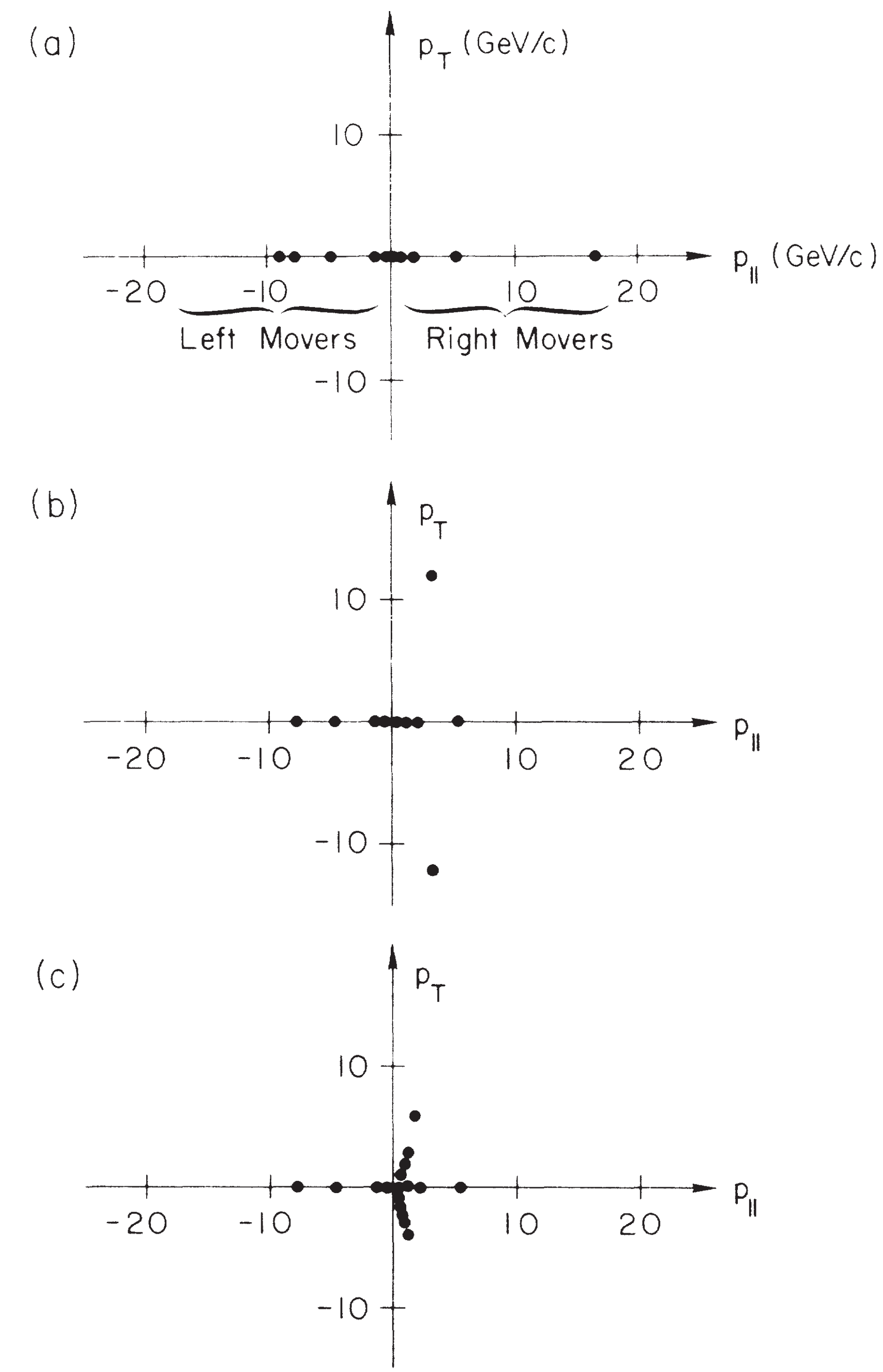


FIG. 4. A momentum-space visualization of hadron-hadron deep-inelastic scattering occurring in three steps.

DESY provided early evidence

- The PETRA accelerator had enough energy to make jets clearly visible.
- The PETRA experiments had 4π detectors, so that one could be convinced that two and three jet events existed with single event displays.

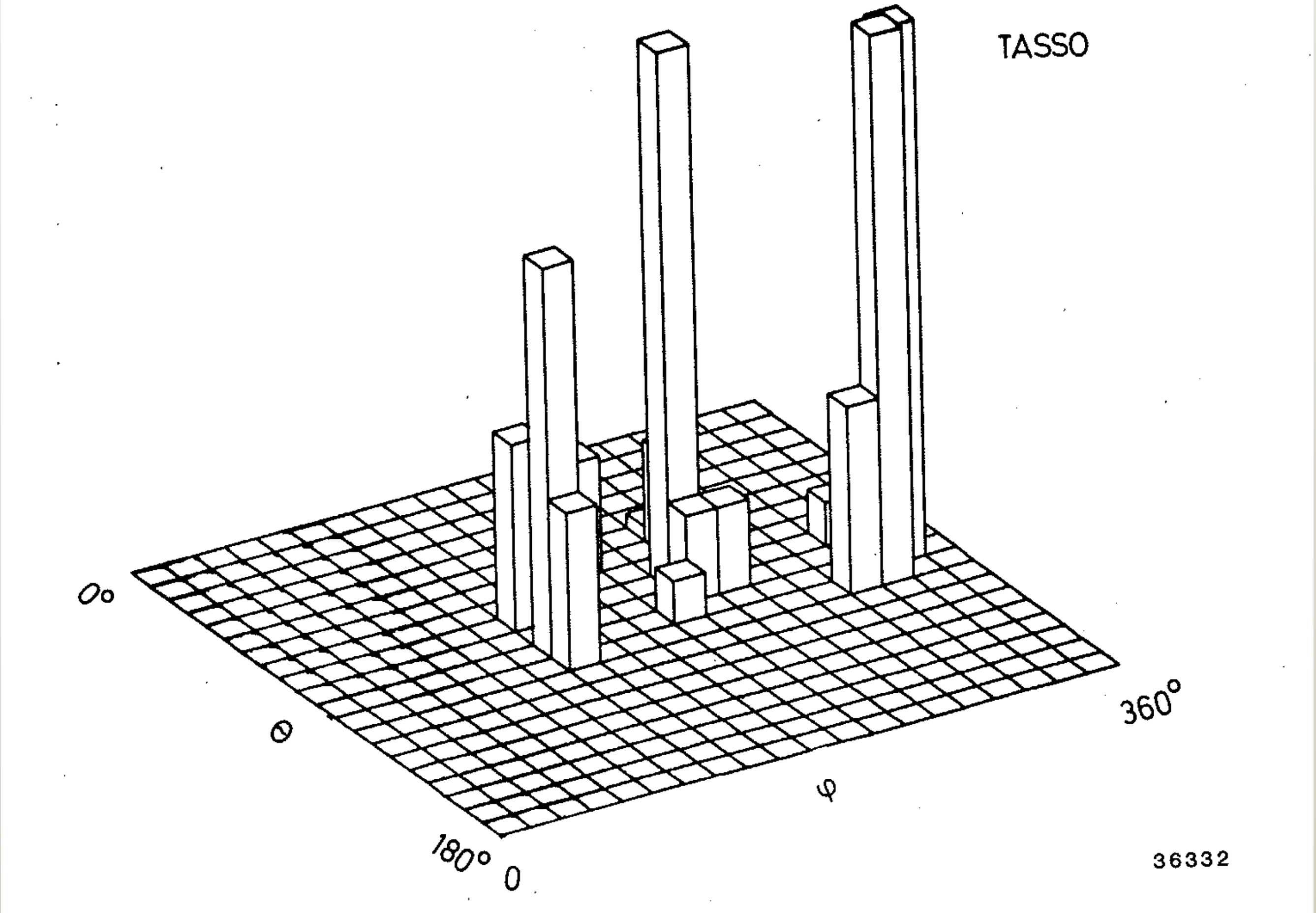
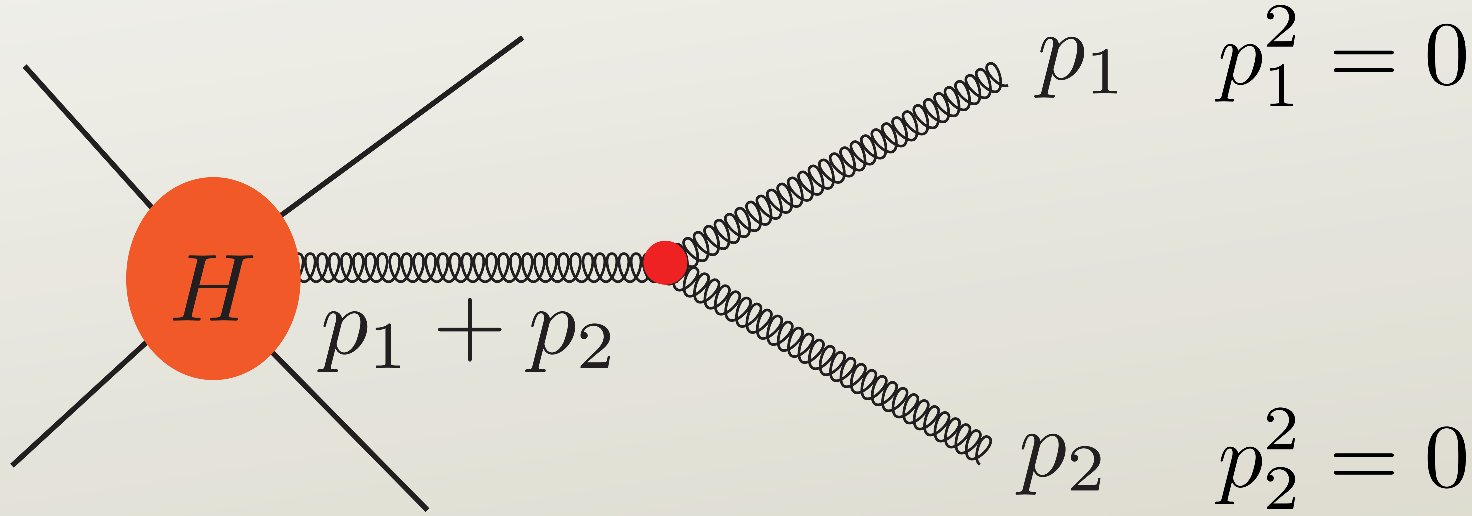


Fig.9 A typical three jet event

from G. Wolf, Multiparticle Conference, 1983

Why are there jets in QCD?

- Consider a gluon that splits into two gluons.



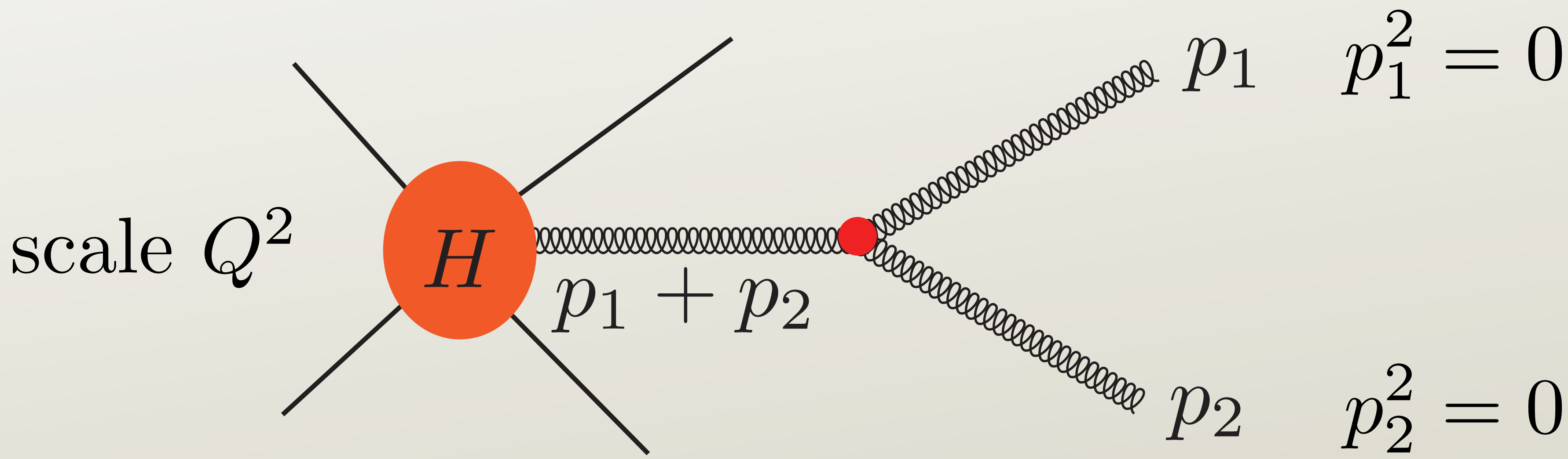
- The amplitude is biggest when $(p_1 + p_2)^2 \approx 0$.

  $\implies$ p_1 and p_2 are nearly collinear, or one is soft.

- Iterating this, we expect to make jets of roughly collinear particles.

How showers work in QCD, with some history

Showers and factorization

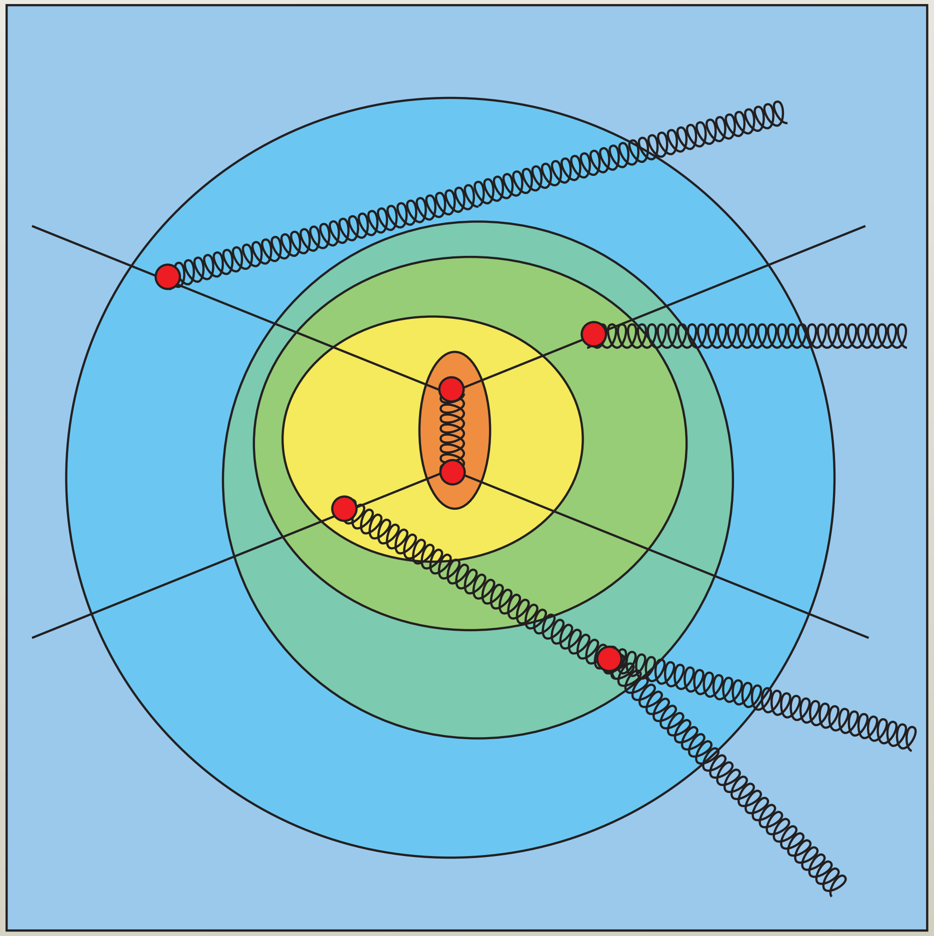


- If $(p_1 + p_2)^2 \ll Q^2$, we can set $(p_1 + p_2)^2 = 0$ in H.
- That is, the amplitude factorizes into

$$H \times \text{ splitting function}$$

Assembling QCD showers

- Think of shower starting at hard interaction and proceeding to softer splittings.
- Most probable: soft and collinear splittings.
- Such splittings from a high P_T parton builds up a jet.
- Very hard interactions happen with probability α_s.

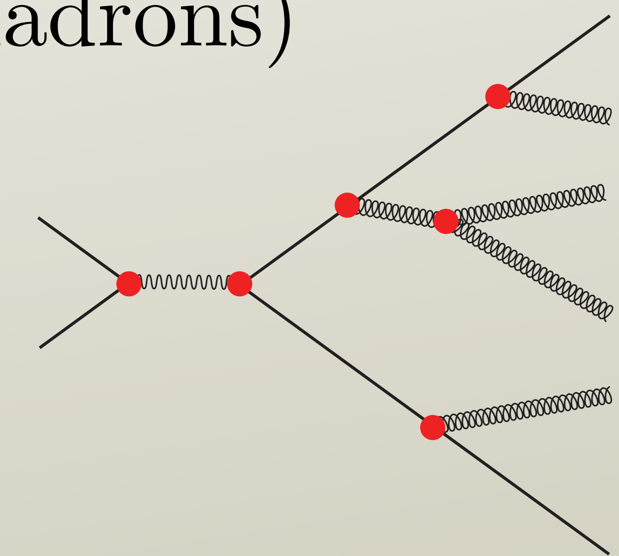


Early event generators

- Late 1970s:

  partonic cross section $\times$ fragmentation functions.

- 1980: iterative splitting (for $e^+e^- \to$ hadrons)
  - Fox and Wolfram
  - Odorico



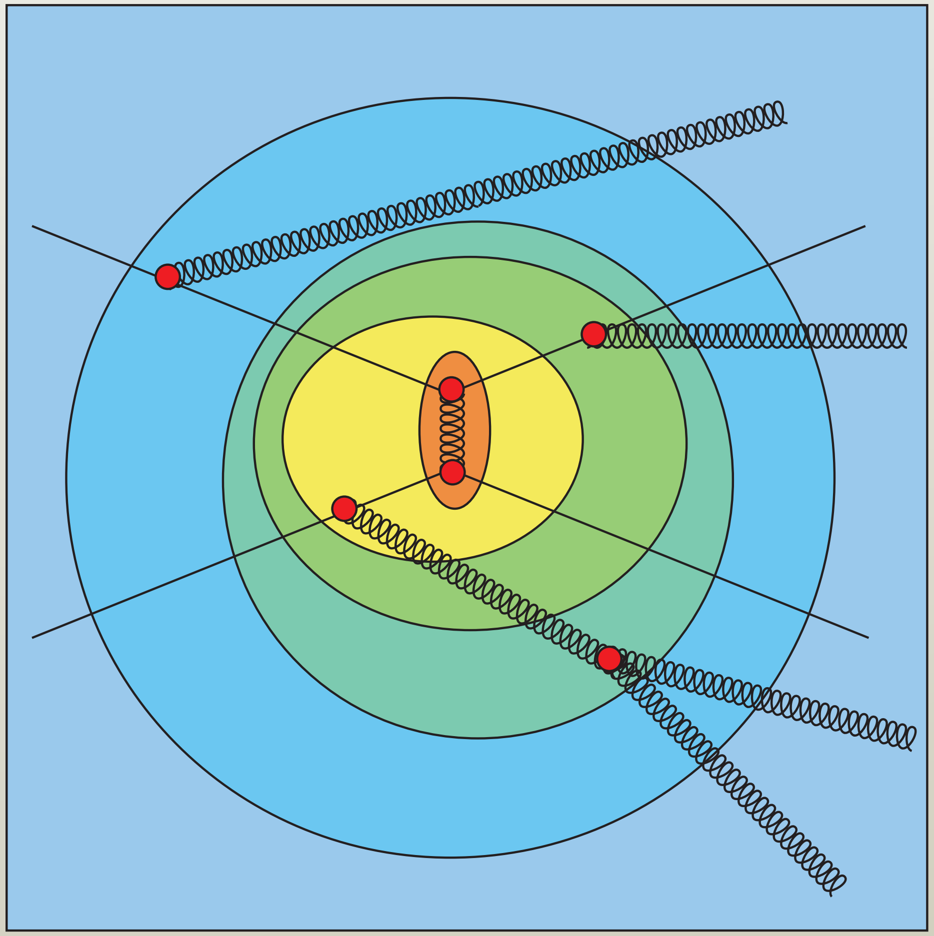
- Early 1980s: work at Lund on event generators.
  - String fragmentation for hadronization.
  - Bo Andersson (with student Torbjörn Sjöstrand).

Idea exchanges

- My university, the University of Oregon, is proud to have organized the Oregon Workshop on Super High Energy Physics, 18 March - 10 August, 1985.
- Event generators were a major subject.
- Participants included R.K. Ellis, R. Field, T. Gottschalk, R. Odorico, F. Paige, S. Protopopescu, T. Sjöstrand, and B. Webber.
- My opinion: this sort of exchange is important.

Backwards evolution

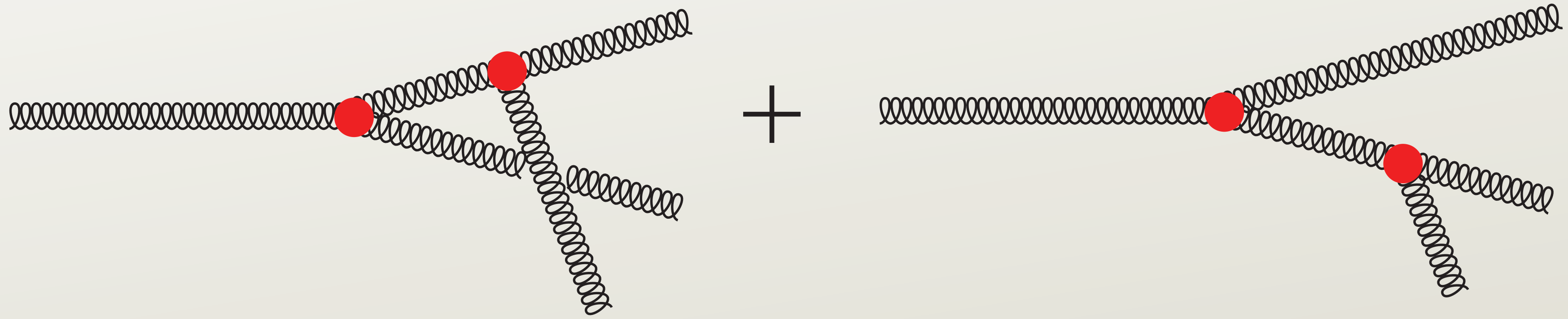
- In an event generator based on factoring soft from hard interactions, we go backwards in time for the initial state.



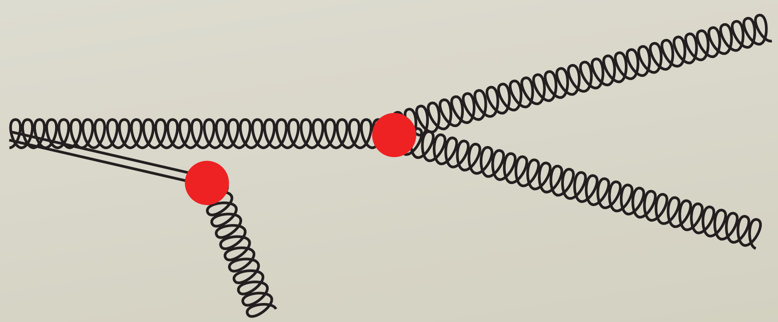
- This is pretty unintuitive.
- It was introduced in 1985 by Sjöstrand.
- A similar version was introduced by Gottschalk.

Color coherence

- Suppose that a gluon splits into two almost collinear gluons.

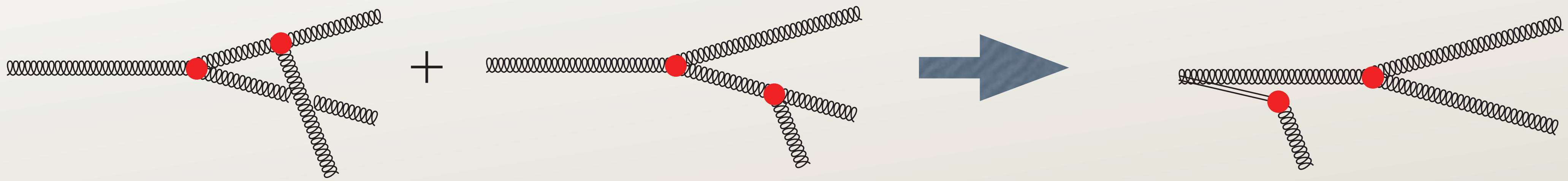


- Then each daughter radiates a soft, wide angle gluon.
- This is as if the soft gluon were emitted from the mother.



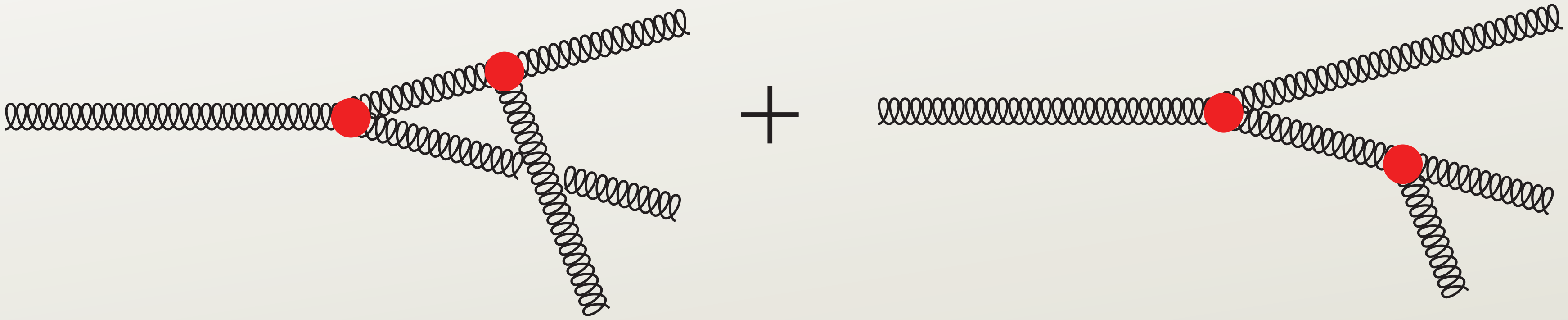
- Or, rather, to an on-shell approximation to the mother.

Implementing color coherence



- Webber and Marchesini (1984) showed how to implement this in an event generator.
- This became the basis of Herwig (Webber, 1984).
- Put the wide angle splittings first.
- This involves an approximation for the azimuthal angle distribution.

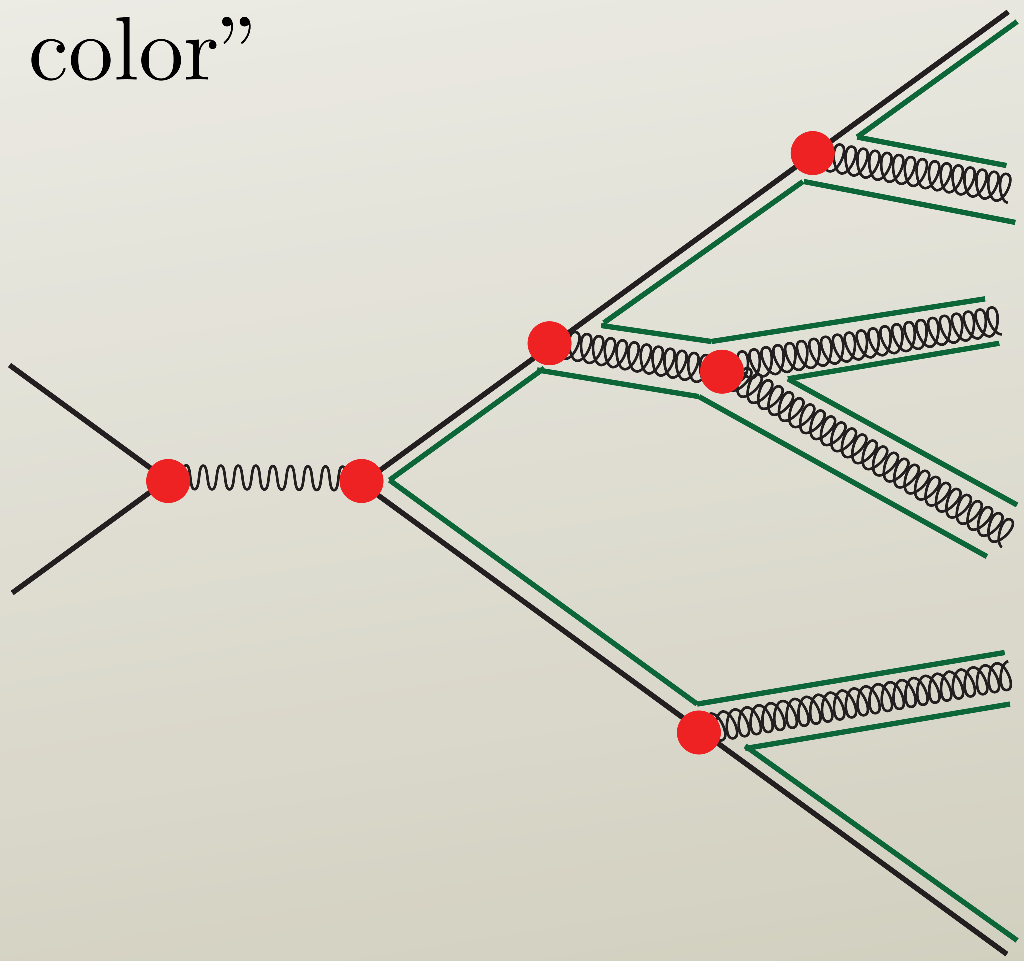
What about Pythia?



- Pythia orders splittings by a measure of hardness.
- Then later in physical time means later in the shower.
- In older versions, used virtuality $(p^2 - m^2)$.
- Now k_T^2.
- (Actually, one can argue that $(p^2 - m^2)/E$ is best.)
- Early Pythia just imposed a cut on angles.
- This roughly simulates the coherence effect.

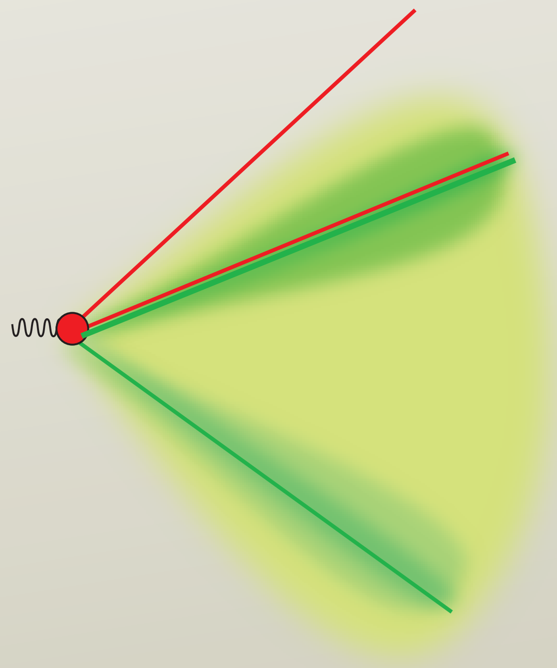
Leading color approximation

- Parton shower event generators track color.
- Mostly they use the "leading color" approximation.
- Gluons carry color $\mathbf{3} \times \bar{\mathbf{3}}$ rather than $\mathbf{8}$.
- Corrections are order $1/N_c^2$ $(N_c = 3)$.
- Improvements on this are part of the workshop "Event Generators and Resummation."

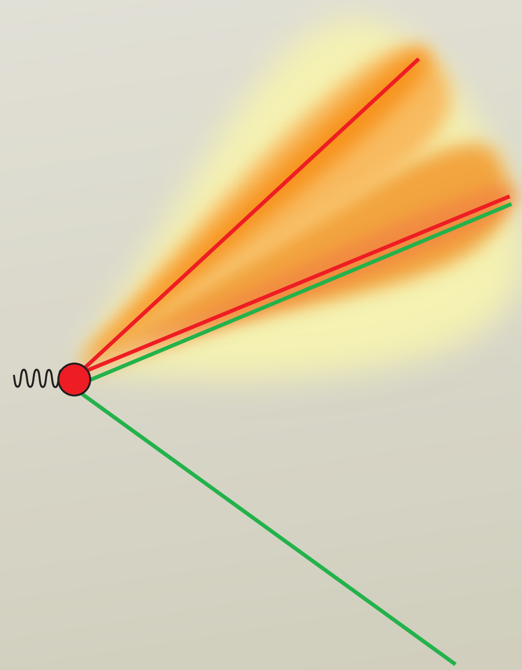


Color coherence with dipoles

- In today's hardness ordered showers, color coherence is achieved based on a dipole picture.
- This is fairly simple within the leading color approximation.
- Consider soft radiation from a qqg system.



The wide angle dipole gives a wide angle pattern.



The narrow angle dipole gives a narrow angle pattern.

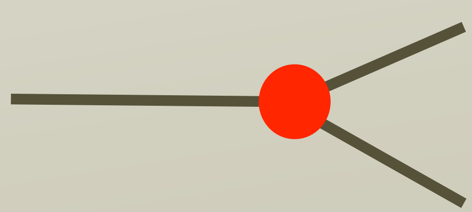
Understanding showers

Structure of shower evolution

- State $\big|\rho\big)$.
- Probability for momenta p and flavors f is $\big(\{p,f\}_m\big|\rho\big)$.
- (Think about color and spin later.)
- Evolution with shower time t: $\big|\rho(t)\big) = \mathcal{U}(t,0)\big|\rho(0)\big)$

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

splitting — no splitting



$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

- Since $\mathcal{V}(t)$ is simple, rewrite as

split

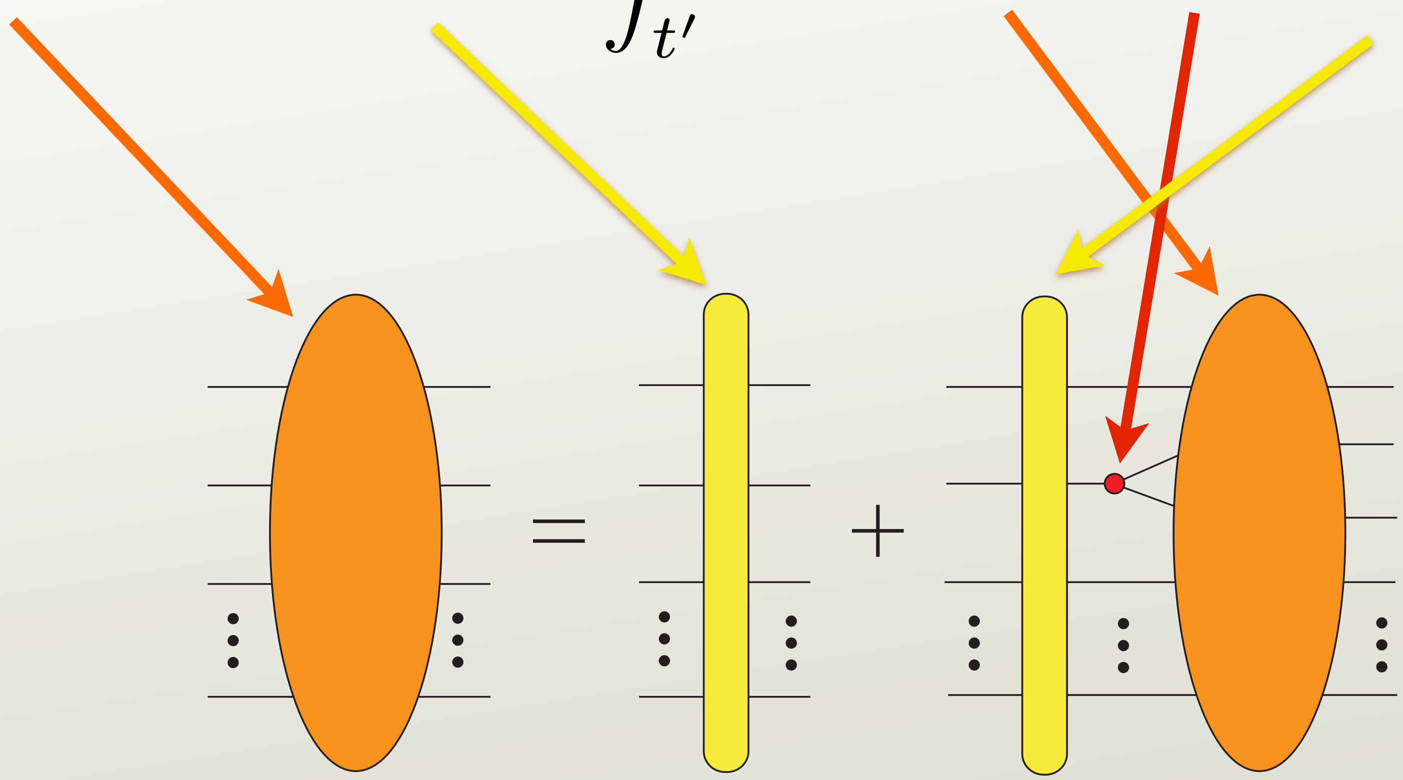
$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\,\mathcal{H}_{\rm I}(\tau)\,\mathcal{N}(\tau,t')$$

exponentiate the probability of not splitting

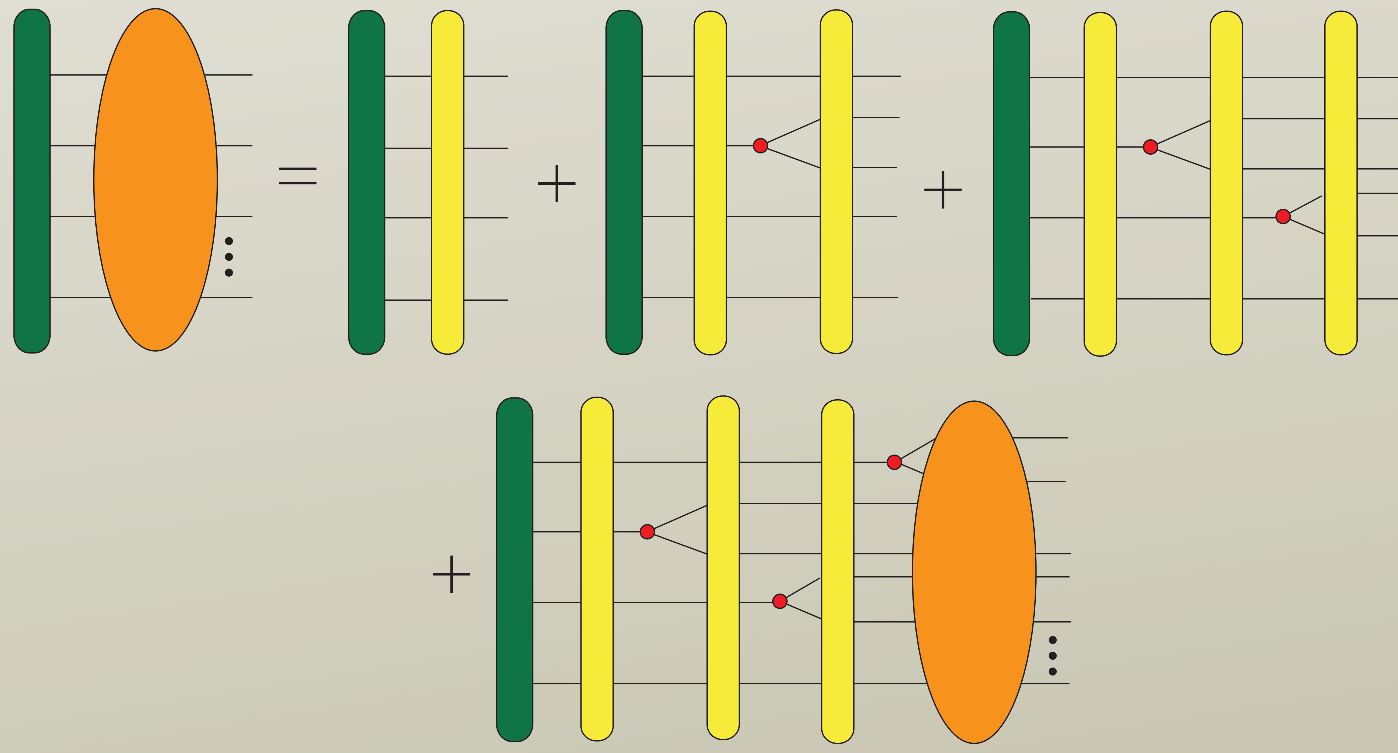
$$\mathcal{N}(t,t') = \mathbb{T}\exp\left\{-\int_{t'}^{t} d\tau\,\mathcal{V}(\tau)\right\}$$

this is the
Sudakov factor

$$\mathcal{U}(t,t') = \mathcal{N}(t,t') + \int_{t'}^{t} d\tau\, \mathcal{U}(t,\tau)\, \mathcal{H}_{\rm I}(\tau)\, \mathcal{N}(\tau,t')$$



- Iterated, gives a picture of what shower evolution does...

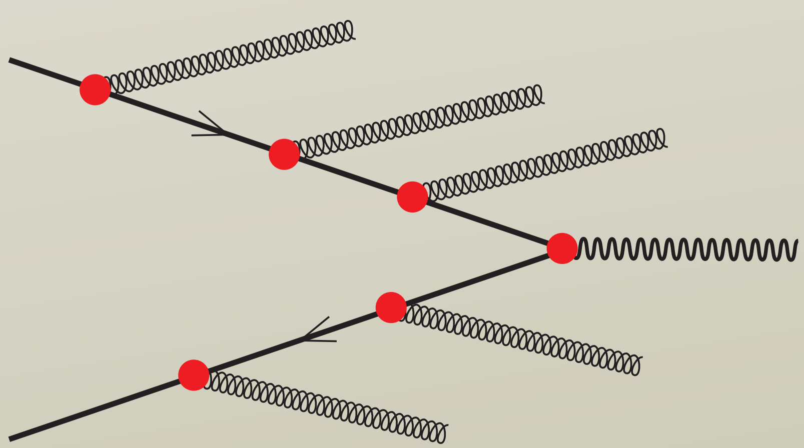


Summing logs

- Consider $A + B \to Z + X$.
- Measure the $p_\perp$ of the Z-boson for $p_\perp^2 \ll M_Z^2$,

$$\frac{d\sigma}{dp_\perp dY}$$

- There are large logarithms $\log(M_Z^2/p_\perp^2)$.
- We know how to sum these in QCD.



The QCD answer,

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$
$$\times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2)$$
$$\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right)$$
$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \quad .$$

$$A(\alpha_\mathrm{s}) = 2\,C_\mathrm{F}\,\frac{\alpha_\mathrm{s}}{2\pi} + 2\,C_\mathrm{F}\,\left\{ C_\mathrm{A}\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_\mathrm{f}}{9}\right\}\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \quad ,$$

$$B(\alpha_\mathrm{s}) = -4\,\frac{\alpha_\mathrm{s}}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f}\pi^2}{27} + \frac{8\zeta(3)}{3}\right]\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \quad ,$$

$$C_{a'a}(z,\alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)\,(\pi^2-8)\right\} + \delta_{ag}\, z(1-z)\right]$$

$$x_\mathrm{A} = \sqrt{\frac{M^2}{s}}e^{Y} \qquad x_\mathrm{B} = \sqrt{\frac{M^2}{s}}e^{-Y} \qquad C = 2e^{-\gamma_E}$$

- The most important part is the exponentiation in b-space.
- In exponent,

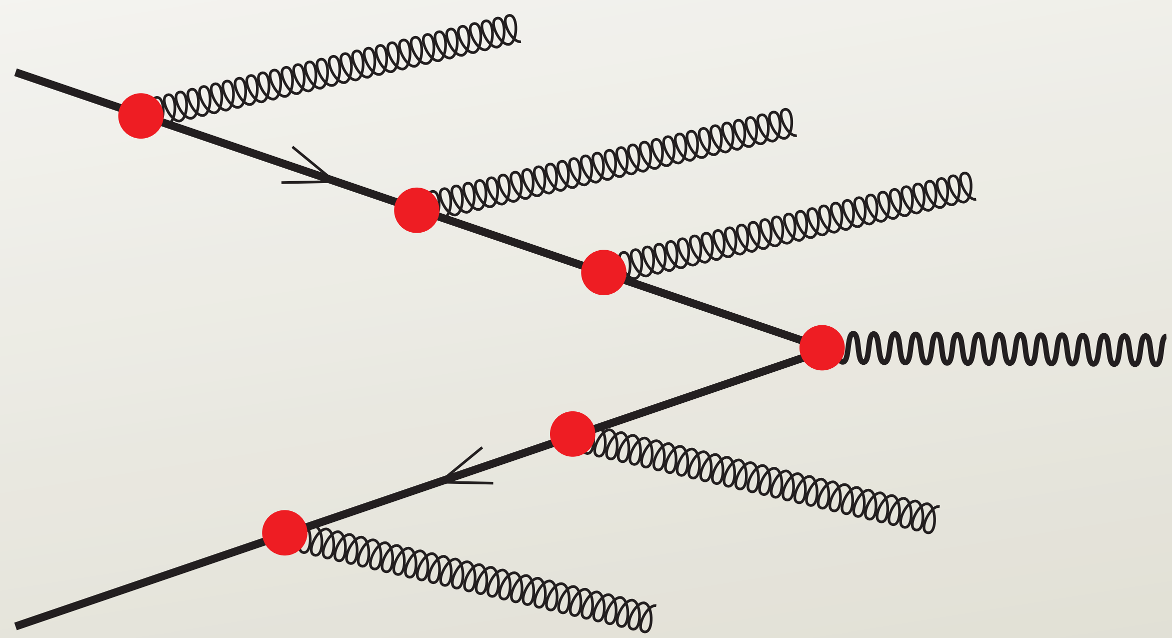
$$\alpha_s(M^2)^n \log(\boldsymbol{b}^2 M^2)^{n+1}$$

not

$$\alpha_s(M^2)^n \log(\boldsymbol{b}^2 M^2)^{2n}$$

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$
$$\times \sum_{a,b} \int_{x_a}^1 \frac{d\eta_a}{\eta_a} \int_{x_b}^1 \frac{d\eta_b}{\eta_b}\, f_{a/A}(\eta_a, C^2/\boldsymbol{b}^2)\, f_{b/B}(\eta_b, C^2/\boldsymbol{b}^2)$$
$$\times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$
$$\times \sum_{a',b'} H^{(0)}_{a'b'}\, C_{a'a}\left(\frac{x_a}{\eta_a}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_b}{\eta_b}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) .$$

- Parton shower event generators can (maybe) do this!



- The Z-boson gets $p_\perp$ because of recoils against initial state radiation. (Parisi & Petronzio.)

- Parton shower splitting functions match QCD for soft and collinear radiation.

How to check

Z. Nagy and DES

- Use parton shower evolution equations.

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

- Fourier transform from $\boldsymbol{k}_\perp$ to $\boldsymbol{b}$.

- Solve the evolution equations analytically with the appropriate approximations.

Result of checking

✓ Exponentiation

$$\frac{d\sigma}{d\boldsymbol{p}_\perp\, dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times \sum_{a,b} \int_{x_a}^{1} \frac{d\eta_a}{\eta_a} \int_{x_b}^{1} \frac{d\eta_b}{\eta_b}\; f_{a/A}(\eta_a, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_b, C^2/\boldsymbol{b}^2)$$

$$\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_s(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_s(\boldsymbol{k}_\perp^2)) \right] \right)$$

$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_a}{\eta_a}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_b}{\eta_b}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \; .$$

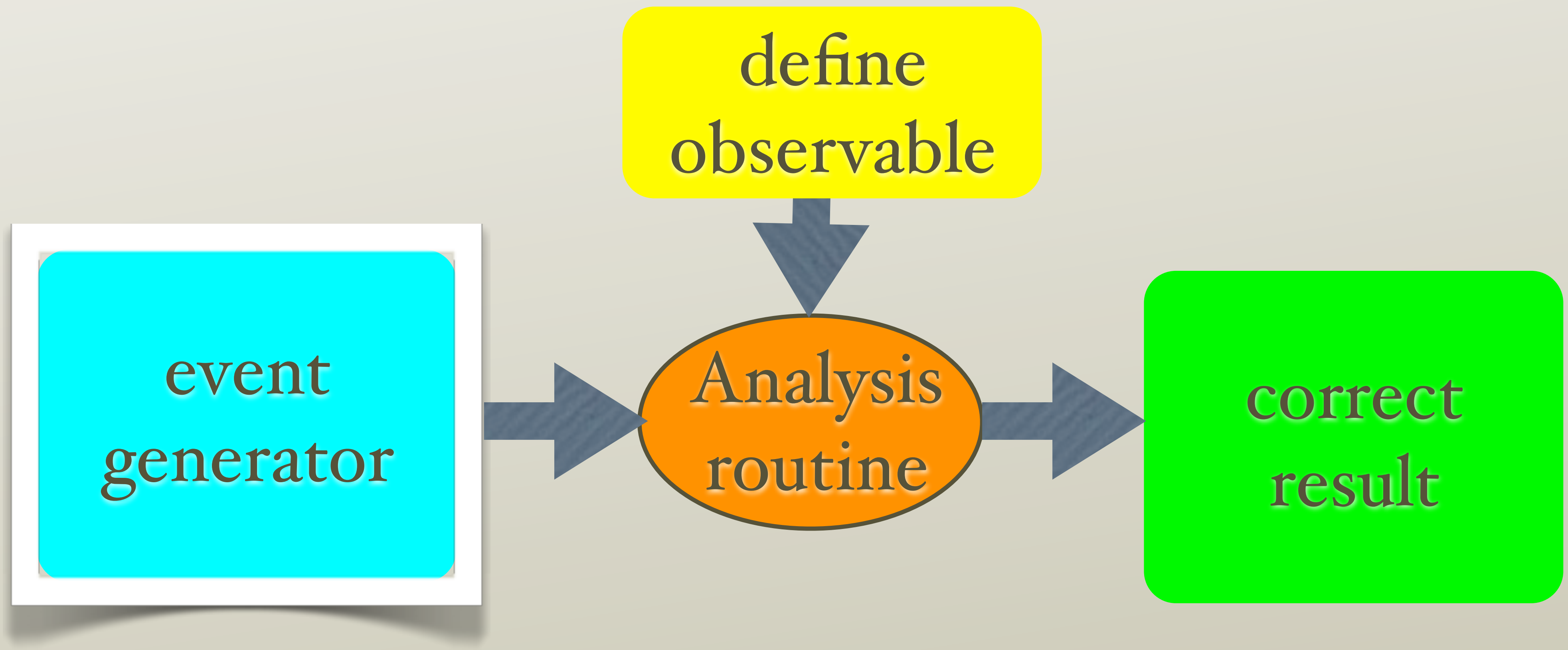
$$A(\alpha_s) = 2\, C_F\, \frac{\alpha_s}{2\pi} + 2\, C_F \left\{ C_A \left[\frac{67}{18} - \frac{\pi^2}{6} \right] - \frac{5\, n_f}{9} \right\} \left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots \; ,$$

$$B(\alpha_s) = -4\, \frac{\alpha_s}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_f}{9} + \frac{20\pi^2}{3} - \frac{8 n_f \pi^2}{27} + \frac{8\zeta(3)}{3} \right] \left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots \; ,$$

$$C_{a'a}(z, \alpha_s) = \delta_{a'a}\delta(1-z) + \frac{\alpha_s}{2\pi} \left[\delta_{a'a} \left\{ \frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)(\pi^2 - 8) \right\} + \delta_{ag}\, z(1-z) \right]$$

Was this inevitable?

- One might imagine that because parton splitting functions are correct in the limits of soft and collinear splittings, all large log summations will come out correctly.



- I will argue that this claim is far from obvious.

- In the case of the $p_\perp$ distribution of Z-bosons:

  * Some "minor" details matter.
  * If we get the "minor" details right, it works.
  * There are some "major" details that are wrong in standard showers: color and spin.
  * These don't matter in this case.

- Other cases are more complicated.

  * One suspects that "superleading" logarithms in cross sections to have gaps between jets are not correctly calculated (Forshaw, Kyrieleis, Seymour).

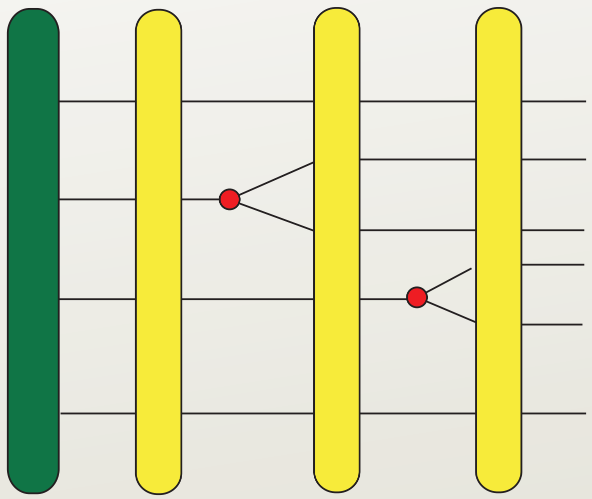
A critique of pure perturbation theory

- Consider a cross section involving N jets at a single scale Q^2.
- Perturbation theory gives

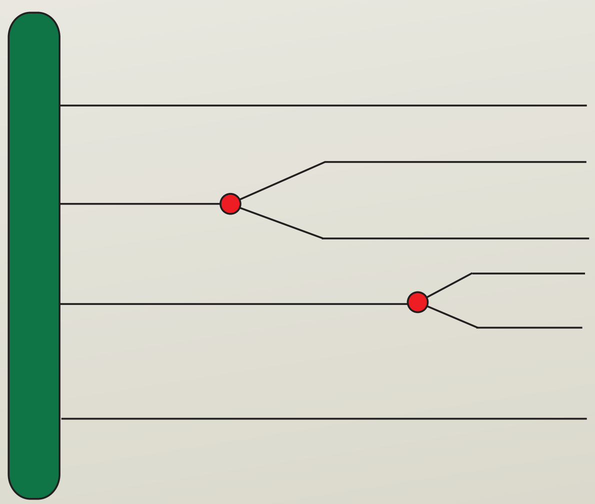
$$\sigma(N \text{ jet}) = \alpha_s^A(Q^2)\left\{C_0 + \alpha_s(Q^2)\, C_1 + \alpha_s^2(Q^2)\, C_2 + \cdots\right\}$$

- At LO, we have C_0, at NLO we have C_0 and C_1.
- But what if we need a $\tilde{\sigma}(N \text{ jet})$ that is infrared sensitive?
- *Eg.* our calorimeter responds differently to $\pi^{\pm}$ and π^0.
- The perturbative formula does not help.
- A shower event generator (with hadronization) does help.

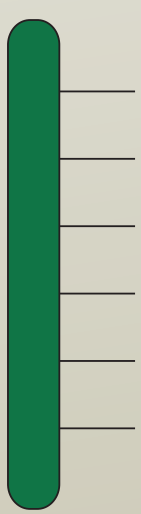
A critique of pure showers



- The standard shower has Sudakov exponentials and small $p_\perp$ approximations for splitting.



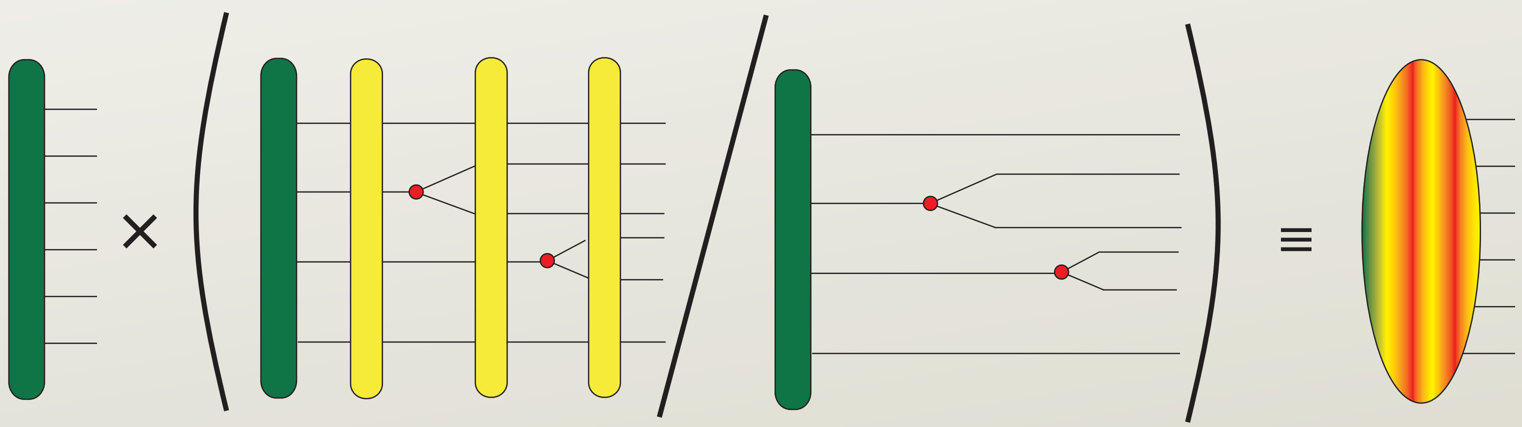
- The small $p_\perp$ approximations.



- Maybe the exact matrix element would be better. But that lacks the Sudakov factors.

An improved version

- Define a Sudakov corrected matrix element,



- This is the essential idea of Catani, Krauss, Kuhn, and Webber for matched showers.
- There is more to it than this.
- There are several methods.
- This has been a subject of discussion at the workshop "Event Generators and Resummation."

This is harder at NLO

- Expanding

$$
\begin{aligned}
\mathcal{U}(t,t') = 1 &+ \int_{t'}^{t} d\tau_1\ [\mathcal{H}_{\rm I}(\tau_1) - \mathcal{V}(\tau_1)] \\
&+ \int_{t'}^{t} d\tau_2 \int_{t'}^{\tau_2} d\tau_1\ [\mathcal{H}_{\rm I}(\tau_2) - \mathcal{V}(\tau_2)][\mathcal{H}_{\rm I}(\tau_1) - \mathcal{V}(\tau_1)] \\
&+ \cdots
\end{aligned}
$$

we see that shower evolution applied to the Born $|\mathcal{M}|^2$ generates perturbative corrections.

- We need to replace the shower $\mathcal{H}_{\rm I} - \mathcal{V}$ for the hardest splitting by the exact NLO correction.
- This has been a subject of discussion at the workshop "Event Generators and Resummation."

The goal

- Using a shower matched to LO or NLO perturbative calculations, we want to produce good approximate results for infrared sensitive measurements.

- At the same time, for an infrared safe measurement with a single scale Q^2, we should match (for LO) C_0 or (for NLO) C_0 and C_1 in

$$\sigma(\text{IR safe}) = \alpha_s^A(Q^2)\left\{C_0 + \alpha_s(Q^2)\,C_1 + \alpha_s^2(Q^2)\,C_2 + \cdots\right\}$$

Color

- In $\big|\rho(t)\big) = \mathcal{U}(t,0)\big|\rho(0)\big)$, what is $\big|\rho(t)\big)$?
- It cannot be simply the probability density for the partons to have certain momenta and flavors.
- Partons carry color (& spin, but I omit that for today.)
- We need quantum statistical mechanics.
- We need the quantum density operator:

$$\sum_{c,c'} \big|\{c\}_m\big\rangle\, \rho(\{p,f,c',c\}_m,t)\, \big\langle\{c'\}_m\big|$$

- $\big|\rho(t)\big)$ represents the function $\rho(\{p,f,c',c\}_m,t)$.

- Now in

$$\frac{d}{dt}\,\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

$\mathcal{H}_{\rm I}(t)$ and $\mathcal{V}(t)$ become matrices in color space.

- With the leading color approximation, this is simple.
- Beyond the leading color approximation, this is not simple.
- $\mathcal{V}(t)$ is a non-trivial matrix in general.
- Then the Sudakov factor

$$\mathcal{N}(t,t') = \mathbb{T}\exp\left\{-\int_{t'}^{t} d\tau\,\mathcal{V}(\tau)\right\}$$

is not nice.

- Progress in this is a subject at the workshop.

Summary

- After 32 years since 1980, developing parton shower ideas is still an active field.
- Progress is slow because this is not easy.
- Progress is happening because this is important.
- In the past few years, there have been substantial improvements in how parton showers work.
- More improvements are coming.