A Stringy Mechanism for a Small Cosmological Constant

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Background

- There is very strong evidence that we are living in a de-Sitter vacuum with a very small cosmological constant.
- There is strong evidence that our universe has gone through an inflationary period, when the vacuum energy is below the Planck scale but much higher than the TeV scale.
- Can string theory accommodate these phenomena ? Or even explain them ?
- Pressing Question : why dark energy contributes 70% of the content of our universe ?
  Why not 99.999999....999999% ?

Cosmic Landscape

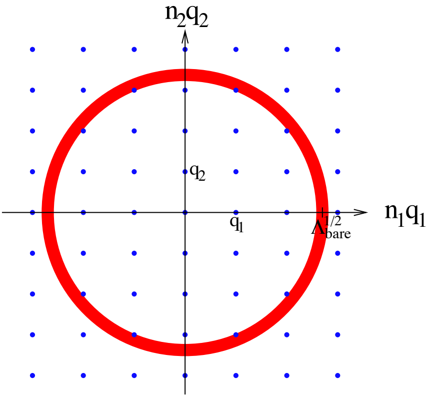
Views of the Cosmos through Strings and Branes :

- String theory is the only viable model for quantum gravity.
- String scale is so high that there is very little hope that laboratory experiments can directly detect its properties.
- Cosmological data and properties hopefully will tell us more about string theory and the specific solution that we live in.

String theory also allows us to ask more philosophical questions

- There are many RR fields that reduce to dozens of types of 4-form fluxes in 4-dim.
- They take constant values so their contributions to the vacuum energy density take constant values.
- Bousso-Polchinski (0004134) pointed out that their quantized values may explain why string theory may have a very small CC as one of its solutions.
- For string scale around GUT scale, $J > 20$ will give a CC comparable to the observed value.
- This explains how string theory can yield such a small CC.

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2\kappa_4^2}R - \lambda_{\rm bare} - \frac{Z}{2\cdot 4!}F_4^2\right) + S_{\rm branes}$$

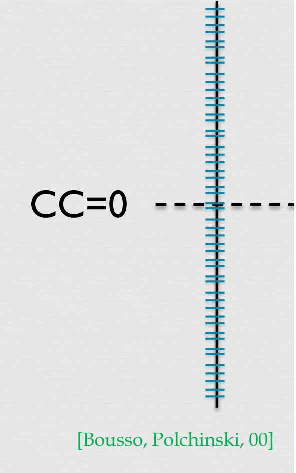


$$F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$$

$$\int_X \mathbf{F_4} = \frac{2\pi n}{e}\ , \quad n \in \mathbf{Z}$$

$$\lambda = \lambda_{\rm bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$$

Bousso & Polchinski, 0004134



$$\lambda = \lambda_{\text{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$$

Pressing Question

- Why nature picks such a very small positive Λ ?
- Stringy Mechanism :
  - Probability distribution
  - Stringy dynamics
- Type IIB :
  - Model with a single Kähler modulus
  - Rummel-Westphal model

Single Modulus Model

- Consider the superpotential

$$W = W_0 - Ae^{-x}$$

where W_0 and A are parameters and x is a Kähler modulus.

- A stable vacuum can exist at $x = x_m$

$$\Lambda \simeq BW_0A(x_m - x_0) \tag{1}$$

where B is a constant.

- Let us treat $W_0 \geq 0$ and $A \geq 0$ as random parameters.
- Then what is the probability distribution of Λ ?

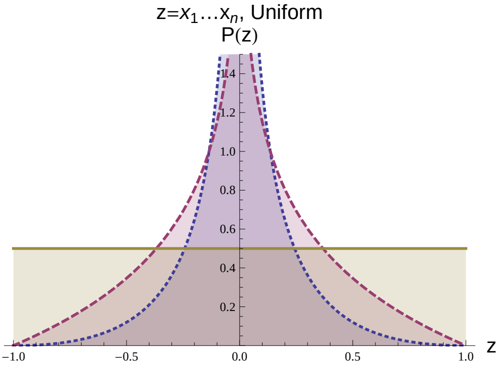
Basic Idea

Let x_j to have a uniform distribution $f(x_j) = 1$ between 0 and 1.
What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln \frac{1}{z}$$

for $0 \leq z \leq 1$

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$



$$P(z) = \frac{1}{2(n-1)!}\left(\ln\frac{1}{|z|}\right)^{n-1} \quad (2)$$

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$

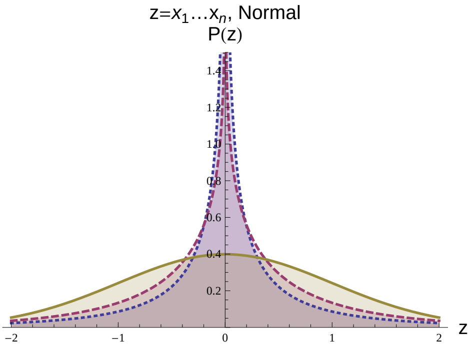
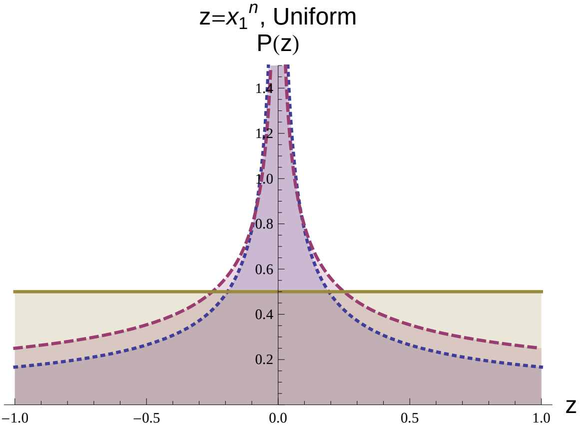


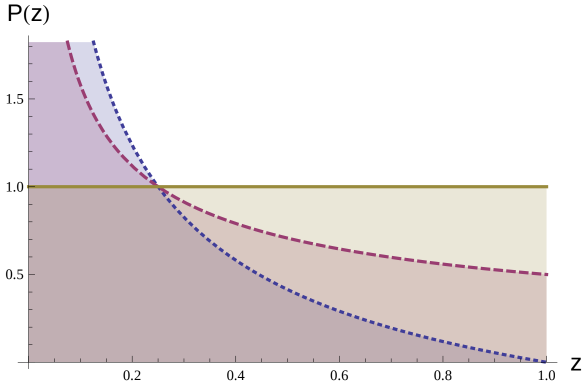
Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.

Probability distribution $P(z)$ for $z = x_1^n$



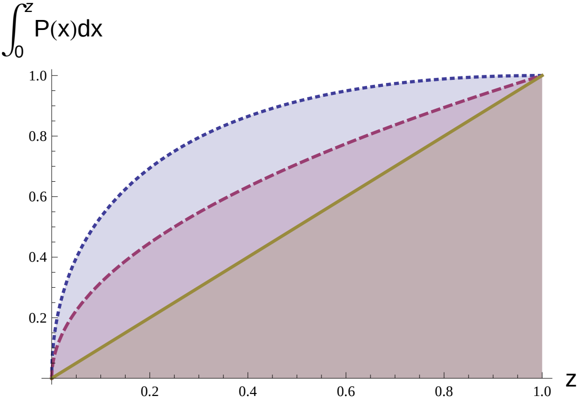
$$P(z) \sim z^{-1+1/n}$$

Probability distribution $P(z)$ for $z = x_1^2$ and $z = x_1^2 x_2$



$z = x_1$ (solid horizontal line), $z = x_1^2$ (red dashed curve), and $z = x_1^2 x_2$ (blue dotted curve)

Cumulative distribution for $z = x_1^2$ and $z = x_1^2 x_2$



$z = x_1$ (solid horizontal line), $z = x_1^2$ (red dashed curve), and $z = x_1^2 x_2$ (blue dotted curve)

Probability distribution $P(z)$

z	Asymptote of $P(z)$ at $z=0$
$x_1 \cdots x_n$	$(\ln(1/\|z\|))^{n-1}$
x_1^n	$z^{-1+1/n}$
$x_1^n \cdots x_m^n$	$z^{-1+1/n}(\ln(1/\|z\|))^{m-1}$
$x_1^m x_2^n$	$(z^{-1+1/m} - z^{-1+1/n})/(m-n)$
$x_1 \cdots x_m \;/\; y_1 \cdots y_n$	$(\ln(1/\|z\|))^{m-1}$
x_1^m / y_1^n	$z^{-1+1/m}$
$x_1^{n_1} + \cdots + x_m^{n_m}$	$z^{-1+1/n_1+\cdots 1/n_m}$
$x_1 x_2$, $0 < c = x_1/x_2 < \infty$	smooth
$x_1 x_2$, $0 \leq c = x_1/x_2$ or $c \leq \infty$	$\ln(1/\|z\|)$

Model in IIB

$$V = e^{\frac{K}{M_P^2}} \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - \frac{3}{M_P^2} |W|^2 \right) ,$$

$$K = -2M_P^2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) ,$$

$$\hat{\xi} = -\frac{\zeta(3)}{4\sqrt{2}(2\pi)^3 g_s^{3/2}} \chi(M) \sim 8.57 \times 10^{-4} \frac{\chi(M)}{g_s^{3/2}},$$

$$W = M_P^3 \left(W_0 + A_1 e^{-a_1 T_1} \right) .$$

Rummel-Westphal Model

$$\frac{V}{M_P^4} \sim \frac{a_1 A_1 e^{-a_1 t_1} W_0}{2\gamma_1^2 t_1^2} + \frac{3 W_0^2 \hat{\xi}}{64\sqrt{2}\gamma_1^3 t^{9/2}}$$

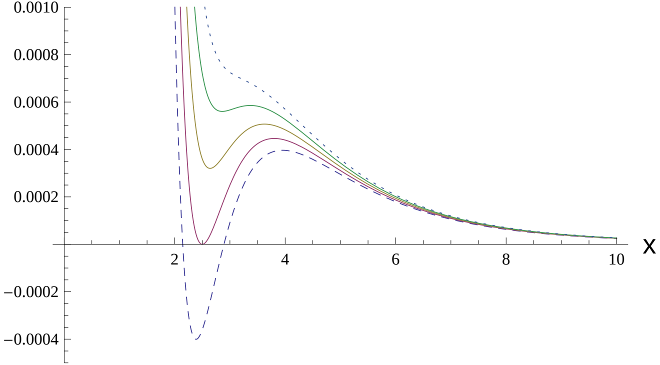
$$= -\frac{W_0 a_1^3 A_1}{2\gamma_1^2}\left(\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}\right),$$

$$C \equiv \frac{-27 W_0 \hat{\xi} a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1}, \quad x \equiv a_1 t_1.$$

$$V' = 0, \qquad V'' \geq 0$$

$$3.65 \lesssim C \lesssim 3.89, \quad 2.50 \leq x \lesssim 3.11.$$

$$\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}$$



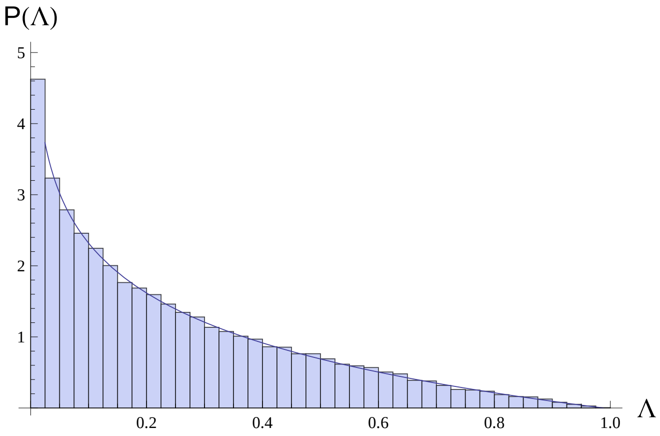
$$W_0 A_1 \leq 0$$

$$\frac{\Lambda}{M_P^4} \sim \frac{e^{-5/2}}{9}\left(\frac{2}{5}\right)^2 \frac{-W_0 a_1^3 A_1}{\gamma_1^2}\left(x - \frac{5}{2}\right) + \cdots$$

$$= \frac{\sqrt{10}}{45}\frac{-W_0 a_1^3 A_1}{\gamma_1^2}\left(C - \frac{225\sqrt{10}}{16 e^{5/2}}\right) + \cdots ,$$

$$C \equiv \frac{-27 W_0 \hat{\xi} a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1} \sim \left(\frac{5}{2}\right)^{5/2} e^{-5/2}(x+2)\cdots .$$

$P(\Lambda)$



Asymptote of $P(\hat{\Lambda})$

Model	Random variables	Asymptote
1	W_0, A_1	$-\ln\hat{\Lambda}$
2	$W_0, A_1, \hat{\xi}$	$-\ln\hat{\Lambda}$
3	a_1, W_0, A_1	$\hat{\Lambda}^{-4/9}$
4	$1/y_m \leq a_1 = 1/y_1 \leq 1, W_0, A_1$	$-y_m^4 \ln\hat{\Lambda}$

Table: A summary of the properties in the constrained product distribution $P(\hat{\Lambda})$ for 4 plausible scenarios.

$$\hat{\Lambda} = \frac{\Lambda}{M_P^4} \sim 10^{-7}(2\pi/N_c)^{9/2}$$

What happens if the distributions of W_0, A_1 are peaked ?

$P(y_1) \sim [\ln(1/y_1)]^{n_1}$ and $P(y_2) \sim [\ln(1/y_2)]^{n_2}$, then the probability distribution of $z = y_1 y_2 \geq 0$ goes like, as $z \to 0$,

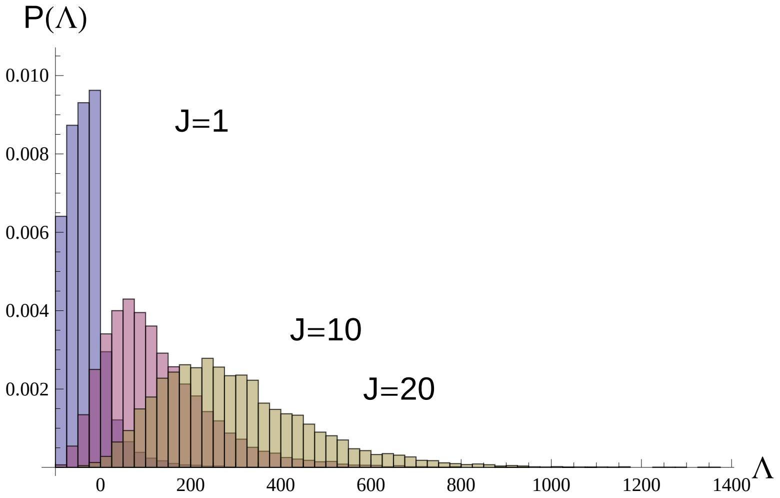
$$P(z) \sim [\ln(1/z)]^{n_1+n_2+1}$$

It is simple and probably natural to have $W_0 = A = 0$, yielding supersymmetric Minkowski solutions. So zeros for them should be very likely. Other values are also possible, so we believe that their probability distributions naturally peak at zero values.

The modulus mass :
$m \sim \sqrt{\hat{\Lambda}} M_P > 1$ Tev $\to \hat{\Lambda} > 10^{-30}$

$$\hat{\Lambda} \simeq B W_0 A (x_m - x_0)$$

Non-interacting case



Non-interacting moduli

- Let us consider $V(\phi_i) = \sum_{j=1}^{N} V_j(\phi_j)$
- Then $V_{min}(\phi_i) = \sum_j V_{j,min}(\phi_j)$. So if V_j has n_j number of minima, then there are Πn_j number of classical minima. For $n_j \sim n$, we have $n^N = e^{N \ln n}$ minima. This is implicit in BP.
- Roughly, V_j has $2n_j$ extrema, half of which are minima. So out of $\Pi(2n_j)$ extrema, we have Πn_j number of minima.
- The probability for an extremum to be a minimum is $\mathcal{P} = 1/2^N = e^{-N \ln 2}$.
- Still, there are $\mathcal{P}\times$ number of extrema $= e^{N \ln n}$ minima.

Multi-moduli case

- We are interested in dS space solutions of string theory.
- However if the various ϕ_j interact with each other, it is very hard to say how many minima there are.
- Typical $V(\phi_j)$ can be very complicated.
- The symmetric mass-squared matrix at an extremum of V, i.e., the Hessian $\mathcal{H} = V_{ij}$ for $V_i = 0$, must be positive definite for the extremum to be classically stable (meta-stable).
- If the Hessian is complicated, what is the probability that an extremum is meta-stable ?

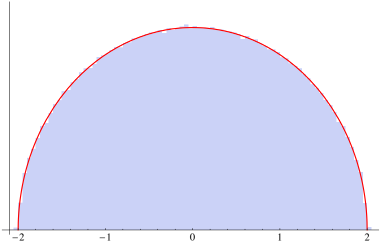
General Case

- Consider the Hessian $\mathcal{H} = V_{ij}$ for $V_i = 0$.
- $\mathcal{H}$ is positive definite if all eigenvalues are positive. By Sylvester Criteria, this is true if all left-upper sub-matrices $\mathcal{H}_J$ of $\mathcal{H}$ have det $\mathcal{H}_J > 0$, for $J = 1, 2, ..., N$. (So there are N conditions.)
- If the Hessian is large and complicated, what is the probability that an extremum is meta-stable ?
- This problem was first studied by Aazami and Easther (0512050). More recently by Chen, Shiu, Sumitomo and me (also Marsh, McAllister and Wrase.)

- The tool to study a large complicated $\mathcal{H}$ is random matrix theory : Wigner, Tracy-Widom, Dean-Majumdar etc.
- Given a random $\mathcal{H}$ that typically has negative eigenvalues, the theory of fluctuation of extreme eigenvalues allows one to compute the probability of drawing a positive-definite matrix from the ensemble.
- The key phenomenon is eigenvalue repulsion: a large fluctuation through which all eigenvalues becomes positive-definite generally requires an increase in the local eigenvalue density, which is statistically costly.
- So $\mathcal{P}$ for no negative eigenvalue is Gaussianly suppressed.

Wigner's semi-circle

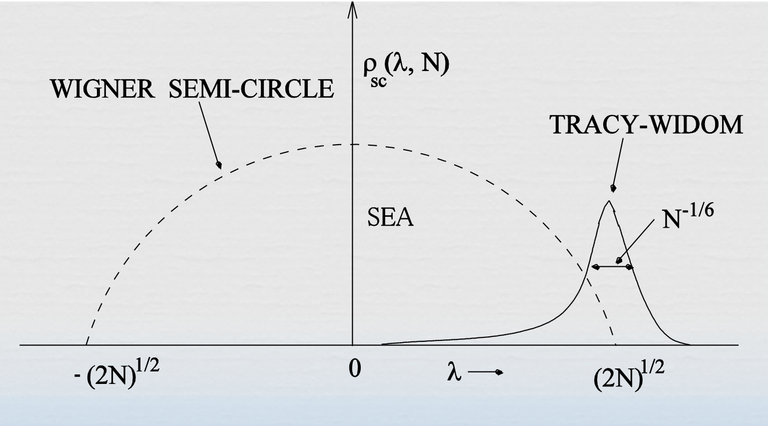
$$M = A + A^\dagger$$



Dyson

Distribution of eigenvalues

Elements of A are independent identically distributed variables drawn from some statistical distribution.



We are interested in the tail that extends pass zero.

- Here is an improved version on what they found : for N number of moduli,

$$\mathcal{P} \sim e^{-\frac{\ln 3}{4}(N+0.7)^2}$$

where $\ln(3)/4 = 0.275$ is obtained analytically by Dean and Majumdar (cond-mat 0609651).
- The number of extrema probably goes like e^{cN}(recall 10^{500}) so if the probability is Gaussianly suppressed, then there is little chance we'll have a meta-stable dS minimum when the number of moduli is large.

Summary

- At high vacuum energies, no stable vacua
- At lower energies, heavy stabilized moduli may be frozen
- At low energies, stable vacua begin to survive

New Picture