A New Way to Physics Beyond the Standard Model

H. Kowalski, L.N. Lipatov, D.A. Ross

The method is based on properties of the BFKL Gluon Density

Outline:

In contrast to DGLAP, BFKL describes the gluon system as a bound state; properties of bound states are very sensitive to BSM effects

Application to HERA data, F_2

determination of the SuperSym scale from HERA data; 10 TeV

Future application to LHC data: DY processes

H. Kowalski, Hamburg, 9th of January 2012

the talk is based on 3 papers

Indirect Evidence for New Physics at the 10 TeV Scale

H. Kowalski, L.N. Lipatov, D.A. Ross, arXiv:1109.0432v1

Using HERA data to determine the infrared behaviour of the BFKL amplitude

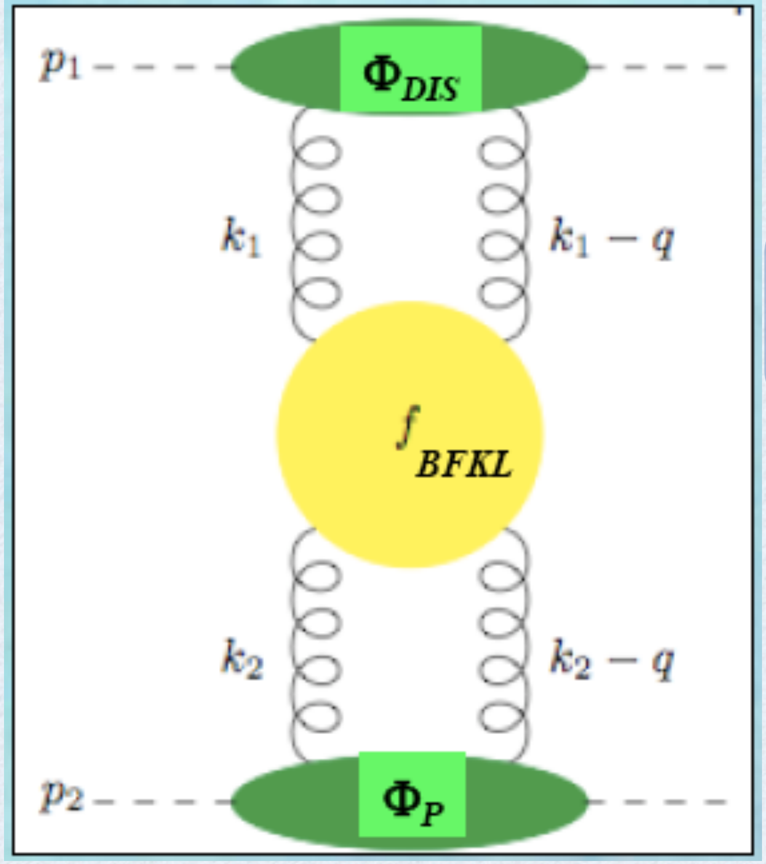
H. Kowalski, L.N. Lipatov, D.A. Ross and G. Watt, EPJC 70: 983, 2010

Evidence for the discrete asymptotically-free BFKL Pomeron from HERA data

J. Ellis, H. Kowalski, D.A. Ross Physics Letters B 668 (2008) 51–56

The dynamics of Gluon Density at low x is determined by **the amplitude for the scattering of a gluon on a gluon**, described by the BFKL equation

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') = \delta(k^2 - k'^2) + \int dq^2 \mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}')$$



which can be solved in terms of the eigenfunctions of the kernel

$$\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}') f_\omega(\mathbf{k}') = \omega f_\omega(\mathbf{k})$$

in LO, with fixed α_s

$$f_\omega(\mathbf{k}) = \left(k^2\right)^{i\nu-1/2}$$
$$\omega = \alpha_s \chi_0(\nu)$$

prevailing intuition (based on DGLAP) - gluon are a gas of particles

BFKL leads to a richer structure - basic feature: oscillations

Properties of the BFKL Kernel

Quasi-locality

$$\mathcal{K}(\mathbf{k},\mathbf{k}') \;=\; \frac{1}{kk'}\sum_{n=0}^{\infty} c_n \delta^{(n)}\left(\ln(\mathbf{k}^2/\mathbf{k}'^{\,2})\right)$$

$$c_n = \int_0^{\infty} dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')\frac{k}{k'}\frac{1}{n!}\left(\ln(\mathbf{k}^2/\mathbf{k}'^{\,2})\right)^n$$

Similarity to the Schroedinger equation

$$k\int dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \sum_{n=0}^{\infty} c_n \left(\frac{d}{d\ln(\mathbf{k}^2)}\right)^n \bar{f}_\omega(\mathbf{k}) \;=\; \omega\bar{f}_\omega(\mathbf{k})$$

Characteristic function

$$k\int dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \chi\left(-i\frac{d}{d\ln k^2},\alpha_s(k^2)\right)\bar{f}_\omega(k) \;=\; \omega\bar{f}_\omega(k)$$

with running α_s, BFKL frequency ν becomes k-dependent, $\nu(k)$

$$\alpha_s(k^2)\chi_0(\nu(\mathbf{k})) + \alpha_s^2(k^2)\chi_1(\nu(\mathbf{k})) \;=\; \omega$$

NLO

ν has to become a function of k because ω is a constant

GS resummation applied

evaluation in diffusion ($\nu \approx 0$) or semiclassical approximation ($\nu > 0$)

For sufficiently large k, there is no longer a real solution for ν.
The transition from real to imaginary $\nu(k)$ singles out a special value of

$k = k_{crit}$, with $\nu(k_{crit}) = 0$.

The solutions below and above this critical momentum k_{crit} have to match. This fixes the phase of ef's.

Near $k{=}k_{crit}$, the BFKL eq. becomes the Airy eq. which is solved by the Airy eigenfunctions (to a very good approximation)

$$k\, f_\omega(k) = \bar{f}_\omega(k) = \mathrm{Ai}\left(-(\frac{3}{2}\phi_\omega(k))^{\frac{2}{3}}\right)$$

with

$$\phi_\omega(k) = 2\int_k^{k_{\mathrm{crit}}} \frac{d\,k'}{k'} |\nu_\omega(k')|$$

instead of

$$f_\omega(\mathbf{k}) \;=\; \left(k^2\right)^{i\nu-1/2};$$

for $k{<<}k_{crit}$ the Airy function has the asymptotic behaviour

$$k\, f_\omega(k) \sim \sin\left(\phi_\omega(k) + \frac{\pi}{4}\right)$$

The two fixed phases at $k{=}k_{crit}$ and at $k{=}k_0$ (near Λ_{QCD}) lead to the quantization condition

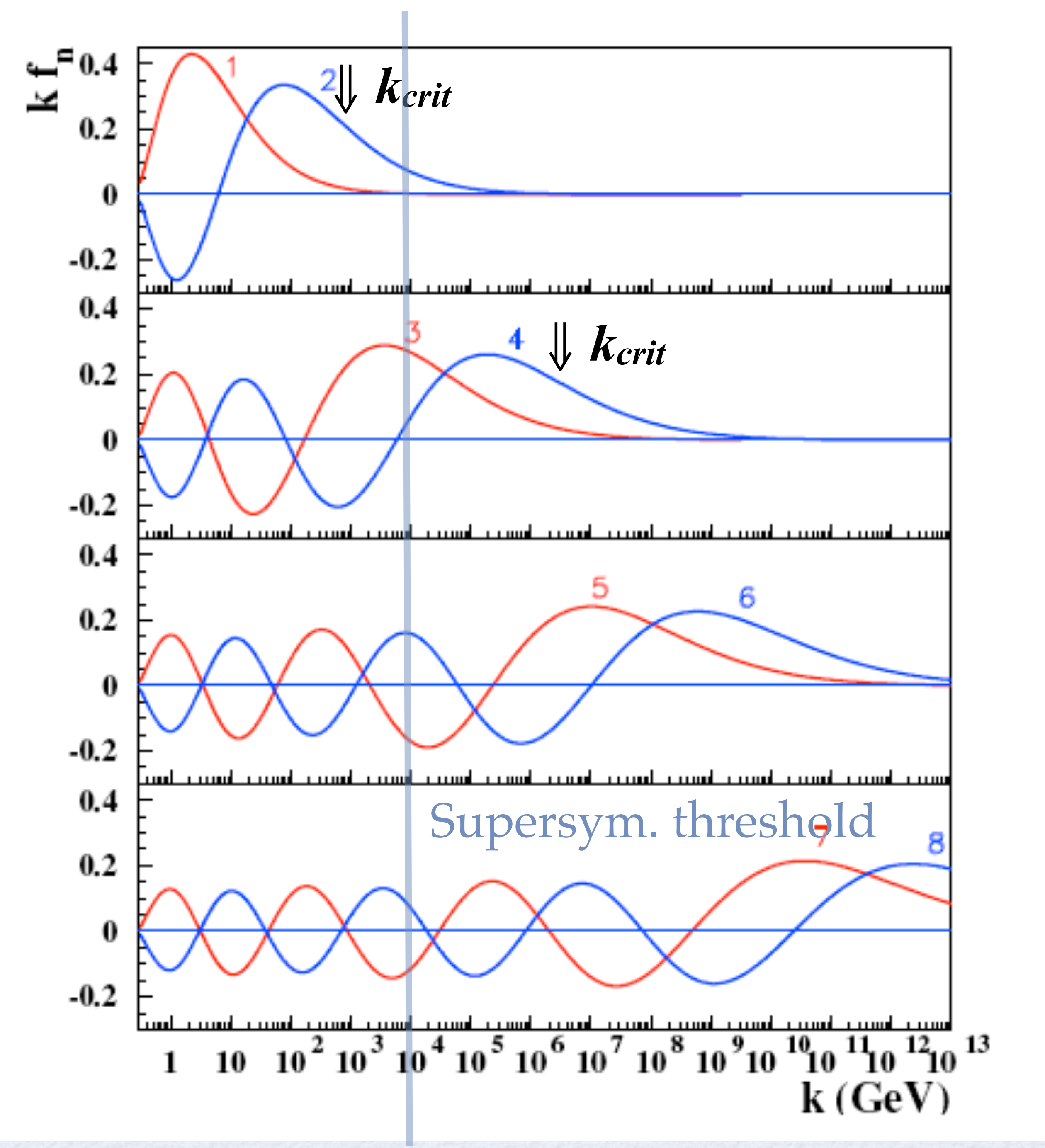
$$\phi_\omega(k_0) = \left(n - \frac{1}{4}\right)\pi + \eta\,\pi$$

Discrete Pomeron Solution of the BFKL eq

The first eight eigenfunctions determined at $\eta=0$

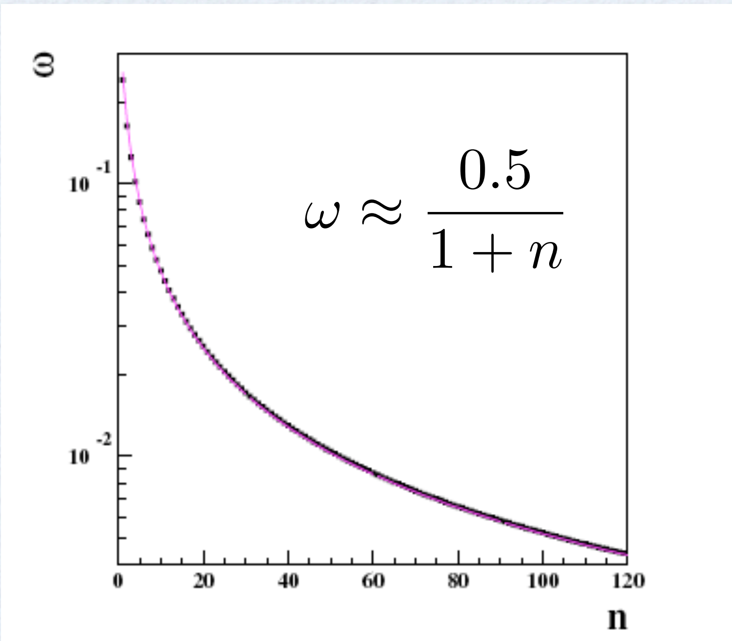
$k_{crit} \simeq c \exp(4n)$

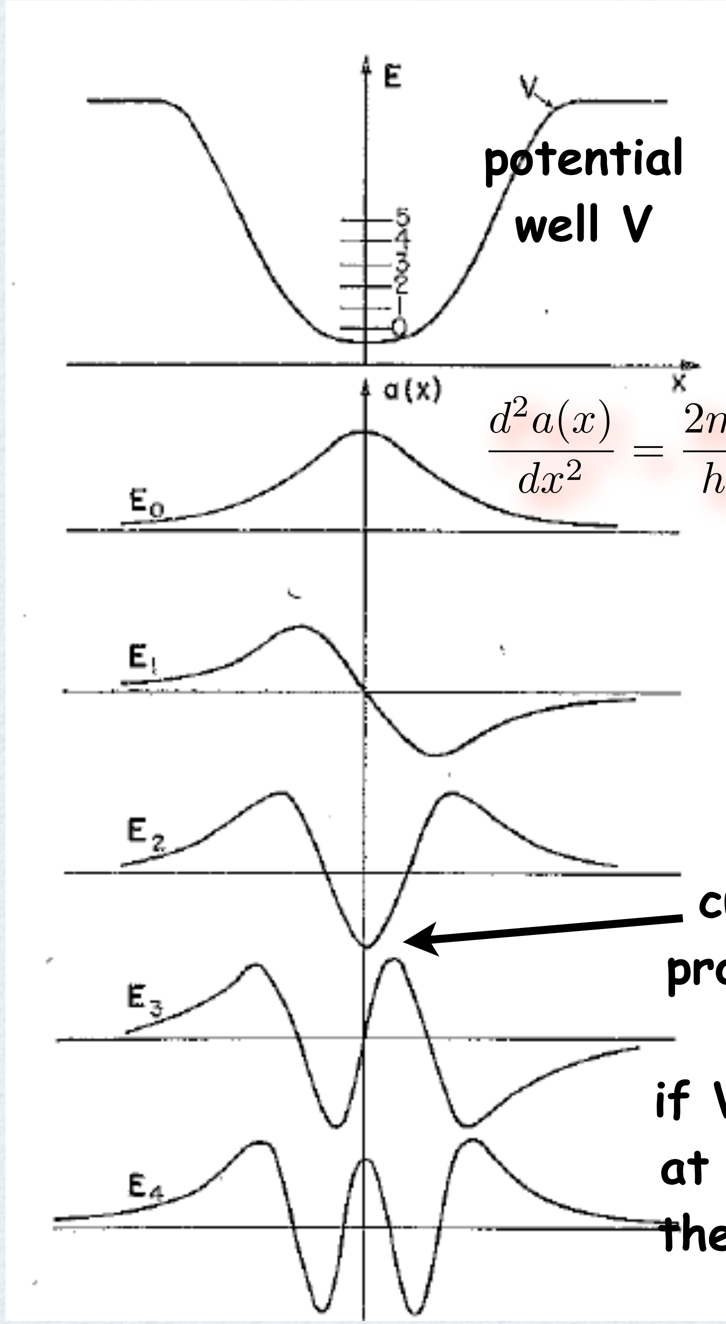
$c \simeq \Lambda_{QCD}$



Similarity with the
Schroedinder eq.
for the potential well
Feynman Lecture III

BFKL eq is similar to S. eq
for the potential well with
dynamically increasing
width

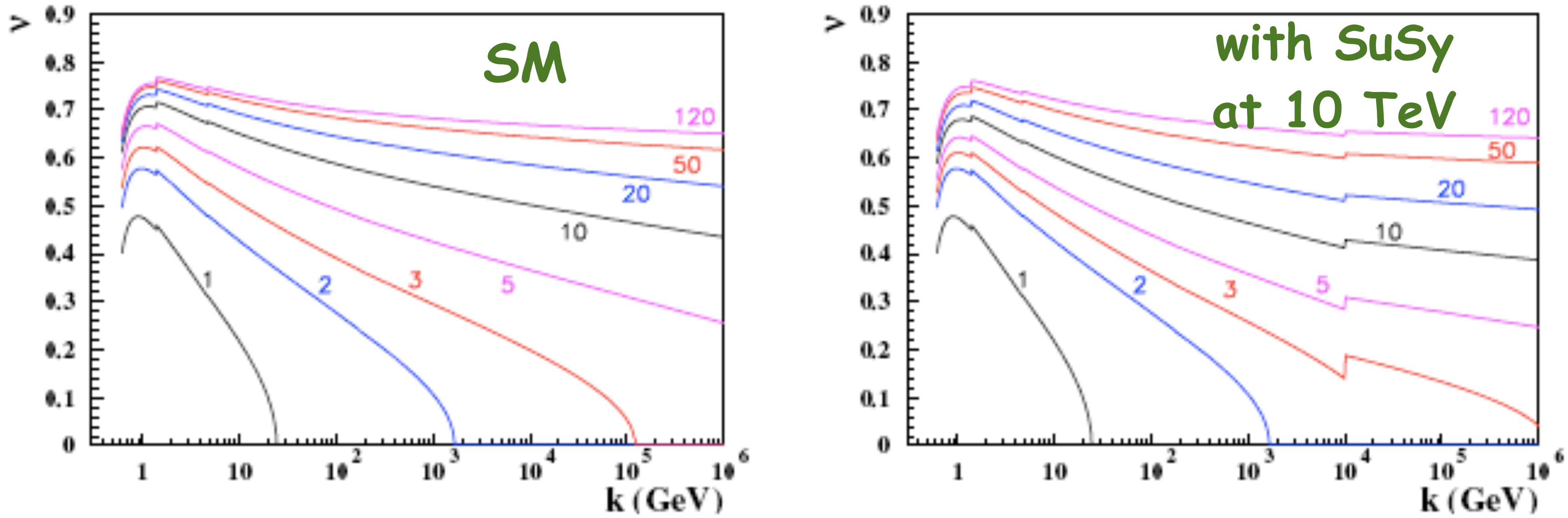




analogy
worked out
with
J.Bartels

curvature is
proportional to
(V-E)
if V is modified
at some x then
the whole wf is
changed

The frequencies *ν(k)*

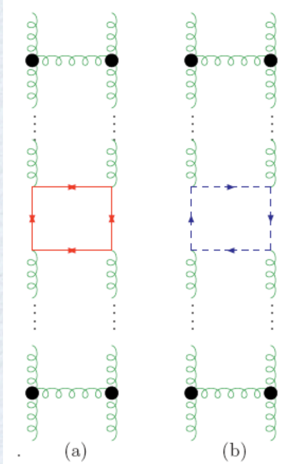


$$\delta_f \chi_1(\nu) = \frac{\pi^2}{32} \frac{\sinh(\pi\nu)}{\nu(1+\nu^2)\cosh^2(\pi\nu)} \left(\frac{11}{4} + 3\nu^2\right)$$

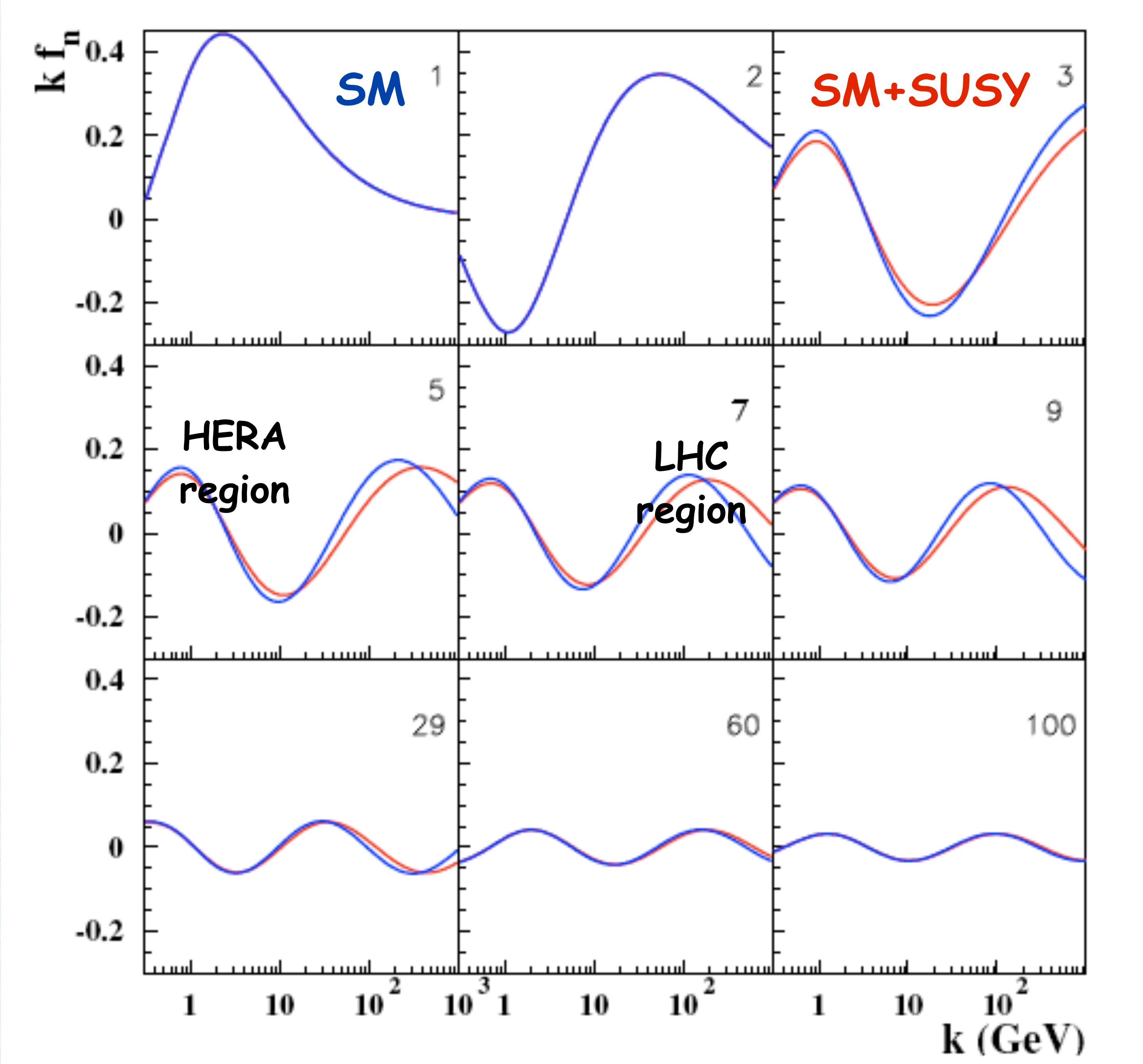
for gluinos

$$\delta_s \chi_1(\nu) = -\frac{\pi^2}{32} \frac{n_f}{C_A^3} \frac{\sinh(\pi\nu)}{\nu(1+\nu^2)\cosh^2(\pi\nu)} \left(\frac{5}{4} + \nu^2\right)$$

for squarks



Kotikov, Lipatov 2003


k f_n
SM
SM+SUSY
HERA region
LHC region
1
2
3
5
7
9
29
60
100
0.4
0.2
0
-0.2
1
10
10^2
10^3
k (GeV)

Some remarks about the decoupling theorem

The theorem states that massive particles decouple from the finite quantities calculated at energy scales below their masses.

With fixed α_s, the "potential box" would have an infinite size.

The contribution to the phase $\phi_\omega(k) = 2 \int_{M_{SUSY}}^{k} \nu(k) dk'/k'$

coming from the integration over $k > M_{SuSy}$ would not alter the phases for $k < M_{SuSy}$ and could be fully compensated, in analogy to the absorption of large logarithms arising in the renormalization of the parameters of a theory from massive particles inside loops. In such cases massive SuSy particles effectively decouple.

Some remarks about the decoupling theorem

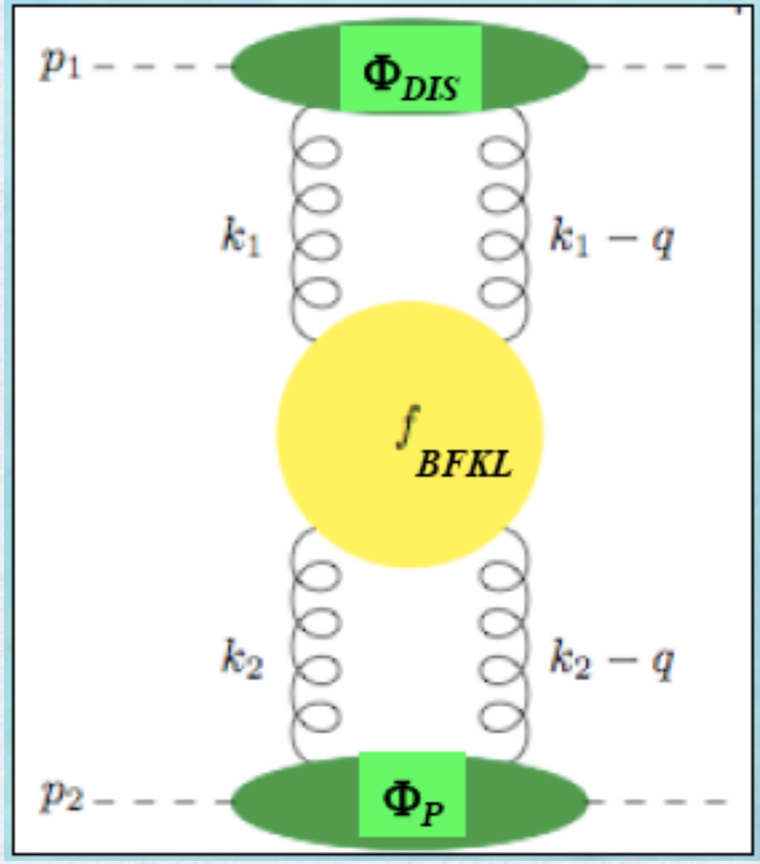
When α_s is running, the "potential box" has to have a finite size. Therefore, the contribution to the phase coming from the integration over $k > M_{SuSy}$ has to alter the phases for $k < M_{SuSy}$ because we have now two fixed points, $\eta(k_{crit})$ and $\eta(k_0)$, which are fixed by the perturbative and non-perturbative QCD respectively

A rough estimate of the phase difference due to SuSy shows that it is not a small number (in any approximation, LO, NLO...)

$$\Delta\phi_\omega \sim \omega \ln\left(\frac{M_{SUSY}}{k_{crit}}\right),$$

$\ln(kcrit) \nearrow n, \qquad \omega \searrow 1/n$

Comparison with HERA data



Discreet Pomeron Green function

$$\mathcal{A}(\mathbf{k},\mathbf{k}') \; = \; \sum_{m,n} f_m(\mathbf{k}) \mathcal{N}^{-1}_{mn} f_n(\mathbf{k}') \left(\frac{s}{kk'}\right)^{\omega_n}.$$

Integrate with the photon and proton impact factors

$$\mathcal{A}^{(U)}_n \; \equiv \; \int_x^1 \frac{d\xi}{\xi} \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(Q^2,k,\xi) \left(\frac{\xi k}{x}\right)^{\omega_n} f_n(\mathbf{k})$$

$$\mathcal{A}^{(D)}_m \; \equiv \; \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{1}{k'}\right)^{\omega_m} f_m(\mathbf{k}').$$

$$F_2(x,Q^2) \; = \; \sum_{m,n} \mathcal{A}^{(U)}_n \mathcal{N}^{-1}_{nm} \mathcal{A}^{(D)}_m$$

the infrared boundary condition

Proton impact factor

$$\Phi_p(\mathbf{k}) = A\,k^2 e^{-bk^2}$$

The fit is not sensitive to the particular form of the impact factor.

The support of the proton impact factor is much smaller than the oscillation period of f_n and because the frequencies ν have a limited range

➤ many eigenfunctions have to contribute and η has to be a function of n. Phase condition at $\tilde{k}_0$, (close to Λ_{QCD})

$$\eta = \eta_0 \left(\frac{n-1}{n_{\max}-1} \right)^{\kappa}$$

additional parameter k_0 which should be in the perturbative region but close to Λ_{QCD}

$$\phi_n(\tilde{k}_0) = \phi_n(k_0) - 2\nu_n^0 \ln\left(\frac{k_0}{\tilde{k}_0}\right),$$

Fits to F_2, $Q^2 > 8\ \mathrm{GeV}^2$, $x > 0.01$ N_{df}=103, (two loop α_s)

SUSY Scale (TeV)	χ^2	κ	$\tilde{k}_0\ (GeV)$	η_0	A	b
3	125.7	0.555	0.288	-0.87	201.2	10.6
6	114.1	0.575	0.279	-0.880	464.8	15.0
10	109.9	0.565	0.275	-0.860	720.1	17.7
15	110.1	0.555	0.279	-0.860	882.2	18.6
30	117.8	0.582	0.278	-0.870	561.6	16.2
50	114.9	0.580	0.279	-0.870	627.4	16.8
90	114.8	0.580	0.279	-0.870	700.2	17.5
∞	122.5	0.600	0.274	-0.800	813.1	17.5

Table 1: Fits for N=1 SUSY at different scales. The bottom row corresponds to the Standard Model. All fits are performed with $n_{max} = 100$.

$\chi 2/N_{df}$= 110/103=1.06

Note: we are partially absorbing the SUSY effects into the free parameters of the boundary conditions: e.g best SuSy fit with η_0, κ of SM gives $\chi 2$=572

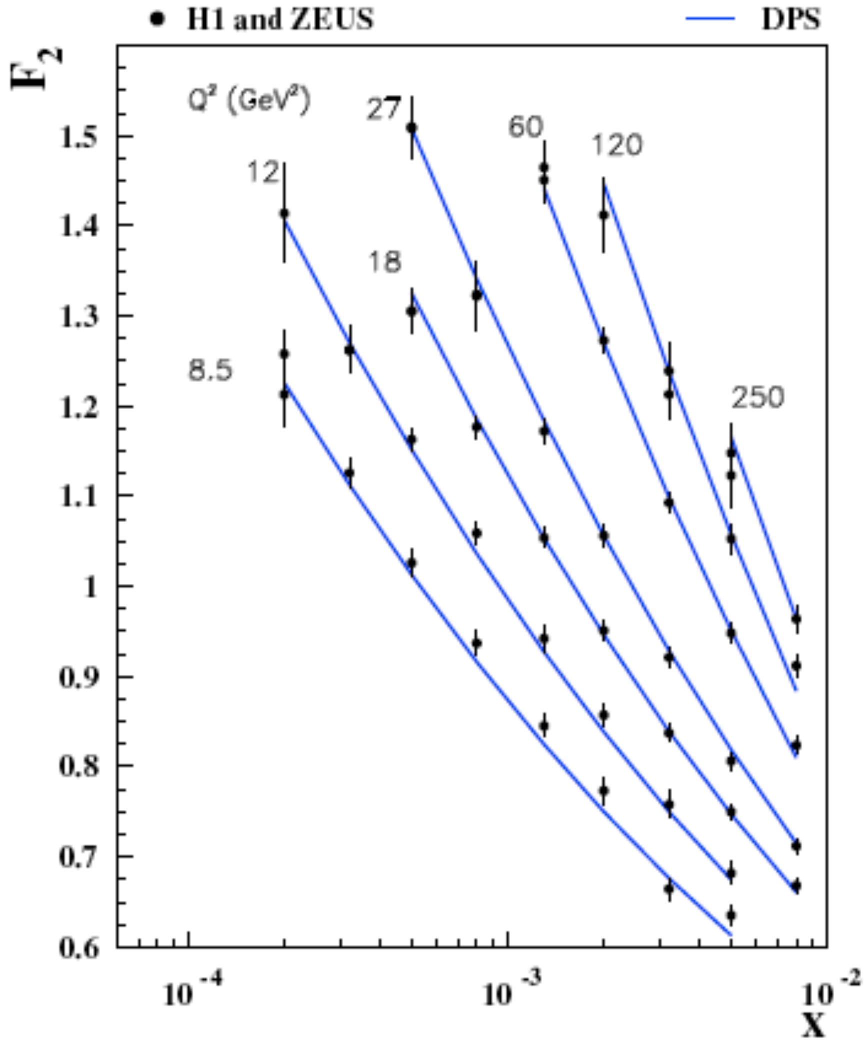
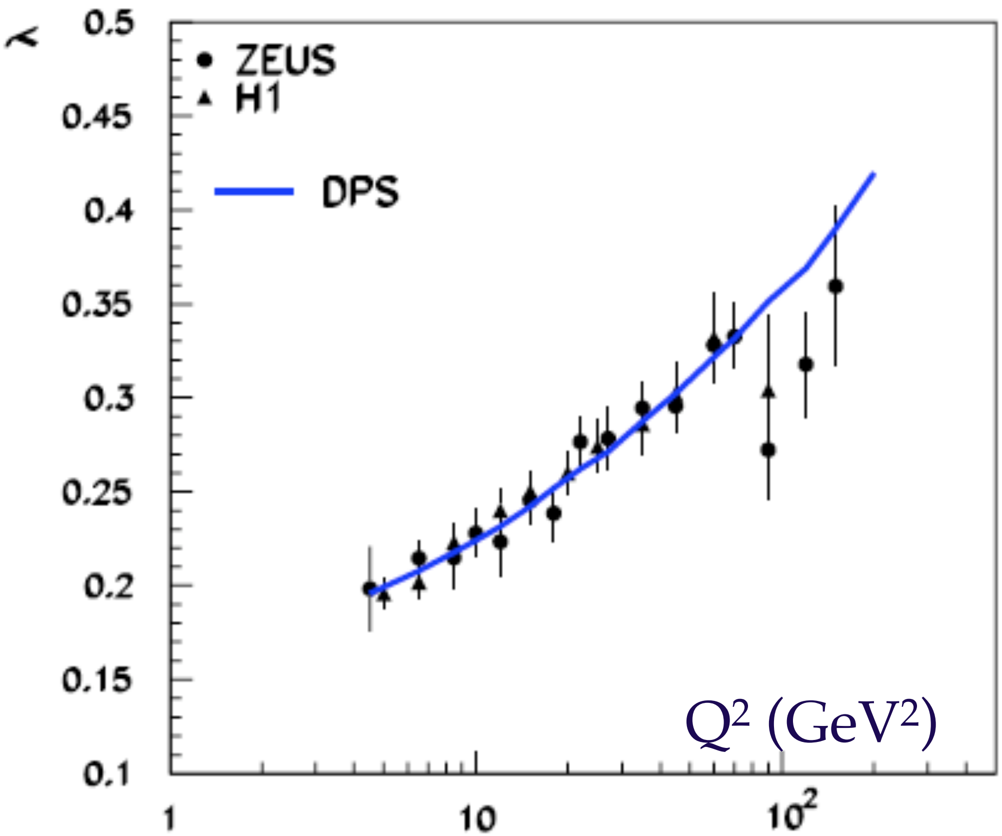


Figure 7: Comparison of the DPS fit with $M_{SUSY} = 10$ TeV with HERA data.

The rate of rise λ

$F_2 \sim (1/x)^\lambda$



The first successful pure BFKL description of the λ plot.

For many years it was claimed that BFKL analysis was not applicable to HERA data because of the observed substantial variation of λ with Q^2

The qualities of fits for various numbers of eigenfunctions, $Q^2 > 4$ GeV^2 (one loop α_s)

$n_{\max}$	χ^2/N_{df}	κ	A	b
1	10811 /125	—	146	30.0
5	350.0 /125	3.78	$3.1 \cdot 10^6$	78.0
20	286.5 /125	0.96	632	15.8
40	193.3 /125	0.84	2315	23.2
60	163.3 /125	0.78	3647	25.6
80	156.5 /125	0.73	3081	24.4
100	149.1 /125	0.69	2414	22.8
120	143.7 /125	0.66	2041	21.8

➤ new data are crucial for finding the right solution

the differences in the fit qualities would be negligible if the errors where more than 2-times larger

Discrete BFKL-Pomeron

Why so many eigenfunctions?

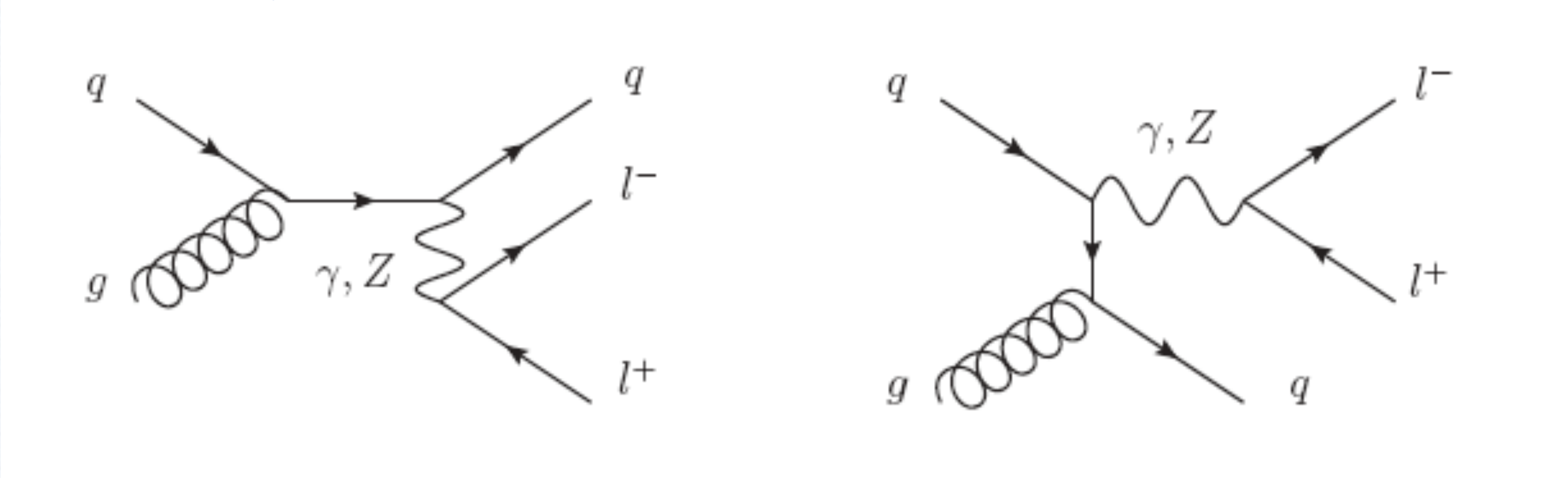
the contribution of large n ef's is only weakly suppressed, enhancement by $(1/x)^{\omega}$ is not very large because

$\omega_1 \approx 0.25, \quad \omega_5 \approx 0.1, \quad \omega_{10} \approx 0.05$

suppression of large n contribution only by the normalization condition $\sim 1/\sqrt{n}$

Drell-Yan processes at LHC

Dominant process at LHC



Additional requirement: add valence quarks contribution, i.e; gluon and sea-quark contribution like in DPS and valence quarks like in DGLAP

investigation is in progress:

Conclusions

The shape of the discrete BFKL states of the gluon density is very sensitive to BSM effects, The analysis of HERA data provides evidence for a SuSy scale at 10 TeV.

LHC Drell-Yan data should substantially improve the sensitivity to BSM effects

the evaluation depends on the infrared boundary condition (ibc), the understanding of ibc can be improved:

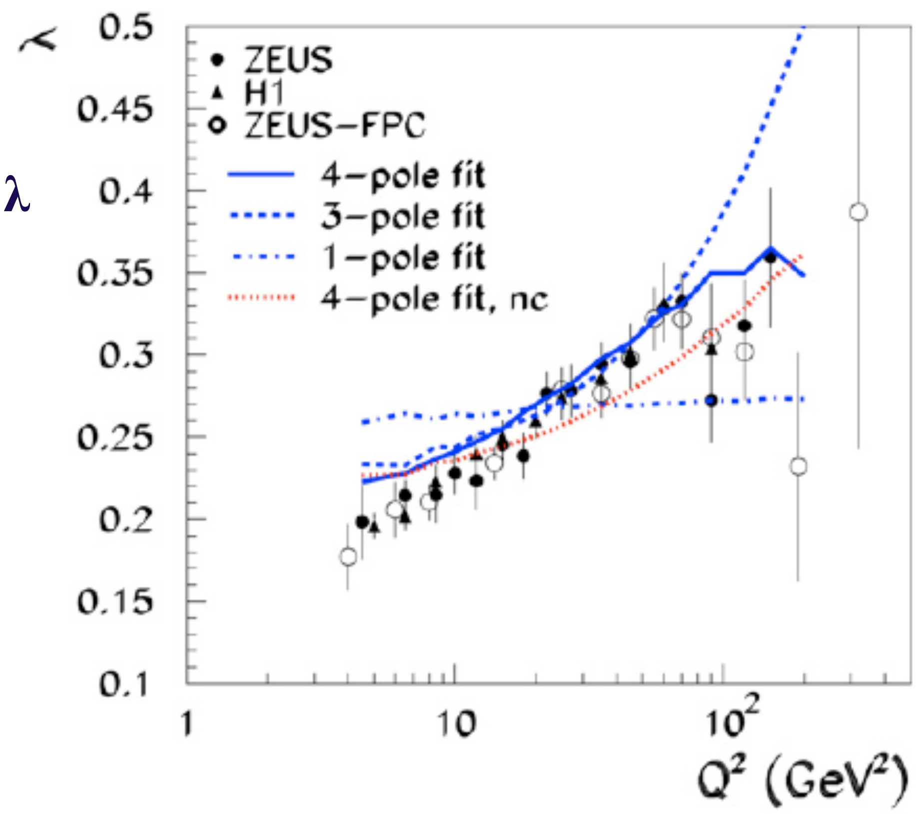
- by analyzing different physics reactions,
  - e.g.: F_2 together with LHC Drell-Yan reactions
- by involving more sophisticated theoretical methods

ultimate goal: understanding of higher symmetry effects up to Planck scale

Back up slides

BFKL eq., with fixed α_s, predicts $F_2 \sim (1/x)^{\omega}$
with $\omega \sim$ constant with Q^2, $\omega \sim 0.5$ in LO and $\omega \sim 0.3$ in NLO
Therefore, the prevailing opinion was that the BFKL analysis is not applicable to HERA data.

The rate of rise λ
$F_2 \sim (1/x)^{\lambda}$



First hints that in BFKL λ can be substantially varying with Q^2 was given in PL 668 (2008) 51 by EKR

Lipatov 86 & EKR 2008: BFKL solutions with the running α_s are substantially different from solutions with the fixed α_s.

$$k\int dk'^{\,2}\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \chi\left(-i\frac{d}{d\ln k^2},\alpha_s(k^2)\right)\bar{f}_\omega(k) \;=\; \omega\bar{f}_\omega(k)$$

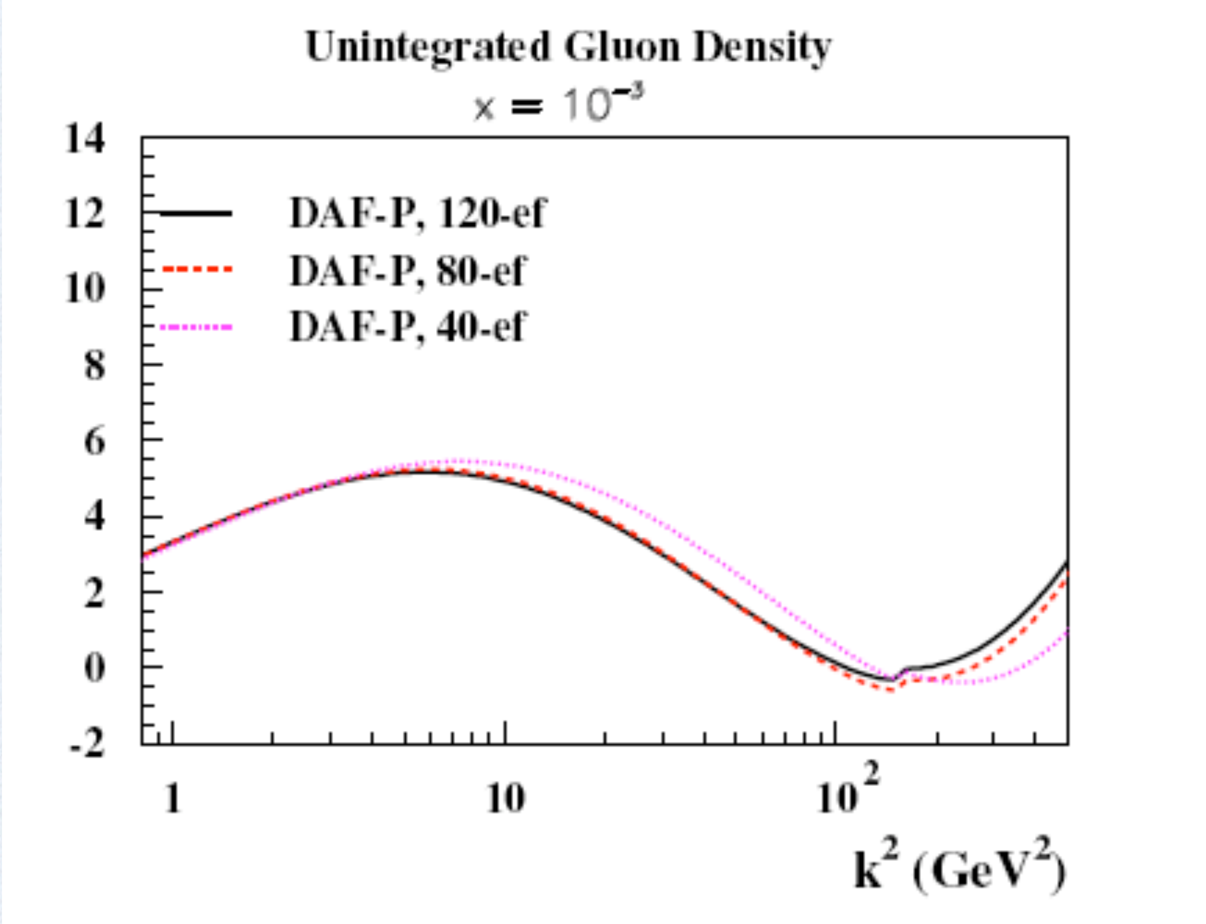
semiclassical approximation

$$\left(\frac{d}{d\ln(k)}\right)^r \bar{f}_\omega(k) \;\approx\; \bar{f}_\omega(k)\left(\frac{d\ln\bar{f}_\omega(k)}{d\ln k}\right)^r$$

$$\chi\left(-i\frac{d\ln\bar{f}_\omega(k)}{d\ln k^2},\alpha_s(k^2)\right) \;=\; \omega$$

$$\frac{d\bar{f}_\omega(k)}{d\ln(k^2)} \;=\; i\nu_\omega(\alpha_s(k^2))\bar{f}_\omega(k)$$

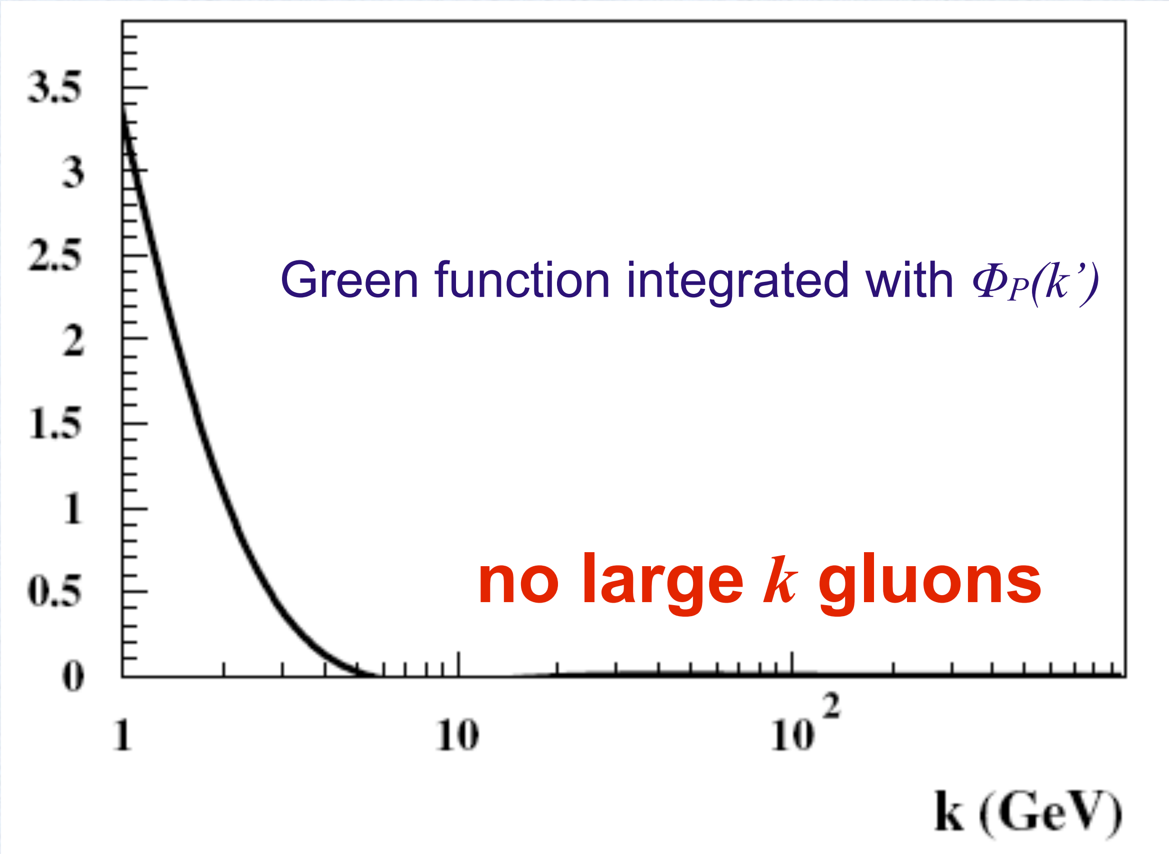
DGLAP


Unintegrated Gluon Density
x = 10^-3
DAF-P, 120-ef
DAF-P, 80-ef
DAF-P, 40-ef
14
12
10
8
6
4
2
0
-2
1
10
10^2
k^2 (GeV^2)

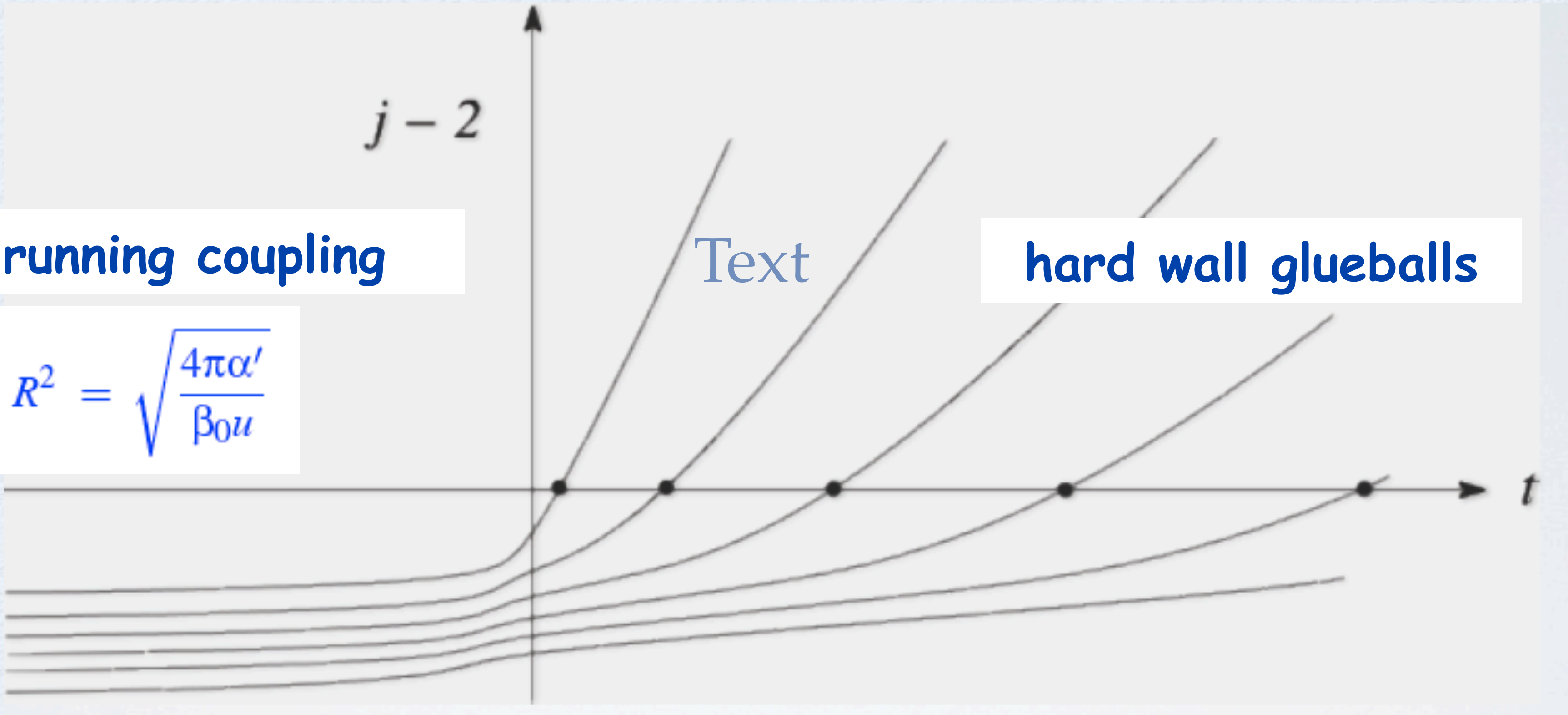
Quasi-locality of the kernel

$$\mathcal{K}(\mathbf{k},\mathbf{k}') = \frac{1}{kk'}\sum_{n=0}^{\infty} c_n \delta^{(n)}\left(\ln(\mathbf{k}^2/\mathbf{k}'^2)\right),$$

and of the Green function



Pomeron Regge trajectories in ADS



running coupling

$$R^2 = \sqrt{\frac{4\pi\alpha'}{\beta_0 u}}$$

hard wall glueballs