Top quark theory for the LHC

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Plan

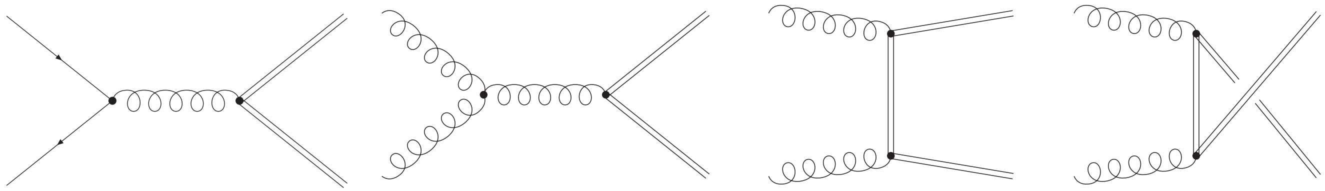
Review of recent theory activities

- This talked is based on work with:
  - M. Aliev , H. Lacker , U. Langenfeld , P. Uwer and M. Wiedermann on arXiv:1007.1327 (top-quark cross section calculator Hathor)
  - S. Alioli and P. Uwer on arXiv:1110.5251 (top-quark pair-production with one jet and parton showering)
  - U. Langenfeld and P. Uwer on arXiv:0907.5273 (top-quark running mass)
  - S. Alekhin and J. Blümlein on arXiv:1202.2281 (benchmark cross sections and parton distributions)
- Goal (for the next 20 minutes)
  - transfer knowledge, initiate discussions, identify open issues, . . .

Top quark production

- Leading order Feynman diagrams

$$\begin{aligned} \mathrm{q} + \bar{\mathrm{q}} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \\ \mathrm{g} + \mathrm{g} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \end{aligned}$$



- QCD factorization $$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$
  - NLO in QCD for $\hat{\sigma}_{ij\to t\bar{t}}$ accurate to $\mathcal{O}(15\%)$ at LHC
    Nason, Dawson, Ellis ‘88; Beenakker, Smith, van Neerven ‘89; Mangano, Nason, Ridolfi ‘92; Bernreuther, Brandenburg, Si, Uwer ‘04; Mitov, Czakon ‘08; . . .

Challenge

- Improve theory predictions and reduce theoretical uncertainty
  - hard scattering cross section $\hat{\sigma}_{ij\to t\bar{t}}$
  - nonperturbative parameters:
    top-quark mass m_t, strong coupling α_s, parton luminosity $f_i \otimes f_j$

Total cross section

- Standard currency for comparisons: $\sigma_{pp \to t\bar{t}}(S, m_t)$
- Simplest observable for theorists
- Ideal playground for theory improvements:
  - higher orders
  - resummation
- Benchmark for dependence on nonperturbative parameters
  - top-quark mass m_t
  - strong coupling α_s
  - parton luminosity $f_i \otimes f_j$

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  - top-quark mass m_t
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  - parton luminosity $f_i \otimes f_j$
- ... and of course, we all know that acceptance cuts are important (from an e-mail)
  `Dear Sven, [...] And a final remark that my colleagues know very well: unfortunately for us, the total cross section [...] is totally useless!! [...] cheers [name undisclosed]`

NLO

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

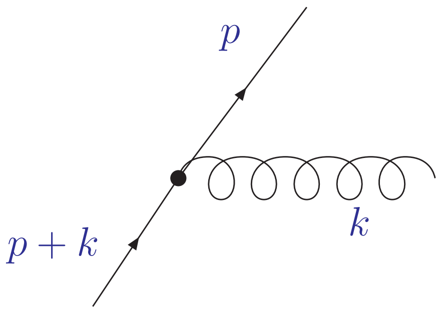
$$\hat{\sigma}_{ij \to t\bar{t}} \quad = \quad \frac{{\alpha_s}^2}{m_t^2} \left\{ f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) \right\}$$

- Perturbative expansion at NLO

Strategy beyond NLO

- Use universal features of soft/collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft θ_{qg} = 0 collinear

$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p \cdot k} \quad = \quad \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$



$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \quad \longrightarrow \quad \alpha_s \int dE_g \, d\sin\theta_{qg} \, \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\longrightarrow \quad \alpha_s \, \ln^2(\dots)$$

Beyond NLO: all-order resummation

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

$$\hat{\sigma}_{ij\to t\bar{t}} = \frac{{\alpha_s}^2}{m_t^2}\Big\{f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f)$$
$$+ f_{ij}^{\text{resummed}}(\alpha_s, N, \mu_f/m_t, \mu_r/\mu_f) + \mathcal{O}(N^{-1}\ln^n N)\Big\}$$

- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
  - Sudakov logarithms in heavy-quark velocity $\beta = \sqrt{1 - 4m^2/s}$
  - resummation in Mellin space (renormalization group equation)
  - long history Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...
- Upshot:
  - $f_{ij}^{\text{resummed}} \simeq \exp\big(\alpha_s \ln^2 N\big) + \mathcal{O}(N^{-1}\ln^n N)$

Beyond NLO: NNLO$_{\rm approx}$

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

$$\hat{\sigma}_{ij\to t\bar{t}} = \frac{{\alpha_s}^2}{m_t^2}\Big\{f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho,\mu_f/m_t,\mu_r/\mu_f) + (4\pi\alpha_s)^2 f_{ij}^{(2)}(\rho,\mu_f/m_t,\mu_r/\mu_f) + \mathcal{O}({\alpha_s}^3)\Big\}$$

- General structure at NNLO
  - dependence on factorization and renormalization scale $L_M = \ln(\mu_f^2/m_t^2)$ and $L_R = \ln(\mu_r^2/\mu_f^2)$

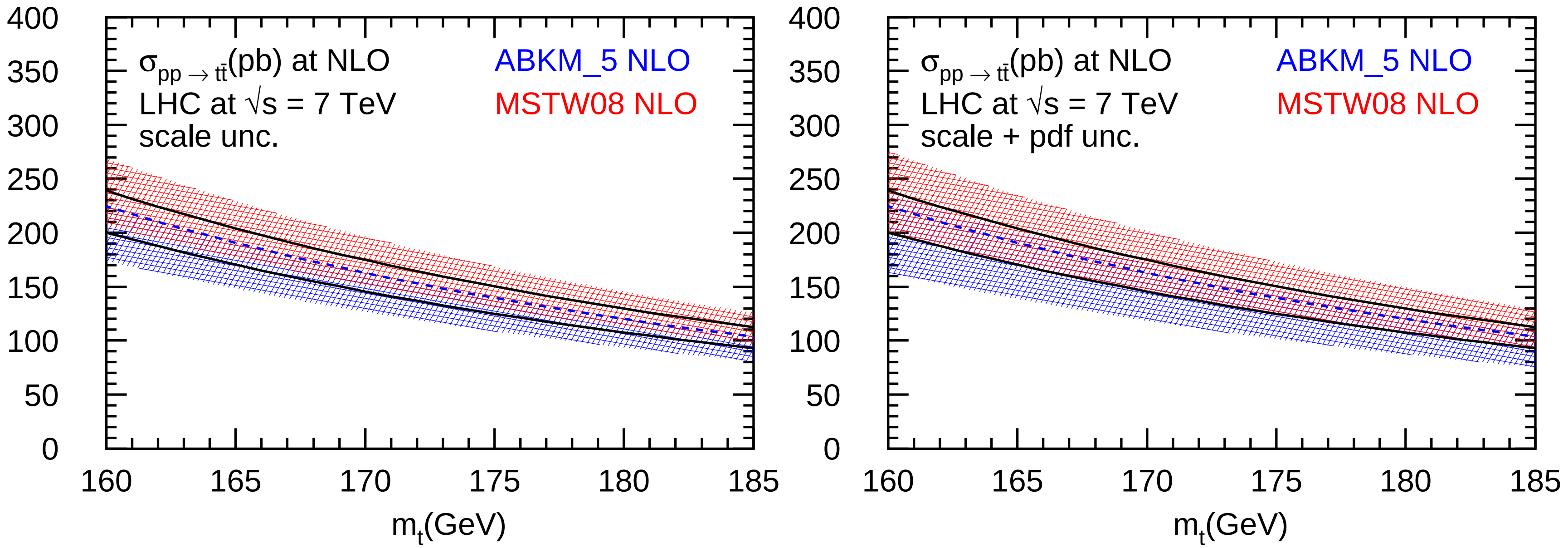
$$f_{ij}^{(1)}(\rho,\mu_f/m_t,\mu_r/m_t) = f_{ij}^{(10)} + L_M f_{ij}^{(11)} + 2\beta_0 L_R f_{ij}^{(0)}\,,$$

$$f_{ij}^{(2)}(\rho,\mu_f/m_t,\mu_r/m_t) = f_{ij}^{(20)} + L_M f_{ij}^{(21)} + L_M^2 f_{ij}^{(22)} + 3\beta_0 L_R f_{ij}^{(10)} + 3\beta_0 L_R L_M f_{ij}^{(11)} + 2\beta_1 L_R f_{ij}^{(0)} + 3\beta_0^2 L_R^2 f_{ij}^{(0)}$$

  - only unknown: $f_{ij}^{(20)}$ (but knowledge of threshold logarithms)
    S.M, Uwer ‘08; Beneke, Czakon, Falgari, Mitov, Schwinn ‘09
  - all other function known through renormalization group equations

Results

- Comparision of NLO and NNLO$_{\mathrm{approx}}$
- Theoretical uncertainty from scale variation: $\mu_r, \mu_f \in [m_t/2, 2m_t]$
  - scale dependence reduces from NLO to NNLO$_{\mathrm{approx}}$
- Sizeable difference between the ABKM09/MSTW08 sets at NNLO well outside the $\pm 1\sigma$ PDF uncertainty
  - due to value of α_s and shape of gluon PDFs at $\langle x \rangle = 2m_t/\sqrt{s}$
  - potential to discriminate with LHC data



Results

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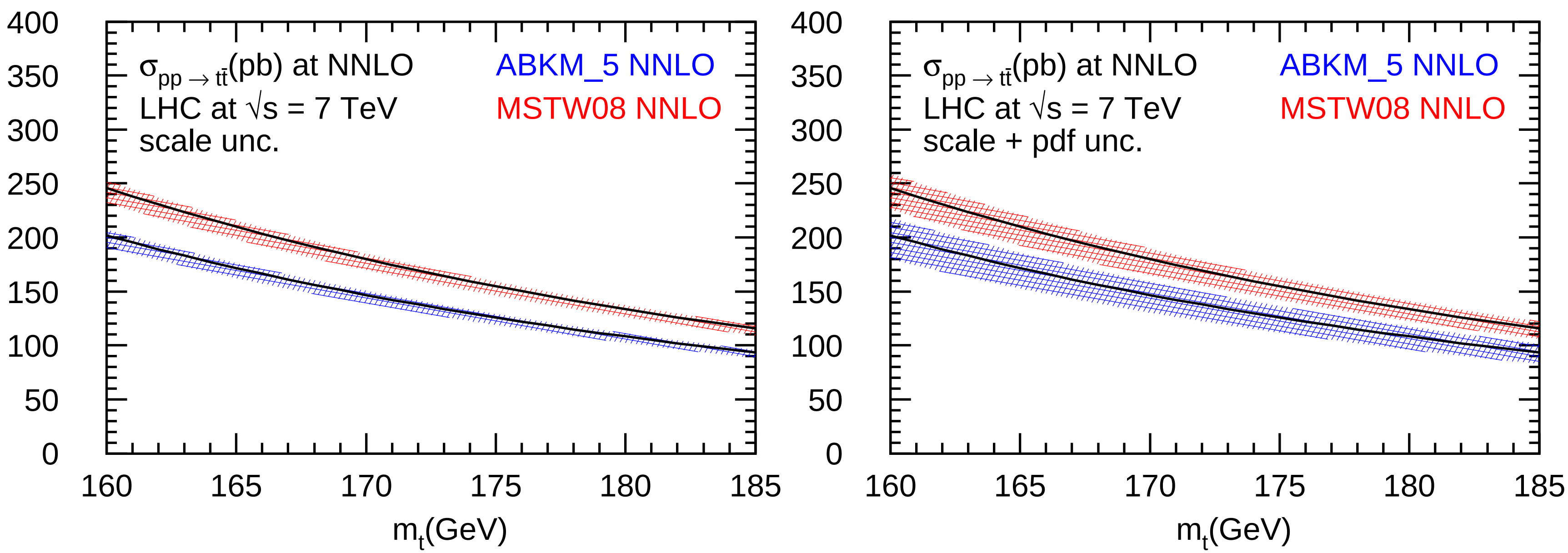
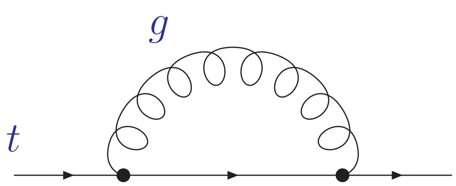


Mass dependence of cross section

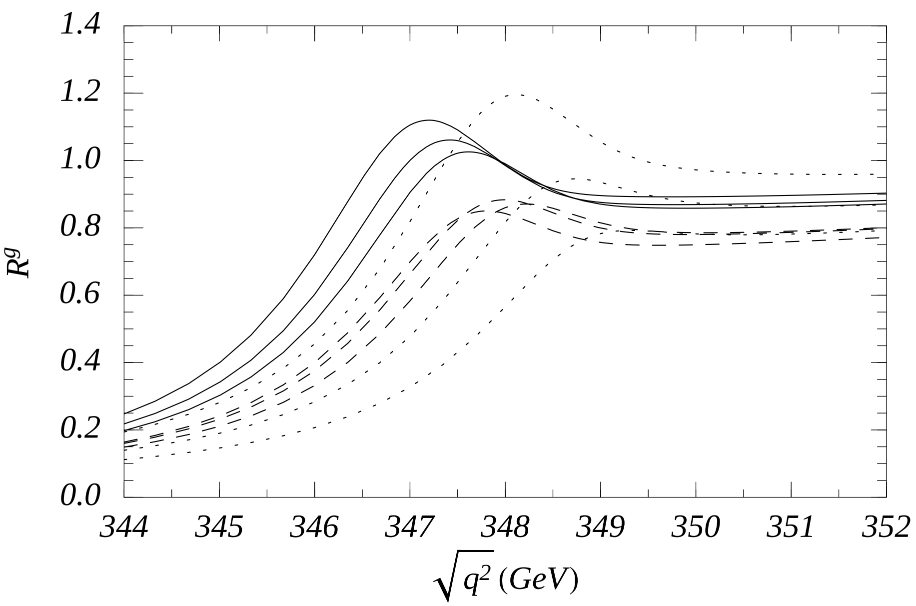
- Pole mass based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_t - \Sigma(p, m_t)\Big|_{p^2=m_t^2}$$



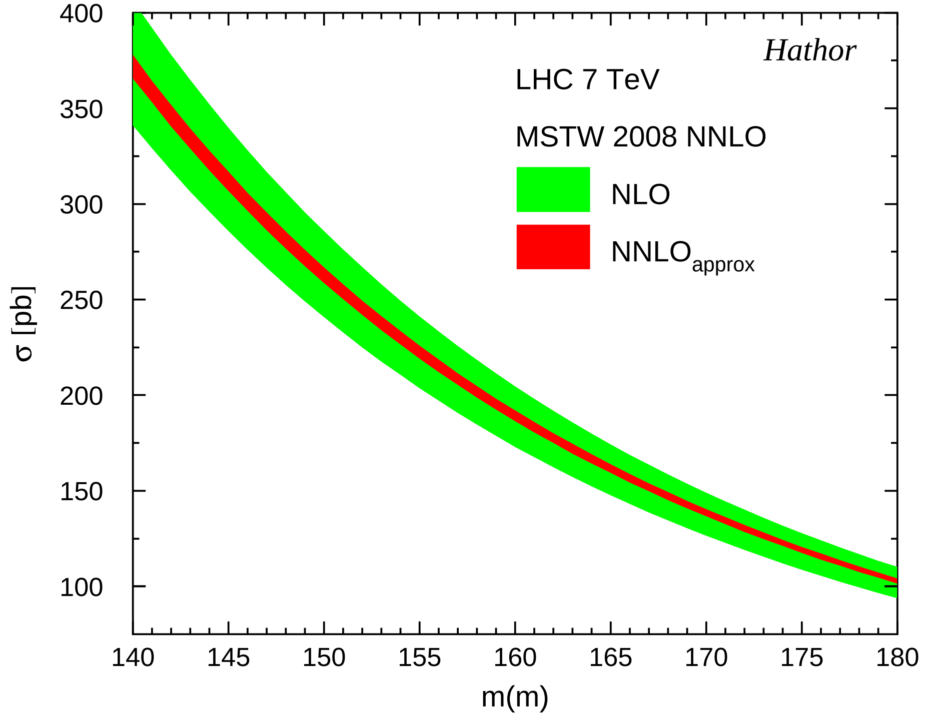
  - heavy-quark self-energy $\Sigma(p, m_t)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{\rm QCD})$

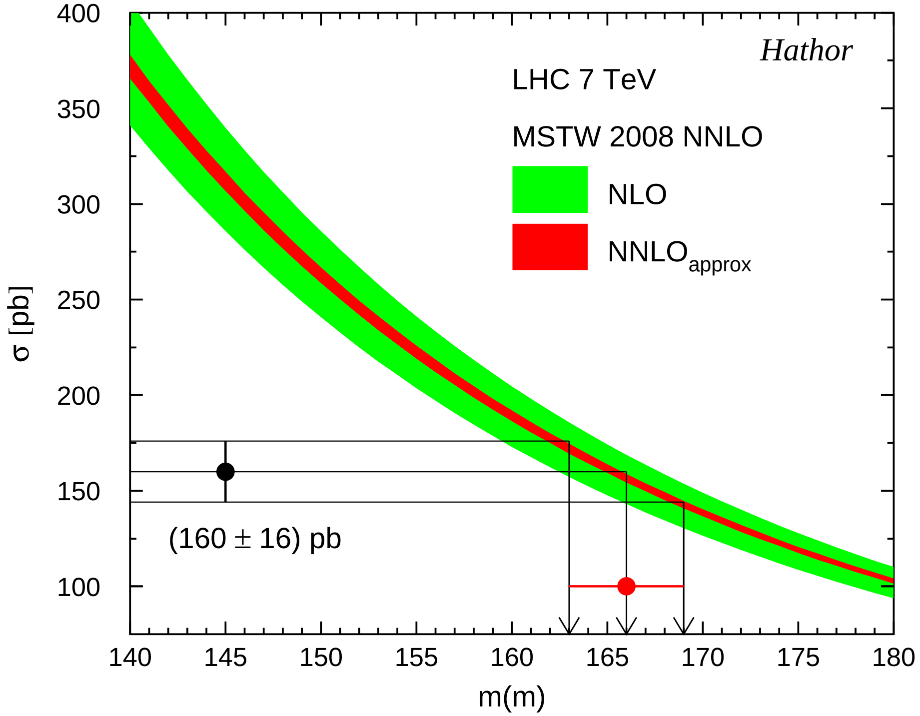
- Pole mass measurements are strongly order-dependent
  - e.g. threshold scan of cross section in e^+e^- collision
    Beneke, Signer, Smirnov '99;
    Hoang, Teubner '99;
    Melnikov, Yelkhovsky '98;
    Penin, Pivovarov '99;
    Yakovlev '99
  - LO (dotted), NLO (dashed), NNLO (solid)



Running top-quark mass

- $\overline{MS}$ mass definition $m(\mu_r)$ realizes running mass (scale dependence)
  - conversion between pole and $\overline{MS}$ mass in perturbation theory
  - scale dependence greatly reduced and convergence of perturbative expansion improved
- Total top-quark cross section as function of $\overline{m} = m(m)$
  - theoretical uncertainity (band) due to variation of $\mu_r \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_f \in \overline{m}/2, \overline{m}, 2\overline{m}$





Cross sections for LHC8 (NNLO$_{\rm approx}$ with pole mass m_t)

m_t	ABM11	ABKM09	JR09	MSTW	NN21
165	$243.8^{+4.6}_{-13.0}\,^{+10.0}_{-10.0}$	$247.5^{+4.4}_{-12.9}\,^{+12.6}_{-12.6}$	$289.3^{+3.9}_{-12.2}\,^{+18.3}_{-18.3}$	$298.1^{+5.0}_{-16.0}\,^{+7.1}_{-7.4}$	$307.5^{+5.8}_{-14.7}\,^{+7.6}_{-7.6}$
166	$236.8^{+4.0}_{-13.1}\,^{+9.7}_{-9.7}$	$239.2^{+5.0}_{-11.8}\,^{+12.3}_{-12.3}$	$279.5^{+5.0}_{-10.6}\,^{+17.9}_{-17.9}$	$289.2^{+4.9}_{-15.5}\,^{+7.0}_{-7.2}$	$298.3^{+5.6}_{-14.2}\,^{+7.4}_{-7.4}$
167	$229.6^{+3.8}_{-12.7}\,^{+9.4}_{-9.4}$	$231.8^{+4.9}_{-11.4}\,^{+12.0}_{-12.0}$	$271.2^{+4.9}_{-10.2}\,^{+17.4}_{-17.4}$	$280.5^{+4.7}_{-15.1}\,^{+6.8}_{-7.0}$	$289.2^{+5.5}_{-13.7}\,^{+7.2}_{-7.2}$
168	$222.5^{+3.7}_{-12.3}\,^{+9.2}_{-9.2}$	$224.9^{+4.8}_{-11.1}\,^{+11.7}_{-11.7}$	$263.2^{+4.8}_{-10.0}\,^{+17.1}_{-17.1}$	$272.2^{+4.6}_{-14.7}\,^{+6.6}_{-6.8}$	$280.6^{+5.4}_{-13.3}\,^{+7.0}_{-7.0}$
169	$215.7^{+3.6}_{-11.9}\,^{+8.9}_{-8.9}$	$218.0^{+4.7}_{-10.7}\,^{+11.3}_{-11.3}$	$255.4^{+4.8}_{-9.6}\,^{+16.6}_{-16.6}$	$264.1^{+4.6}_{-14.2}\,^{+6.4}_{-6.6}$	$272.2^{+5.4}_{-12.8}\,^{+6.8}_{-6.8}$
170	$209.2^{+3.5}_{-11.7}\,^{+8.7}_{-8.7}$	$211.5^{+4.6}_{-10.4}\,^{+11.0}_{-11.0}$	$248.0^{+4.7}_{-9.3}\,^{+16.2}_{-16.2}$	$256.4^{+4.5}_{-13.8}\,^{+6.2}_{-6.4}$	$264.1^{+5.2}_{-12.4}\,^{+6.6}_{-6.6}$
171	$202.9^{+3.4}_{-11.3}\,^{+8.4}_{-8.4}$	$205.1^{+4.5}_{-10.0}\,^{+10.7}_{-10.7}$	$240.7^{+4.6}_{-8.9}\,^{+15.8}_{-15.8}$	$248.8^{+4.4}_{-13.3}\,^{+6.1}_{-6.2}$	$256.3^{+5.2}_{-11.9}\,^{+6.5}_{-6.5}$
172	$196.8^{+3.3}_{-10.9}\,^{+8.2}_{-8.2}$	$199.0^{+4.3}_{-9.8}\,^{+10.4}_{-10.4}$	$233.7^{+4.5}_{-8.7}\,^{+15.5}_{-15.5}$	$241.6^{+4.2}_{-13.0}\,^{+5.9}_{-6.1}$	$248.8^{+5.0}_{-11.6}\,^{+6.3}_{-6.3}$
173	$190.9^{+3.3}_{-10.6}\,^{+8.0}_{-8.0}$	$193.1^{+4.2}_{-9.5}\,^{+10.1}_{-10.1}$	$227.0^{+4.4}_{-8.4}\,^{+15.1}_{-15.1}$	$234.6^{+4.1}_{-12.6}\,^{+5.8}_{-5.9}$	$241.5^{+4.9}_{-11.2}\,^{+6.1}_{-6.1}$
174	$185.3^{+3.1}_{-10.3}\,^{+7.8}_{-7.8}$	$187.4^{+4.1}_{-9.2}\,^{+9.9}_{-9.9}$	$220.4^{+4.3}_{-8.1}\,^{+14.7}_{-14.7}$	$227.8^{+4.0}_{-12.2}\,^{+5.6}_{-5.8}$	$234.5^{+4.8}_{-10.8}\,^{+6.0}_{-6.0}$
175	$179.7^{+3.2}_{-9.9}\,^{+7.6}_{-7.6}$	$181.9^{+4.0}_{-8.9}\,^{+9.6}_{-9.6}$	$214.0^{+4.3}_{-7.9}\,^{+14.4}_{-14.4}$	$221.3^{+4.0}_{-11.8}\,^{+5.5}_{-5.6}$	$227.9^{+4.6}_{-10.6}\,^{+5.8}_{-5.8}$
176	$174.5^{+3.0}_{-9.7}\,^{+7.4}_{-7.4}$	$176.6^{+4.0}_{-8.6}\,^{+9.4}_{-9.4}$	$207.9^{+4.2}_{-7.6}\,^{+14.2}_{-14.2}$	$214.9^{+3.9}_{-11.5}\,^{+5.4}_{-5.5}$	$221.3^{+4.5}_{-10.3}\,^{+5.7}_{-5.7}$
177	$169.4^{+3.0}_{-9.4}\,^{+7.2}_{-7.2}$	$171.5^{+3.8}_{-8.5}\,^{+9.2}_{-9.2}$	$202.0^{+4.0}_{-7.4}\,^{+13.8}_{-13.8}$	$208.8^{+3.7}_{-11.2}\,^{+5.2}_{-5.4}$	$215.0^{+4.2}_{-10.1}\,^{+5.6}_{-5.6}$
178	$164.4^{+2.9}_{-9.1}\,^{+7.0}_{-7.0}$	$166.5^{+3.7}_{-8.2}\,^{+8.9}_{-8.9}$	$196.2^{+4.0}_{-7.1}\,^{+13.5}_{-13.5}$	$202.9^{+3.7}_{-10.9}\,^{+5.1}_{-5.2}$	$208.9^{+4.2}_{-9.8}\,^{+5.4}_{-5.4}$
179	$159.6^{+2.8}_{-8.9}\,^{+6.8}_{-6.8}$	$161.8^{+3.5}_{-8.1}\,^{+8.7}_{-8.7}$	$190.9^{+3.6}_{-7.0}\,^{+13.1}_{-13.1}$	$197.2^{+3.5}_{-10.7}\,^{+5.0}_{-5.1}$	$203.1^{+3.9}_{-9.7}\,^{+5.3}_{-5.3}$
180	$155.0^{+2.7}_{-8.7}\,^{+6.6}_{-6.6}$	$157.1^{+3.5}_{-7.8}\,^{+8.5}_{-8.5}$	$185.6^{+3.6}_{-6.8}\,^{+12.8}_{-12.8}$	$191.7^{+3.5}_{-10.4}\,^{+4.8}_{-4.9}$	$197.3^{+3.9}_{-9.3}\,^{+5.1}_{-5.1}$

HATHOR

C++ package

Aliev, Lacker, Langenfeld, S.M., Uwer, Wiedermann ‘10

- Cross section evaluation done in Hathor class (now version 1.2)

```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;
double mt = 171., muf=171., mur=171.;
double val,err,chi2a,up,down;

Lhapdf pdf("abm11_5n_nnlo");
Hathor XS(pdf)

XS.setSqrtShad(8000); XS.setPrecision(Hathor::MEDIUM);
XS.getXsection(mt,mur,muf);
XS.getResult(0,val,err,chi2a);

XS.setScheme(scheme | Hathor::PDF_SCAN);
XS.setSqrtShad(8000); XS.setPrecision(Hathor::LOW);
XS.getXsection(mt,mur,muf);
XS.getPdfErr(up,down);
```

`Lhapdf pdf("abm11_5n_nnlo");` $\longleftarrow$ **PDF choice**

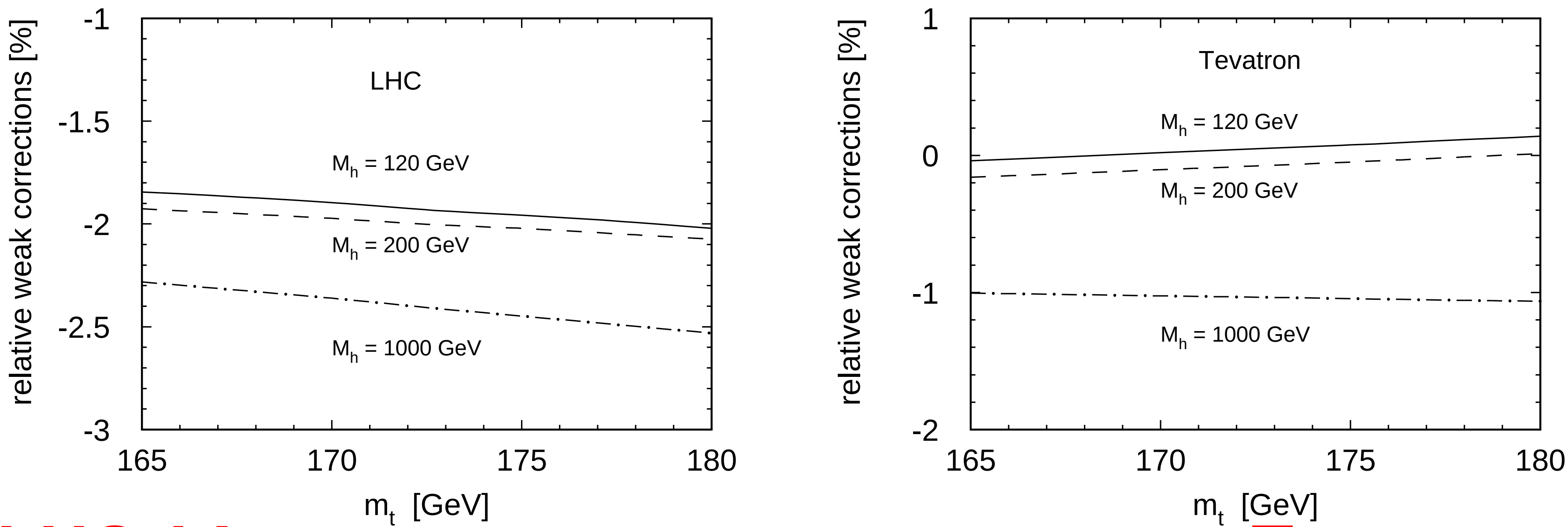
`XS.getXsection(mt,mur,muf);` $\longrightarrow \sigma = 202.9^{+3.5}_{-11.3}\,\mathrm{pb}$ **(scale unc.)**

`XS.setScheme(scheme | Hathor::PDF_SCAN);` $\longleftarrow$ **with PDF error scan**

`XS.getXsection(mt,mur,muf); XS.getPdfErr(up,down);` $\Big\} \longrightarrow \sigma = 202.9^{+3.5}_{-11.3}\,\mathrm{pb}$ **(sc.)** $^{+8.4}_{-8.4}\,\mathrm{pb}$ **(PDF unc.)**

Electroweak corrections

- Electroweak corrections (ratio of $\sigma_{\mathrm{EW}}/\sigma_{\mathrm{LO}}$) Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth '94; Bernreuther, Fücker '05; Kühn, Uwer, Scharf '06
- Effect depends on Higgs mass
  (choices $m_H = 120$GeV, $m_H = 200$GeV, $m_H = 1000$GeV)



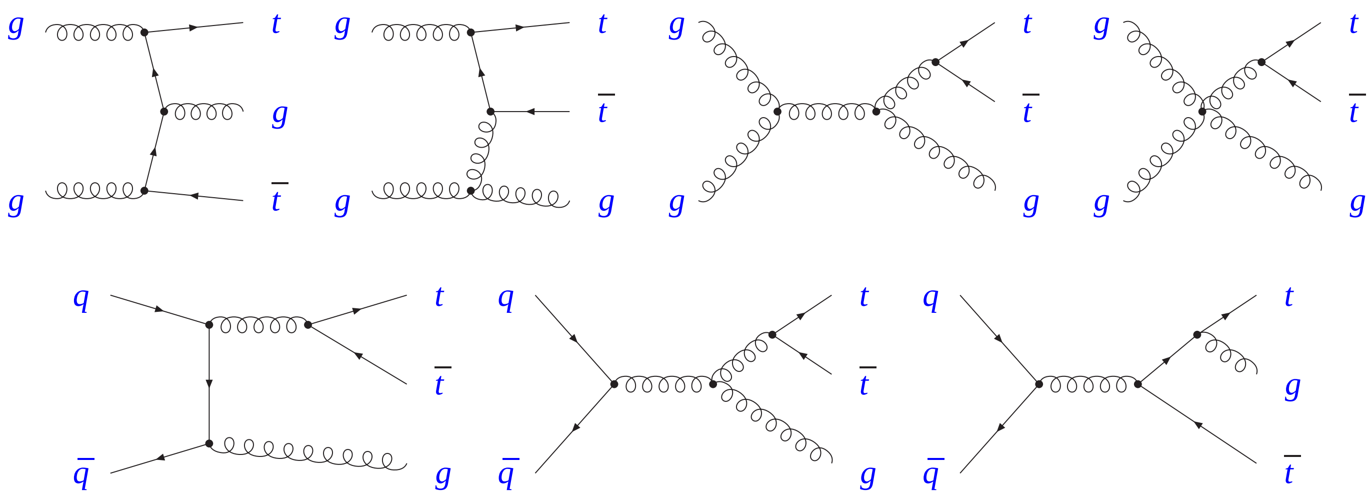
LHC 14 **Tevatron**

- LHC: $\mathcal{O}(2\%)$ with respect to σ_{LO} negative contribution to total cross section, e.g. at LHC14 $\Delta\sigma_{\mathrm{EW}} \simeq \mathcal{O}(10-15)$ pb
- Electroweak corrections to be implemented in Hathor soon

Top-quark pairs with one jet

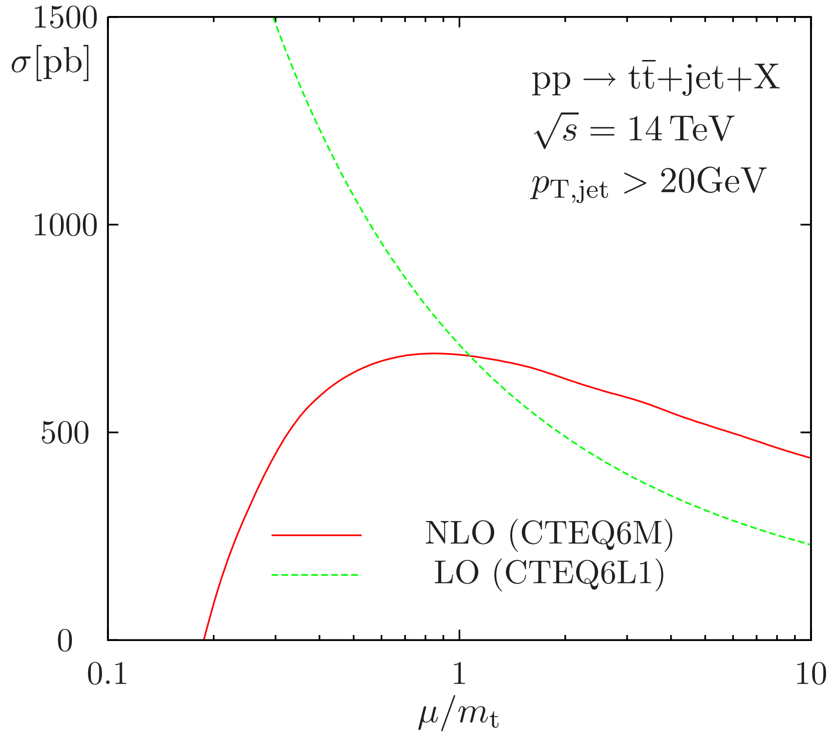
Production of $t\bar{t}$+jet at fixed order

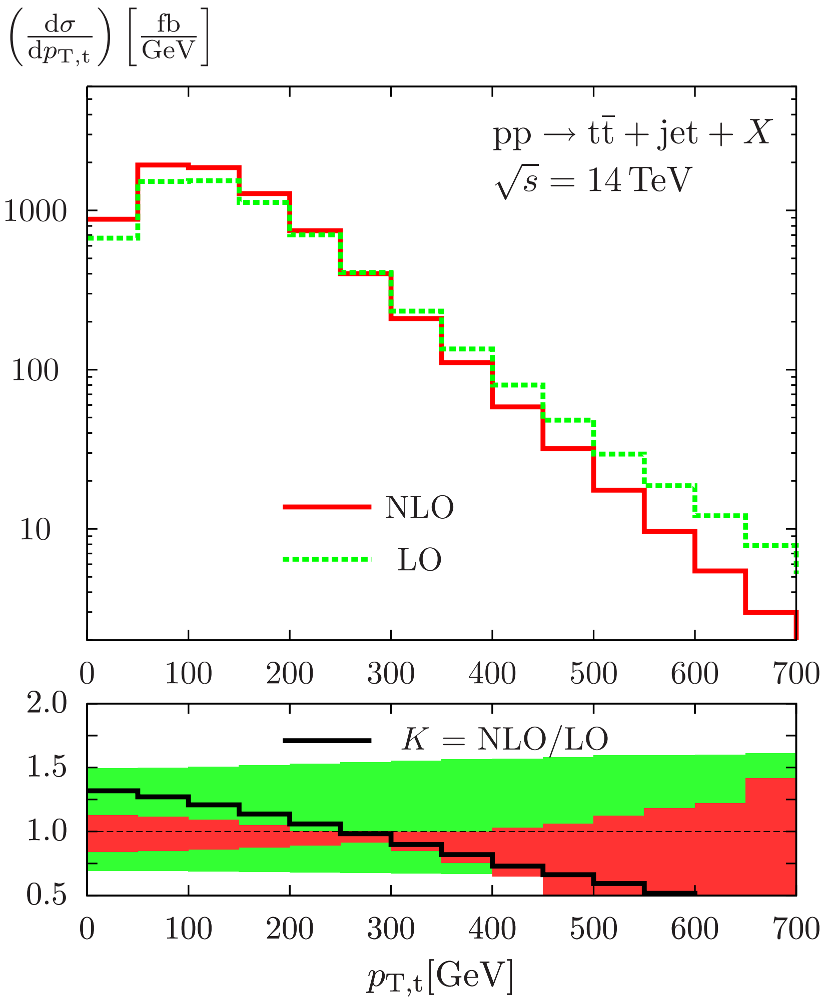
- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- Scale dependence at LO large



- Feynman diagrams (sample) for $t\bar{t}+$ jet production at LO

Production of $t\bar{t}+$ jet at NLO





- NLO QCD corrections Dittmaier, Uwer, Weinzierl ‘07-‘08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero
  - transverse-momentum distributions of top-quark $p_{T,t}$ along with K-factor and scale variation $m_t/2 \leq \mu \leq 2m_t$

Monte Carlo and parton showers at NLO

- Merging of fixed order NLO with parton shower Monte Carlo
  Frixione, Webber '02, Nason '04
  - combining accuracy of exact hard matrix elements for large angle scattering at NLO with soft/collinear emission of parton shower
- POWHEG BOX as standard interface to parton shower programs PYTHIA or HERWIG Alioli, Nason, Oleari, Re '10
- Production of $t\bar{t}+$ jet and parton showers
  Kardos, Papadopoulos, Trocsanyi '11, Alioli, S.M., Uwer '11

Implementation

- Event generation with cut on $p_t^{\rm gen} \simeq 1$ GeV
- Alternative option for soft and collinear divergences at Born level: generation of weighted events with Born suppression factor
  $\bar{B}_{\rm supp} = \bar{B} \times F(p_t)$ Alioli, Nason, Oleari, Re '10

$$F(p_t) = \left(\frac{p_t^2}{p_t^2 + (p_t^{\rm supp})^2} \right)^n$$

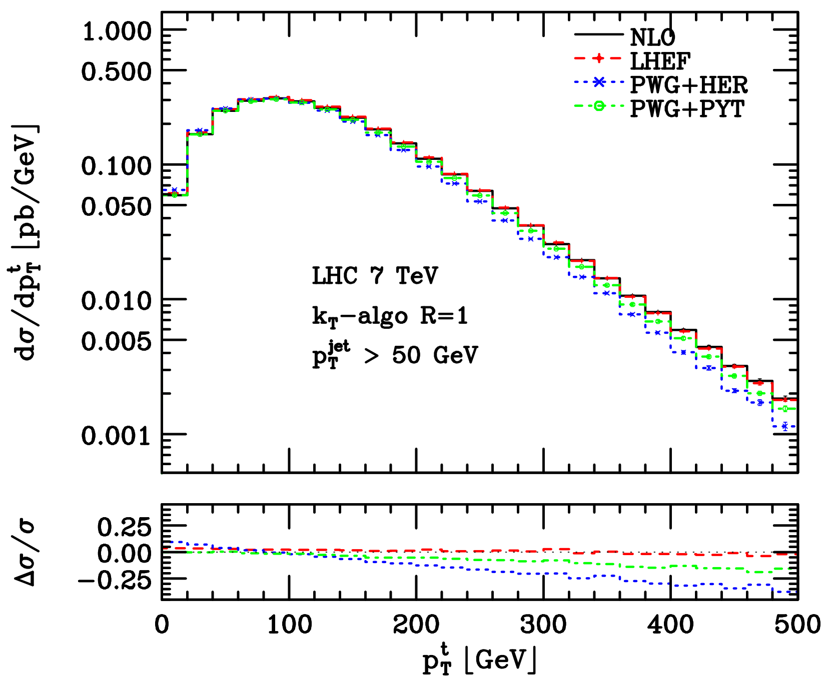
Quality check

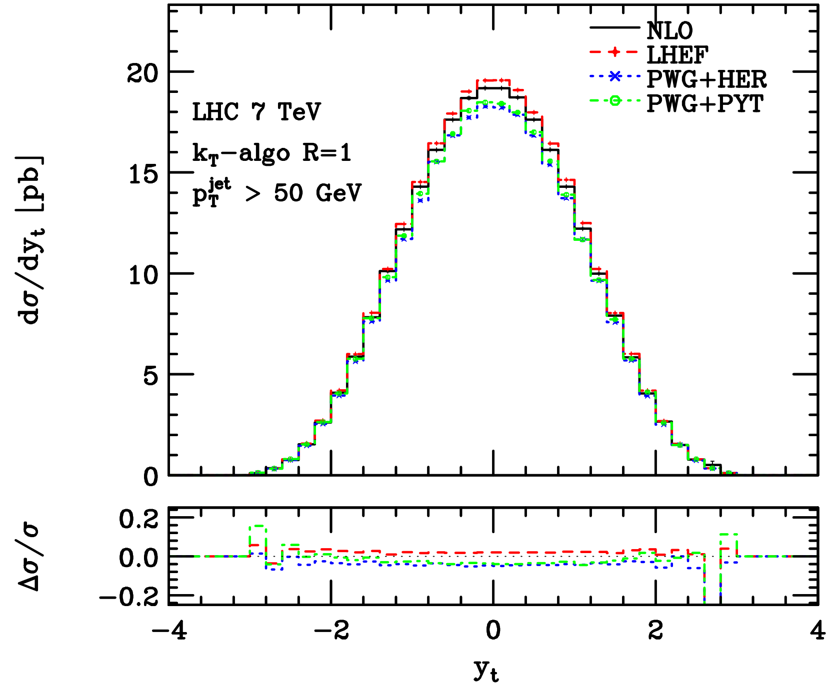
- Independence of NLO cross section on generation cut $p_t^{\rm gen}$ and on Born suppression factor $p_t^{\rm supp}$ for analysis cut $p_t^{\rm an}$ Alioli, S.M., Uwer '11
  - inclusive k_t-algorithm with $R = 1$ for $\mu_r = \mu_f = m_t = 174$ GeV and CTEQ6M PDFs

	$p_t^{\rm gen}$ [GeV]	$p_t^{\rm supp}$ [GeV]	$p_t^{\rm an}$ [GeV]	$\sigma^{\rm NLO}$ [pb]
TEV 1.96 TeV	0	20	20	1.793 ± 0.002
	2	0	20	1.790 ± 0.001
	2	20	20	1.791 ± 0.002
	2	200	20	1.793 ± 0.002
	5	0	20	1.782 ± 0.001
	5	20	20	1.785 ± 0.001
LHC 7 TeV	0	400	50	52.6 ± 0.5
	5	400	50	52.7 ± 0.5
	5	100	50	53.1 ± 0.2
	10	0	50	52.9 ± 0.4
	10	400	50	52.5 ± 0.1
	15	0	50	52.6 ± 0.4
LHC 14 TeV	0	400	50	379.8 ± 1.6
	5	100	50	376.1 ± 0.2
	5	400	50	377.2 ± 1.6

Production $t\bar{t}+$ jet and parton shower

- Differential distributions in top-quark's transverse momentum p_T^t and rapidity y_t at LHC7
  - comparision of NLO, LHEF for POWHEG hardest emission without showering, and POWHEG with shower/hadronization with HERWIG or PYTHIA





Asymmetries for $t\bar{t}+$ jet

- Top-quark charge asymmetry A_C in $t\bar{t}+$ jet sample at LHC7
  - inclusive k_t-algorithm with $R = 1$ and $p_t^{\text{jet}} > 50$ GeV minimum jet cut
  - $\Delta|x| = |x_t| - |x_{\bar{t}}|$ and $x = \eta$ (pseudo-rapidity) or $x = y$ (rapidity)
    ATLAS-CONF-2011-106, CMS-PAS-TOP-11-014

$$A_C^x = \frac{1}{\sigma}\left(\int\limits_{\Delta|x|>0} d\sigma - \int\limits_{\Delta|x|<0} d\sigma \right)$$

LHC 7 TeV	NLO [%]	LHEF [%]	PWG+HER [%]	PWG+HER+UE [%]	PWG+PYT[%]	PWG+PYT+MPI [%]
A_C^η	0.19 ± 0.09	0.18 ± 0.06	0.46 ± 0.10	0.26 ± 0.11	0.40 ± 0.11	0.57 ± 0.11
A_C^y	0.51 ± 0.09	0.47 ± 0.06	0.73 ± 0.10	0.52 ± 0.11	0.66 ± 0.11	$0.76 + 0.11$

- Sizeable effects on asymmetries in $t\bar{t}+$ jet sample due to parton showers

Summary

- Top quark theory
  - much recent progress for LHC phenomenology
  - improved understanding of theory and application of new concepts
- $t\bar{t}$ production
  - NNLO$_{\mathrm{approx}}$ prediction for total cross section controls theoretical uncertainty
  - Electroweak corrections are important for precision predictions
- Sizeable (correlated) differences due to non-perturbative parameters
  - top-quark mass m_t (advantage of $\overline{MS}$ mass definition)
  - strong coupling α_s and gluon PDF
- $t\bar{t}$+jet production
  - merging with parton showers shows good stability
  - important effects for asymmetries