Introduction to

Statistics

Blockkurs Krippen 2012

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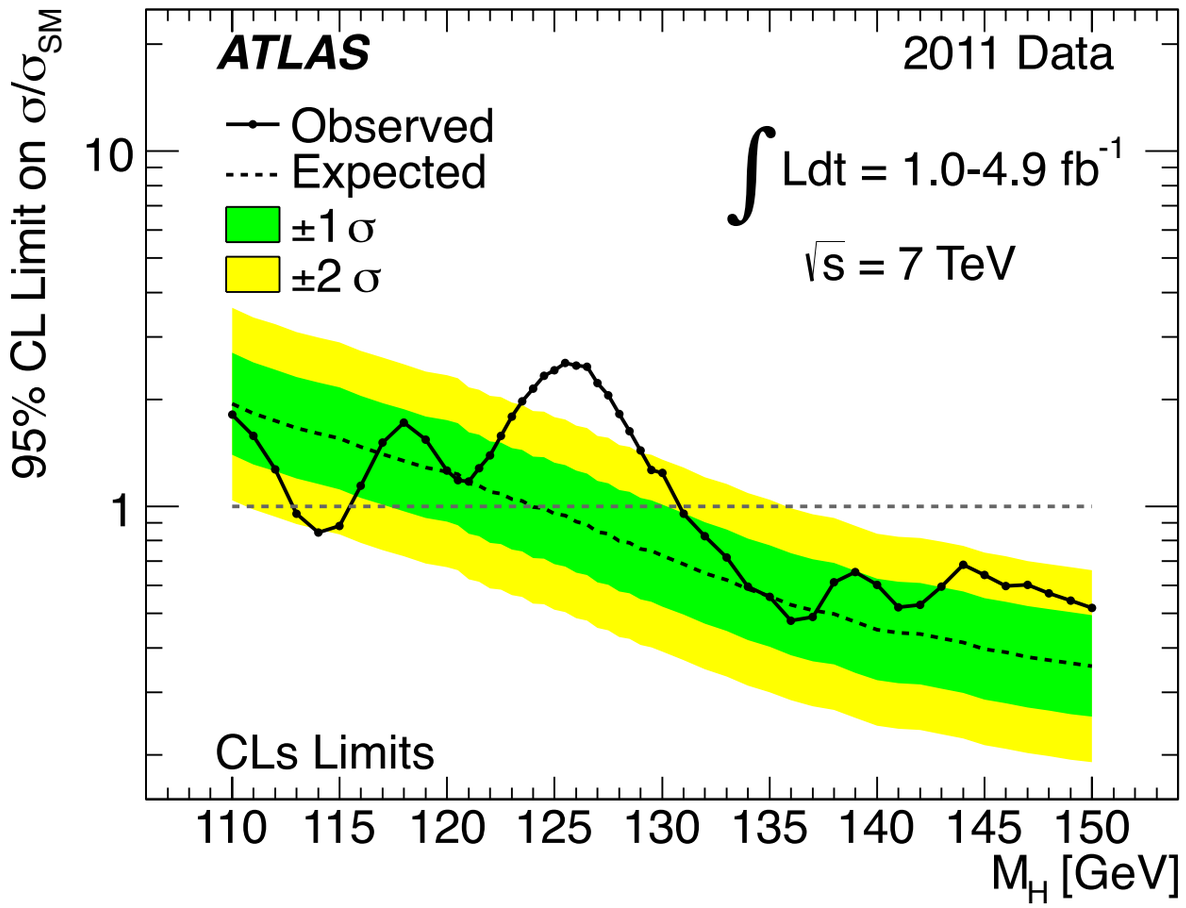
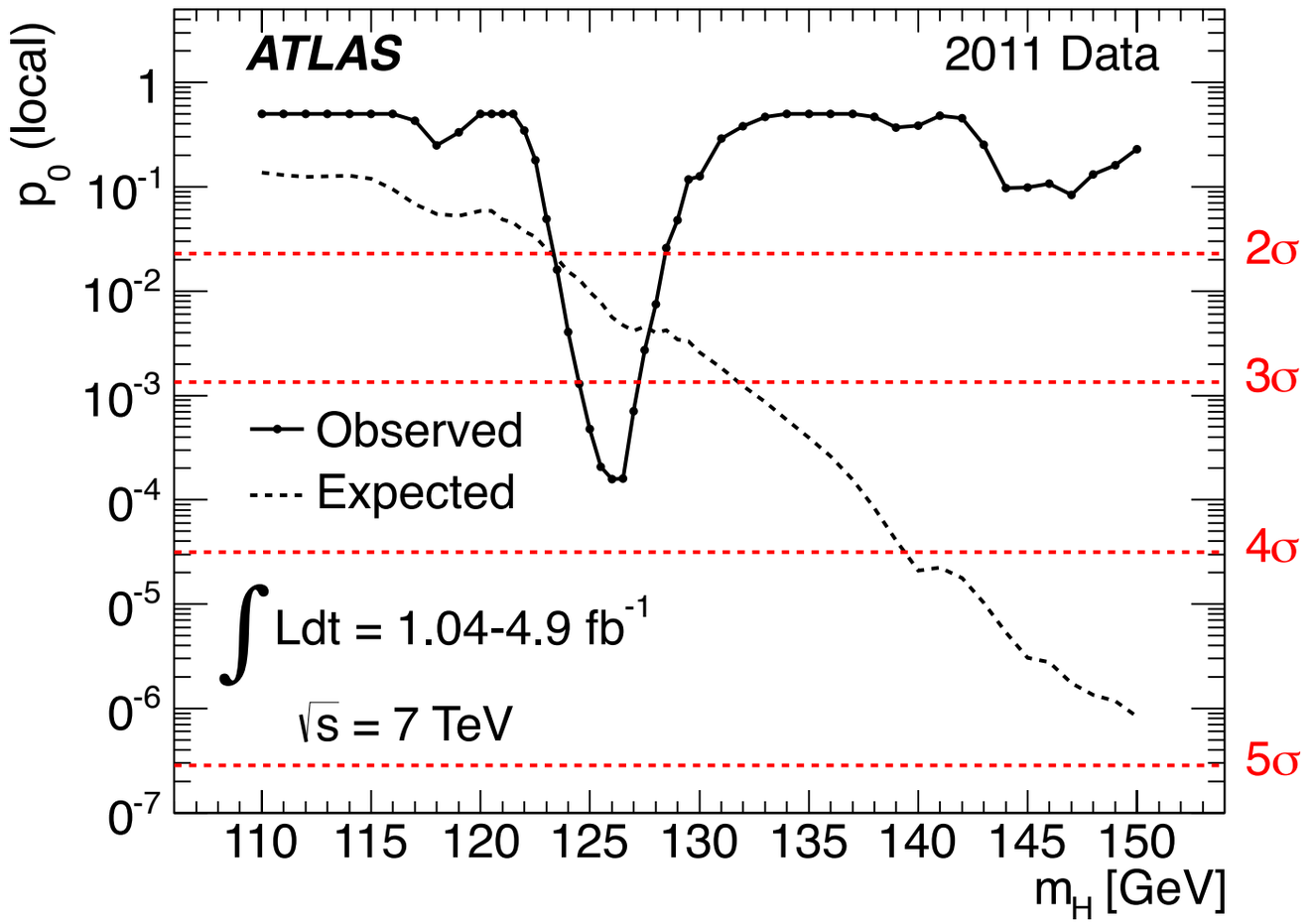
Introduction

- What is a hypothesis in statistics?
- What is a *p*-value?
- What means *significance*?
- What is the Likelihood Ratio?
- What is a statistical test?
- How to claim discovery and exclusion?

Mainly taken from "*Asymptotic formulae for likelihood-based tests of new physics*", Glen Cowan, Kyle Cranmer, Eilam Gross, Ofer Vitells
http://arxiv.org/pdf/1007.1727.pdf

Introduction





Hypotheses

	Discovery	Exclusion
Null Hypothesis H0	known processes, Standard Model, background-only	signal and background
Alternative Hypothesis H1	background and signal	background-only

Main Goal: exclude H0

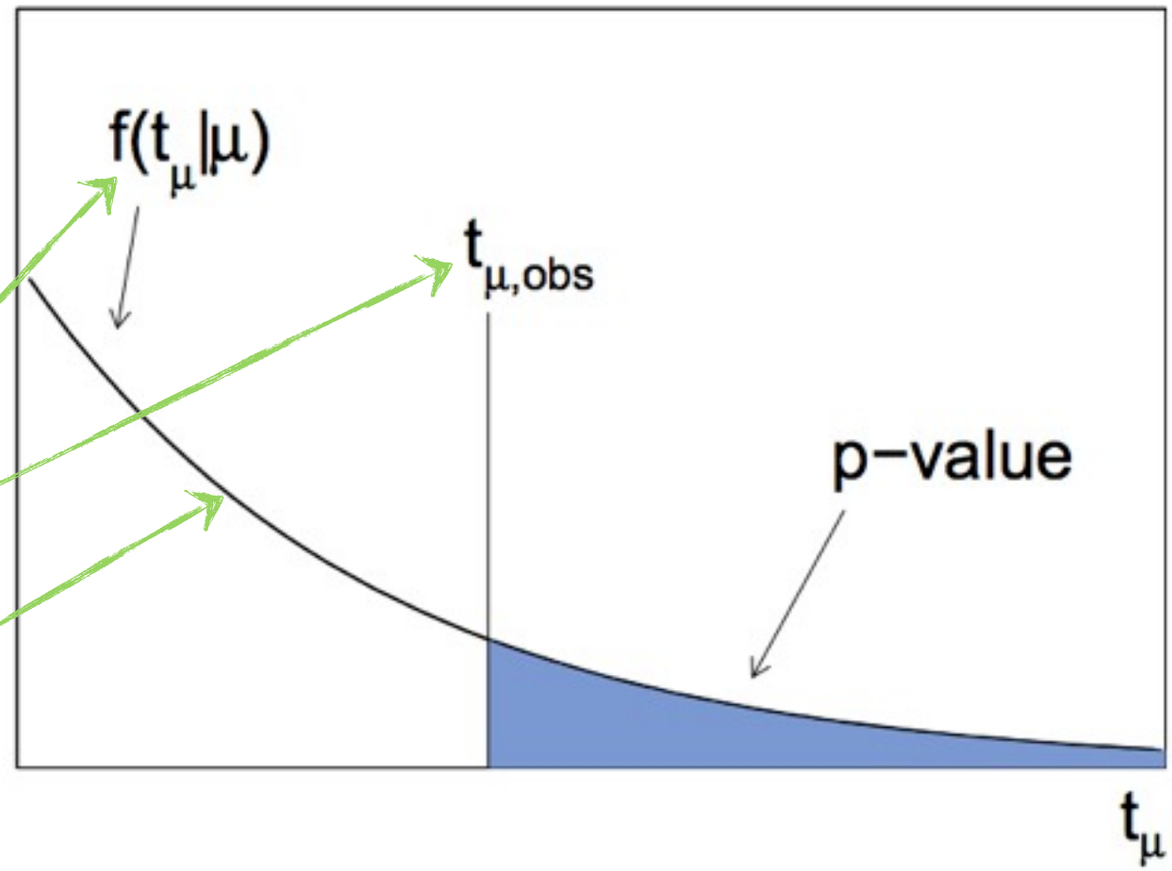
p-value and Significance

Goal: exclude H0

test statistic (see later how to get it)

observed value in experiment

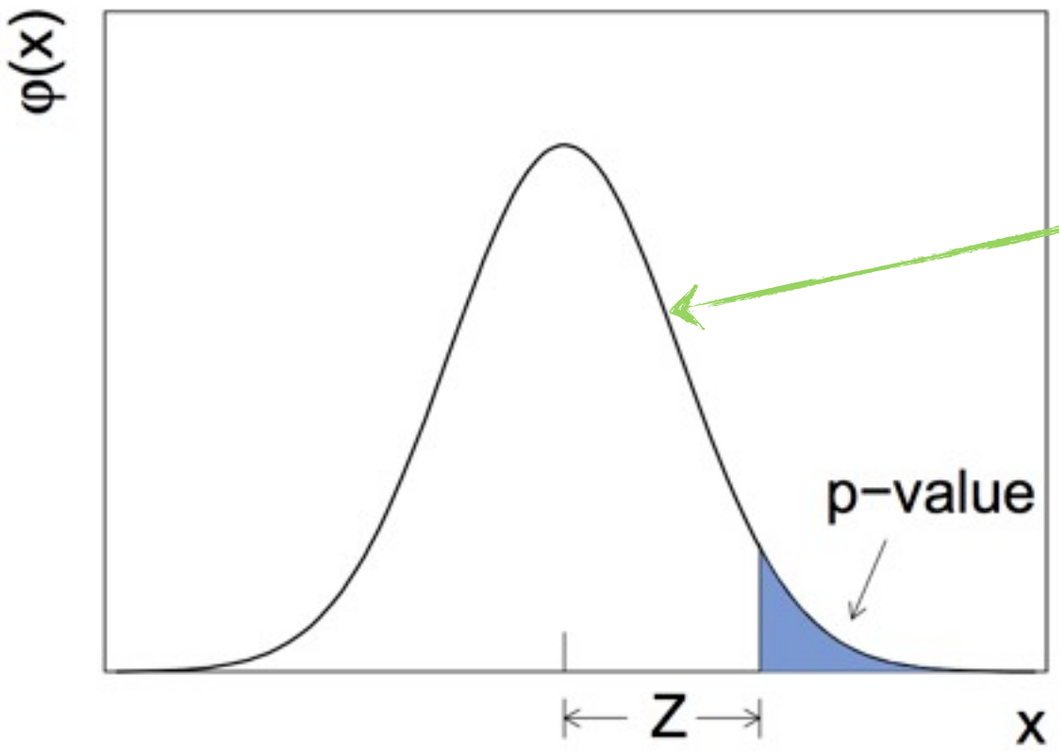
Hypothesis



p-value:

“(...) probability, under assumption of H0, of finding data of equal or greater incompatibility with the predictions of H0 (...)” than has been observed by experiment.

p-value and Significance



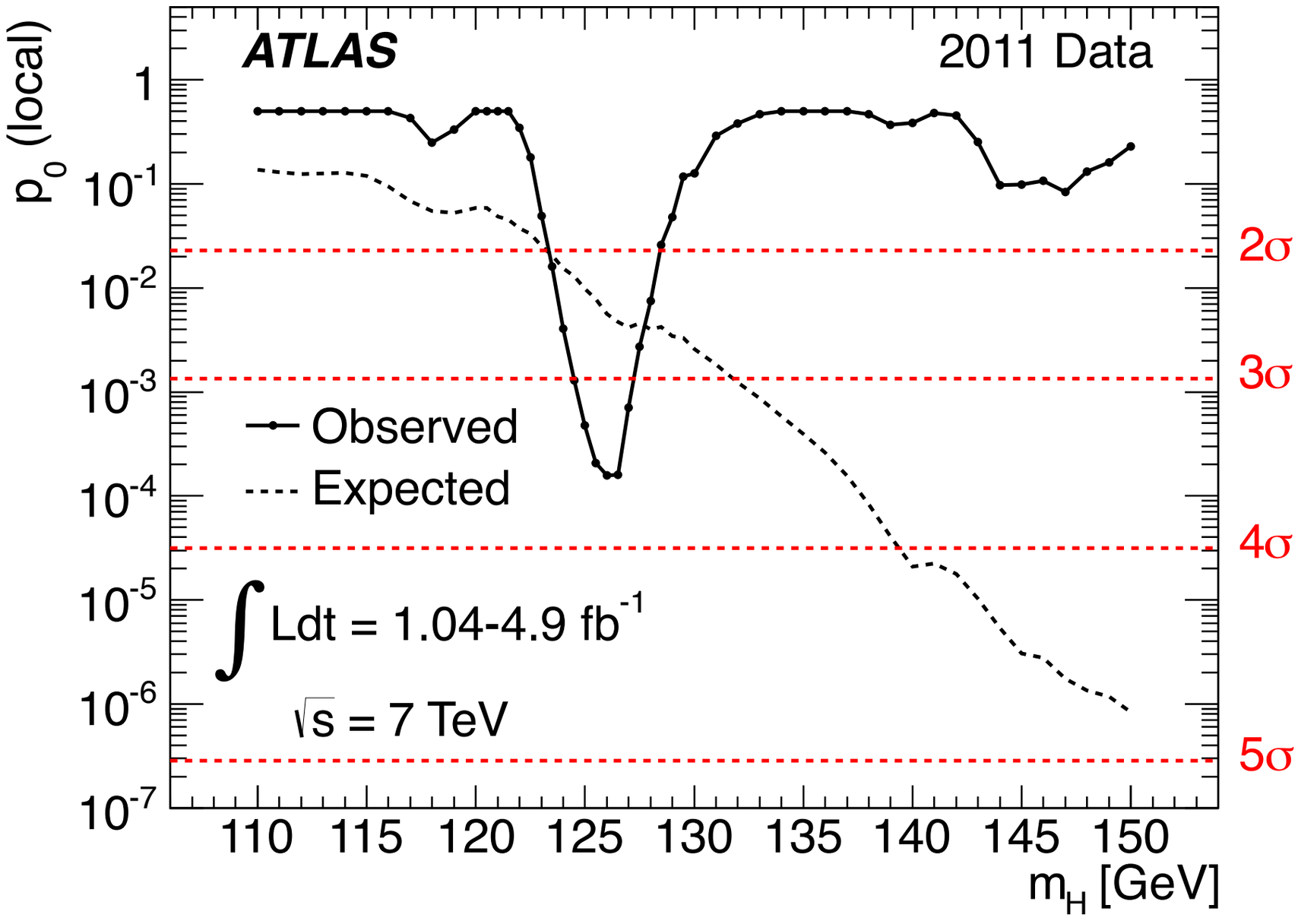
Standard Gaussian ($\sigma = 1$)

Significance

Cummulative Gaussian
(inverse of Standard Gaussian)

$$Z = \Phi^{-1}(1 - p)$$

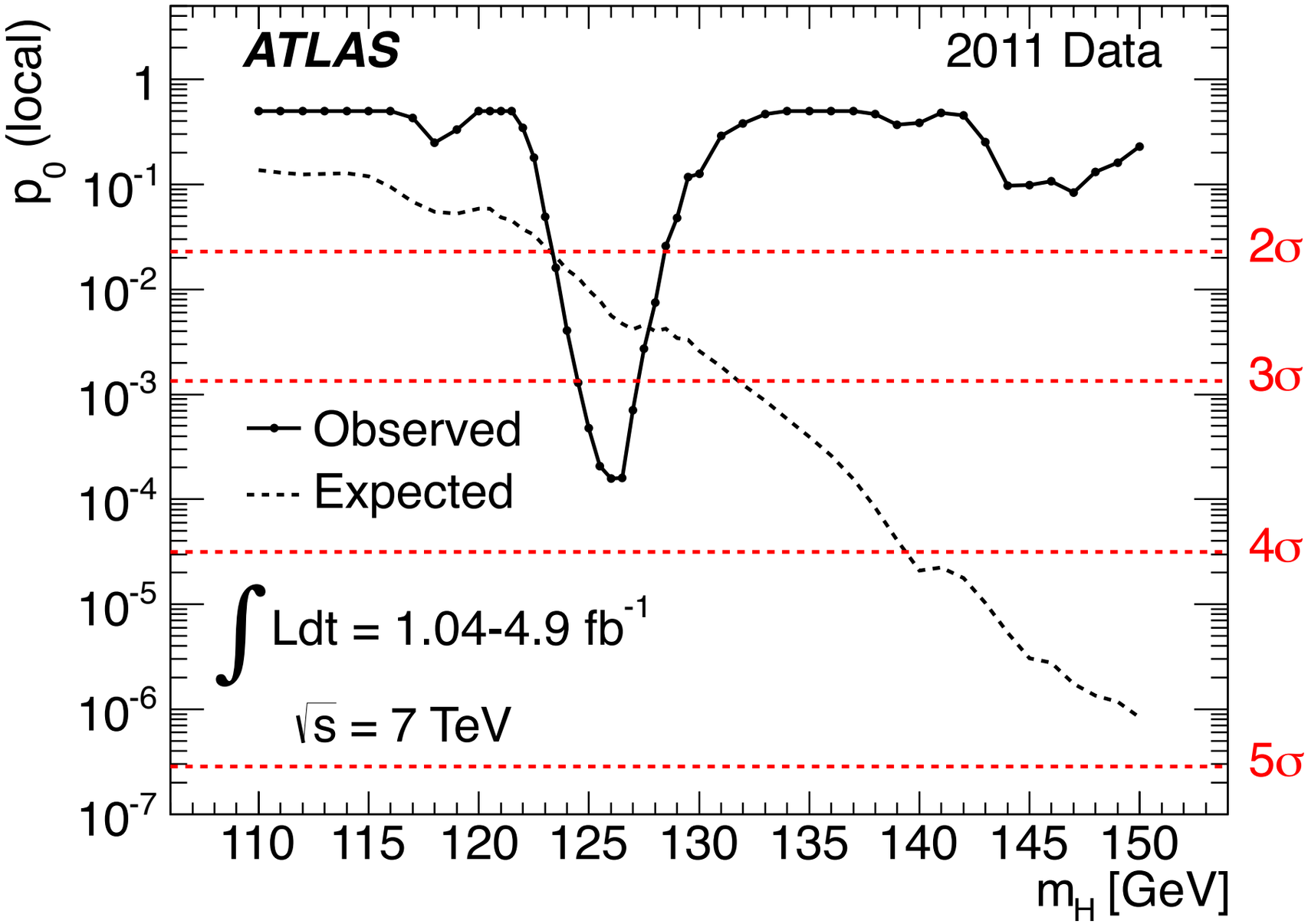
p-value and Significance



Combined search for the Standard Model Higgs boson using up to 4.9 fb−1 of pp collision data at sqrt(s) = 7 TeV with the ATLAS detector at the LHC

p-value and Significance

- How do I get the observed significance/*p*-value from data?
- How do I get the expected (median) significance/*p*-value?



p-value and Significance

$$p(\theta) = \int_{\hat{\theta}_0}^{\infty} f(\hat{\theta}|\theta) d\hat{\theta}$$

p-value
probability of obtaining a result as compatible or less with theta

true value of parameter
(e.g. Higgs mass)
not known and will be never known

"measured" estimator
(in data or MC)

hypthesis' pdf
(later)

estimator

p-value and Significance

for only **one** true-value point:

$$p_\mu = \int_{t_{\mu,0}}^{\infty} f(t_\mu|\mu)\mathrm{d}t_\mu$$

signal strength parameter to separate H0 from H1
0 if no signal, 1 if “full” signal

test statistic for the pdf
(this is the hardest part and will follow now)

(1) What to do with statistical uncertainties?
(2) What to do with nuisance parameters?

any parameter that is not of
direct interest in the final result

Profile Likelihood Ratio as Test Statistic

signal strength parameter to separate H0 from H1
0 if no signal, 1 if “full” signal

conditional maximum-likelihood estimator for given value of μ

unconditional maximum-likelihood estimator

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\mathbf{b}}})}{L(\hat{\mu}, \hat{\mathbf{b}})}$$

$$t_\mu = -2\ln\lambda(\mu)$$

$$L(\mu, \theta) = \frac{(\mu s + b)^n}{n!} \mathrm{e}^{-(\mu s + b)} \prod_{k=1}^{M} \frac{u_k^{m_k}}{m_k!} \mathrm{e}^{-u_k}$$

likelihood function, more precisely $L(n, \mathbf{m}|\mu, \mathbf{b})$

Poisson probabilities for “signal” and background estimation

Mean number of signal/background

- Expectation value of number of events: $E[n] = \mu s + b$
- Sidebands/Subsidiary measurements: (e.g. to determine background b) $E[m] = u(\theta)$

p-value and Significance

for only **one** true-value point:

$$p_\mu = \int_{t_{\mu,0}}^{\infty} f(t_\mu|\mu)\mathrm{d}t_\mu$$

signal strength parameter to separate H0 from H1
0 if no signal, 1 if “full” signal

?

test statistic for the pdf
(this is the hardest part and will follow now)

✓

Sampling distribution

With some appoximations and assumptions

[Wald approximation, Wilks approximation, Asimov dataset, see paper]

the result is a simple **chi-square distribution**:

$$f(t_\mu|\mu) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{t_\mu}} \mathrm{e}^{-t_\mu/2}$$

Discovery Significance

Calculations [see paper] result in the **easy** formula

$$Z_0 = \sqrt{t_0}$$

remember: discovery → exclude
background-only → μ = 0

with [see last slides]:

$$t_\mu = -2\ln\lambda(\mu)$$

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\mathbf{b}}})}{L(\hat{\mu}, \hat{\mathbf{b}})}$$

$$L(\mu, \theta) = \frac{(\mu s + b)^n}{n!} \mathrm{e}^{-(\mu s + b)} \prod_{k=1}^{M} \frac{u_k^{m_k}}{m_k!} \mathrm{e}^{-u_k}$$

Upper Limit Significance

Calculations [see paper] result in the **easy** formula

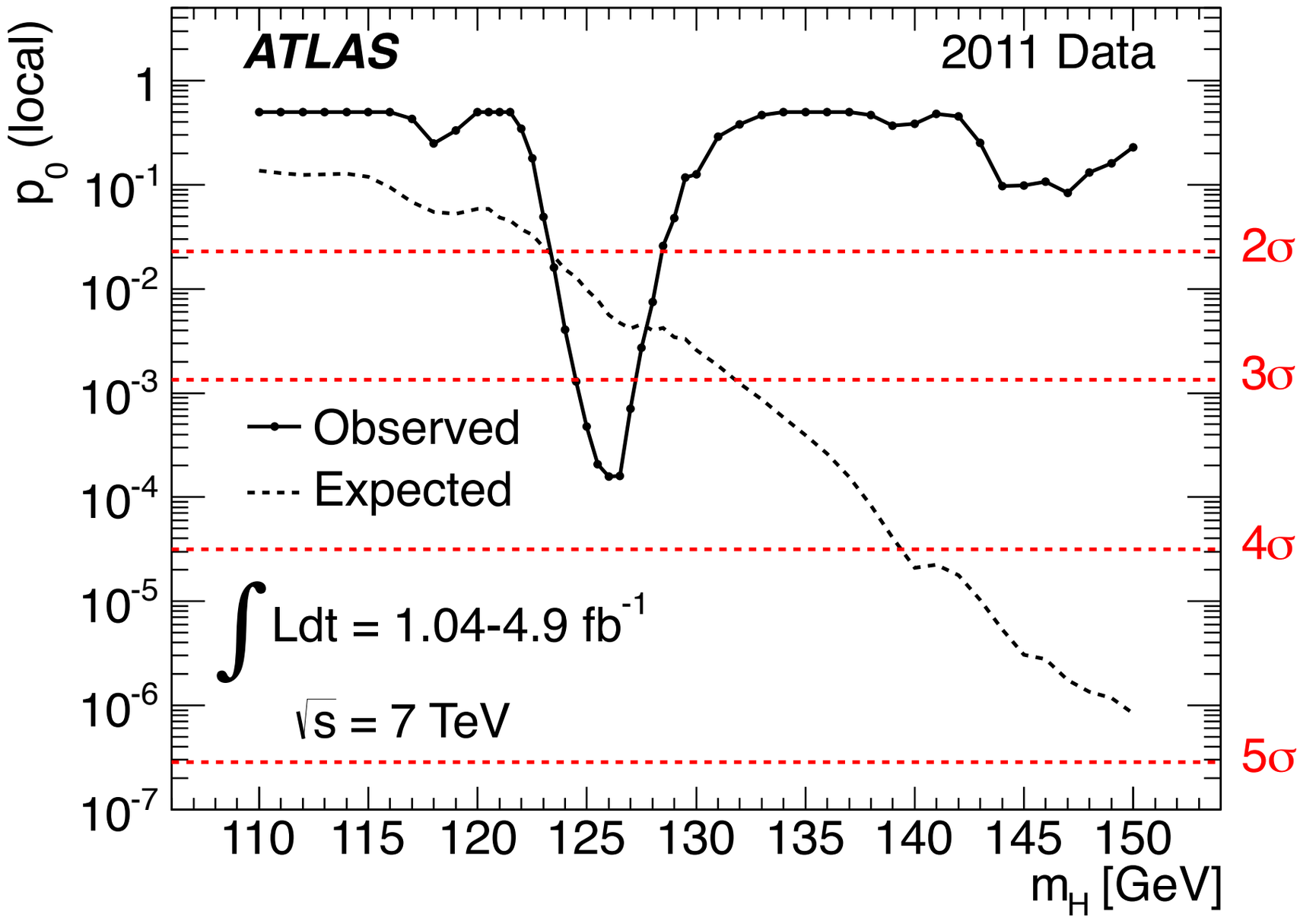
$$Z_\mu = \sqrt{t_\mu}$$

If Z_μ is found above a specific value (1.64; $p_\mu < 5\%$) one says
"**μ** is excluded"

Now it's easy to calculate upper limit on **μ**...

p-value and Significance

- ☑ How do I get the observed significance/*p*-value from data?
- ☐ How do I get the expected (median) significance/*p*-value?



Experimental Sensitivity

What is the expected (more precisely, median) significance of an experiment?

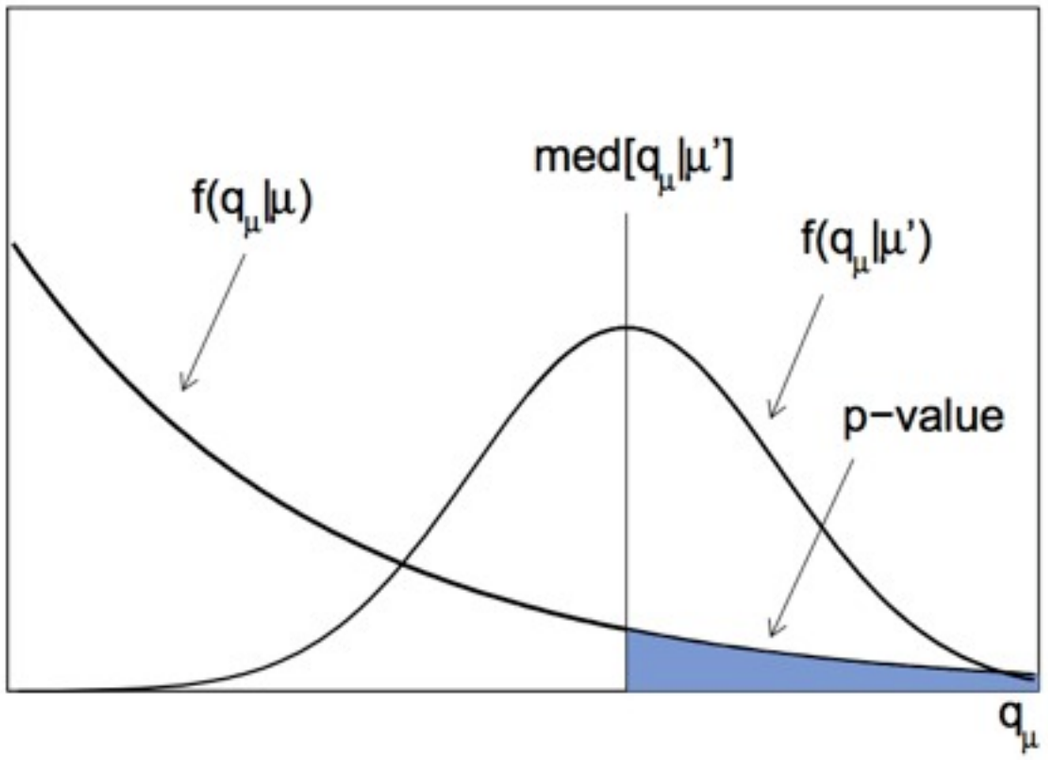
This means: No data available → no observed value $t_{\mu,0}$ available

Discovery: $\mathrm{med}[Z_0|\mu'] = \sqrt{q_{0,\mathrm{A}}}$

Limit: $\mathrm{med}[Z_\mu|0] = \sqrt{q_{\mu,\mathrm{A}}}$

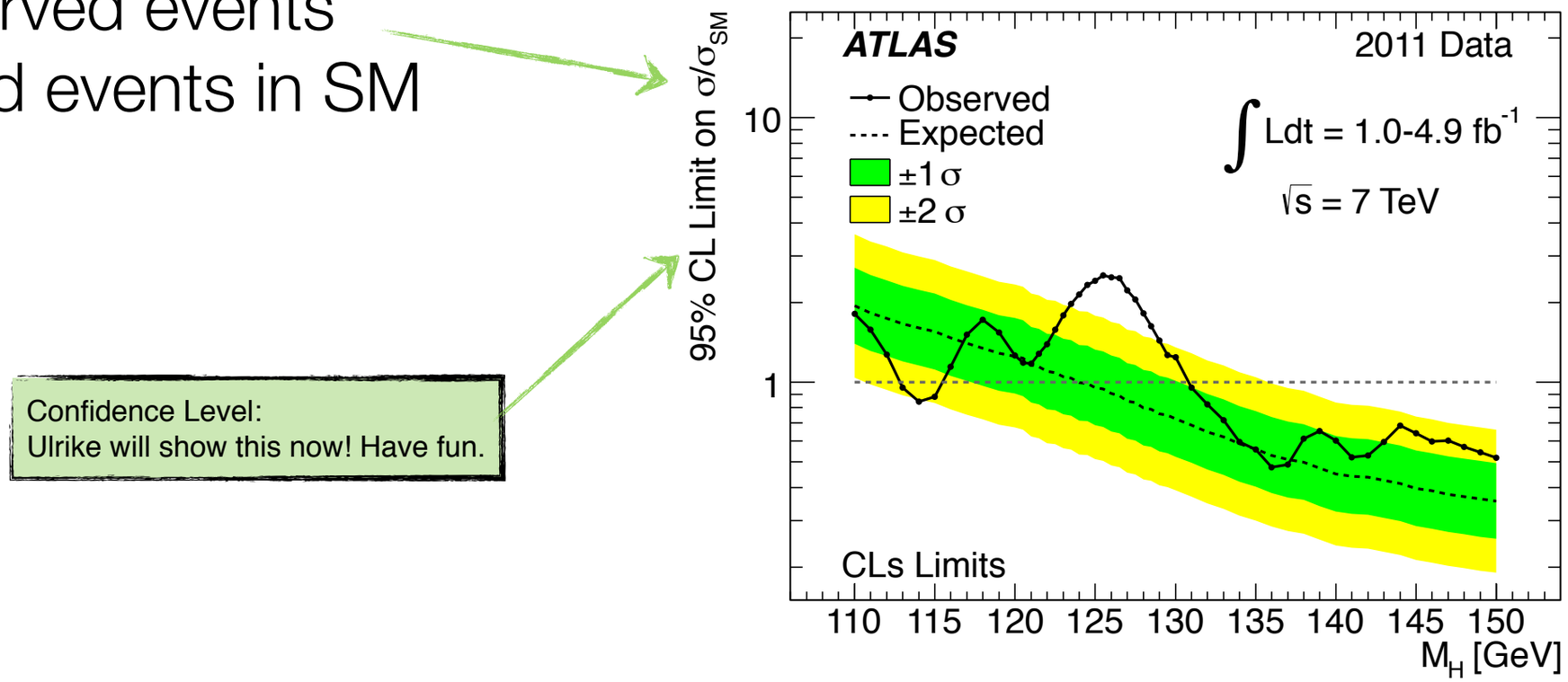
expecting Asimov data set:
Use the estimated values from MC as measured values.

$$n_{i,\mathrm{A}} = E[n_i] = \nu_i = \mu' s_i(\boldsymbol{\theta}) + b_i(\boldsymbol{\theta}) ,$$
$$m_{i,\mathrm{A}} = E[m_i] = u_i(\boldsymbol{\theta}) .$$



Last Slide
Famous Higgs Exclusion Plot

upper limit on observed events
divided by expected events in SM



Confidence Level:
Ulrike will show this now! Have fun.

hypothetical data example:

ATLAS has found 100 Higgs candidates for a masspoint
SM predicts 200
Quotient = 0.5
Upper Limit = 0.8
ATLAS could have meassured a maximum of 160 Higgs (95%)
160 < 200 -> Exclusion

Thank you