Application of Feldman and Cousins

"Sense and sensitivity of double beta decay experiments (2011)"

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About this talk

Sense and sensitivity of double beta decay experiments

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- Small Double Beta Decay (DBD) introduction
- Sensitivities with Feldman Cousins
- Comparing DBD experiments

DBD? Why is it cool?

$$(Z, A) \rightarrow (Z + 2, A) + 2e^-$$

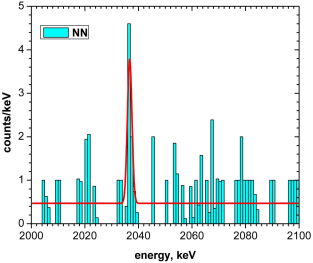
- Lepton number violated ($\Delta L = 2$)
  - Considered the most fundamental implication
  - Baryogenesis through B-L symmetry
- Neutrino is a Majorana particle
- Coupling strength of LNV process with $0\nu\beta\beta$ half-life
  - In the standard interpretation: Determination of effective Majorana neutrino mass m_{ee}
  - ν-mass scale and mass hierarchy
- Determination of Majorana phases

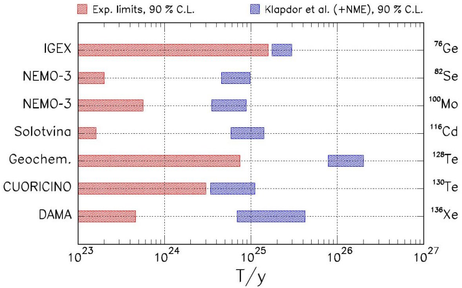
DBD? Why is it cool NOW?

Claim of $0\nu\beta\beta$ by subset of Heidelberg-Moscow experiment (2002)

$$T_{1/2}^{0\nu} = 2.23^{+0.44}_{-0.31} \cdot 10^{25}\,\mathrm{yr} \qquad \rightarrow \qquad |m_{ee}| = (0.11..0.56)\,\mathrm{eV}$$

Mod. Phys. Let. A, Vol.21, 1547 (2006)



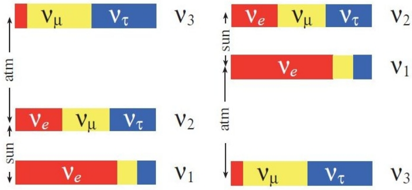


Test of claim by this generation experiments within $<$ 1 years!

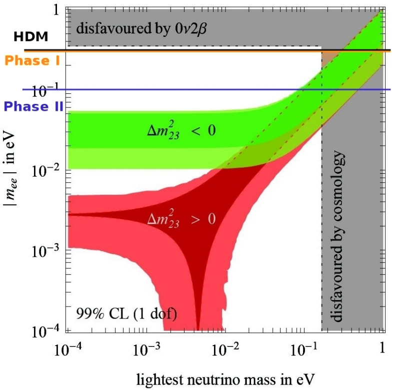
Neutrino Property: Effective Majorana Neutrino Mass

$$|m_{ee}|^2 = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i(\alpha_2 - \alpha_1)} + m_3 |U_{e3}|^2 e^{-i(\alpha_1 + 2\delta)} \right|^2$$

m_{ee}	Effective Majorana neutrino mass
$m_{1..3}$	Mass of neutrino mass eigenstates
$U_{e1..e3}$	PMNS matrix elements



normal hierarchy | inverted hierarchy



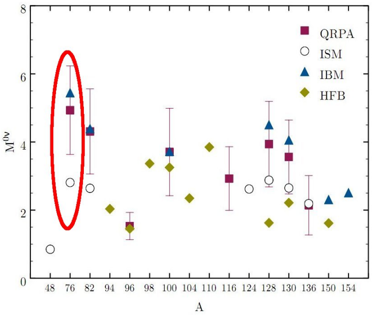
How to get $|m_{ee}|$?

Standard interpretation:
(Light Majorana neutrino propagator)

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}(Q,Z) \cdot |\mathcal{M}^{0\nu}|^2 \cdot |m_{ee}|^2$$

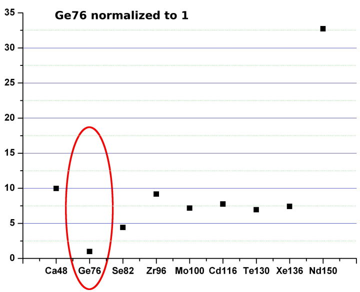
$T_{1/2}^{0\nu}$	$0\nu\beta\beta$ half-life
$G^{0\nu}(Q,Z)$	Phase Space Factor
$\mathcal{M}^{0\nu}$	Nuclear Matrix Element
$\lvert m_{ee}\rvert^2$	Effective Majorana neutrino mass

Nuclear matrix element



ArXiv:hep-ex/1010.5112v3, 2010

Phase space factor



How to get $T_{1/2}$?

Simple decay law:

$$T_{1/2} = \log 2 \cdot \frac{N_A}{A} \cdot \epsilon \cdot \frac{M \cdot t}{n_{\mathrm{obs}}}$$

N_A Avogadros number
A Atomic weight
ϵ Detection efficiency
M Target mass
t Measuring time

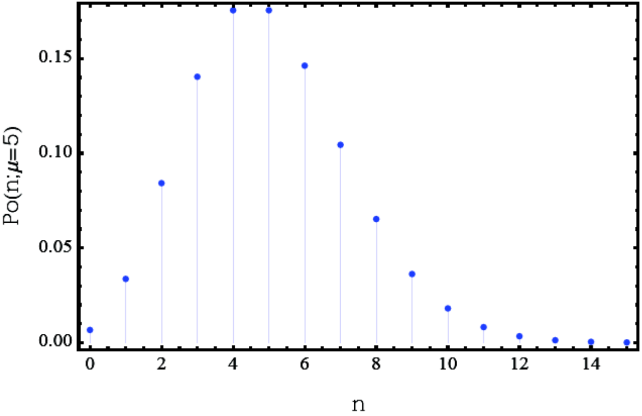
Upper limit for count rate is lower limit for half-life:

$$T_{1/2} > \log 2 \cdot \frac{N_A}{A} \cdot \epsilon \cdot \frac{M \cdot t}{n_{\mathrm{ul}}}$$

Old method for large backgrounds:

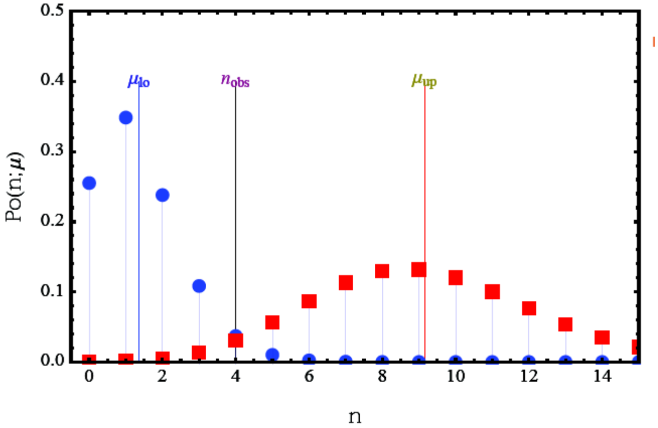
Simple sensitivity estimation (no signal): $n_{\mathrm{ul}} = \sqrt{b}$
(Signal counts could hide in a 1σ gaussian background fluctuation)

Ideal Experiment: $b = 0$, $\mu = 5$



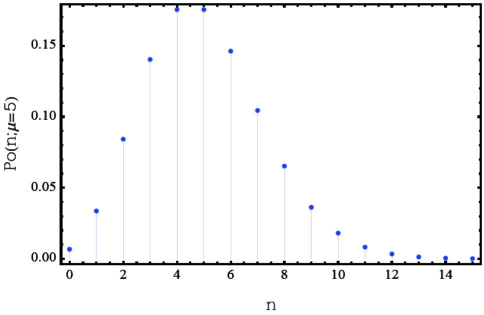
- Poisson distribution: $Po(n;\mu) = \frac{\mu^n}{n!} e^{-\mu}$
- Only 17 % of experiments measure $n = 5$
- Same amount of experiment measure $n = 4$

Ideal Experiment: $b = 0$, $\mu = 5$



- Lower limit $\mu_{\rm lo}$: $\alpha = \sum_{n_{\rm obs}}^{\infty} Po(n;\mu_{\rm lo})$
- Upper limit $\mu_{\rm up}$: $\beta = \sum_{0}^{n_{\rm obs}} Po(n;\mu_{\rm up})$
- For $\alpha = \beta = 0.05$: $\mu_{\rm lo} = 1.37$ and $\mu_{\rm up} = 9.15$

Ideal Unlucky Experiment: $b = 0$, $\mu = 5$, $n = 0$



- Chance of being unclucky: 0.7 %
- Report upper limit: $\beta = \sum_0^{n_{\rm obs}=0} Po(n; \mu_{\rm up})$ $\quad\rightarrow\quad$ $\mu_{\rm up} = -\ln(\beta)$
- $\mu_{\rm up} = 3$ @ 95 % CL

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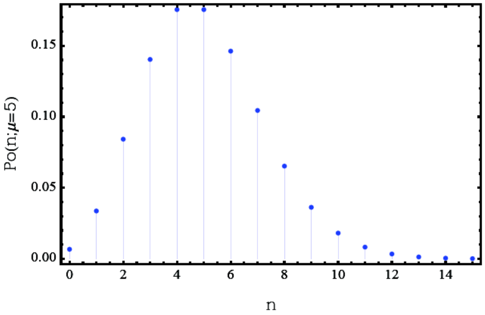

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For no background and no signal the upper limit is fix at a specific CL.

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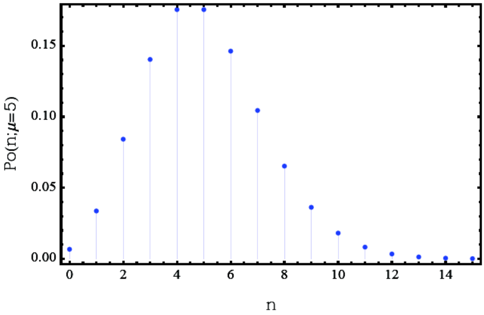

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For no background and no signal the upper limit is fix at a specific CL.

Definition of sensitivity:
The average upper limit from an ensemble of experiments with no signal ($n = b$)

Sensitivity of an Ideal Experiment

Roadmap:

$$N \rightarrow T_{1/2} \rightarrow |m_{ee}|$$

count rate → half life → neutrino mass

From previous equations: (K_1 - isotope dependend constant)

$$|m_{ee}| = K_1 \sqrt{\frac{N}{\epsilon M t}}$$

Sensitivity of an ideal experiment: $N = 3$ @ 95 % CL

$$|m_{ee}| \propto \sqrt{\frac{1}{Mt}}$$

Realistic Experiment: $b \neq 0$

Poisson pdf with mean $\mu + b$:

$$Po(n; \mu + b) = \frac{(\mu + b)^n}{n!} e^{-\mu + b}$$

Lower limit

$$\alpha = \sum_{n_{\rm obs}}^{\infty} \frac{(\mu_{\rm lo} + b)^n}{n!} e^{-\mu_{\rm lo} + b}$$

Upper limit

$$\beta = \sum_{0}^{n_{\rm obs}} \frac{(\mu_{\rm up} + b)^n}{n!} e^{-\mu_{\rm up} + b}$$

An Example

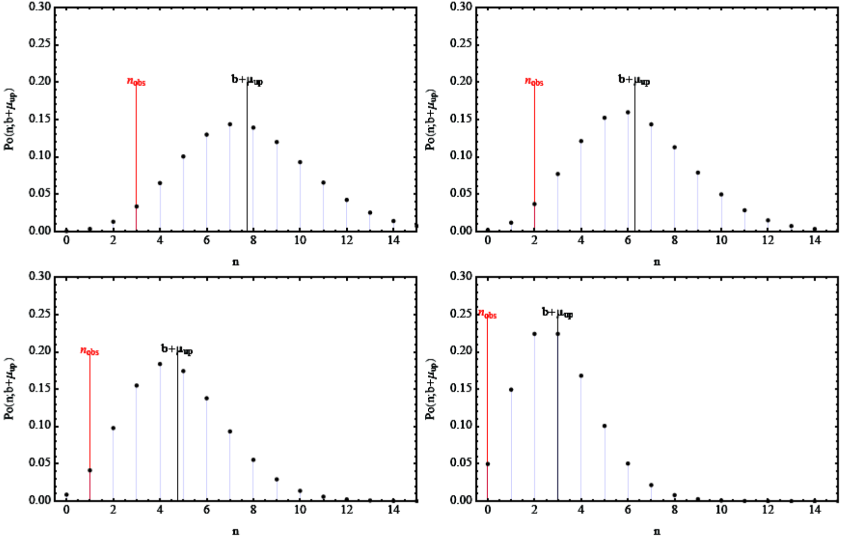
Background index 0.001 $\mathrm{cts}/(\mathrm{kg} \cdot \mathrm{yr} \cdot \mathrm{keV})$ with
$\Delta E = 5\,\mathrm{keV}$, $M = 100\,\mathrm{kg}$ and $t = 10\,\mathrm{yr}$

$b = 5$, $n = 5$

For $\beta = \alpha = 0.05$ solve numerically: $\mu_{\rm up} = 5.51$, $\mu_{\rm lo} = -3.03$

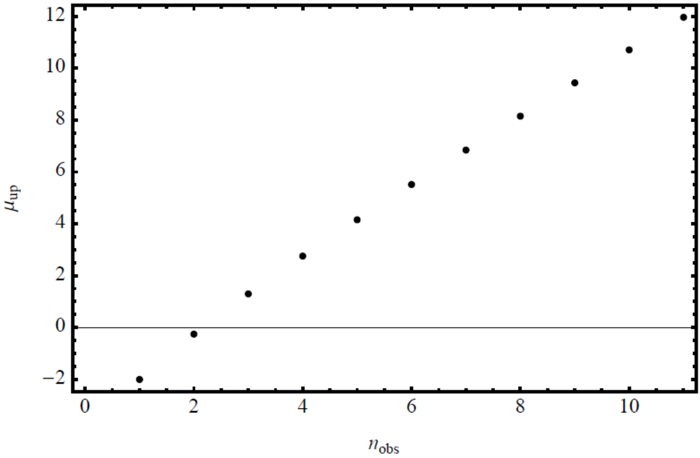
Publish: Experiments yields only upper limit for μ: $\mu_{\rm up} = 5.51$

Example with Background: ... Repeat for $n_{\text{obs}} = 4, 3, 2, 1, 0$ and $b = 5$



$n_{\text{obs}} =$	4	3	2	1	0
$\mu_{\text{up}} =$	4.15	2.75	1.29	-0.25	-2

Example with Background: $b = 5$



Classical limit fails for small n with larger background b

Escape: Unified Approach of Feldman and Cousins

Sensitivity with the Unified Approach

Definition of sensitivity:
The average upper limit from an ensemble of experiments with no signal ($n = b$)

Function for upper limit

Feldman Cousins: $FC(n, b, CL)$

Ensemble of experiments with no signal

Poisson distribution with expectation b
Different n possible with probability $Po(n, b)$

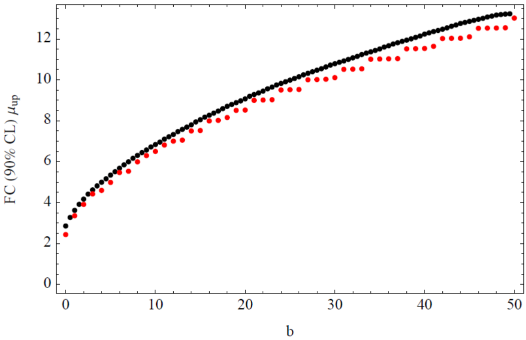
Sensitivity for different backgrounds b

$$S(b) = \sum_{n=0}^{\infty} Po(n, b) FC(n, b, CL)$$

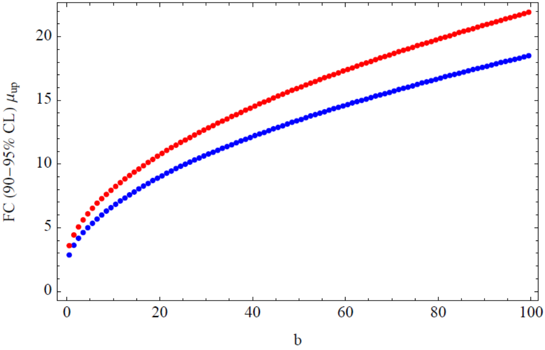
Example $b = 5$

$$S(5) = \sum_{n=0}^{\infty} Po(n,5) FC(n,5,90\,\%) \quad (1)$$
$$= 0.175 \cdot 4.99 + 0.175 \cdot 3.66 + \ldots \quad (2)$$
$$= 5.17 \quad (3)$$

Not correct: $S(5) = FC(n,5,90\,\%) = 4.99$



Different CLs and Limit for Large Backgrounds



$S(b)$ for 90 % CL and 95 % CL

Limit for large backgrounds

Sensitivity approaches classical limit:
$S(b) \approx a \cdot \sqrt{b}$
$a = 1.64$ for 90 % CL and $a = 1.96$ for 95 % CL

Implementation in ROOT

ROOT class: TFeldmanCousins

Exampled: root/tutorials/math/FeldmanCousins.C

```
TFeldmanCousins FC;
FC.SetCL(0.90);

Double_t Nobserved = 5.0;
Double_t Nbackground = 5.0;

Double_t ul = FC.CalculateUpperLimit(Nobserved, Nbackground);
Double_t ll = FC.GetLowerLimit();

cout<<"UL: "<<ul<<endl;
cout<<"LL: "<<ll<<endl;
```

Output:

```
UL: 4.99
LL: 0
```

Comparing Experiments

From before: set N to the classical limit $\sqrt{b}$

$$|m_{ee}| = K_1 \sqrt{\frac{\sqrt{b}}{\epsilon M t}}$$

b is the total background in the analysis bin.

Better to compare is the Background Index B in $\mathrm{cts}/(\mathrm{kg} \cdot \mathrm{yr} \cdot \mathrm{keV})$
$b = B \cdot M \cdot t \cdot \Delta E$

$$|m_{ee}| = K_2 \cdot \sqrt{1/\epsilon} \cdot \left(\frac{B \cdot \Delta E}{M \cdot t} \right)^{-1/4}$$

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$|m_{ee}|$ improves very slowly with exposure $(M \cdot t)^{-1/4}$

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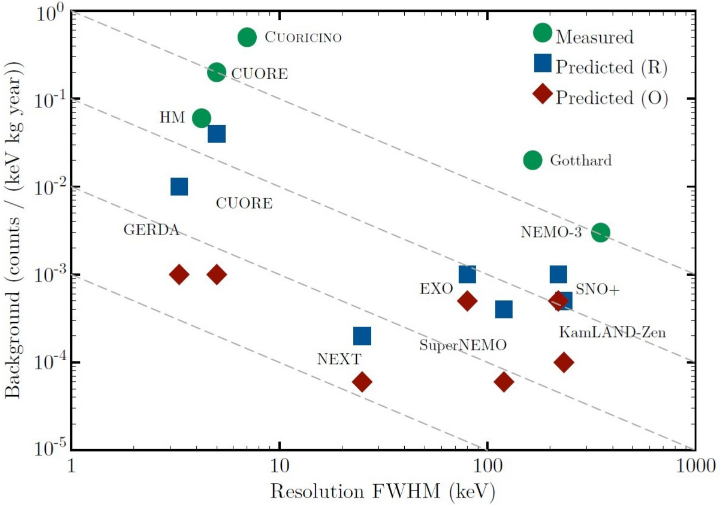
$|m_{ee}|$ improves very slowly with exposure $(M \cdot t)^{-1/4}$

$B \cdot \Delta E$ is absolutely crucial and determined by the technology of the experiment

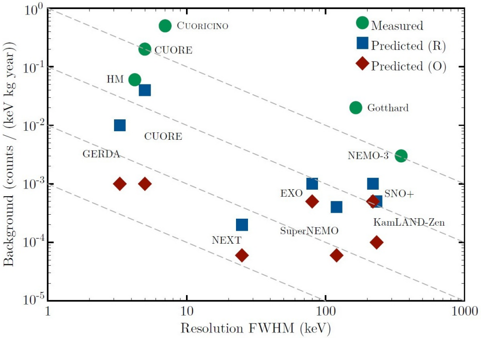
This is compared now...

Comparing Experiments

arXiv: hep-ex:1010.5112v3



Some Comments



- Two population of experiments: Semiconducter and scintillation
- No mass, detection efficiency and isotope properties considered
- All this-generation experiments have to proof if they can reach their goals

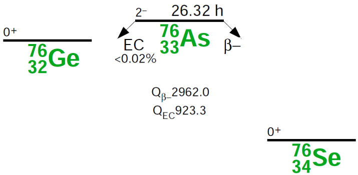
- Personal opinion: This plot is less a benchmark of this-generation experiments but rather a benchmark of technologies for next-generation experiments

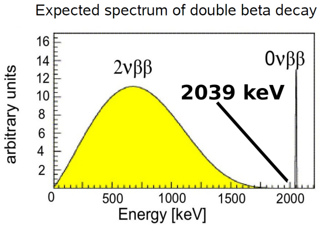
Conclusions

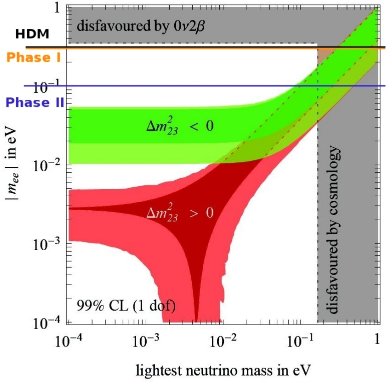
- Double beta decay is cool
  - Neutrino property: Effective Majorana neutrino mass
  - LNV, Majorana neutrinos
- Feldman Cousins
  - Used in a sensitivity study for this-generation DBD experiments
  - Main usage: Treatment of unphysical reagion
- Comparing experiment
  - Comparison of experiment technology only
  - Two main kind: Semiconducter experiments and scinillation experiments

BACKUP

Neutrinoless Double Beta Decay - $0\nu\beta\beta$







$2\nu\beta\beta$

$(Z, A) \rightarrow (Z+2, A) + 2e^- + 2\bar{\nu}_e$

- SM process

$0\nu\beta\beta$

$(Z, A) \rightarrow (Z+2, A) + 2e^-$

- Lepton number violated: $\Delta L = 2$
- Neutrino is Majorana particle
- Constraints / measurement on neutrino mass

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu} \cdot |\mathcal{M}^{0\nu}|^2 \cdot |m_{ee}|^2$$