The backtracking algorithm:

The algorithm used to create the following plots works as follows:
For any given point $\boldsymbol{x}$ in the detector calculate:

$$h(\boldsymbol{x},t) = \sum_{i=1}^{N_{\mathrm{PMT}}} \Theta(q_i - q_{\mathrm{th}}) \sum_{l=1}^{N_\gamma} f(t_{il} - t_i^{\mathrm{TOF}}, t) \tag{1}$$

where N_{PMT} is the number of PMTs, q_i is the charge on the i^{th} PMT, q_{th} is the minimal charge for a PMT to be used for the analysis, N_γ is the number of hits used per PMT, t_{il} is the l^{th} hit on the i^{th} PMT and t_i^{TOF} is the expected time of flight between the PMT i and $\boldsymbol{x}$. $f(\Delta t, t)$ is a function used to characterize how well the events cluster. The plotted quantity for each point is:

$$\int_{-\infty}^{\infty} |h(\boldsymbol{x},t)|^2 \; dt \tag{2}$$

As long as N_γ is small with respect to q_{th} the expected distribution of the PMT-hits around the expected hit time is approximately Gaussian. Hence the first idea was to use a Gaussian distribution with a standard deviation equaling the hits on PMT transit time spread for f. To get a better suppression of random hits, a bipolar function would be beneficial though, as this would allow for cancellation of the signals of random hits. Hence, the derivative of a Gaussian was used for f:

$$f(\Delta t, t) \propto (t - \Delta t) \exp\left[-\frac{(\Delta t - t)^2}{2\sigma_{\mathrm{tts}}}\right] \tag{3}$$

The following plots present a few examples showing the capabilities of the algorithm.

parameter	value
number of PMTs	13472
PMT diameter	20"
Winston cones	no
quantum efficiency	100%
light yield	2000 MeV^{-1}
dark noise	0 Hz
PMT resolution type	Gaussian
PMT tts	1 ns
Scintillator	LAB
optical parameters	standard in simulation rev. 485

Tab. 1: Parameters used in simulation and analysis

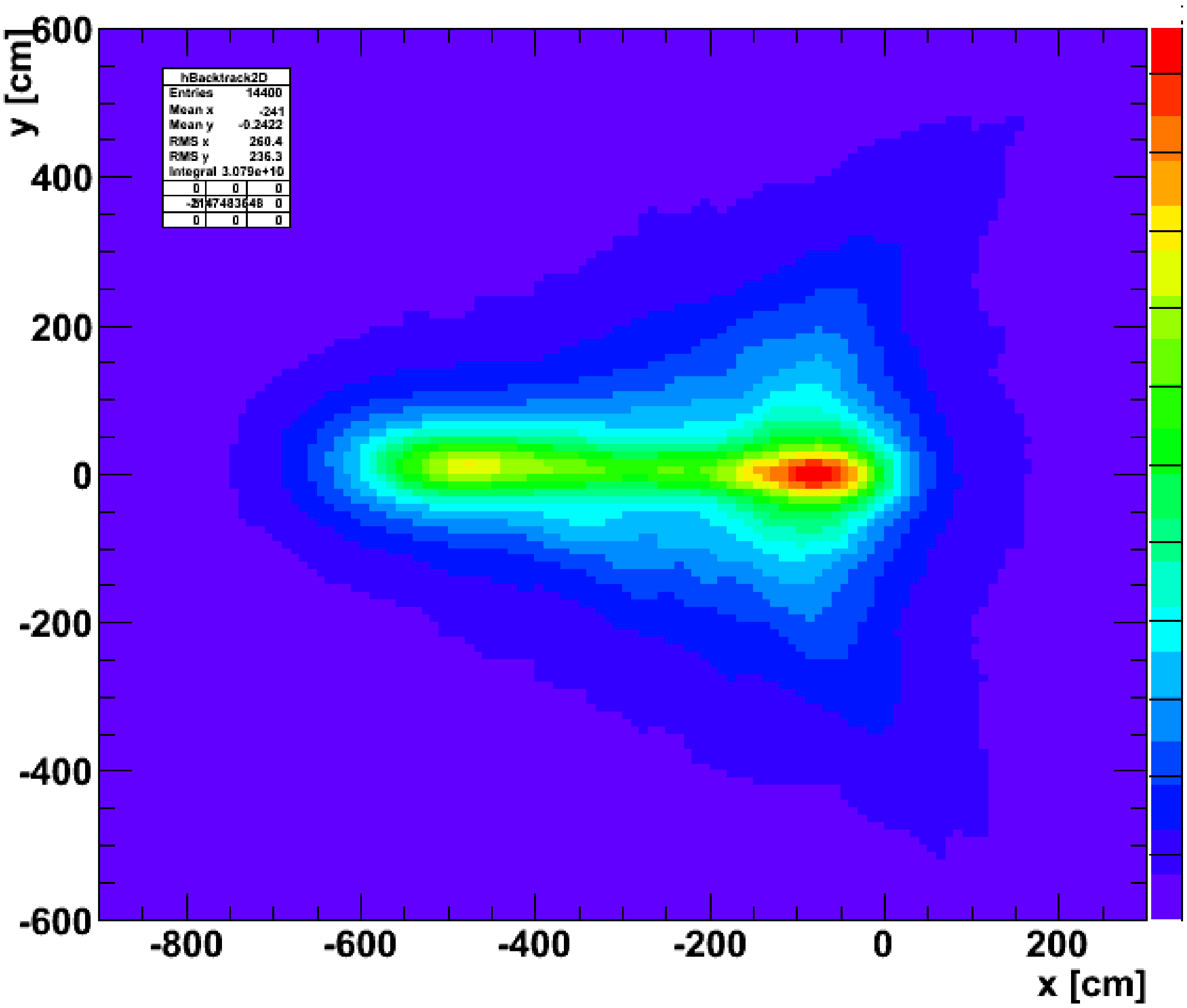

Fig. 1: Results of backtracking for a single 1-GeV muon, taking into account the first three hits on each PMT. The muon was started in the center of the detector and traveled along the negative x-axis.

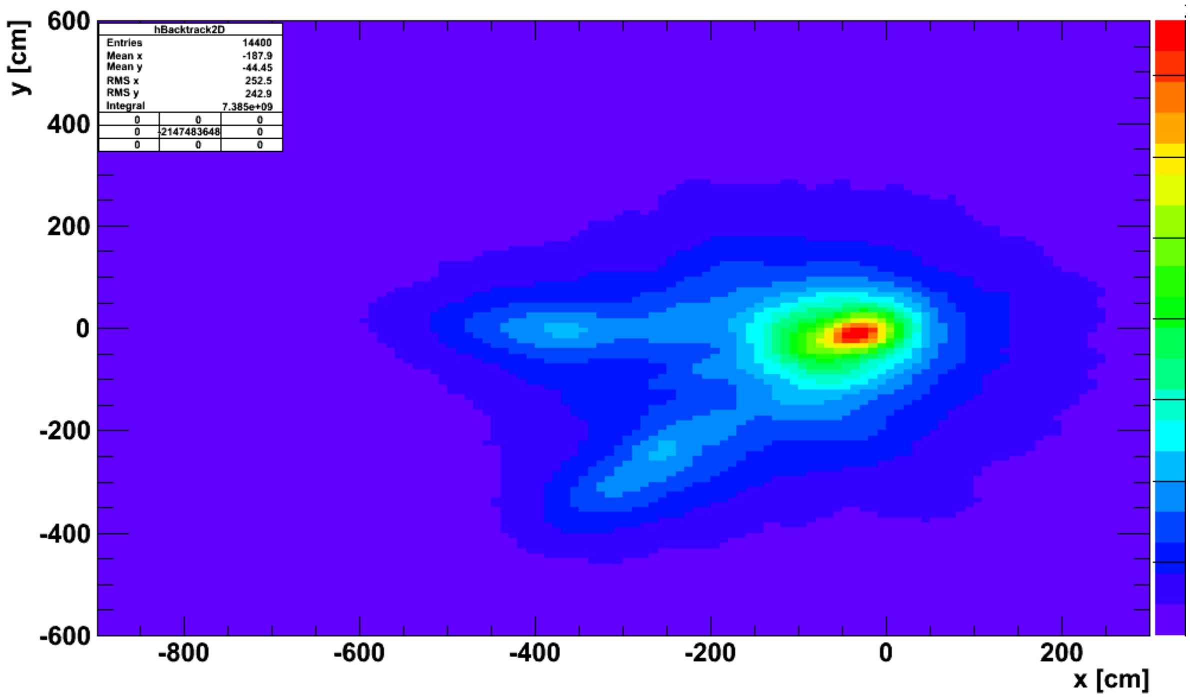

Fig. 2: Results of backtracking for a double muon event, taking into account only the first hit on each PMT. Both muons have an energy of 1 GeV. The directions of the initial muon impulse were $(-1, 0, 0)$ and $\frac{1}{\sqrt{2}}(-1, -1, 0)$.

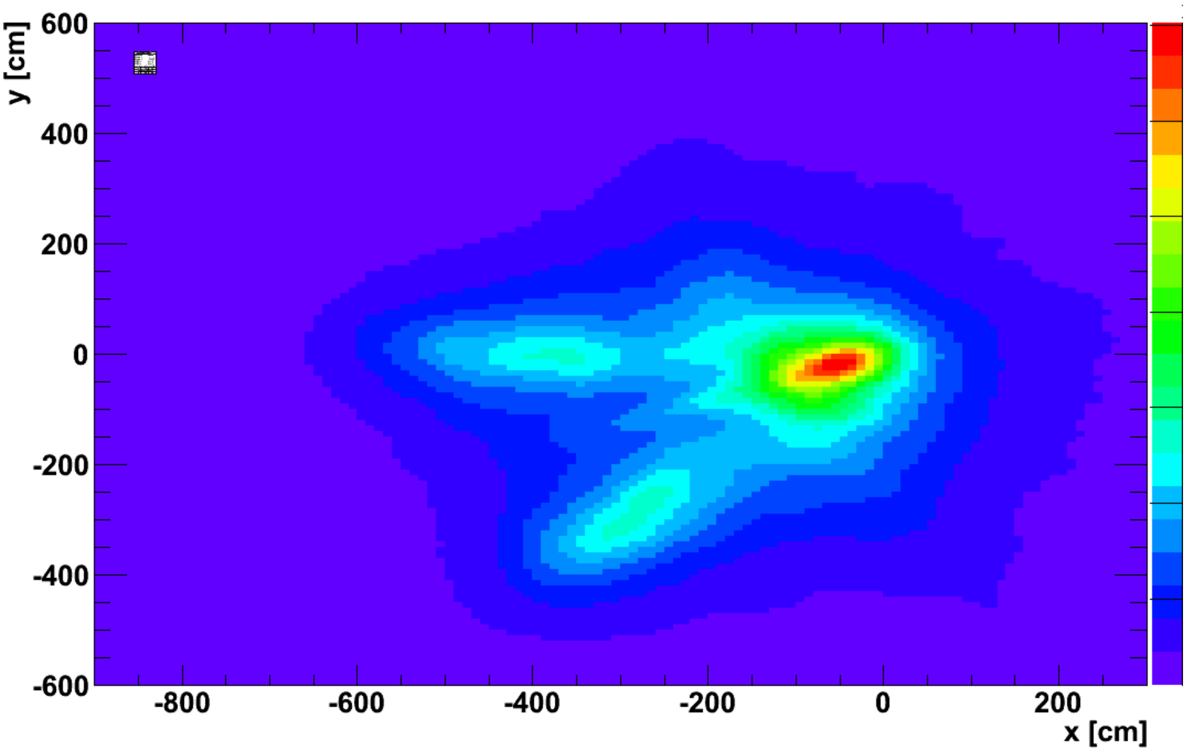

Fig. 3: Results of backtracking for a double muon event, taking into account the first 7 hits on each PMT. Both muons have an energy of 1 GeV. The directions of the initial muon impulse were $(-1, 0, 0)$ and $\frac{1}{\sqrt{2}}(-1, -1, 0)$.