The S_3 flavour symmetry: quarks, leptons and Higgs

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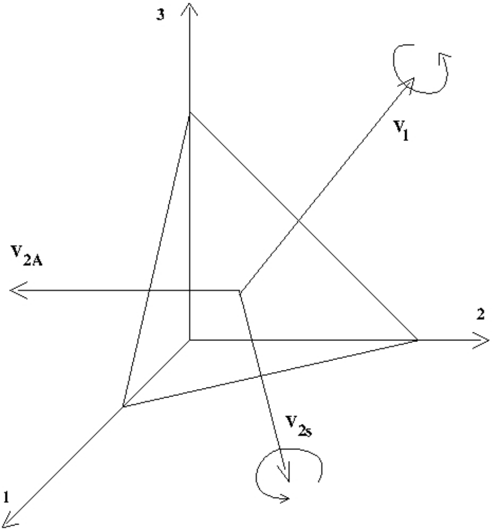
How do we choose a flavour symmetry?

- Find the smallest possible flavour symmetry suggested by the data
- Follow it to the end

The S_3 symmetry group: permutations of 3 objects.



Permutations | Rotations

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \iff$$ a rotation of 120° around the invariant vector $\mathbf{V}_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \iff$$ a rotation of 180° around the invariant vector $\mathbf{V}_{2S}$

The irreps of the group are:

- 1 dimension: $\mathbf{1}_A$, $\mathbf{1}_S$
- 2 dimensions: $\mathbf{2}$

The direct products between irreps are:

- $\mathbf{1}_S \otimes \mathbf{1}_S = \mathbf{1}_S$
- $\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_S$
- $\mathbf{1}_A \otimes \mathbf{1}_S = \mathbf{1}_A$
- $\mathbf{1}_S \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{2} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{1}_S \oplus \mathbf{1}_A$

The tensor product of two doublets:

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \qquad \textit{and} \qquad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

we have two singlets, r_S and r_A, and one doublet $\mathbf{r}_D$, where:

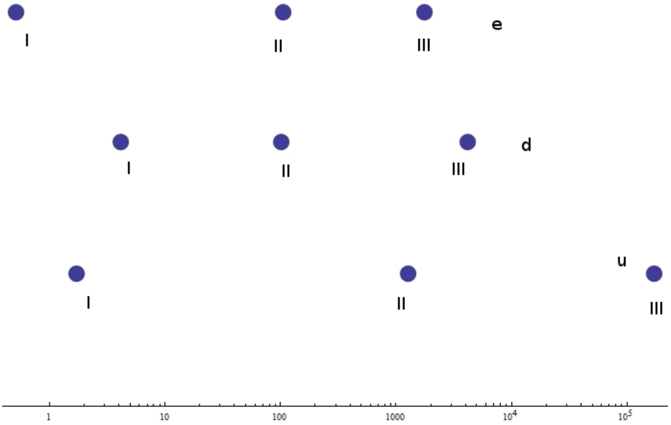
$r_S = p_{D1}q_{D1} + p_{D2}q_{D2}$ is invariant, $r_A = p_{D1}q_{D2} - p_{D2}q_{D1}$ is not invariant

and

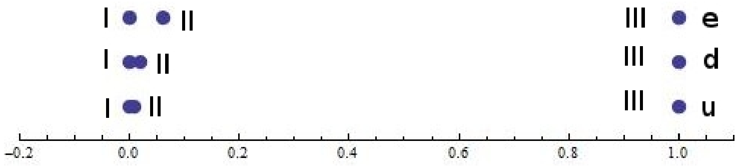
$$\mathbf{r}_D = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

is invariant.

Logarithmic plot of fundamental known fermion masses



Plot of fundamental fermion mass ratios



Fundamental fermions normalized by the heaviest of each type

Suggests $\mathbf{2} \oplus \mathbf{1}$ structure

Assignment between fermion fields and irreps:

$$\Phi \rightarrow F \quad = \quad F(\Phi_1, \Phi_2, \Phi_3)$$

F is a S_3 reducible representation $\mathbf{1_S} \oplus \mathbf{2}$

$$F_s \quad = \quad \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big)$$

$$F_D \quad = \quad \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Quarks and Leptons in the S_3 flavour model

Some references of works with an S_3 symmetry...

- S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)
- E. Derman, Phys. Rev. D19, 317 (1979)
- D. Wyler, Phys. Rev. D19, 330 (1979)
- R. Yahalom, Phys. Rev. D29, 536 (1984)
- A. Mondragón et al, Phys. Rev. D59, 093009, (1999)
- J. Kubo, A. Mondragón, et al, Prog. Theor. Phys. 109, 795 (2003)
- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)
- A. Mondragón et al, Phys. Rev. D76, 076003, (2007)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- T. Teshima, Phys.Rev. D85 105013 (2012)
- And many more... I apologize for those references I didn't include.

Facts

Some aspects of the flavour problem:

- Quark weak mixing angles (PDG 2010):
  - $\theta_{12} \approx 13.0^o$
  - $\theta_{23} \approx 2.4^o$
  - $\theta_{13} \approx 0.2^o$
- Lepton weak mixing angles
  best fit of recent experimental data
  (Schwetz, Tórtola, & Valle 2011, Forero, Tórtola, Valle, 2012):
  - $\Theta_{12} \approx 33.9^o$
  - $\Theta_{23} \approx 46.1^o$
  - $\Theta_{13} \approx 9.27^o (IH\ 9.45^o)$
    $\Rightarrow \Theta_{13} \neq 0$
- CP-violation occurs in the weak sector
- The mass hierarchy

- In the MS with S_3, if we give masses to the particles the flavor symmetry has to be broken

  (A. Mondragón, E. Rodríguez-Jáuregui, 1999, 2000)

- Possible to classify texture zeroes in equivalence classes, simplify analysis (A. Mondragón and F. González, 2011)

- If S_3 it's seen as conserved then... $\Rightarrow$

  we need to introduce additionally two Higgs weak-doublets more to the SM to preserve the permutational symmetry

  (J. Kubo, A. Mondragón, M. Mondragón, E. Rodríguez-Jáuregui, 2003)

- And, as a consequence, we get for every Dirac fermion the generic mass matrix:

$$\mathcal{M}_f = \begin{pmatrix} \mu_1 + \mu_2 & \mu_4 & \mu_5 \\ \mu_4 & \mu_1 - \mu_2 & \mu_6 \\ \mu_7 & \mu_8 & \mu_3 \end{pmatrix}$$

- Furthermore, the concepts of flavours and generations are taken to a more fundamental level.

The Lagrangian $\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}$

$$
\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\
&\quad -Y_2^d [\, \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \,] \\
&\quad -Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\
&\quad -Y_2^u [\, \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\
&\quad -Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\
&\quad -Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\
&\quad -Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\
&\quad -Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\
&\quad -Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.},
\end{aligned}
$$

and

$$
\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
$$

Quarks

Numerical study of quarks showed compatibility with data (Kubo, Mondragón, M, Rodríguez-Jáuregui, 2003)

FCNC's in quark sector are suppressed (Teshima, 2012)

A general study of the parameterization of the quark mass matrices and numerical analsys with recent data is in progress

(F. González, A. Mondragón, U. Saldaña, L. Velasco, 2012)

It is possible to parametrize the mixing matrices in terms of masses.

Starting from the matrix

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \sqrt{2}Y^f_2 v_S + Y^f_3 w_2 & Y^f_3 w_1 & \sqrt{2}Y^f_5 w_1 \\ Y^f_3 w_1 & \sqrt{2}Y^f_2 v_S - Y^f_3 w_2 & \sqrt{2}Y^f_5 w_2 \\ \sqrt{2}Y^f_6 w_1 & \sqrt{2}Y^f_6 w_2 & 2Y^f_1 v_S \end{pmatrix}$$

where w_1 and w_2 are the vev's of the Higgs doublets and v_S is the one of the singlet

For the Hermitian case

$$\mathcal{M}_H^f = \begin{pmatrix} 0 & |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta) & 0 \\ |\mu_2^f|\sin\theta\cos\theta(3-\tan^2\theta) & -2|\mu_2^f|\cos^2\theta(1-3\tan^2\theta) & \mu_8^f\sec\theta \\ 0 & \mu_8^{f*}\sec\theta & |\mu_3^f|-\Delta_f \end{pmatrix}$$

where $\Delta_f = |\mu_1^f| + |\mu_2^f|\cos^2\theta(1-3\tan^2\theta) << 1$, $\tan\theta = \dfrac{w_1}{w_2}$ and

$$\mu_1^f \equiv \sqrt{2}Y_2^f v_S,\;\; \mu_2^f \equiv Y_3^f w_2,\;\; \mu_3^f \equiv 2Y_1^f v_S,\;\; \mu_4^f \equiv Y_3^f w_1,\;\; \mu_5^f \equiv \sqrt{2}Y_4^f v_A,$$

$$\mu_6^f \equiv \sqrt{2}Y_5^f w_1,\;\; \mu_7^f \equiv \sqrt{2}Y_5^f w_2,\;\; \mu_8^f \equiv \sqrt{2}Y_6^f w_1,\;\; \mu_9^f \equiv \sqrt{2}Y_6^f w_2,$$

$$c\theta = \cos\theta \text{ and } s\theta = \sin\theta.$$

For the non-Hermitian case

$$\mathcal{M}^f_{NNI} = \begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu^f_2 & 0 \\ +\frac{2}{\sqrt{3}}\mu^f_2 & 0 & \frac{2}{\sqrt{3}}\mu^f_7 \\ 0 & \frac{2}{\sqrt{3}}\mu^f_8 & \mu^f_3 - \Delta_f \end{pmatrix}$$

where $|\Delta_f| = |\mu^f_1| << 1$.

F. González, A. Mondragón, U. Saldaña, L. Velasco,
work in progress 2012

See talks of L. Velasco-Sevilla "What's nu?" and U. Saldaña in PASCOS2012

best overall fit: three Higgs doublet model à la Kubo, Mondragón et al

Leptons

- In the leptonic sector we add a Z_2 symmetry
- FCNC's are strongly suppressed by the $S_3 \times Z_2$ symmetry and the strong mass hierarchy of the charged leptons
- Predictions for neutrino masses and mixings
- S_3 gives $\neq \Theta_{13}$

  A. Mondragón, M. Mondragón, E. Peinado, 2007,2008
- Compatible with recent data

  A. Mondragón, M. Mondragón, F. González, arXiv:1205.4755

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

The mass matrix of the charged leptons takes the form

$$\mathbf{M}_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}.$$

The resulting expression for $\mathbf{M}_e$, reparametrized in terms of its eigenvalues and written to order $\left(m_\mu m_e / m_\tau^2\right)^2$ and $x^4 = (m_e/m_\mu)^4$, is

$$\mathbf{M}_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

The mass matrix of the Dirac neutrinos takes the form

$$\mathbf{M}_{\nu_D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix},$$

The right handed neutrino mass matrix is

$$\mathbf{M}_{\nu_R} = \mathrm{diag}(M_1, M_2, M_3)$$

Then, the mass matrix $\mathbf{M}_{\nu_L}$ takes the form

$$\mathbf{M}_{\nu_L} = \begin{pmatrix} \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} \\ \frac{2\lambda\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\left(\mu_2^\nu\right)^2}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu \lambda}{\overline{M}} \\ \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu \lambda}{\overline{M}} & \frac{2\left(\mu_4^\nu\right)^2}{\overline{M}} + \frac{\left(\mu_3^\nu\right)^2}{M_3} \end{pmatrix}, \quad \begin{array}{l} \lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \text{ and} \\ \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}. \end{array}$$

We diagonalize this matriz in the following way

$$\mathbf{M}_{\nu_L} = \mathbf{Q}\mathcal{U}_{\frac{\pi}{4}} \left(\mu_0 \mathbf{I}_{3\times 3} + \widehat{\mathbf{M}}\right) \mathcal{U}^{\dagger}_{\frac{\pi}{4}} \mathbf{Q}, \tag{1}$$

where $\mathbf{Q} = e^{i\phi_2}\mathrm{diag}\left\{1, 1, e^{i\delta_\nu}\right\}$ with $\delta_\nu = \phi_4 - \phi_2 = \arg\{\mu_4^\nu\} - \arg\{\mu_2^\nu\}$,

$$\mathcal{U}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}, \mu_0 = \frac{2\left|\mu_2^\nu\right|^2}{\left|\overline{M}\right|}\left(1 - |\lambda|\right), \widehat{\mathbf{M}} = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & 2d \end{pmatrix} \tag{2}$$

with $A = \sqrt{2}\frac{\left|\mu_2^\nu\right|\left|\mu_4^\nu\right|}{\left|\overline{M}\right|}\left(1 - |\lambda|\right)$, $B = \frac{2\left|\mu_4^\nu\right|^2}{\left|\overline{M}\right|} + \frac{\left|\mu_3^\nu\right|^2}{M_3} - \frac{2\left|\mu_2^\nu\right|^2}{\left|\overline{M}\right|}\left(1 - |\lambda|\right)$,
$C = \sqrt{2}\frac{\left|\mu_2^\nu\right|\left|\mu_4^\nu\right|}{\left|\overline{M}\right|}\left(1 + |\lambda|\right)$ and $d = \frac{2|\lambda|\left|\mu_2^\nu\right|^2}{\left|\overline{M}\right|}$.

Then, the mass matrix $\mathbf{M}_{\nu_L}$ for a normal [inverted] hierarchy in the mass spectrum takes the form

$$\mathbf{M}_{\nu_L}^{N[I]} = \begin{pmatrix} \mu_0 + d & d & \frac{1}{\sqrt{2}}\left(C^{N[I]} + A^{N[I]}\right) \\ d & \mu_0 + d & \frac{1}{\sqrt{2}}\left(C^{N[I]} - A^{N[I]}\right) \\ \frac{1}{\sqrt{2}}\left(C^{N[I]} + A^{N[I]}\right) & \frac{1}{\sqrt{2}}\left(C^{N[I]} - A^{N[I]}\right) & m_{\nu_1} + m_{\nu_2} + m_{\nu_3} - 2\left(\mu_0 + d\right) \end{pmatrix} \quad (3)$$

with $C^{N[I]} = \sqrt{\frac{\left(2d+\mu_0-m_{\nu_1}\right)\left(2d+\mu_0-m_{\nu_{2[3]}}\right)\left(m_{\nu_{3[2]}}-\mu_0-2d\right)}{2d}}$ and $A^{N[I]} = \sqrt{\frac{\left(m_{\nu_2}-\mu_0\right)\left(m_{\nu_{3[1]}}-\mu_0\right)\left(\mu_0-m_{\nu_{1[3]}}\right)}{2d}}$.

The values allowed for the parameters μ_0 and $2d + \mu_0$ are in the following ranges: $m_{\nu_{2[1]}} > \mu_0 > m_{\nu_{1[3]}}$ and $m_{\nu_{3[2]}} > 2d + \mu_0 > m_{\nu_{2[1]}}$.

This way it is possible to write the mixing angles in terms of the lepton masses

(A. Mondragón, M. M., E. Peinado, 2007, 2008
Review: F. González, A. Mondragón, M.M., 2012)

For the reactor mixing angle θ^l_{13} and for an inverted neutrino mass hierarchy $(m_{\nu_2} > m_{\nu_1} > m_{\nu_3})$ we obtain:

$$\sin^2\theta^l_{13} \approx \frac{(\mu_0 + 2d - m_{\nu_3})(\mu_0 - m_{\nu_3})}{(m_{\nu_1} - m_{\nu_3})(m_{\nu_2} - m_{\nu_3})}.$$

Preliminary analysis for the reactor mixing angle θ^l_{13} and for a normal neutrino mass hierarchy

$m_{\nu_1} = 3.22 \times 10^{-3}\ \mathrm{eV}$, $m_{\nu_2} = 9.10 \times 10^{-3}\ \mathrm{eV}$, $m_{\nu_3} = 4.92 \times 10^{-2}\ \mathrm{eV}$

and the parameter values

$\delta_l = \pi/2$, $\mu_0 = 0.049$ eV and $d = 8 \times 10^{-5}$ eV,

gives

$$\sin^2\theta^l_{13} \approx 0.029 \longrightarrow \theta^l_{13} \approx 10.8^\circ,$$

in good agreement with experimental data.

The solar and atmospheric mixing angles:

$$\theta^{l^{th}}_{12} = 35^\circ, \quad \theta^{l^{th}}_{23} = 46^\circ,$$

When $d = 0 \Rightarrow M_1 = M_2$ gives lower bound for θ_{13}

recover previous results of Mondragón, M, Peinado

Complete analysis of data with χ^2 underway

(F. González, A. Mondragón)

Multi-Higgs models and flavour symmetries

- Interesting work has been done on stability of multi-Higgs models with flavour symmetries
- Also in their properties concerning CP violation
- Of immediate interest to us: adding discrete symmetries can imply continuos symmetries $\Rightarrow$ Goldstone bosons

Lavoura et al, 1994; Barroso et al; 2004, 2006, 2007, Branco et al, 2005; Ferreira et al, 2005, 2010,2011;

The Higgs potential in the S_3 flavour model

Some references of works with an S_3 invariant Higgs potential...

- S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978)
- E. Derman, Phys. Rev. D19, 317 (1979)
- D. Wyler, Phys. Rev. D19, 330 (1979)
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- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- S. Chen et al, Phys. Rev. D70, 073008 (2004)
- O. Félix-Beltrán, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)
- Again, there are many more, I apologize for those not included.

In what sense are we asking: Which is the **most general** S_3-invariant Higgs potential?

- It has the **highest** level of flavour symmetry.
- It has the **highest** arbitrariness without breaking the flavour symmetry.
- Crucial to phenomenology $\Rightarrow$ consistency is essential

Two essential things to work it out were:

The tensorial products between irreps:

To carefully carry the weak ($SU(2)_L$) index.

Follow the symmetry,
Ulises...

1. Find out all the **l.i.** S_3-invariant terms for **2** and **4** scalar fields.

$n = 2$:

- $\mathbf{1_S} \otimes \mathbf{1_S}$
- $[\mathbf{2} \otimes \mathbf{2}]_{\mathbf{S}}$

$n = 4$:

- $\mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{1_S} \otimes \mathbf{2})]_{\mathbf{S}}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{2} \otimes \mathbf{2})_{\mathbf{2}}]_{\mathbf{S}}$
- $(\mathbf{2} \otimes \mathbf{2})_{\mathbf{A}} \otimes (\mathbf{2} \otimes \mathbf{2})_{\mathbf{A}}$
- $(\mathbf{2} \otimes \mathbf{2})_{\mathbf{S}} \otimes (\mathbf{2} \otimes \mathbf{2})_{\mathbf{S}}$
- $[(\mathbf{2} \otimes \mathbf{2})_{\mathbf{2}} \otimes (\mathbf{2} \otimes \mathbf{2})_{\mathbf{2}}]_{\mathbf{S}}$

2. Take an explicit convention for the whole theory (Yukawa Lagrangian and Higgs potential) of where to place the symmetric and antisymmetric doublet components.

$$H_D = \begin{pmatrix} H_{DA} \\ H_{DS} \end{pmatrix}$$

$$(f_{DA}, f_{DS})^T \otimes (g_{DA}, g_{DS})^T = \frac{1}{\sqrt{2}}(f_{DA}g_{DA} + f_{DS}g_{DS})_{\mathbf{1}_S}$$
$$\oplus \frac{1}{\sqrt{2}}(f_{DA}g_{DS} - f_{DS}g_{DA})_{\mathbf{1}_A}$$

$$\oplus \frac{1}{\sqrt{2}} \begin{pmatrix} f_{DA}g_{DS} + f_{DS}g_{DA} \\ f_{DA}g_{DA} - f_{DS}g_{DS} \end{pmatrix}_{\mathbf{2}}$$

3. For each S_3-invariant term make all the different independent weak indexes contractions. For example:

- $(\mathbf{2}\otimes\mathbf{2})_\mathbf{S}\otimes(\mathbf{2}\otimes\mathbf{2})_\mathbf{S}=\mathbf{1}_{\mathbf{S_3}}$
  - $(\mathbf{2_w}\otimes\mathbf{2_w})_\mathbf{S}\otimes(\mathbf{2_{w'}}\otimes\mathbf{2_{w'}})_\mathbf{S}=\mathbf{1}_{\mathbf{S_3\otimes G_{SM}}}$
  - $(\mathbf{2_w}\otimes\mathbf{2_{w'}})_\mathbf{S}\otimes(\mathbf{2_w}\otimes\mathbf{2_{w'}})_\mathbf{S}=\mathbf{1}_{\mathbf{S_3\otimes G_{SM}}}$
  - $(\mathbf{2_w}\otimes\mathbf{2_{w'}})_\mathbf{S}\otimes(\mathbf{2_{w'}}\otimes\mathbf{2_w})_\mathbf{S}=\mathbf{1}_{\mathbf{S_3\otimes G_{SM}}}$
- In terms of the Higgs fields:
  - $\frac{1}{2}(H^\dagger_{1w}H_{1w}+H^\dagger_{2w}H_{2w})^2$
  - $\frac{1}{2}[(H^\dagger_{1w}H_{1w})^2+(H^\dagger_{2w}H_{2w})^2+(H^\dagger_{1w}H_{2w})^2+(H^\dagger_{2w}H_{1w})^2]$
  - $\frac{1}{2}[(H^\dagger_{1w}H_{1w})^2+(H^\dagger_{2w}H_{2w})^2+(H^\dagger_{1w}H_{2w})^2+(H^\dagger_{2w}H_{1w})^2]$

4. Assign the same self-coupling parameter for each different contraction of the same S_3-invariant term.

We define

$$x_1 = H^{\dagger_1} H_1, \quad x_4 = \mathcal{R}\left(H^{\dagger_1} H_2\right), \quad x_7 = \mathcal{I}\left(H^{\dagger_1} H_2\right),$$
$$x_2 = H_2^{\dagger} H_2, \quad x_5 = \mathcal{R}\left(H^{\dagger_2} H_S\right), \quad x_8 = \mathcal{I}\left(H^{\dagger_1} H_S\right),$$
$$x_3 = H_S^{\dagger} H_S, \quad x_6 = \mathcal{R}\left(H^{\dagger_1} H_S\right), \quad x_9 = \mathcal{I}\left(H^{\dagger_2} H_S\right),$$

then the potential is

$$\begin{aligned} V = \; & \mu_D^2 (x_1 + x_2) + \mu_s^2 x_3 + a x_3^2 + \\ & b[x_3 (x_1 + x_2) + 3 (x_6^2 + x_5^2) - (x_8^2 + x_9^2)] \\ & + 2c (2 x_4 x_6 + x_5 (x_1 - x_2)) \\ & + d[2(x_1 + x_2)^2 + (x_1 - x_2)^2 + 4(x_4^2 - x_7^2)] \\ & - 4 e x_7^2 + f[(x_1 - x_2)^2 + 2(x_1 + x_2)^2] + 4(x_4^2 - 2x_7^2)] \end{aligned}$$

The three Higgs doublets H_1, H_2 and H_S can be written as

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \; H_2 = \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \; H_S = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix},$$

where S is the flavour index for the S_3 Higgs field singlet.

Stationary points

The potential has three types of stationary points

- The normal minimum with the following field configuration:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9$$

- The stationary point which breaks electric charge, here two of the charged fields ϕ acquire non zero vev's :

$$\phi_7 = v_1', \phi_8 = v_2', \phi_9 = v_3', \phi_1 = \alpha, \phi_3 = \beta,$$

- The *CP* breaking minimum, where two imaginary components of the neutral fields ϕ acquire non zero vev's.

$$\phi_7 = v_1'', \phi_8 = v_2'', \phi_9 = v_3'', \phi_{10} = \delta, \phi_{11} = \gamma,$$

- After the electroweak symmetry breaking we have the following massive Higgses:
- 4 charged ones
- 3 neutral ones
- 2 neutral pseudoscalar ones
- 3 Goldstone bosons to give mass to the $W^{\pm}$ and Z

- We look at the normal minimum, i.e. no CP or charge breaking
- For the case $c = 0$, i.e. no mixing between the singlet and doublet Higgs:
  We find a rotational symmetry in this case at the minimum $w_1^2 + w_2^2 = r^2$
- Accidental S_2 symmetry (consistent with Sugawara and Pakvasa)
- Stability of this potential and study of the minima already analyzed (D. Emmanuel-Costa, O. Félix, M.M., E. Rodríguez-Jáuregui, 2006)
- The case with $c \neq 0$ gives a relation between the three vev's seems consistent with more general quark mass matrices analysis
- We derive the mass matrices and find the eigenvalues crucial for phenomenology
- The other two minima have to be analyzed and compared too (work in progress)

Conclusions

- The permutational symmetry S_3 with extended Higgs sector accomodates very well the quark and lepton masses, reducing the number of free parameters
- Allows a "unified" treatment of quark, lepton and Higgs sectors
- Possible to find analytical expressions for mixing matrices in terms of masses
- Gives predictions in the neutrino sector
  mixing angles in terms of masses
  in particular $\Theta_{13} \neq 0$
  and consistent with experimental data

Conclusions

- Further predictions will come from the Higgs sector
- Essential to define correctly the potential
- In our case: maximum degree of symmetry
- The normal minimum, without mixing of singlet and doublet, has an accidental S_2 symmetry
- Mixing of doublet and singlet appears consistent with more general analysis of quark masses
- Compare the three stationary points: normal, CP and charge breaking ones
- Look at the constraints that are imposed on the vev's and couplings from internal consistency and experiment
- Leptogenesis (with Arturo Alvarez) possible and consistent with above
- SUSY $SU(5) \times Q_6$ (with J.C. Gomez and F. Gonzalez) also gives good results