UNIVERSITY OF
Southampton
School of Physics
and Astronomy

Flavour Symmetry Models after Daya Bay and RENO

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Dortmund 1st July, 2012

Theory Road Map

Daya Bay/RENO

Family Symmetry

Anarchy

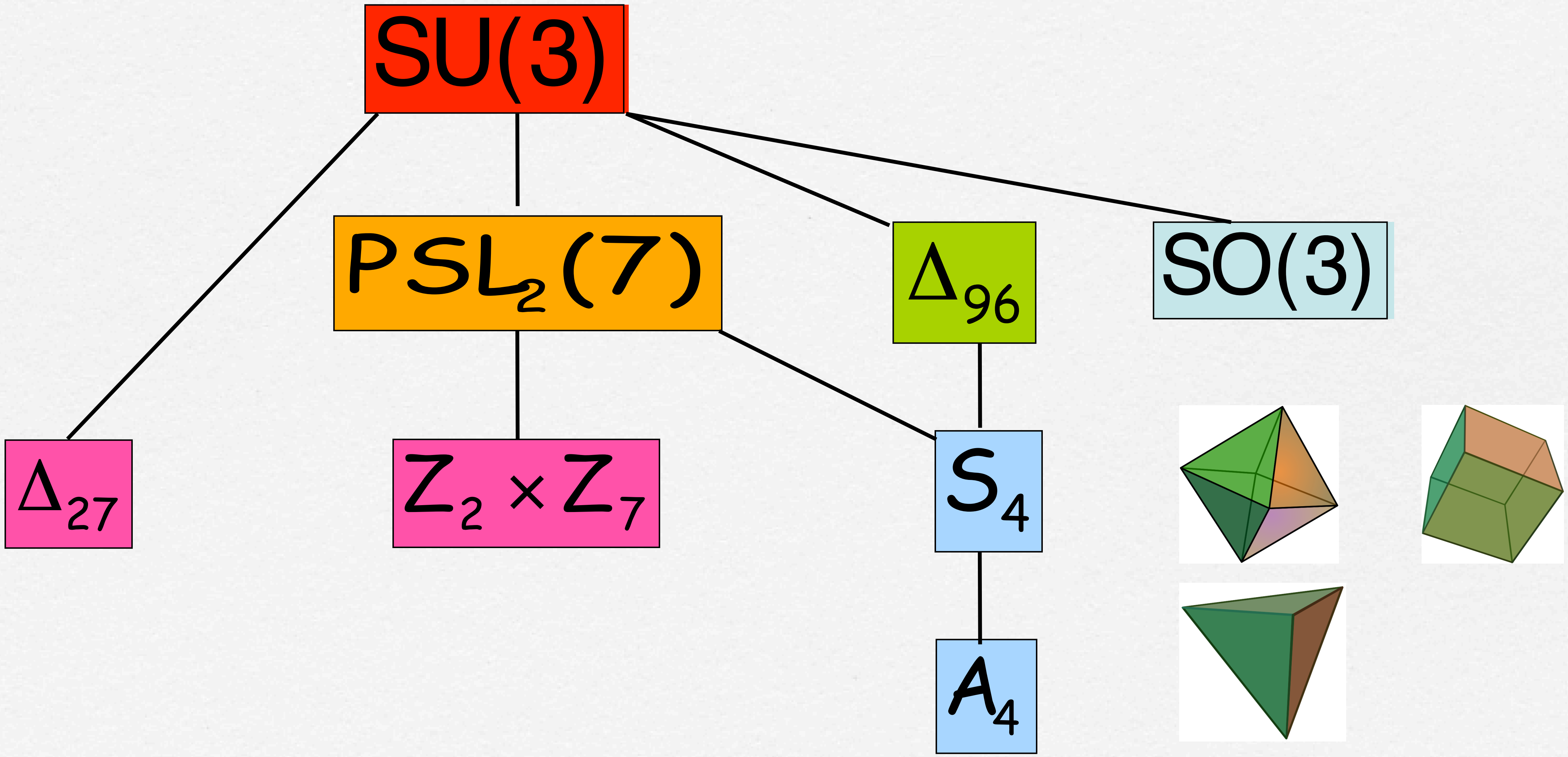
Direct

Indirect

Landscape

Family Symmetry

Family Symmetries G_F
which contain triplet reps
(three families in a triplet)



Partial list of authors who have worked on symmetry approach to large θ_{13}

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(sincere apologies for incompleteness)

Altarelli, Feruglio, Ma, Hagedorn, Merlo, Luhn, ...

The Direct Approach

Family Symmetry G_F broken in special way

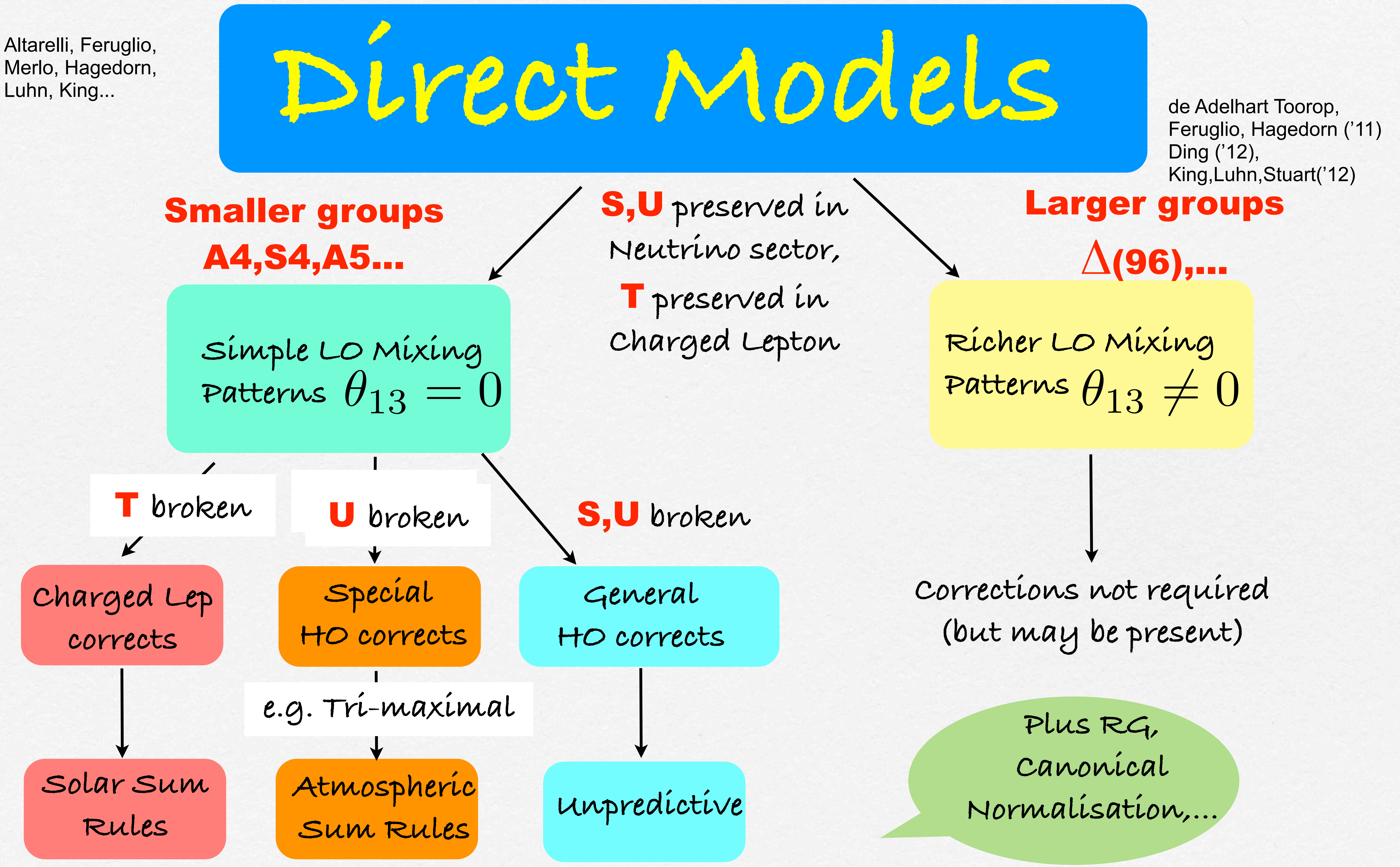
Subgroups preserved in neutrino/charged lepton sectors

$G_F \supset S,T,U$ ← Group generators

"Klein symmetry" — **S,U** only preserved → **Neutrino Mass Matrix**

T only preserved → **Charged lepton Mass Matrix**

Neutrino Mass Matrix ← U_{PMNS} → **Charged lepton Mass Matrix**



Altarelli, Feruglio, Merlo, Hagedorn, Luhn, King...
Direct Models
de Adelhart Toorop, Feruglio, Hagedorn ('11) Ding ('12), King,Luhn,Stuart('12)
Smaller groups
A4,S4,A5...
S,U preserved in Neutrino sector,
T preserved in Charged Lepton
Larger groups
Δ(96),...
Simple LO Mixing Patterns $\theta_{13} = 0$
Richer LO Mixing Patterns $\theta_{13} \neq 0$
T broken
U broken
S,U broken
Charged Lep corrects
Special HO corrects
General HO corrects
Corrections not required (but may be present)
e.g. Tri-maximal
Solar Sum Rules
Atmospheric Sum Rules
Unpredictive
Plus RG, Canonical Normalisation,...

Simple LO Mixing Patterns

$$\theta_{13} = 0 \quad \theta_{23} = 45^o$$

- Bimaximal

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

V. Barger, S. Pakvasa, T. Weiler and K. Whisnant

- Tri-bimaximal

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\theta_{12} = 35.26^o$$

Harrison, Perkins and Scott

- Golden ratio

$$U_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

Datta, Ling, Ramond; Kajirama, Raidal, Strumia; Everett, Stuart,Ding: Feruglio, Paris

$$\phi = \frac{1+\sqrt{5}}{2} \qquad \tan\theta_{12} = \frac{1}{\phi} \qquad \theta_{12} = 31.7^o$$

Large Charged Lepton Corrections to the rescue

$\theta_{12}^e \approx \theta_C$

- BM, TBM, GR might only apply to neutrino mixing and $U_{PMNS} = U_e U_\nu^\dagger$ implies $\theta_{13} \approx \frac{\theta_{12}^e}{\sqrt{2}}$

- Solar Sum Rules

Sum Rule: King ('05); Masina ('05); Antusch, King ('05)

Charged Lepton Corrections: King ('02), Frampton, Petcov, Rodejohann ('04), Altarelli, Feruglio, Masina ('04), Antusch, King ('04), Ferrandis, Pakvasa ('04), Feruglio ('05), Datta, Everett, Ramond ('05), Mohapatra, Rodejohann ('05) Antusch, Maurer ('11) Mazocca, Petcov, Romanino, Spinrath ('11)

- Bimaximal $\theta_{12} = 45^o + \theta_{13} \cos\delta \to \delta \approx \pi$

- Tri-bimaximal $\theta_{12} = 35^o + \theta_{13} \cos\delta \to \delta \approx \pm\frac{\pi}{2}$

- Golden ratio $\theta_{12} = 32^o + \theta_{13} \cos\delta$

N.B. $\theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9.2^o$

King 1205.0506

See also: Antusch,Gross, Maurer, Sluka 1205.1051; Relation also appears in "Quark Lepton Complementarity" ... : Minakata, Smirnov ('04); and "Cabibbo Haze" Everett, Ramond ('05)

King; Parke; Pakvasa, Rodejohann, Weiler

Tri-Bimaximal Parametrisation

$$U_{\rm PMNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix} P$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)\ , \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)\ , \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

$$s = -0.03 \pm 0.03 \qquad a = -0.02 \pm 0.10 \qquad r = 0.22 \pm 0.02$$

s = solar a = atmospheric r = reactor

Allows for deviations from TB mixing

E.g. TB solar sum rule recast as $s=r.\cos\delta$

Tri-bimaximal Hydras

- Tri-bimaximal (s=a=r=0)

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

Harrison, Perkins, Scott

- Tri-bimaximal-reactor (s=a=0)

$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

King; Antusch,Boudjemaa,King; Morisi,Patel,Peinado; Luhn, King

- Tri-maximal 1 (s=0, a=r.cosδ)

$$U_{\mathrm{TM}_1} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1-\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+re^{-i\delta}) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1+\frac{3}{2}re^{i\delta}) & -\frac{1}{\sqrt{2}}(1-re^{-i\delta}) \end{pmatrix} P$$

Lam; Albright,Rodejohann; Antusch, King, Luhn, Spinrath

- Tri-maximal 2 (s=0, a=-r/2.cosδ)

Haba,Watanabe,Yoshioka; He,Zee; Grimus,Lavoura; Albright,Rodejohann; King, Luhn

$$U_{\mathrm{TM}_2} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(1-\frac{1}{2}re^{-i\delta}) \\ -\frac{1}{\sqrt{6}}(1-\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(1+\frac{1}{2}re^{-i\delta}) \end{pmatrix} P$$

N.B. Atmospheric sum rules: a=r.cosδ, a=-r/2.cosδ

Indirect Models

King, Ross, de Medeiros Varzielas, Antusch, Malinsky,...

Starting point is type I see-saw

$$m_{LR} = \begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix} \quad M_{RR} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \longrightarrow m^{\nu} = \frac{AA^T}{M_1} + \frac{BB^T}{M_2} + \frac{CC^T}{M_3}$$

$$A^T = (A_1, A_2, A_3) \quad B^T = (B_1, B_2, B_3)$$

Promote the columns (A,B,C) to dynamical fields

G_F yields special vacuum alignments, for example:

- (A,B,C) proportional to columns of PMNS called Form Dominance (FD) — Chen, King('09)
- $AA^T/M_1 \ll BB^T/M_2 \ll CC^T/M_3$ called Sequential Dominance (SD) — King('98,'02)
- SD with B=b(1,1,-1) and C=c(0,1,1) called Constrained SD gives TB Mixing — King('05)
- SD with B=b(1,1,-1) and C=c(r,1,1) called Partially CSD gives TBR mixing — King('09), King,Luhn('11)
- SD with B=b(1,2,0) and C=c(0,1,1) called CSD2 gives TM1 mixing — Antusch, King, Luhn, Spinrath ('11)

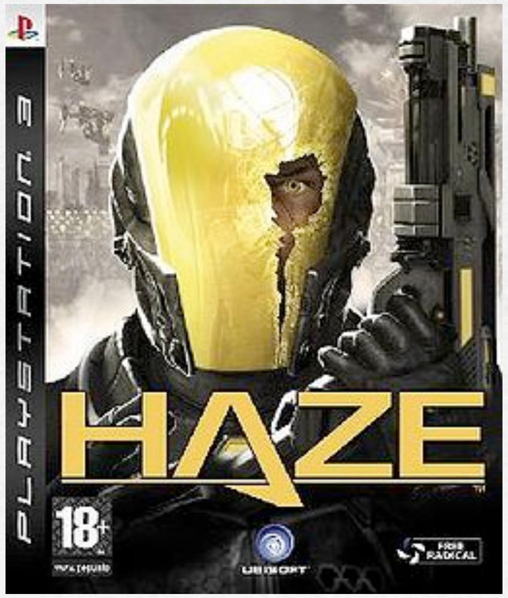
King 1205.0506

Tri-bimaximal-Cabibbo Mixing

Combine TB mixing with $\theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9.2^o$

$$s_{13} = \frac{\lambda}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}, \quad s_{23} = \frac{1}{\sqrt{2}} \qquad \lambda = 0.2253 \pm 0.0007$$

$$U_{TBC} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{2}}\lambda e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \\ \frac{1}{\sqrt{6}}(1-\lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \end{pmatrix}$$



Describes all current data!

Obtained from PCSD with B=b(1,1,-1) and C=c(λ,1,1)

Conclusions

- With large θ_{13}, still two theory approaches: Symmetry or Anarchy
- Family Symmetry may be implemented Directly or Indirectly
- Simplest Direct models A4,S4,A5 with S,U and T conservation predict Bimaximal, Tri-bimaximal, Golden Ratio at LO
- However T broken in GUT models due to Charged Lepton corrections, (Cabibbo-like charged lepton angle required) imply solar sum rules
- U breaking at HO leads to TM2 mixing, atmospheric sum rules
- Larger Finite Groups such as Delta(96) predict e.g. $\theta_{13} \sim 12^{o}$ at LO
- Indirect family symmetry models can lead to TM1 or TBC mixing
- Vital to measure the mixing angles and CP phase delta to good precision to test the sum rules, hence discriminate between models, decide if the Universe is based on Symmetry or if Anarchy Rules

Summary of Sum Rule Predictions

- Quark-Lepton Complementarity $\theta_{12} + \theta_C = 45^o$
- Solar sum rules
  - Bimaximal $\theta_{12} = 45^o + \theta_{13} \cos\delta$
  - Tri-bimaximal $\theta_{12} = 35^o + \theta_{13} \cos\delta$
  - Golden Ratio $\theta_{12} = 32^o + \theta_{13} \cos\delta$

Plus HO corrections...

- Atm. sum rules
  - Tri-bimaximal-Cabibbo $\theta_{12} = 35^o \quad \theta_{23} = 45^o$, $\theta_{13} = \theta_C/\sqrt{2} = 9.2^o$
  - Trimaximal1 $\theta_{23} = 45^o + \sqrt{2}\theta_{13} \cos\delta$
  - Trimaximal2 $\theta_{23} = 45^o - \frac{\theta_{13}}{\sqrt{2}} \cos\delta$

Plus Charged Lepton Corrections...

Now that θ_{13} is measured these predict $\cos\delta$