TFH Mixing Patterns, Large θ_{13} and $\Delta(96)$ Flavor Symmetry

Gui-Jun Ding

Department of Modern Physics, University of Science and Technology of China

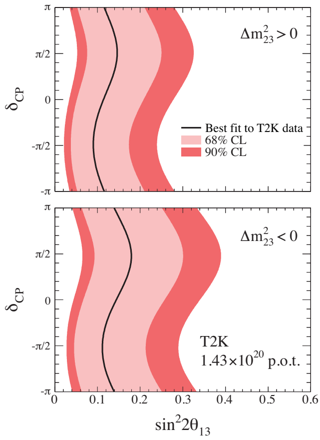
based on Nucl.Phys. B862 (2012) 1-42, arXiv:1201.3279 [hep-ph]

FLASY2012, Jul 1, 2012, Dortmund

Outline

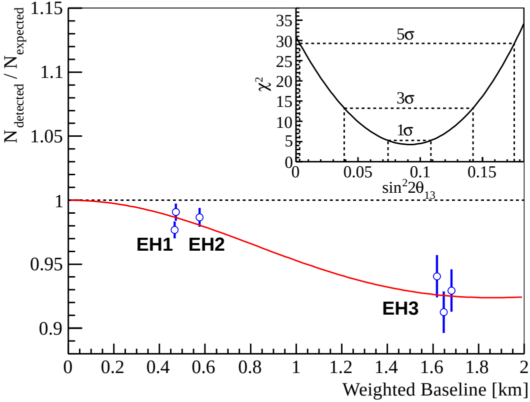
Results from T2K (on Jun,2011)

- At 90% C.L., $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for $\delta_{CP} = 0$ and normal (inverted) hierarchy.



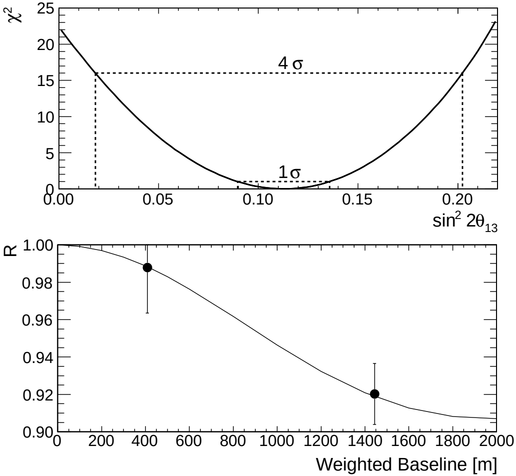
Results from Daya Bay (on Mar,2012)

- The best-fit($\pm 1\sigma$) result is $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\mathrm{stat}) \pm 0.005(\mathrm{syst})$, θ_{13} is non-zero at 5.2σ.



Results from RENO (on Apr,2012)

- The best-fit($\pm 1\sigma$) result is $\sin^2 2\theta_{13} = 0.113 \pm 0.013(\mathrm{stat}) \pm 0.019(\mathrm{syst})$, θ_{13} is non-zero at 4.9σ.



- Present neutrino oscillation parameters

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34-2.69$ $-(2.25-2.59)$	$2.26-2.77$ $-(2.15-2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$\left(0.83^{+0.54}_{-0.64}\right)\pi$ 0.07π	$0-2\pi$	$0-2\pi$

Taken from Forero,Tortola and Valle, arXiv:1205.4018.

Fogli, Lisi *et al.*, arXiv:1205.5254 : $0.0149(0.015) \leq \sin^2\theta_{13} \leq 0.0344(0.0347)$ at 3σ

How to understanding largish θ_{13} from flavor symmetry?

- Some LO textures outside the range of the global fitting values + largish subleading corrections (G.Altarelli et el,JHEP 0905 (2009) 020)

$$U_0 = U_{BM} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \delta\theta_{12} \sim \lambda_c, \quad \delta\theta_{13} \sim \lambda_c, \quad \delta\theta_{23} \sim \lambda_c^2$$

- Starting from some new textures with large θ_{13}(F.Feruglio et al.,Phys.Lett. B703 (2011) 447-451,Nucl.Phys. B858 (2012))

$$U_0 = U_{TFH} \equiv \begin{pmatrix} \frac{1}{6}(3+\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(-3+\sqrt{3}) \\ \frac{1}{6}(-3+\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(3+\sqrt{3}) \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix} \Longleftarrow \Delta(96)$$

$$\sin^2\theta_{12} = (8-2\sqrt{3})/13, \sin^2\theta_{23} = (5+2\sqrt{3})/13, \sin^2\theta_{13} = (2-\sqrt{3})/6$$

Group theory of $\Delta(96)$

- $\Delta(96)$ is a non-abelian finite subgroup of SU(3) of order 96, it is isomorphic to $(Z_4 \times Z_4) \rtimes S_3$, and it can be conveniently defined by four generators a, b, c and d obeying the relations:

$$
\begin{aligned}
S_3: &\quad a^3 = b^2 = (ab)^2 = 1 \\
Z_4 \times Z_4: &\quad c^4 = d^4 = 1, \quad cd = dc \\
\rtimes: &\quad aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c \\
&\quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}\,.
\end{aligned}
$$

  The generator d is not independent.

- $\Delta(96)$ has 10 irreducible representations:

  Two singlets: $\mathbf{1}$ and $\mathbf{1}'$

  One doublet: $\mathbf{2}$

  Six triplets: $\mathbf{3_1}$, $\mathbf{3_1'}$, $\overline{\mathbf{3}}_\mathbf{1}$, $\overline{\mathbf{3}}_\mathbf{1}'$, $\mathbf{3_2}$ and $\mathbf{3_2'}$

  One sextet: $\mathbf{6}$

Pathway to TFH mixing within $\Delta(96)$

- In the minimalist framework(A. Zee, Phys. Lett. B 630, 58 (2005)), the charged lepton masses are generated by the operator:

$$O_\ell = E^c \ell h_d \phi_\ell / \Lambda$$

ϕ_ℓ is the flavon field breaking $\Delta(96)$ in the charged lepton sector at LO. Neutrino masses are generated by the effective Weinberg operator:

$$O_\nu = \ell h_u \ell h_u \phi_\nu / \Lambda^2$$

$$\boxed{\text{The mismatch between the symmetry breaking induced by } \phi_\ell \text{ and } \phi_\nu}$$
$$\Updownarrow$$
$$\boxed{\text{TFH Mixing}}$$

There are numerous ways to produce the TFH mixing in $\Delta(96)$.

- Possible realizations of TFH mixing

$$\langle\phi_\ell\rangle=\left\{\begin{array}{ll}(0,0,v), & \phi_\ell\sim\mathbf{3_1},\mathbf{3'_1},\mathbf{\overline{3}_1},\mathbf{\overline{3}'_1},\mathbf{3_2},\mathbf{3'_2}\\(0,0,v_3,0,0,v_6), & \phi_\ell\sim\mathbf{6}\end{array}\right\}\textit{break }\Delta(96)\to Z_3$$

$$\langle\phi_\nu\rangle=\left\{\begin{array}{ll}(1,1,1)u, & \phi_\nu\sim\mathbf{3'_1},\mathbf{\overline{3}'_1}\\(u_1,u_2,(u_1+u_2)/2), & \phi_\nu\sim\mathbf{3'_2}\end{array}\right\}\textit{break }\Delta(96)\to K_4$$

ℓ	E^c	ϕ_ℓ	ϕ_ν
$\mathbf{3_1}$	$\tau^c\sim\mathbf{1}$, $(\mu^c,e^c)\sim\mathbf{2}$	$\mathbf{\overline{3}_1}$, $\mathbf{\overline{3}'_1}$	$\mathbf{3'_1}$, $\mathbf{3'_2}$
	$\tau^c\sim\mathbf{1'}$, $(\mu^c,e^c)\sim\mathbf{2}$	$\mathbf{\overline{3}_1}$, $\mathbf{\overline{3}'_1}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3_1}$	$\mathbf{3_1}$, $\mathbf{3'_1}$, $\mathbf{3'_2}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3'_1}$	$\mathbf{3_1}$, $\mathbf{3'_1}$, $\mathbf{3_2}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{\overline{3}_1}$	$\mathbf{1}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{\overline{3}'_1}$	$\mathbf{1'}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3_2}$	$\mathbf{3'_1}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3'_2}$	$\mathbf{3_1}$, $\mathbf{6}$	
$\mathbf{3'_1}$	$\tau^c\sim\mathbf{1}$, $(\mu^c,e^c)\sim\mathbf{2}$	$\mathbf{\overline{3}_1}$, $\mathbf{\overline{3}'_1}$	$\mathbf{3'_1}$, $\mathbf{3'_2}$
	$\tau^c\sim\mathbf{1'}$, $(\mu^c,e^c)\sim\mathbf{2}$	$\mathbf{\overline{3}_1}$, $\mathbf{\overline{3}'_1}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3_1}$	$\mathbf{3_1}$, $\mathbf{3'_1}$, $\mathbf{3_2}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3'_1}$	$\mathbf{3_1}$, $\mathbf{3'_1}$, $\mathbf{3'_2}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{\overline{3}_1}$	$\mathbf{1'}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{\overline{3}'_1}$	$\mathbf{1}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3_2}$	$\mathbf{3_1}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{3'_2}$	$\mathbf{3'_1}$, $\mathbf{6}$	

ℓ	E^c	ϕ_ℓ	ϕ_ν
$\mathbf{\bar{3}_1}$	$\tau^c \sim \mathbf{1}$, $(e^c,\mu^c)\sim\mathbf{2}$	$\mathbf{3_1}$, $\mathbf{3'_1}$	$\mathbf{\bar{3}'_1}$, $\mathbf{3'_2}$
	$\tau^c \sim \mathbf{1'}$, $(e^c,\mu^c)\sim\mathbf{2}$	$\mathbf{3_1}$, $\mathbf{3'_1}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3_1}$	$\mathbf{1}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3'_1}$	$\mathbf{1'}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{\bar{3}_1}$	$\mathbf{\bar{3}_1}$, $\mathbf{\bar{3}'_1}$, $\mathbf{3'_2}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{\bar{3}'_1}$	$\mathbf{\bar{3}_1}$, $\mathbf{\bar{3}'_1}$, $\mathbf{3_2}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3_2}$	$\mathbf{\bar{3}'_1}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3'_2}$	$\mathbf{\bar{3}_1}$, $\mathbf{6}$	
$\mathbf{\bar{3}'_1}$	$\tau^c \sim \mathbf{1}$, $(e^c,\mu^c)\sim\mathbf{2}$	$\mathbf{3_1}$, $\mathbf{3'_1}$	$\mathbf{\bar{3}'_1}$, $\mathbf{3'_2}$
	$\tau^c \sim \mathbf{1'}$, $(e^c,\mu^c)\sim\mathbf{2}$	$\mathbf{3_1}$, $\mathbf{3'_1}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3_1}$	$\mathbf{1'}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3'_1}$	$\mathbf{1}$, $\mathbf{6}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{\bar{3}_1}$	$\mathbf{\bar{3}_1}$, $\mathbf{\bar{3}'_1}$, $\mathbf{3_2}$	
	$(\mu^c,e^c,\tau^c)\sim\mathbf{\bar{3}'_1}$	$\mathbf{\bar{3}_1}$, $\mathbf{\bar{3}'_1}$, $\mathbf{3'_2}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3_2}$	$\mathbf{\bar{3}_1}$, $\mathbf{6}$	
	$(e^c,\mu^c,\tau^c)\sim\mathbf{3'_2}$	$\mathbf{\bar{3}'_1}$, $\mathbf{6}$	

Charged lepton masses and fine-tuning

For example, matter fileds: $L=(\ell_1,\ell_2,\ell_3)\sim\mathbf{3_1}$, $E^c=(\mu^c,e^c,\tau^c)\sim\mathbf{3_1}$, flavon fields: $\phi_{\ell_1}\sim\mathbf{3_1}$, $\phi_{\ell_2}\sim\mathbf{3'_1}$ and $\phi_{\ell_3}\sim\mathbf{3'_2}$ with $\langle\phi_{\ell_i}\rangle=(0,0,v_i)$.
Charged lepton masses are described by the superpotential

$$w_\ell=\frac{y_{\ell_1}}{\Lambda}((E^cL)_{\mathbf{\bar{3}_1}}\phi_{\ell_1})h_d+\frac{y_{\ell_2}}{\Lambda}((E^cL)_{\mathbf{\bar{3}'_1}}\phi_{\ell_2})h_d+\frac{y_{\ell_3}}{\Lambda}((E^cL)_{\mathbf{3'_2}}\phi_{\ell_3})h_d$$

Charged lepton mass matrix is diagonal

$$\begin{aligned} m_e&=(-y_{\ell_1}\frac{v_1}{\Lambda}+y_{\ell_2}\frac{v_2}{\Lambda}+y_{\ell_3}\frac{v_3}{\Lambda})v_d\,,\\ m_\mu&=(y_{\ell_1}\frac{v_1}{\Lambda}+y_{\ell_2}\frac{v_2}{\Lambda}+y_{\ell_3}\frac{v_3}{\Lambda})v_d\,,\\ m_\tau&=(-2y_{\ell_2}\frac{v_2}{\Lambda}+y_{\ell_3}\frac{v_3}{\Lambda})v_d\,,\end{aligned}$$

We have $y_{\ell_1}v_1:y_{\ell_2}v_2:y_{\ell_3}v_3\approx0:-1:1\Longrightarrow$ fine-tuning is required!

Model for TFH Mixing

- Flavor symmetry and Field contents

Fields	ℓ	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	χ	ϕ	η	ξ	ρ	φ	ψ
$\Delta(96)$	$\mathbf{3_1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{\bar{3}_1}$	$\mathbf{1}$	$\mathbf{3_1}$	$\mathbf{\bar{3}_1}$	$\mathbf{2}$	$\mathbf{3'_1}$	$\mathbf{2}$	$\mathbf{\bar{3}'_1}$	$\mathbf{3'_2}$
Z_3	0	2	2	2	0	0	1	1	0	0	0	0	0
Z_3	0	1	2	0	0	0	0	0	1	1	0	0	0

- Symmetry breaking

Fields	σ^0	ζ^0	χ^0	ϕ^0	ρ^0	φ^0
$\Delta(96)$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3_1}$	$\mathbf{3'_2}$	$\mathbf{2}$	$\mathbf{3_1}$
Z_3	0	1	2	1	0	0
Z_3	1	0	2	0	0	0

$$\langle\chi\rangle=(0,0,v_\chi),\quad \langle\phi\rangle=(0,0,v_\phi),\quad \langle\eta\rangle=(v_\eta,0),\quad \langle\xi\rangle=(v_\xi,0,0)\Longrightarrow \Delta(96)\to\mathbf{1}$$
$$\langle\rho\rangle=(1,\omega)v_\rho,\quad \langle\varphi\rangle=(1,1,1)v_\varphi,\quad \langle\psi\rangle=(v_1,v_2,(v_1+v_2)/2)\Longrightarrow \Delta(96)\to K_4$$

- Charged lepton

$$\begin{aligned}w_\ell=&\frac{y_\tau}{\Lambda}\tau^c(\ell\phi)h_d+\frac{y_{\mu_1}}{\Lambda^2}\mu^c(\ell(\chi\xi)_{\bar{\mathbf{3}}'_1})'h_d+\frac{y_{\mu_2}}{\Lambda^2}\mu^c(\ell(\eta\phi)_{\bar{\mathbf{3}}'_1})'h_d+\frac{y_{e_1}}{\Lambda^3}e^c(\ell\phi)(\eta\eta)h_d\\&+\frac{y_{e_2}}{\Lambda^3}e^c(\ell((\eta\eta)_{\mathbf{2}}\phi)_{\bar{\mathbf{3}}_1})h_d+\frac{y_{e_3}}{\Lambda^3}e^c(\ell(\chi(\eta\xi)_{\mathbf{3}_1})_{\bar{\mathbf{3}}_1})h_d+\frac{y_{e_4}}{\Lambda^3}e^c(\ell(\chi(\eta\xi)_{\mathbf{3}'_1})_{\bar{\mathbf{3}}_1})h_d\\&+\frac{y_{e_5}}{\Lambda^3}e^c(\ell(\chi(\xi\xi)_{\mathbf{3}'_2})_{\bar{\mathbf{3}}_1})h_d\end{aligned}$$

Charged lepton mass matrix is diagonal

$$m_e=\omega^2 y_{e_2}\frac{v_\eta^2 v_\phi}{\Lambda^3}+(y_{e_4}-y_{e_3})\frac{v_\eta v_\xi v_\chi}{\Lambda^3}+y_{e_5}\frac{v_\xi^2 v_\chi}{\Lambda^3},\quad m_\mu=y_{\mu_1}\frac{v_\xi v_\chi}{\Lambda^2}+y_{\mu_2}\frac{v_\eta v_\phi}{\Lambda^2},\quad m_\tau=y_\tau\frac{v_\phi}{\Lambda}$$

Mass hierarchies among the charged leptons are produced for $VEV/\Lambda\sim\lambda_c^2$

- Neutrino

$$w_\nu=y(\nu^c\ell)h_u+x_{\nu_1}((\nu^c\nu^c)_{\mathbf{3}_1'}\varphi)+x_{\nu_2}((\nu^c\nu^c)_{\mathbf{3}_2'}\psi)$$

Dirac and Majorana neutrino mass matrices read

$$m_D=yv_u\mathbb{1}$$

$$m_M=\left(\begin{array}{ccc}-4x_{\nu_1}v_\varphi+2x_{\nu_2}v_2 & 2x_{\nu_1}v_\varphi+x_{\nu_2}(v_1+v_2) & 2x_{\nu_1}v_\varphi+2x_{\nu_2}v_1\\ 2x_{\nu_1}v_\varphi+x_{\nu_2}(v_1+v_2) & -4x_{\nu_1}v_\varphi+2x_{\nu_2}v_1 & 2x_{\nu_1}v_\varphi+2x_{\nu_2}v_2\\ 2x_{\nu_1}v_\varphi+2x_{\nu_2}v_1 & 2x_{\nu_1}v_\varphi+2x_{\nu_2}v_2 & -4x_{\nu_1}v_\varphi+x_{\nu_2}(v_1+v_2)\end{array}\right)$$

Light neutrino mass matrix is exactly diagonalized by the TFH mixing matrix

$$m_\nu=-m_D^T m_M^{-1}m_D=U_{TFH}\,\mathrm{diag}(m_1,m_2,m_3)U_{TFH}^T$$

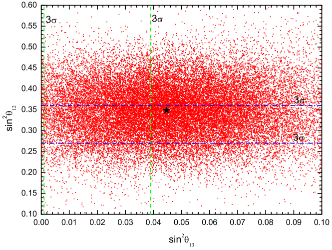
with

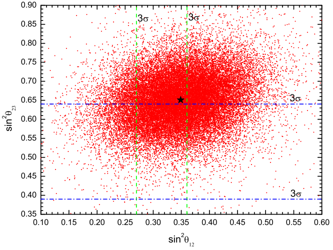
$$m_1=\frac{y^2v_u^2}{6x_{\nu_1}v_\varphi+\sqrt{3}x_{\nu_2}(v_1-v_2)},\quad m_2=-\frac{y^2v_u^2}{3x_{\nu_2}(v_1+v_2)},\quad m_3=\frac{y^2v_u^2}{6x_{\nu_1}v_\varphi-\sqrt{3}x_{\nu_2}(v_1-v_2)}$$

No sum rule among the light neutrino masses is found, and the neutrino mass spectrum can be normal or inverted hierarchy.

- Next to leading order corrections
  Both the charged lepton and neutrino receive corrections from the shifted vacuum and the higher dimensional operators in the Yukawa superpotentials.The NLO corrections to the three leptonic mixing angle are of order λ_c^2.

$$\delta\sin^2\theta_{12} \sim \lambda_c^2, \quad \delta\sin^2\theta_{23} \sim \lambda_c^2, \quad \delta\sin^2\theta_{13} \sim \lambda_c^2$$





Possible mixing textures within $\Delta(96)$

$$
\begin{tikzcd}
 & & \Delta(96) \arrow[dl] \arrow[dr] & & \\
 & G_\ell \arrow[dl] & & G_\nu \arrow[dr] & \\
g_\ell^\dagger m_\ell^\dagger m_\ell g_\ell = m_\ell^\dagger m_\ell & & & & g_\nu^T m_\nu g_\nu = m_\nu
\end{tikzcd}
$$

$$\boxed{G_\ell,\, G_\nu} \Longleftrightarrow \boxed{\text{leptonic MNSP matrix}}$$

- Light neutrinos are Majorana particles, the residual symmetry G_ν is $K_4 \cong Z_2 \times Z_2$.
- G_ℓ is taken to be the cyclic Z_N subgroup, non-abelian would result in a complete or partial degeneracy of the charged lepton mass spectrum.
- The left-handed leptons transform as faithful triplet of $\Delta(96)$.

★ $\Delta(96)$ has *seven* K_4 subgroups, *sixteen* Z_3 subgroups, *twelve* Z_4 subgroups and *six* Z_8 subgroups.

- The allowed leptonic mixing patterns in $\Delta(96)$

	$\|\|U_{PMNS}\|\|$	Symmetry breaking	Generated group
I	$\frac{1}{\sqrt{3}}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$	$(Z_3^{(1)},K_4^{(1)})$, $(Z_3^{(2)},K_4^{(1)})$, $(Z_3^{(3)},K_4^{(1)})$, $(Z_3^{(4)},K_4^{(1)})$, $(Z_3^{(5)},K_4^{(1)})$, $(Z_3^{(6)},K_4^{(1)})$, $(Z_3^{(7)},K_4^{(1)})$, $(Z_3^{(8)},K_4^{(1)})$, $(Z_3^{(9)},K_4^{(1)})$, $(Z_3^{(10)},K_4^{(1)})$, $(Z_3^{(11)},K_4^{(1)})$, $(Z_3^{(12)},K_4^{(1)})$, $(Z_3^{(13)},K_4^{(1)})$, $(Z_3^{(14)},K_4^{(1)})$, $(Z_3^{(15)},K_4^{(1)})$, $(Z_3^{(16)},K_4^{(1)})$	A_4
II	$\begin{pmatrix}\sqrt{\frac{2}{3}}&\frac{1}{\sqrt{3}}&0\\\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{6}}&\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{2}}\end{pmatrix}$	$(Z_3^{(1)},K_4^{(2)})$, $(Z_3^{(1)},K_4^{(4)})$, $(Z_3^{(1)},K_4^{(6)})$, $(Z_3^{(2)},K_4^{(3)})$, $(Z_3^{(2)},K_4^{(4)})$, $(Z_3^{(2)},K_4^{(7)})$, $(Z_3^{(3)},K_4^{(2)})$, $(Z_3^{(3)},K_4^{(4)})$, $(Z_3^{(3)},K_4^{(6)})$, $(Z_3^{(4)},K_4^{(3)})$, $(Z_3^{(4)},K_4^{(4)})$, $(Z_3^{(4)},K_4^{(7)})$, $(Z_3^{(5)},K_4^{(3)})$, $(Z_3^{(5)},K_4^{(5)})$, $(Z_3^{(5)},K_4^{(6)})$, $(Z_3^{(6)},K_4^{(2)})$, $(Z_3^{(6)},K_4^{(4)})$, $(Z_3^{(6)},K_4^{(6)})$, $(Z_3^{(7)},K_4^{(3)})$, $(Z_3^{(7)},K_4^{(5)})$, $(Z_3^{(7)},K_4^{(6)})$, $(Z_3^{(8)},K_4^{(2)})$, $(Z_3^{(8)},K_4^{(5)})$, $(Z_3^{(8)},K_4^{(7)})$, $(Z_3^{(9)},K_4^{(3)})$, $(Z_3^{(9)},K_4^{(4)})$, $(Z_3^{(9)},K_4^{(7)})$, $(Z_3^{(10)},K_4^{(2)})$, $(Z_3^{(10)},K_4^{(5)})$, $(Z_3^{(10)},K_4^{(7)})$, $(Z_3^{(11)},K_4^{(3)})$, $(Z_3^{(11)},K_4^{(5)})$, $(Z_3^{(11)},K_4^{(6)})$, $(Z_3^{(12)},K_4^{(2)})$, $(Z_3^{(12)},K_4^{(4)})$, $(Z_3^{(12)},K_4^{(6)})$, $(Z_3^{(13)},K_4^{(3)})$, $(Z_3^{(13)},K_4^{(5)})$, $(Z_3^{(13)},K_4^{(6)})$, $(Z_3^{(14)},K_4^{(2)})$, $(Z_3^{(14)},K_4^{(5)})$, $(Z_3^{(14)},K_4^{(7)})$, $(Z_3^{(15)},K_4^{(3)})$, $(Z_3^{(15)},K_4^{(4)})$, $(Z_3^{(15)},K_4^{(7)})$, $(Z_3^{(16)},K_4^{(2)})$, $(Z_3^{(16)},K_4^{(5)})$, $(Z_3^{(16)},K_4^{(7)})$	S_4

	$\|\|U_{PMNS}\|\|$	Symmetry breaking	Generated group
III	$\begin{pmatrix} \frac{1}{6}(3+\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(3-\sqrt{3}) \\ \frac{1}{6}(3-\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(3+\sqrt{3}) \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$	$(Z_3^{(1)},K_4^{(3)})$, $(Z_3^{(1)},K_4^{(5)})$, $(Z_3^{(1)},K_4^{(7)})$, $(Z_3^{(2)},K_4^{(2)})$, $(Z_3^{(2)},K_4^{(5)})$, $(Z_3^{(2)},K_4^{(6)})$, $(Z_3^{(3)},K_4^{(3)})$, $(Z_3^{(3)},K_4^{(5)})$, $(Z_3^{(3)},K_4^{(7)})$, $(Z_3^{(4)},K_4^{(2)})$, $(Z_3^{(4)},K_4^{(5)})$, $(Z_3^{(4)},K_4^{(6)})$, $(Z_3^{(5)},K_4^{(2)})$, $(Z_3^{(5)},K_4^{(4)})$, $(Z_3^{(5)},K_4^{(7)})$, $(Z_3^{(6)},K_4^{(3)})$, $(Z_3^{(6)},K_4^{(5)})$, $(Z_3^{(6)},K_4^{(7)})$, $(Z_3^{(7)},K_4^{(2)})$, $(Z_3^{(7)},K_4^{(4)})$, $(Z_3^{(7)},K_4^{(7)})$, $(Z_3^{(8)},K_4^{(3)})$, $(Z_3^{(8)},K_4^{(4)})$, $(Z_3^{(8)},K_4^{(6)})$, $(Z_3^{(9)},K_4^{(2)})$, $(Z_3^{(9)},K_4^{(5)})$, $(Z_3^{(9)},K_4^{(6)})$, $(Z_3^{(10)},K_4^{(3)})$, $(Z_3^{(10)},K_4^{(4)})$, $(Z_3^{(10)},K_4^{(6)})$, $(Z_3^{(11)},K_4^{(2)})$, $(Z_3^{(11)},K_4^{(4)})$, $(Z_3^{(11)},K_4^{(7)})$, $(Z_3^{(12)},K_4^{(3)})$, $(Z_3^{(12)},K_4^{(5)})$, $(Z_3^{(12)},K_4^{(7)})$, $(Z_3^{(13)},K_4^{(2)})$, $(Z_3^{(13)},K_4^{(4)})$, $(Z_3^{(13)},K_4^{(7)})$, $(Z_3^{(14)},K_4^{(3)})$, $(Z_3^{(14)},K_4^{(4)})$, $(Z_3^{(14)},K_4^{(6)})$, $(Z_3^{(15)},K_4^{(2)})$, $(Z_3^{(15)},K_4^{(5)})$, $(Z_3^{(15)},K_4^{(6)})$, $(Z_3^{(16)},K_4^{(3)})$, $(Z_3^{(16)},K_4^{(4)})$, $(Z_3^{(16)},K_4^{(6)})$	$\Delta(96)$

	$\|\|U_{PMNS}\|\|$	Symmetry breaking	Generated group
IV	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$(Z_4^{(7)},K_4^{(4)})$, $(Z_4^{(7)},K_4^{(5)})$, $(Z_4^{(7)},K_4^{(6)})$, $(Z_4^{(7)},K_4^{(7)})$, $(Z_4^{(8)},K_4^{(4)})$, $(Z_4^{(8)},K_4^{(5)})$, $(Z_4^{(8)},K_4^{(6)})$, $(Z_4^{(8)},K_4^{(7)})$, $(Z_4^{(9)},K_4^{(2)})$, $(Z_4^{(9)},K_4^{(3)})$, $(Z_4^{(9)},K_4^{(6)})$, $(Z_4^{(9)},K_4^{(7)})$, $(Z_4^{(10)},K_4^{(2)})$, $(Z_4^{(10)},K_4^{(3)})$, $(Z_4^{(10)},K_4^{(6)})$, $(Z_4^{(10)},K_4^{(7)})$, $(Z_4^{(11)},K_4^{(2)})$, $(Z_4^{(11)},K_4^{(3)})$, $(Z_4^{(11)},K_4^{(4)})$, $(Z_4^{(11)},K_4^{(5)})$, $(Z_4^{(12)},K_4^{(2)})$, $(Z_4^{(12)},K_4^{(3)})$, $(Z_4^{(12)},K_4^{(4)})$, $(Z_4^{(12)},K_4^{(5)})$,	S_4
		$(Z_8^{(1)},K_4^{(4)})$, $(Z_8^{(1)},K_4^{(5)})$, $(Z_8^{(1)},K_4^{(6)})$, $(Z_8^{(1)},K_4^{(7)})$, $(Z_8^{(2)},K_4^{(4)})$, $(Z_8^{(2)},K_4^{(5)})$, $(Z_8^{(2)},K_4^{(6)})$, $(Z_8^{(2)},K_4^{(7)})$, $(Z_8^{(3)},K_4^{(2)})$, $(Z_8^{(3)},K_4^{(3)})$, $(Z_8^{(3)},K_4^{(6)})$, $(Z_8^{(3)},K_4^{(7)})$, $(Z_8^{(4)},K_4^{(2)})$, $(Z_8^{(4)},K_4^{(3)})$, $(Z_8^{(4)},K_4^{(6)})$, $(Z_8^{(4)},K_4^{(7)})$, $(Z_8^{(5)},K_4^{(2)})$, $(Z_8^{(5)},K_4^{(3)})$, $(Z_8^{(5)},K_4^{(4)})$, $(Z_8^{(5)},K_4^{(5)})$, $(Z_8^{(6)},K_4^{(2)})$, $(Z_8^{(6)},K_4^{(3)})$, $(Z_8^{(6)},K_4^{(4)})$, $(Z_8^{(6)},K_4^{(5)})$	$\Delta(96)$

	$\|\|U_{PMNS}\|\|$	Symmetry breaking	Generated group
V	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$(Z_4^{(4)},K_4^{(1)})$, $(Z_4^{(5)},K_4^{(1)})$, $(Z_4^{(6)},K_4^{(1)})$, $(Z_4^{(7)},K_4^{(3)})$, $(Z_4^{(8)},K_4^{(2)})$, $(Z_4^{(9)},K_4^{(5)})$, $(Z_4^{(10)},K_4^{(4)})$, $(Z_4^{(11)},K_4^{(7)})$, $(Z_4^{(12)},K_4^{(6)})$	$Z_2\times Z_4$
VI	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$(Z_4^{(4)},K_4^{(2)})$, $(Z_4^{(4)},K_4^{(3)})$, $(Z_4^{(5)},K_4^{(4)})$, $(Z_4^{(5)},K_4^{(5)})$, $(Z_4^{(6)},K_4^{(6)})$, $(Z_4^{(6)},K_4^{(7)})$, $(Z_4^{(7)},K_4^{(1)})$, $(Z_4^{(7)},K_4^{(2)})$, $(Z_4^{(8)},K_4^{(1)})$, $(Z_4^{(8)},K_4^{(3)})$, $(Z_4^{(9)},K_4^{(1)})$, $(Z_4^{(9)},K_4^{(4)})$, $(Z_4^{(10)},K_4^{(1)})$, $(Z_4^{(10)},K_4^{(5)})$, $(Z_4^{(11)},K_4^{(1)})$, $(Z_4^{(11)},K_4^{(6)})$, $(Z_4^{(12)},K_4^{(1)})$, $(Z_4^{(12)},K_4^{(7)})$	D_4
		$(Z_4^{(4)},K_4^{(4)})$, $(Z_4^{(4)},K_4^{(5)})$, $(Z_4^{(4)},K_4^{(6)})$, $(Z_4^{(4)},K_4^{(7)})$, $(Z_4^{(5)},K_4^{(2)})$, $(Z_4^{(5)},K_4^{(3)})$, $(Z_4^{(5)},K_4^{(6)})$, $(Z_4^{(5)},K_4^{(7)})$, $(Z_4^{(6)},K_4^{(2)})$, $(Z_4^{(6)},K_4^{(3)})$, $(Z_4^{(6)},K_4^{(4)})$, $(Z_4^{(6)},K_4^{(5)})$,	$(Z_4\times Z_4)\rtimes Z_2$
		$(Z_8^{(1)},K_4^{(1)})$, $(Z_8^{(2)},K_4^{(1)})$, $(Z_8^{(3)},K_4^{(1)})$, $(Z_8^{(4)},K_4^{(1)})$, $(Z_8^{(5)},K_4^{(1)})$, $(Z_8^{(6)},K_4^{(1)})$	$Z_8\rtimes Z_2$
VII	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2-\sqrt{2}}}{2} & \frac{\sqrt{2+\sqrt{2}}}{2} \\ 0 & \frac{\sqrt{2+\sqrt{2}}}{2} & \frac{\sqrt{2-\sqrt{2}}}{2} \end{pmatrix}$	$(Z_8^{(1)},K_4^{(2)})$, $(Z_8^{(1)},K_4^{(3)})$, $(Z_8^{(2)},K_4^{(2)})$, $(Z_8^{(2)},K_4^{(3)})$, $(Z_8^{(3)},K_4^{(4)})$, $(Z_8^{(3)},K_4^{(5)})$, $(Z_8^{(4)},K_4^{(4)})$, $(Z_8^{(4)},K_4^{(5)})$, $(Z_8^{(5)},K_4^{(6)})$, $(Z_8^{(5)},K_4^{(7)})$, $(Z_8^{(6)},K_4^{(6)})$, $(Z_8^{(6)},K_4^{(7)})$	$(Z_4\times Z_4)\rtimes Z_2$

- If we require that the elements of G_ℓ and G_ν generate the full group $\Delta(96)$, only the TFH and bimaximal mixing patterns are admissible.
- Deformed tri-bimaximal mixing: exchanging the order of the rows and columns of the tri-bimaximal mixing matrix, we find

$$||U_{PMNS}|| = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

$$\Rightarrow \sin^2\theta_{13} = \frac{1}{6}, \qquad \sin^2\theta_{12} = \frac{2}{5}, \qquad \sin^2\theta_{23} = \frac{4}{5}$$

If all the three mixing angles should undergo large corrections of order $0.1 \sim 0.2$, the resulting leptonic mixing could be compatible with experimental data. Deformed tri-bimaximal can be used as a leading order mixing to understand largish θ_{13}, and it can be naturally derived from the S_4 flavor symmetry(work in progress).

Conclusions

- TFH mixing is a good starting point to understanding large θ_{13}, and $\Delta(96)$ is the approximate family symmetry to derive it.
- The possible ways to reproduce TFH mixing with $\Delta(96)$ are studied in the "minimalist" framework, and consistent model is constructed explicitly.
- The allowed mixing patterns within $\Delta(96)$ are analyzed from group theory, we suggest the deformed tri-bimaximal mixing is an alternative to explaining largish θ_{13}, and it can be produced from S_4 flavor symmetry at leading order.
- $\Delta(96)$ have 2-dimensional irreducible representation, it could be used to describe the quark mass hierarchies and CKM mixing.

Conclusions

- TFH mixing is a good starting point to understanding large θ_{13}, and $\Delta(96)$ is the approximate family symmetry to derive it.
- The possible ways to reproduce TFH mixing with $\Delta(96)$ are studied in the "minimalist" framework, and consistent model is constructed explicitly.
- The allowed mixing patterns within $\Delta(96)$ are analyzed from group theory, we suggest the deformed tri-bimaximal mixing is an alternative to explaining largish θ_{13}, and it can be produced from S_4 flavor symmetry at leading order.
- $\Delta(96)$ have 2-dimensional irreducible representation, it could be used to describe the quark mass hierarchies and CKM mixing.

Thank you very much!