A SUSY $SU(5) \times T'$ Unified Model of Flavour with large θ_{13}

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In collaboration with
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arXiv:1205.5241

Outline of the talk

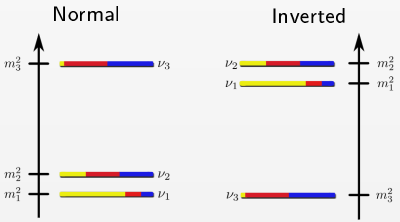
- Motivations for a $SU(5) \otimes T'$ Unified Model of Flavour
- General Setup of the Model
- Neutrino Sector
- Conclusions

Global fit analysis including the Daya Bay and RENO results
Fogli et al. ArXiV: 1205.5254.

Parameter		Best fit	3σ range
$\Delta m^2_\odot$ $[10^{-5}\ \text{eV}^2]$	(NH or IH)	7.54	6.99 – 8.18
$\|\Delta m^2_A\|$ $[10^{-3}\ \text{eV}^2]$	(NH)	2.43	2.15 – 2.66
	(IH)	2.42	2.14 – 2.65
$\sin^2\theta_{12}/10^{-1}$	(NH or IH)	3.07	2.59 – 3.59
$\sin^2\theta_{13}/10^{-2}$	(NH)	2.45	1.49 – 3.44
	(IH)	2.46	1.50 – 3.47
$\sin^2\theta_{23}/10^{-1}$	(NH)	3.98	3.30 – 6.38
	(IH)	4.08	3.35 – 6.58

Open Questions in Neutrino Physics

- **Majorana or Dirac**?
- Absolute values of neutrino masses
- Hierarchy (normal $m_1 < m_2 < m_3$ or inverted $m_3 < m_1 < m_2$)
- CP-phases: δ, α_{31} and α_{21}



GUT: $SU(5)$

* $SU(5)$, Unified picture of quarks and leptons
* possibly leads to sizeable θ_{13}

discrete symmetry: T'

* treats quarks and lepton mixing **simultaneously**
* can allow spinorial unitary irreducible reps
* "geometrical" CP violation
* corrections to U_{TBM} via lepton sector

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Literature

A model based on $SU(5) \otimes T' \otimes Z_{12} \otimes Z_{12}$ was developed in M.C. Chen, K.T. Mahanthappa Phys. Lett. B652,34 (2007)

- Too small $\theta_{13} \sim \theta^c/(3\sqrt{2}) \Rightarrow \sin^2\theta_{13} \sim 0.003 \rightarrow$ RULED OUT!

- T' is the Double-valued group of $T \sim A_4$ (even permutations of 4 objects)
- inequivalent UIRs : $\underbrace{\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}}_{\text{TBM for Neutrinos}} + \underbrace{\mathbf{2}, \mathbf{2}', \mathbf{2}''}_{\text{2+1 assignments for q and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !

Clebsch-Gordan coefficients can be **complex** $\Rightarrow$ geometrical origin of **CP** violation!

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{2(2')} \otimes \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}_{2(2'')} = \left(\frac{x_1 x'_2 - x_2 x'_1}{\sqrt{2}} \right)_1 \oplus \begin{pmatrix} -\frac{(1+i)}{2}(x_1 x'_2 + x_2 x'_1) \\ x_1 x'_1 \\ -i x_2 x'_2 \end{pmatrix}_3$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_3 \sim \left[\begin{pmatrix} (1+i) x_2 u_2 + x_1 u_1 \\ (1-i) x_1 u_3 - x_2 u_1 \end{pmatrix}_2 \oplus \begin{pmatrix} (1+i) x_2 u_3 + x_1 u_2 \\ (1-i) x_1 u_1 - x_2 u_2 \end{pmatrix}_{2'} \oplus \begin{pmatrix} (1+i) x_2 u_1 + x_1 u_3 \\ (1-i) x_1 u_2 - x_2 u_3 \end{pmatrix}_{2''} \right]$$

M.-C. Chen, K.T. Mahanthappa Phys.Lett. B652 (2007)
F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl.Phys. B775 (2007)

Conventions and Assignments: Matter and Higgs fields

Fundamental Symmetries	$SU(5) \otimes T'$ (SUSY!)
Shaping symmetry	$Z_{12} \otimes Z_8^3 \otimes Z_6^2 \otimes Z_4$
$U(1)_R$	Continuous version of the usual R-parity
Matter Content	$\mathbf{10}(Q, u^c, e^c)_L$: ten-plets into $(T1, T2) \sim \mathbf{2}$, $T3 \sim \mathbf{1}$
	$\bar{\mathbf{5}}(d^c, \ell)_L$: five-plets into $(F1, F2, F3) \sim \mathbf{3}$
	Three Heavy RH Majorana N_k, $(N_1, N_2, N_3) \sim \mathbf{3}$
	light active neutrinos via Type I See-Saw
	Higgs sector: copies of $\mathbf{5}$, $\bar{\mathbf{5}}$ and $\mathbf{24}$ assigned to $\mathbf{1}$, $\mathbf{1}$"

Conventions and assignments: Flavon fields

up and down quark sector $\langle\tilde{\psi}'\rangle \sim \langle\psi'\rangle \propto \begin{pmatrix}1\\0\end{pmatrix}$, $\langle\tilde{\psi}''\rangle \sim \langle\psi''\rangle \propto \begin{pmatrix}0\\1\end{pmatrix}$

$\langle\tilde{\phi}\rangle \sim \langle\phi\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix}$, singlets e.g. $\langle\tilde{\zeta}'\rangle$

in the ν sector $\langle\xi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}\xi_0$, $\langle\rho\rangle = \rho_0$, $\langle\tilde{\rho}\rangle = \tilde{\rho}_0$

Flavons fields take all real vevs when T' is broken!

Flavon Alignment problem $\Leftrightarrow$ "driving fields" $(S_\rho, S_\xi, D_\xi, \ldots)$
for example in the neutrino sector

$$\mathcal{W}_{\xi,\rho,\tilde{\rho}} = \frac{D_\xi}{\Lambda}\left(\xi^2\epsilon_9 + \xi\rho\epsilon_9 + \xi\tilde{\rho}\epsilon_9\right) + S_\xi\left(\xi^2 - M_\xi^2\right) + S_\rho\left(\rho^2 + \tilde{\rho}^2 - M_\rho^2\right) .$$

$$\xi_i = \xi_0 \neq 0 \quad \text{if} \quad \rho_0 = -\tilde{\rho}_0$$

S. Antusch, S. King, C. Luhn, M. Spinrath ArXiv: 1103.5930

Yukawa couplings: Effective Operators - arXiv: 1205.5241

RL convention, i.e. $-\mathcal{L} = Y_{ij}\overline{f_R^i} f_L^j H + \text{H.c.}$

$$Y_u = \begin{pmatrix} (1-\mathrm{i})a_u & \mathrm{i}\, b_u & 0 \\ \mathrm{i}\, b_u & c_u & (1+\mathrm{i})d_u \\ 0 & (1+\mathrm{i})d_u & e_u \end{pmatrix},$$

$$Y_d = \begin{pmatrix} (1+\mathrm{i})\, a_d & \mathrm{i}\, b'_d & 0 \\ (1-\mathrm{i})\, b_d & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix} \quad \text{and} \quad Y_\ell = \begin{pmatrix} -\frac{3}{2}\,(1+\mathrm{i})\, a_d & (1-\mathrm{i})\, b_d & 0 \\ 6\ \mathrm{i}\ b'_d & 6\, c_d & 0 \\ 0 & 0 & -\frac{3}{2}\, d_d \end{pmatrix}$$

S. Antusch, M. Spinrath arXiv:0902.4644

$$|V_{us}| \sim \left|\frac{b_d}{c_d}\right|$$

$$\theta_{12}^e \sim= \left|\frac{6\, i b'_d}{6\, c_d}\right| \sim \left|\frac{b'_d}{b_d}\right| \theta^c, \quad (b'_d = 0.9\, b_d)$$

$$\theta_{13} = \frac{1}{\sqrt{2}}\theta_{12}^e = \frac{0.9}{\sqrt{2}}\theta^c$$

Talk by M. Spinrath

D. Marzocca, S. T. Petcov, A. Romanino, M. Spinrath arXiv:1108.0614

Antusch, Maurer arXiv:1107.3728

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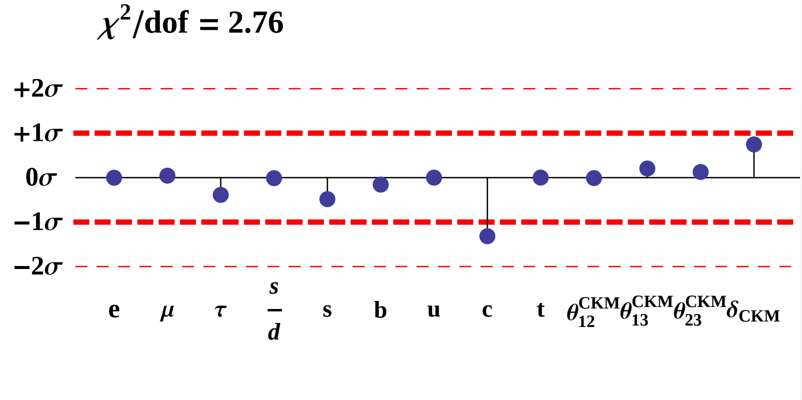


A. M., S. T. Petcov, M. Spinrath arXiv:1205.5241

$$\mathcal{W}_\nu = \underbrace{\lambda_1 NN\xi + NN(\lambda_2\rho + \lambda_3\tilde{\rho})}_{\text{RH Majorana Mass } M_R} + \underbrace{\frac{y_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\rho)_1 + \frac{\tilde{y}_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\tilde{\rho})_1}_{\text{Dirac Yukawa coupling } M_D}$$

$$\langle\xi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}\xi_0, \quad \langle\rho\rangle = \rho_0\ , \quad \langle\tilde{\rho}\rangle = \tilde{\rho}_0$$

$$M_R = \begin{pmatrix}2Z+X & -Z & -Z\\ -Z & 2Z & -Z+X\\ -Z & -Z+X & 2Z\end{pmatrix}\ , \quad M_D = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\frac{\rho'}{\Lambda}$$

X, Z and ρ' are real parameters

M_R is form diagonalizable

$$U_{\mathbf{TBM}}^T\, M_R\, U_{\mathbf{TBM}} = \mathrm{Diag}(3Z+X, X, 3Z-X)$$

Diagonal Majorana Mass Matrix

$$\begin{pmatrix} 3Z+X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & 3Z-X \end{pmatrix} \longrightarrow |X| \begin{pmatrix} |1+\alpha e^{\mathrm{i}\,\phi}|e^{\mathrm{i}\,\phi_1} & 0 & 0 \\ 0 & e^{\mathrm{i}\,\phi_2} & 0 \\ 0 & 0 & |1-\alpha e^{\mathrm{i}\,\phi}|e^{\mathrm{i}\,\phi_3} \end{pmatrix}$$

$$\alpha \equiv |3Z/X| > 0, \quad \phi \equiv \arg(Z) - \arg(X)$$

$$\phi_i = 0, \pi$$

Light neutrino Majorana mass term via type I see-saw mechanism:

$$M_\nu \;=\; -M_D^T\, M_R^{-1}\, M_D = U_\nu^*\, \mathrm{Diag}\,(m_1, m_2, m_3)\, U_\nu^\dagger$$

$$\boldsymbol{U_\nu} = \mathrm{i}\;\boldsymbol{U_{\mathrm{TBM}}}\;\mathrm{Diag}\left(e^{\mathrm{i}\,\phi_1/2}, e^{\mathrm{i}\,\phi_2/2}, e^{\mathrm{i}\,\phi_3/2}\right)$$

$$m_i = \left(\frac{\rho'}{\Lambda}\right)^2 \frac{1}{M_i}\,, \quad i = 1,2,3 \quad m_i > 0$$

$$(remember : \alpha > 0 \quad \cos\phi = \pm 1!)$$

$$\Delta m^2_{31} \equiv \Delta m^2_A = \frac{1}{|X|^2} \left(\frac{\rho'}{\Lambda}\right)^4 \frac{4\alpha \cos\phi}{|1+\alpha\, e^{\mathrm{i}\,\phi}|^2\,|1-\alpha\, e^{\mathrm{i}\,\phi}|^2}\,.$$

$$\begin{cases} \text{for } \cos\phi = +1 & \text{mass spectrum with NO} \\ \text{for } \cos\phi = -1 & \text{mass spectrum with IO} \end{cases}$$

$$\Delta m^2_{21} \equiv \Delta m^2_{\odot} = \frac{1}{|X|^2} \left(\frac{\rho'}{\Lambda}\right)^4 \frac{\alpha\,(\alpha + 2\cos\phi)}{|1+\alpha\, e^{\mathrm{i}\,\phi}|^2}\,.$$

$$\mathbf{r} = \frac{\Delta m^2_{\odot}}{|\Delta m^2_A|} = \frac{1}{4}\,(\alpha + 2\cos\phi)\left(1 - 2\alpha\cos\phi + \alpha^2\right) = \mathbf{0.032} \pm \ \mathbf{0.006} \quad .$$

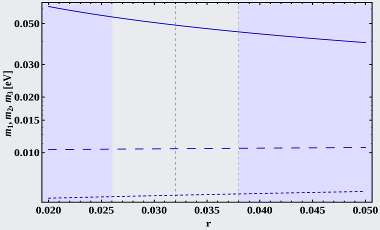
NO Solution A

$$m_1 \cong 4.44 \times 10^{-3}\ \mathrm{eV}$$
$$m_2 \cong 9.77 \times 10^{-3}\ \mathrm{eV}$$
$$m_3 \cong 4.89 \times 10^{-2}\ \mathrm{eV}$$

RH Majorana N_k: $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 11.0$$

$$M_2/M_3 \cong 5.0.$$



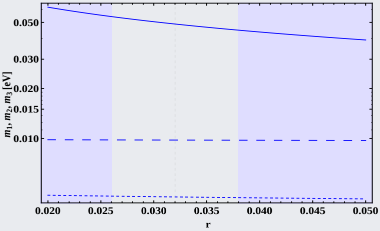
NO Solution B

$$m_1 \cong 5.89 \times 10^{-3}\ \mathrm{eV}$$
$$m_2 \cong 1.05 \times 10^{-2}\ \mathrm{eV}$$
$$m_3 \cong 4.90 \times 10^{-2}\ \mathrm{eV}$$

RH Majorana N_k: $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 8.33$$

$$M_2/M_3 \cong 4.67.$$



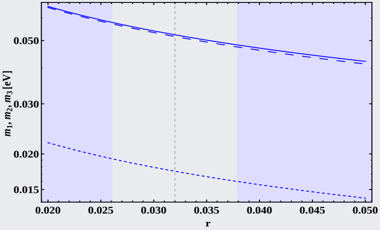
IO

$$m_1 \cong 5.17 \times 10^{-2}\ \mathrm{eV}$$
$$m_2 \cong 5.24 \times 10^{-2}\ \mathrm{eV}$$
$$m_3 \cong 1.74 \times 10^{-2}\ \mathrm{eV}$$

RH Majorana N_k: $M_1 \cong M_2 < M_3$

$$M_1/M_2 \cong 1.014$$

$$M_3/M_2 \cong 3.01.$$



$$U^{\text{PMNS}} = U_{eL}^{\dagger} U_{\nu}, \quad U_{eL} = \text{diag}(1, e^{i\varphi}, 1)\, R_{12}(\theta_{12}^{e}), \quad U_{\nu} = U_{\text{TBM}}\, \tilde{Q}$$

$$U_{appx}^{PMNS} = \begin{pmatrix} \frac{e^{-i\frac{\beta_1}{2}}}{\sqrt{6}} & \frac{e^{-i\frac{\beta_2}{2}}}{\sqrt{3}} & \frac{\sin\theta_{12}^{e}\; e^{-i(\pi-\theta_{12}^{e})/2}}{\sqrt{2}} \\ * & * & * \\ * & * & * \end{pmatrix}$$

$$\theta_{12} = \arcsin(1/\sqrt{3}) + \sqrt{2}/8(\theta_{12}^{e})^2$$
$$\theta_{13} = \theta_{12}^{e}/\sqrt{2}$$
$$\theta_{23} = \pi/4 - 1/4(\theta_{12}^{e})^2$$

$$\delta = \frac{\pi}{2} - \frac{1}{2}\theta_{12}^{e}$$
$$\beta_1 = 2\pi - 2\theta_{12}^{e} + \phi_3$$
$$\beta_2 = 2\pi + \theta_{12}^{e} + \phi_3 - \phi_2$$

Quantity	Experiment (2σ ranges)	Model
$\sin^2\theta_{12}$	0.275 – 0.342	0.340
$\sin^2\theta_{23}$	0.36 – 0.60	0.490
$\sin^2\theta_{13}$	0.015 – 0.032	0.020
δ	-	84.3°
β_1	-	$337.1^\circ + \phi_3$
β_2	-	$11.5^\circ + \phi_3$ - ϕ_2
J_{CP}	-	0.0324

RG corrections checked: no large contributions!

Predictions for Mass spectrum and $\beta\beta 0\nu$Decay - arXiv: 1205.5241

NO Solution A

$$\sum_{k=1}^{3} m_k = 6.31 \times 10^{-2} \text{ eV}$$

$$|<m>| = 4.90 \times 10^{-3} \text{ eV}$$

NO Solution B

$$\sum_{k=1}^{3} m_k = 6.54 \times 10^{-2} \text{ eV}$$

$$|<m>| = 7.95 \times 10^{-3} \text{ eV}$$

IO

$$\sum_{k=1}^{3} m_k = 12.1 \times 10^{-2} \text{ eV}$$

$$|<m>| = 2.17 \times 10^{-2} \text{ eV}$$

- We construct a unified model of flavor with large θ_{13} based on $S(5) \otimes T'$ with type I See-Saw
- T' leads to TBM in the neutrino sector + corrections coming from the charged lepton sector
- Geometrical CP violation via CGs of the T' group
  - Essential ingredient: real flavon vev alignment
- Predictions:
  - $\sin^2\theta_{13} \sim 0.02$
  - $\delta \cong \pi/2 - 0.45\theta^c \sim 84.3^\circ$
  - Neutrino mass spectra: NO and IO possible
  - unambiguous predictions for $\beta\beta0\nu$-Decay $|\langle m \rangle|$

Looking forward to see upcoming exp results!


Thank you

Back up slides

$$SU(5) \otimes T' \otimes Z_{12} \otimes Z_{12}$$

was developed in

M.C. Chen, K.T. Mahanthappa Phys. Lett. B652,34 (2007);
PoS ICHEP2010:407,2010. Phys. Lett. B681, 444 (2009)

- Too small $\theta_{13} \sim \frac{\theta^c}{3\sqrt{2}} \Rightarrow \sin^2\theta_{13} \sim 0.003$
- Different flavour structure → different values for observables!
- Only NO possible
- Flavon vacuum alignment problem
- Messenger sector

Auxiliary Fields: ϵ_i — $SU(5) \otimes T' \sim \mathbf{1}$ and $Q_{U(1)_R} = 0$
transform in a non trivial way under the shaping symmetries
they appear only in the flavon superpotential

$$\mathcal{W}_\epsilon = S_{\epsilon_i}\left(\epsilon_i^2 - M_{\epsilon_i}^2\right) + S_{\epsilon_j}\left(\frac{1}{\Lambda}\epsilon_j^3 - M_{\epsilon_j}^2\right)$$

Driving Fields: $\{S_x \sim \mathbf{1},\, D_x \sim \mathbf{3}\}$ — $SU(5) \sim \mathbf{1}$ and $Q_{U(1)_R} = 2$
transform in a non trivial way under T' and shaping symmetries

for example in the neutrino sector

$$\mathcal{W}_{\xi,\rho,\tilde{\rho}} = \frac{D_\xi}{\Lambda}\left(\xi^2\epsilon_9 + \xi\rho\epsilon_9 + \xi\tilde{\rho}\epsilon_9\right) + S_\xi\left(\xi^2 - M_\xi^2\right) + S_\rho\left(\rho^2 + \tilde{\rho}^2 - M_\rho^2\right)$$

$$\xi_i = \xi_0 \neq 0 \quad \text{if} \quad \rho_0 = -\tilde{\rho}_0$$

- T' is the Double-valued group of $T \sim A_4$ (even permutations of 4 objects)
- inequivalent UIRs : $\underbrace{\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}}_{\text{TBM for Neutrinos}} + \underbrace{\mathbf{2}, \mathbf{2}', \mathbf{2}''}_{\text{2+1 assignments for q and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !
- 3 generators: $s^2 = r, \quad r^2 = t^3 = (st)^3 = e, \quad rt = tr$

$$\mathbf{3}: \quad r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad s = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$$

where $1 + \omega + \omega^2 = 0$ conventionally.

$$\begin{array}{c}
\mathbf{1} \otimes \Gamma^p = \Gamma^p, \quad \mathbf{1} \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{1}'(\mathbf{1}''), \quad \mathbf{1}' \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{1}''(\mathbf{1}), \quad \mathbf{1}'(\mathbf{1}'') \otimes \mathbf{1}'' = \mathbf{1}(\mathbf{1}') \\
\mathbf{2} \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{2}'(\mathbf{2}''), \quad \mathbf{2}' \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{2}''(\mathbf{2}), \quad \mathbf{2}'' \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{2}(\mathbf{2}') \\
\mathbf{2}(\mathbf{2}') \otimes \mathbf{2}(\mathbf{2}'') = \mathbf{1} \oplus \mathbf{3}, \quad \mathbf{2}'(\mathbf{2}) \otimes \mathbf{2}'(\mathbf{2}'') = \mathbf{1} \oplus \mathbf{3}, \quad \mathbf{2}''(\mathbf{2}) \otimes \mathbf{2}''(\mathbf{2}') = \mathbf{1} \oplus \mathbf{3} \\
\mathbf{3} \otimes \mathbf{1}'(\mathbf{1}'') = \mathbf{3}, \quad \mathbf{3} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{2}' \oplus \mathbf{2}'', \quad \mathbf{3} \otimes \mathbf{2}'(\mathbf{2}'') = \mathbf{2}'(\mathbf{2}'') \oplus \mathbf{2}''(\mathbf{2}) \oplus \mathbf{2}(\mathbf{2}') \\
\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}
\end{array}$$

T' group: Clebsch-Gordan series $\Gamma^p \otimes \Gamma^q = \Gamma^q \otimes \Gamma^p$. Here $\Gamma^p = \mathbf{1}, \mathbf{2}, \mathbf{3}$

Table of Charge Assignments: Matter and Higgs fields

	T_3	T_a	$\bar{F}$	N	$H_5^{(1)}$	$H_5^{(2)}$	$H_5^{(3)}$	$\bar{H}_5^{(1)}$	$\bar{H}_5^{(2)}$	$\bar{H}_5^{(3)}$	$\bar{H}_5''$	H_{24}''	$\tilde{H}_{24}''$
$SU(5)$	**10**	**10**	$\bar{5}$	**1**	**5**	**5**	**5**	$\bar{5}$	$\bar{5}$	$\bar{5}$	$\bar{5}$	**24**	**24**
T'	**1**	**2**	**3**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**1**$''$	**1**$''$	**1**$''$
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_{12}^u	2	11	1	9	8	8	2	9	3	6	3	0	3
Z_8^d	4	0	2	6	0	4	0	1	4	7	7	4	2
Z_8^ν	7	6	2	0	2	6	4	1	1	5	7	4	0
Z_8	0	5	2	2	0	0	6	0	0	6	6	4	2
Z_6	5	0	1	0	2	5	2	2	0	2	2	0	0
Z_6'	2	3	1	0	2	5	2	5	0	2	2	0	0
Z_4	3	3	0	0	2	0	2	0	1	1	0	0	1

Table of Charge Assignments: Flavons

	$\tilde{\phi}$	$\tilde{\psi}''$	$\tilde{\psi}'$	$\tilde{\zeta}''$	$\tilde{\zeta}'$	ϕ	ψ''	ψ'	ζ''	ζ'	ξ	ρ	$\tilde{\rho}$
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T'	$\mathbf{3}$	$\mathbf{2}''$	$\mathbf{2}'$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{2}''$	$\mathbf{2}'$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_R$	0	0	0	0	0	0	0	0	0	0	0	0	0
Z_{12}^u	0	3	9	0	0	6	3	9	6	0	6	6	6
Z_8^d	0	0	0	0	0	2	1	7	6	4	4	4	4
Z_8^ν	4	1	7	0	0	2	7	1	6	4	0	0	0
Z_8	4	7	5	4	0	2	5	3	6	4	4	4	4
Z_6	4	4	2	4	2	0	3	3	0	0	0	0	0
Z_6'	4	4	2	4	2	3	0	0	0	0	0	0	0
Z_4	0	2	2	0	0	0	3	1	2	0	0	0	0

$$
\begin{aligned}
\mathcal{W}_{Y_u} = {} & y_{33} H_5^{(1)} T_3 T_3 + \frac{y_{23}}{\Lambda^2}(T_a\tilde{\phi})_{2'} H_5^{(2)} (T_3\tilde{\psi}'')_{2''} + \frac{y_{22}}{\Lambda^3}(T_a\tilde{\psi}'')_3 (H_5^{(1)}\tilde{\zeta}')_{1'} (T_a\tilde{\psi}'')_3 \\
& + \frac{y_{21}}{\Lambda^4}(T_a\tilde{\phi})_{2'}(H_5^{(1)}\tilde{\zeta}')_{1'}(\tilde{\psi}'(T_a\tilde{\psi}')_3)_{2'} + \frac{y_{11}}{\Lambda^4}((T_a\tilde{\phi})_2\tilde{\zeta}'')_{2''} H_5^{(3)}\tilde{(}\zeta''(T_a\tilde{\phi})_{2''})_{2'} \ ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{W}_{Y_{d},\ell} = {} & \frac{y_{33}}{\Lambda^2}((\bar{H}_5^{(2)}\bar{F})_3\phi)_{1'}(H''_{24}T_3)_{1''} + \frac{y_{22}}{\Lambda^3}((\phi T_a)_{2'} H''_{24})_2(\psi'(\bar{H}_5^{(1)}\bar{F})_3)_2 \\
& + \frac{y_{12}}{\Lambda^4}(((T_a\tilde{H}''_{24})_{2''}(\bar{F}\psi')_{2''})_3\psi')_{2''}(\bar{H}_5^{(3)}\psi')_{2'} + \frac{y_{21}}{\Lambda^4}((\bar{F}\psi')_{2''}(\zeta''\bar{H}_5^{(1)})_{1''}\zeta'')_2(T_a\phi)_2 \\
& + \frac{y_{11}}{\Lambda^4}((\bar{F}\psi'')_{2'}(H''_{24}\psi'')_{2'}\bar{H}''_5)_{1'}(T_a\psi'')_{1''} \ ,
\end{aligned}
$$