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Fakultät Physik
Theoretische Physik III

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# Discrete flavor symmetries and geometrical CP violation

I. de Medeiros Varzielas, D. Emmanuel-Costa, PL
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# Discrete symmetries

- Prominently and effectively used to explain masses and mixings of neutrinos, charged leptons and quarks.
- Can also be used to generate CP violation in a similar manner!
- Have looked at $\Delta(27)$ and $\Delta(54)$ symmetric Lagrangians to achieve that.

# CP violation from scalars

- Start with a CP-conserving Lagrangian.
- Break CP through VEVs of scalar fields that break the gauge symmetry spontaneously (SCPV).
- Geometrical CP violation (GCPV):
  - CP phases emerge from symmetry structure of scalar sector rather than arbitrary parameters.
  - Phases are calculable.

# GCPV from $\Delta(27)$

- The groups $\Delta(3n^2)$ are non-Abelian subgroups of SU(3). ($\Delta(3) = Z_3$, $\Delta(12) \simeq A_4$ )
- $\Delta(3n^2) \sim (Z_n \times Z_n) \rtimes Z_3$
- Three generators:
- $a$, with $a^3 = 1$, generating $Z_3$.
- $c, d$, with $c^n = d^n = 1$ and $cd = dc$, generating $Z_3 \times Z_3$.
- 2 triplets, 9 singlets.

# The scalar potential

$H = (H^1, H^2, H^3) \in \mathbf{3}_{01}$
$H^\dagger \in \mathbf{3}_{02}$

- Renormalizable potential invariant under $\Delta(27)$:

$$V_{\text{ren}} = H^i H_i^\dagger + (H^i H_i^\dagger)(H^j H_j^\dagger) +$$

sum over indices
$\rightarrow Z_3$ structure

$$(H^i H_i^\dagger H^i H_i^\dagger) + c_\theta \left[ \sum_{i \neq j \neq k} (H^i)^2 H_j^\dagger H_k^\dagger + \text{h.c.} \right]$$

- We parametrize VEVs as

$$\langle H^1 \rangle = v_1 e^{i\phi_1} \quad \langle H^2 \rangle = v_2 e^{i\phi_2} \quad \langle H^3 \rangle = v_3 e^{i\phi_3}$$

# Minimization

| | + | - |
|---|---|---|
| $v_i^n$ | $\boxed{(1,1,1)}$ | $(0,0,1)$ |
| $v_i^m v_j^n$ | $(0,0,1)$ | $(0,1,1)$ |
| $v_1^l v_2^m v_3^n$ | $(0,0,1)/(0,1,1)$ | $\boxed{(1,1,1)}$ |

- Minimizing the potential gives classes of VEVs:

$$\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2) \qquad \langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$$

$$\omega \equiv e^{2\pi i/3}$$

Phases due to discrete symmetry → geometrical

- Also allowed: cyclic permutations or swapping of powers.
  - $(1, 1, \omega^2)$, $(\omega, 1, 1)$ etc.

# Stability of the solutions

- For improved fermion mass structures, higher order terms might be needed in the Lagrangian.
- Thus, higher order terms cannot be avoided in the scalar potential.
- Do the solutions including the calculable phases hold at higher orders?

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# Stability of the solutions

- There are three types of phase independent terms in the invariants that favor different VEVs:

$$\left.\begin{aligned}&(H^1 H_1^\dagger)^2 + \text{c.p.}\\&(H^1 H_1^\dagger)(H^2 H_2^\dagger) + \text{c.p.}\end{aligned}\right\}\quad \text{renormalizable}$$

$$(H^1 H_1^\dagger)(H^2 H_2^\dagger)(H^3 H_3^\dagger) \longleftarrow \quad \text{appears at order 6}$$

- Potential can be sorted in terms of these expressions.

(H301 ⊗ H301) ⊗ (H302 ⊗ H302),
(H301 ⊗ H302) ⊗ (H301 ⊗ H302),
(H301 ⊗ H302) ⊗ (H302 ⊗ H301)

((H301 ⊗ (H301 ⊗ H301)) ⊗ H302 ⊗ H302 ⊗ H302),
(((H301 ⊗ H301) ⊗ H302) ⊗ H301 ⊗ H302 ⊗ H302),
(H301 ⊗ H302 ⊗ H301 ⊗ H301 ⊗ H302 ⊗ H302),
(H302 ⊗ H301 ⊗ H301 ⊗ H301 ⊗ H302 ⊗ H302),
(((H302 ⊗ (H301 ⊗ H301)) ⊗ H302) ⊗ H301 ⊗ H302),
((H302 ⊗ H301) ⊗ (H302 ⊗ H301) ⊗ H301 ⊗ H302),
(H302 ⊗ H302 ⊗ H301 ⊗ H301 ⊗ (H301 ⊗ H302)),
(((H302 ⊗ H302) ⊗ (H301 ⊗ H301)) ⊗ (H302 ⊗ H301)),
(H302 ⊗ H301 ⊗ H302 ⊗ H301 ⊗ H302 ⊗ H301),
(H302 ⊗ H301 ⊗ H301 ⊗ H302 ⊗ H302 ⊗ H301)

((H301 ⊗ H301) ⊗ (H301 ⊗ H301)) ⊗ ((H302 ⊗ H302) ⊗ (H302 ⊗ H302)),
(H301 ⊗ (H301 ⊗ H301)) ⊗ (H302 ⊗ H301) ⊗ (H302 ⊗ (H302 ⊗ H302)),
(H301 ⊗ (H301 ⊗ H301)) ⊗ ((H302 ⊗ H302) ⊗ (H301 ⊗ (H302 ⊗ H302))),
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((H301 ⊗ H301) ⊗ (H302 ⊗ H302)) ⊗ (H302 ⊗ H301) ⊗ (H302 ⊗ H301),
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(H301 ⊗ H302) ⊗ ((H302 ⊗ H302) ⊗ (H301 ⊗ H301)) ⊗ (H302 ⊗ H301),
(H301 ⊗ H302) ⊗ ((H302 ⊗ H302) ⊗ H301) ⊗ (H302 ⊗ (H301 ⊗ H301)),
(H301 ⊗ H302) ⊗ ((H302 ⊗ H302) ⊗ H302) ⊗ (H301 ⊗ (H301 ⊗ H301))

- Use multiplication rules to expand these expressions.
- Filter for invariants.
- Sort them in a useful way!
- Great simplification:
  - Order 6: 110 → 34
  - Order 8: 1066 → 75

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# Stability of the solutions

| | sign of corresponding coefficient + | - |
|---|---|---|
| $v_i^n$ | $(1, 1, 1)$ | $(0, 0, 1)$ |
| $v_i^m v_j^n$ | $(0, 0, 1)$ | $(0, 1, 1)$ |
| $v_1^l v_2^m v_3^n$ | $(0, 0, 1)/(0, 1, 1)$ | $(1, 1, 1)$ |

- (1, 1, 1) and (0, 0, 1) appear naturally due to the dominance of $v_i^n$.
- Other VEVs need fine-tuning of the parameters.

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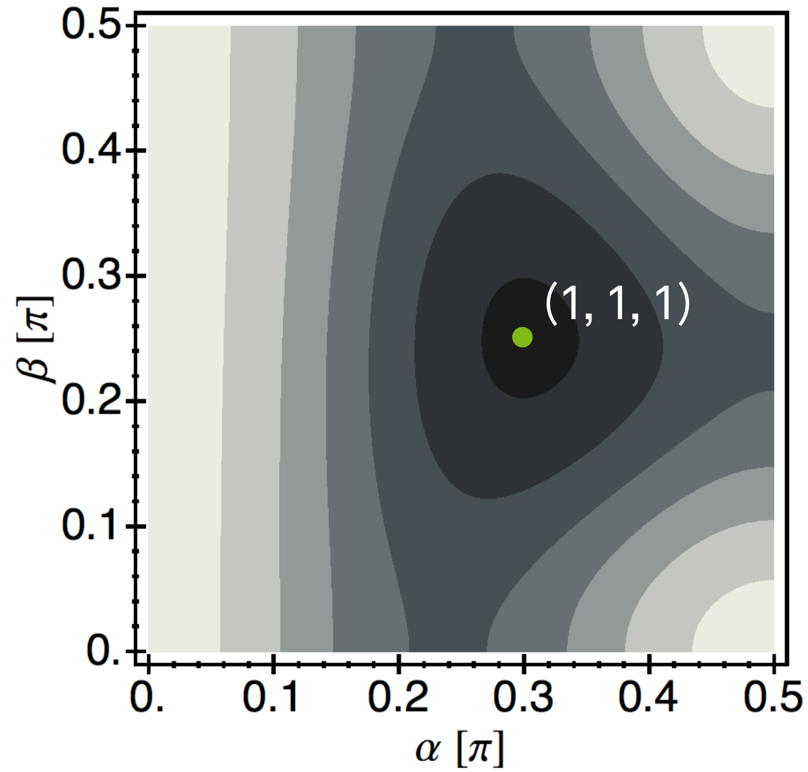
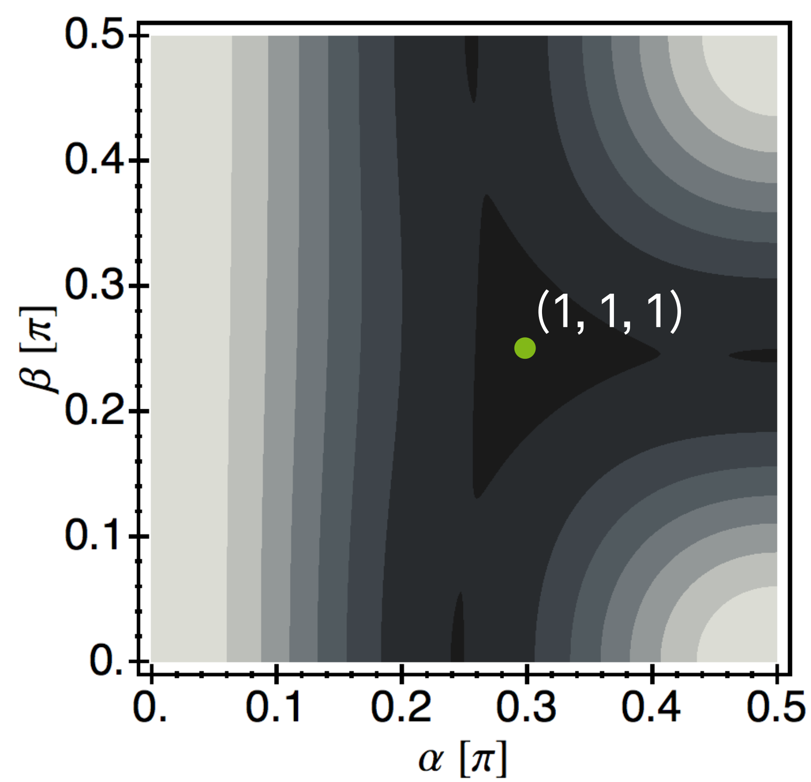
$v_1 = \sin\alpha\cos\beta$
$v_2 = \sin\alpha\sin\beta$
$v_3 = \cos\alpha$

# order 6

- $v_i^n$ (+) and $v_1^l v_2^m v_3^n$ (−) work together to easily produce (1, 1, 1).
- $v_i^n$ (+) overpowers $v_1^l v_2^m v_3^n$ (+), even though $v_i^n$ coefficient is numerically much smaller.
- Reverse sign to get (0, 0, 1).



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# Phase dependent terms

- At renormalizable order one phase combination:

$$\theta_i \equiv -2\phi_i + \phi_j + \phi_k$$

(+) favors $(\omega, 1, 1)$
(−) favors $(1, \omega, \omega^2)$

- At higher orders, additional patterns arise:

$$\theta_i^n \equiv -2n\phi_i + n\phi_j + n\phi_k$$

(+) favors $(\omega, 1, 1)$
(−) favors $(1, \omega, \omega^2)$

$$\eta_i^n \equiv 3n\phi_i - 3n\phi_j + 0\phi_k$$

no preference

suppressed by higher scale

- If one is willing to choose appropriate signs, VEVs with calculable phases can be preserved to arbitrarily high order.

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# Conclusion

- $\Delta(27)$ and $\Delta(54)$ can lead to a scalar sector with VEVs that have geometrical phases determined by the symmetry.
- We have classified the possible invariants and reordered them in a way that greatly reduces the number of relevant parameters.
- All terms appearing at higher orders can be classified into groups according to their effect on the VEVs.
- The geometrical phases can be naturally preserved to arbitrarily high order.

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# Group generators

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

2 triplets

$$\eta \equiv e^{2\pi i/n}$$

$$c = \begin{pmatrix} \eta^{l} & 0 & 0 \\ 0 & \eta^{k} & 0 \\ 0 & 0 & \eta^{-k-l} \end{pmatrix}$$

$$d = \begin{pmatrix} \eta^{-k-l} & 0 & 0 \\ 0 & \eta^{l} & 0 \\ 0 & 0 & \eta^{k} \end{pmatrix}$$

9 singlets

$$a = \omega^{r}$$

$$c = d = \omega^{s}$$

# (1, ω, ω²) is not CP-violating

- If the VEV is self-conjugate, it does not violate CP:
- Moreover, consider this transformation:

$$U\left(\qquad\right) = \left(\qquad\right)^{*}$$

- If $U$ is a symmetry of the Lagrangian, the VEV is not CP-violating either. This applies to $(1, \omega, \omega^2)$, because $U$ is one of the generators.
- The VEV $(1, 1, \omega)$ does not have this problem.