# Neutrino Mass from Higher-Dimensional Effective Operators in GUTs



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#### New Physics at the TeV Scale

The usual type I seesaw implies new physics close to the GUT scale.

 $\rightarrow$  not observable in experiments

#### Generating neutrino mass @TeV

- Radiative generation of neutrino mass. (Suppression via loop integrals)
- Small lepton number violating contribution. (Inverse seesaw, SUSY with RpV, ...)
- No contribution from d = 5 operator, neutrino mass generated by higher dimensional operator



#### Effective Operators

BSM physics can be parameterized as tower of effective operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm eff}^{d=5} + \mathcal{L}_{\rm eff}^{d=6} + \cdots, \quad {\rm with} \quad \mathcal{L}_{\rm eff}^d \propto \frac{1}{\Lambda_{\rm NP}^{d-4}} \, \mathcal{O}^d$$

suppressed by powers of the new physics scale  $\Lambda_{\mathrm{NP}}^{d-4}$ 

Seesaw mechanism: Weinberg operator  $\mathcal{O}_W = (\overline{L^c}i\tau^2 H)(Hi\tau^2 L)$  at d = 5.



After EWSB  $\frac{Y_N^2}{m_N} \langle H \rangle^2 \bar{\nu}^c \nu \rightarrow \text{generates neutrino mass } m_{\nu}^{\text{eff}} \propto \frac{v^2}{\Lambda}$ , with  $\Lambda = m_N$ 



#### Higher-Dimensional Effective Operators

Possible in theories with additional scalars (THDM, MSSM, NMSSM, ....)



- Neutrino mass is generated by higher dimensional operators
- New physics scale can be at lower energy
- Lower dimensional operators must be forbidden



#### Possible Effective Operators in the NMSSM

	Op.#	Effective interaction	Charge	Same as
<i>d</i> = 5	1	LLH <sub>u</sub> H <sub>u</sub>	$2q_L + 2q_{H_{u}}$	
<i>d</i> = 6	2	LLH <sub>u</sub> H <sub>u</sub> S	$2q_L + q_{H_u} - q_{H_d}$	
<i>d</i> = 7	3	$LLH_uH_uH_dH_u$	$2q_L + 3q_{H_{\mu}} + q_{H_d}$	
	4	LLH <sub>u</sub> H <sub>u</sub> SS	$2q_L - 2q_{H_d}$	
d = 8	5	LLH <sub>u</sub> H <sub>u</sub> H <sub>d</sub> H <sub>u</sub> S	$2q_L + 2q_{H_{11}}$	#1
	6	LLH <sub>u</sub> H <sub>u</sub> SSS	$2q_L + 2q_{H_u}$	#1
<i>d</i> = 9	7	LLH <sub>u</sub> H <sub>u</sub> H <sub>d</sub> H <sub>u</sub> H <sub>d</sub> H <sub>u</sub>	$2q_L + 4q_{H_{II}} + 2q_{H_d}$	
	8	$LLH_uH_uH_dH_uSS$	$2q_L + q_{H_u} - q_{H_d}$	#2
	9	LLH <sub>u</sub> H <sub>u</sub> SSSS	$2q_L + q_{H_u} - q_{H_d}$	#2

#### **Characteristics**

- Condition for discrete charges of fields from neutrality of superpotential
- Rules out some operators as leading contribution to neutrino mass
- Several possible fundamental theories can lead to the same effective operator at low energies

(c.f. Bonnet, Hernandez, Ota, Winter (2009); JHEP 0910, 076 for a study in the THDM)



#### <u>A d = 7 Example</u>

#### Superpotential

$$W = W_{ ext{NMSSM}} + Y_N \hat{N} \hat{L} \cdot \hat{H}_u - \kappa_1 \hat{N}' \hat{\xi} \cdot \hat{H}_d + \kappa_2 \hat{N}' \hat{\xi}' \cdot \hat{H}_u + m_N \hat{N} \hat{N}' + m_{\xi} \hat{\xi} \cdot \hat{\xi}'$$



#### New fields:

- SM singlets N, N'
- SU(2)<sub>L</sub> doublets  $\xi$ ,  $\xi'$



#### Mass Matrix

• In the basis  $f^0 = (\nu, N, N', {\xi^0}, {{\xi'}^0})$  we obtain the mass matrix

$$M_f^0 = \begin{pmatrix} 0 & Y_N v_u & 0 & 0 & 0 \\ Y_N v_u & 0 & m_N & 0 & 0 \\ 0 & m_N & 0 & \kappa_1 v_d & \kappa_2 v_u \\ 0 & 0 & \kappa_1 v_d & 0 & m_\xi \\ 0 & 0 & \kappa_2 v_u & m_\xi & 0 \end{pmatrix}$$

By integrating out the heavy fields we obtain an effective mass matrix for the three SM neutrinos at low energies

$$m_
u = v_u^3 v_d Y_N^2 rac{\kappa_1 \kappa_2}{m_\xi m_N^2}$$

Masses at TeV scale for couplings  $\mathcal{O}(10^{-3})$ 

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## **Phenomenology**

- Can the new particles be produced?
  - $\hfill\square$  Rare production of  $\hat{N}$  and  $\hat{N}'$  due to small Yukawa couplings
  - □ SU(2)<sub>L</sub> doublets can be produced in Drell-Yan processes ( $\sigma \sim 10^2$  fb) (similarly to charginos and neutralinos)



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### Phenomenology II

#### **Characteristic Signals**

- Displaced vertices due to small mixing between heavy and light neutrinos
- Lepton number violating processes
  - □ LNC cross-section for  $pp \rightarrow W\ell\ell$  of  $\mathcal{O}(10^2)$  fb LNV processes suppressed due to pseudo-Dirac pairs ( $< \mathcal{O}(10^{-9})$  fb)
  - □ For  $pp \rightarrow W\ell W\ell$  LNV processes larger than naively expected ( $\mathcal{O}(10^{-2})$  fb)

(Numerical analysis with WHIZARD)



#### MBK, Ota, Porod, Winter (2011); PRD 84, 115023



## GUT Completion (work in progress...)

- Additional particles modify running of the gauge couplings
- Spoils unification
- Add complete SU(5) multiplets to avoid this

$$\begin{split} \bar{\mathbf{5}}_{M} &= \begin{pmatrix} d_{1}^{c} \\ d_{2}^{c} \\ e^{-} \\ -\nu_{e} \end{pmatrix}_{L} = \begin{pmatrix} d_{L}^{c} \\ L \end{pmatrix} \qquad \bar{\mathbf{5}}_{\xi'} = \begin{pmatrix} d_{1}^{c} \\ d_{2}^{c} \\ d_{3}^{c} \\ \xi^{--} \\ -\xi'^{0} \end{pmatrix}_{L} = \begin{pmatrix} d_{L}^{\prime c} \\ d_{L}^{\prime} \end{pmatrix} \qquad \mathbf{5}_{\xi} = \begin{pmatrix} d_{1}^{\prime } \\ d_{2}^{\prime} \\ d_{3}^{\prime} \\ \xi^{+} \\ -\xi^{0} \end{pmatrix}_{R} = \begin{pmatrix} d_{N}^{\prime } \\ d_{2}^{\prime} \\ d_{3}^{\prime} \\ \xi^{+} \\ -\xi^{0} \end{pmatrix}_{R} \\ H_{5} = \begin{pmatrix} H_{1} \\ H_{2} \\ H_{3} \\ H_{4}^{\prime} \\ H_{0}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{2} \\ H_{3}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{2} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \\ H_{d}^{\prime} \end{pmatrix} = \begin{pmatrix} H_{col} \\ H_{d}^{\prime} \\ H_{d}^{\prime}$$



#### MSSM vs. NMSSM

#### Extension of MSSM

$$W = y_1 N 5_{\xi} H_5 + y_2 N \overline{5}_{\xi'} H_5 + y_3 N \overline{5}_M H_5 + y_1', N' 5_{\xi} H_5 + y_2' N' \overline{5}_{\xi'} H_5 + y_3' N' \overline{5}_M H_5 + m_{\xi'} \overline{5}_M 5_{\xi} + m_{\xi} \overline{5}_{\xi'} 5_{\xi} + m_N N' N + m_{NN} NN + m_{N'N'} N' N' + y_d \overline{5}_M 10 H_5 + y_d' \overline{5}_{\xi'} 10 H_5 + y_u 10 10 H_5.$$

#### Extension of NMSSM

$$\begin{split} W &= y_1 \, N \, 5_{\xi} \, H_{\bar{5}} + y_2 \, N \, \bar{5}_{\xi'} \, H_5 + y_3 \, N \, \bar{5}_M \, H_5 + \\ & y_1', \, N' \, 5_{\xi} \, H_{\bar{5}} + y_2' \, N' \, \bar{5}_{\xi'} \, H_5 + y_3' \, N' \, \bar{5}_M \, H_5 + \\ & \lambda_{\xi'} \, S \, \bar{5}_M \, 5_{\xi} + \lambda_{\xi} \, S \, \bar{5}_{\xi'} \, 5_{\xi} + \lambda_N S \, N' N + \lambda_{NN} S \, NN + \lambda_{N'N'} S \, N' N' + \\ & y_d \, \bar{5}_M \, 10 \, H_{\bar{5}} + y_d' \, \bar{5}_{\xi'} \, 10 \, H_{\bar{5}} + y_u \, 10 \, 10 \, H_5 \, . \end{split}$$



#### GUT Extensions of the NMSSM

 $LLH_uH_u$  must have different charge than  $(LLH_uH_u)(H_uH_d)$ 

# MSSMNMSSMThe term $\mu H_u H_d$ in the superpotential<br/>breaks the symmetry explicitly $\mu$ -term replaced by $\lambda SH_u H_d$ Mass terms set to TeV scale by handMasses generated by the<br/>VEV of S

 ${\bf BUT}$  if all masses generated by  $\langle S \rangle$  the effective operators become

$$\frac{1}{\langle S \rangle} LLH_u H_u \qquad \frac{1}{\langle S \rangle^3} (LLH_u H_d) (H_u H_d)$$

 $\Rightarrow \langle {\it S} 
angle$  breaks discrete symmetry



#### Heavy d-Quarks



Interactions of d'

#### From F-terms:

The only scalar term for the superpartners of the additional d-like quarks is their mass term  $m_{\ell}^{2} \widetilde{d}'^{\dagger} \widetilde{d}''.$ 

#### From D-terms:

Kinetic terms for the 5-plets in SU(5) GUT is

$$\mathcal{L}_{\mathsf{kin}}^{[5]} = \overline{\Psi}^{[5]} D \!\!\!/ \Psi^{[5]}$$

We obtain:

$$\begin{split} \bar{\mathbf{5}}_{\boldsymbol{\xi}} \not{\mathcal{D}}\mathbf{5}_{\boldsymbol{\xi}} &= \bar{d}' \not{\partial} d' + \bar{\xi}' \not{\partial} \boldsymbol{\xi} \\ &- \mathrm{i} g_{\mathbf{5}} \left( \bar{d}' \sum_{a=1}^{9} G_{\mu}^{a} \tilde{\lambda}_{a} d' + \bar{d}' B_{\mu} \tilde{\lambda}_{24} d' + \bar{\xi}' \sum_{a=21}^{23} A^{a} \tilde{\lambda}_{a} \boldsymbol{\xi}' + \bar{\xi}' B_{\mu} \tilde{\lambda}_{24} \boldsymbol{\xi}' + \mathrm{lq. int.} \right) \end{split}$$



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# Cosmological constraints

Stable d' bound in heavy nuclei  $\rightarrow$  Conflicts with Cosmological constraints!

#### From Big Bang Nucleosynthesis

- Neutral hadrons
  - Pion production alters p-n equilibrium
  - □ heavy nucleon-nucleon scattering produces cascades of light nuclei altering abundances  $({}^{2}H, {}^{3}H, {}^{3}He)$
- Charged hadrons
  - Catalyzed nucleosynthesis
  - $\hfill\square$  Changes properties of nuclear processes  $\rightarrow$  alters abundances of light elements

e.g. locco et. al. (2009); Phys.Rept. 472

#### From other heavy element searches

- Search for heavy hadrons in water
- Stability of neutron stars



# Annihilation of d'





- d' in thermal equilibrium until freeze-out
- $\blacksquare$  Freeze out point depends on  $\langle \sigma \mathbf{v} \rangle$
- Expected relic density far above experimental bounds
- Additional effects due to confinement, but still not in accordance with bounds



Nardi, Roulet (1990); Phys. Lett. B 245, 105

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#### Leptoquarks

- d' can decay via leptoquarks into ξ components, which are unstable
- Decay width  $\propto (100 \text{ GeV})^5/M_{GUT}^4$ larger than for proton decay  $(\propto (1 \text{ GeV})^5/M_{GUT}^4)$
- Lifetime must be small enough to fit cosmological constraints
- Further contributions from decays via colored Higgses



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# <u>Summary</u>

- Possible to use effective operators with d > 5 to generate neutrino masses
- New physics at TeV scale, phenomenological implications at LHC
- Full SU(5) multiplets necessary to not spoil unification
- Additional d-quarks in conflict with cosmological constraints
- NMSSM realization not possible with discrete symmetry

#### **Open Issues**

- d' decay via leptpoquarks
- Systematic study of GUT decompositions of the d = 7 operator



# **Backup-Slides**

#### Decompositions

#	Operator	Mediators	SU(5) multiplets
1	$(H_u i \tau^2 \overline{L^c})(H_u i \tau^2 L)(H_d i \tau^2 H_u)$	$1_0^R$ , $1_0^L$ , $1_0^s$	1, 1, 1
2	$(H_u \mathrm{i} \tau^2 \vec{\tau} \overline{L^c}) (H_u \mathrm{i} \tau^2 L) (H_d \mathrm{i} \tau^2 \vec{\tau} H_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ $1_{0}^{R}$ , $1_{0}^{L}$ , $3_{0}^{s}$	24, 24, (1), (1), 24
3	$(H_u i \tau^2 \vec{\tau} \overline{L^c}) (H_u i \tau^2 \vec{\tau} L) (H_d i \tau^2 H_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $1_{0}^{s}$	24, 24, 1
4	$(-\mathrm{i}\epsilon^{abc})(H_{u}\mathrm{i}\tau^{2}\tau^{a}\overline{L^{c}})(H_{u}\mathrm{i}\tau^{2}\tau^{b}L)(H_{d}\mathrm{i}\tau^{2}\tau^{c}H_{u})$	$3_{0}^{R}$ , $3_{0}^{L}$ , $3_{0}^{s}$	24, 24, 24
5	$(L^{c}\mathrm{i}\tau^{2}\vec{\tau}L)(H_{d}\mathrm{i}\tau^{2}H_{u})(H_{u}\mathrm{i}\tau^{2}\vec{\tau}H_{u})$	$3_{\pm 1}^{s}$ , $3_{\pm 1}^{s}$ , $1_{0}^{s}$	15, 15, 1
6	$(-i\epsilon_{abc})(\overline{L^{c}}i\tau^{2}\tau_{a}L)(H_{d}i\tau^{2}\tau_{b}H_{u})(H_{u}i\tau^{2}\tau_{c}H_{u})$	$3_{+1}^{s}$ , $3_{+1}^{s}$ , $3_{0}^{s}$	15, 15, 24
7	$(H_u \mathrm{i} \tau^2 \overline{L^c}) (L \mathrm{i} \tau^2 \vec{\tau} H_d) (H_u \mathrm{i} \tau^2 \vec{\tau} H_u)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $3_{-1}^{R}$ , $3_{-1}^{L}$ , $3_{+1}^{s}$	$1, 1, 15, \overline{15}, 15$
8	$(-i\epsilon^{abc})(H_ui\tau^2\tau^a\overline{L^c})(Li\tau^2\tau^bH_d)(H_ui\tau^2\tau^cH_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $3_{-1}^{R}$ , $3_{-1}^{L}$ , $3_{+1}^{s}$	$24, 24, 15, \overline{15}, 15$
9	$(H_u \mathrm{i} \tau^2 \overline{L^c})(\mathrm{i} \tau^2 H_u)(L)(H_d \mathrm{i} \tau^2 H_u)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $1_{0}^{s}$	$1, 1, 5, \overline{5}, 1$
10	$(H_u \mathrm{i} \tau^2 \vec{\tau} \overline{L^c}) (\mathrm{i} \tau^2 \vec{\tau} H_u) (L) (H_d \mathrm{i} \tau^2 H_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $1_{0}^{s}$	$24,24,5,\overline{5},1$
11	$(H_u i \tau^2 \overline{L^c})(i \tau^2 H_u)(\tau L)(H_d i \tau^2 \tau H_u)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $3_{0}^{s}$	1, 1, 5, 5, 24
12	$(H_u \mathrm{i} \tau^2 \tau^a \overline{L^c}) (\mathrm{i} \tau^2 \tau^a H_u) (\tau^b L) (H_d \mathrm{i} \tau^2 \tau^b H_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $3_{0}^{s}$	$24, 24, 5, \mathbf{\overline{5}}, 24$
13	$(H_u \mathrm{i} \tau^2 \overline{L^c})(L)(\mathrm{i} \tau^2 H_u)(H_d \mathrm{i} \tau^2 H_u)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{+1/2}^{s}$ , $1_{0}^{s}$	1, 1, 5, 1
14	$(H_u \mathrm{i} \tau^2 \vec{\tau} \overline{L^c})(\vec{\tau} L)(\mathrm{i} \tau^2 H_u)(H_d \mathrm{i} \tau^2 H_u)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $2_{+1/2}^{s}$ , $1_{0}^{s}$	24, 24, 5, 1
15	$(H_u \mathrm{i} \tau^2 \overline{L^c})(L)(\mathrm{i} \tau^2 \vec{\tau} H_u)(H_d \mathrm{i} \tau^2 \vec{\tau} H_u)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{+1/2}^{s}$ , $3_{0}^{s}$	1, 1, 5, 24
16	$(H_u \mathrm{i} \tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i} \tau^2 \tau^b H_u)(H_d \mathrm{i} \tau^2 \tau^b H_u)$	$3_0^R$ , $3_0^L$ , $2_{+1/2}^s$ , $3_0^s$	24, 24, 5, 24
17	$(H_u \mathrm{i} \tau^2 \overline{L^c})(H_d)(\mathrm{i} \tau^2 H_u)(H_u \mathrm{i} \tau^2 L)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$	$1, 1, 5, \overline{5}$
18	$(H_u \mathrm{i} \tau^2 \tau \overline{L^c})(\tau H_d)(\mathrm{i} \tau^2 H_u)(H_u \mathrm{i} \tau^2 L)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $1_{0}^{R}$ , $1_{0}^{L}$	$24, 24, 5, \overline{5}, (1), (1)$
19	$(H_u \mathrm{i} \tau^2 \overline{L^c})(H_d)(\mathrm{i} \tau^2 \vec{\tau} H_u)(H_u \mathrm{i} \tau^2 \vec{\tau} L)$	$1_{0}^{R}$ , $1_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ , $3_{0}^{R}$ , $3_{0}^{L}$	$(1), (1), 5, \overline{5}, 24, 24$
20	$(H_u \mathrm{i} \tau^2 \tau^a \overline{L^c})(\tau^a H_d)(\mathrm{i} \tau^2 \tau^b H_u)(H_u \mathrm{i} \tau^2 \tau^b L)$	$3_{0}^{R}$ , $3_{0}^{L}$ , $2_{-1/2}^{R}$ , $2_{-1/2}^{L}$ ,	$24,24,5,\overline{5}$
21	$(\overline{L^c}i\tau^2\tau^a I)(H;\tau^2\tau^a)(\tau^b H)(H;\tau^2\tau^b H)$	25 25 25	15 5 15



#### Possible Terms

• Terms from  $\overline{\mathbf{5}} \otimes \mathbf{5}(\otimes \mathbf{1})$ 

$$\begin{split} & N \left( 5_{\xi} H_{\overline{5}} + \overline{5}_{\xi'} H_{5} + \overline{5}_{M} H_{5} \right) + (N \leftrightarrow N') \\ & m_{\xi'} \overline{5}_{M} 5_{\xi} \\ & m_{\xi} \overline{5}_{\xi'} 5_{\xi} \end{split}$$

 $\blacksquare \ \, {\sf Terms} \ \, {\sf from} \ \, \overline{{\bf 5}} \otimes {\bf 10} \otimes \overline{{\bf 5}}$ 

$$\bar{5}_{M} 10 H_{\bar{5}}$$
  
 $\bar{5}_{\xi'} 10 H_{\bar{5}}$ 

• Terms from  $10 \otimes 10 \otimes 5$ The usual one which generates up-quark masses (5 =  $H_5$ ).