A Minimal Model of Neutrino Flavor

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Work in progress w/Christoph Luhn (IPPP, Durham) &
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Neutrino Mixing Matrix

- Neutrinos have mass and the different flavors can mix

  Super-Kamiokande Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, hep-ex/9807003

  SNO Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, nucl-ex/0204008

- Charged lepton and neutrino mass matrices cannot be simultaneously diagonalized

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

- Pontecorvo-Maki-Nakagawa-Sakata matrix

  B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429

  Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{D_L U_L^\dagger}_{U_{\mathrm{PMNS}}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino Mixing Matrix

What we know about the mixing angles ...

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{13} s_{23} c_{12} e^{i\delta} & c_{23} c_{12} - s_{13} s_{23} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - s_{13} c_{23} c_{12} e^{i\delta} & -s_{23} c_{12} - s_{13} c_{23} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

T. Schwetz, M. Tortola, and J. Valle, "Where we are on θ_{13}: addendum to 'Global neutrino data and recent reactor fluxes: status of three-flavour oscillation parameters'," *New J.Phys.* **13** (2011) 109401, 1108.1376

Parameter	Best Fit	1σ range	3σ range
θ_{12}	33.96°	$33.02^\circ - 35.0^\circ$	$31.31^\circ - 36.87^\circ$
θ_{23}	46.15°	$42.13^\circ - 49.6^\circ$	$38.65^\circ - 53.13^\circ$
θ_{13}	6.55°	$5.13^\circ - 8.13^\circ$	$1.81^\circ - 10.78^\circ$

Tribimaximal Mixing

➢ Until recently, our best guess was tribimaximal mixing (TBM)

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, hep-ph/0202074

$$U_{\mathrm{PMNS}} \stackrel{?}{=} U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\hookrightarrow \quad \theta_{12} = 35.26^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ$$

➢ Agreement still quite good for θ_{12}, θ_{23}, but $\theta_{13} = 0^\circ$ excluded @5σ

Schwetz et al, 1108.1376, **DAYA-BAY** Collaboration, 1203.1669, **RENO** Collaboration, 1204.0626

Parameter	Tribimaximal	Global fit 1σ	Daya Bay	Reno	
θ_{12}	35.26°	$33.02^\circ - 35.0^\circ$	-	-	✓
θ_{23}	45.00°	$42.13^\circ - 49.6^\circ$	-	-	✓
θ_{13}	0.00°	$5.13^\circ - 8.13^\circ$	8.8°	9.8°	✗

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Post-Daya Bay Confusion

- Regular patter $U_{\rm PMNS}$ is suggestive of a family symmetry

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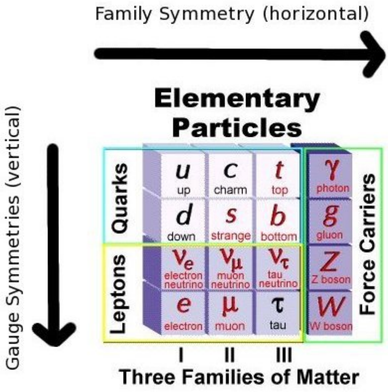
- Daya bay and Reno rule out tribimaximal Mixing
- What are our options?
  - Give up family symmetries? Some remarks towards the end . . .
  - Look for groups that give $\theta_{13} \neq 0^\circ$
    R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," *Phys.Lett.* **B703** (2011) 447–451, 1107.3486
  - Keep TBM and calculate higher corrections → This talk!

Discrete Flavor Symmetries

- ➢ Introduce relations between families of quarks and leptons
- ➢ But which discrete group do we take for the family symmetry?



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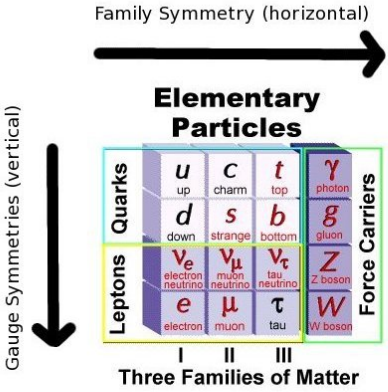
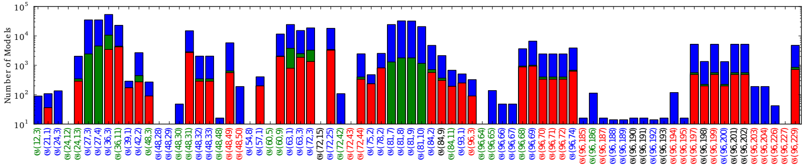


Discrete Flavor Symmetries

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

- We scanned 76 discrete groups
- Many papers on $A_4 \times \mathbb{Z}_n$, $n \geq 3$. Connection between A_4 and TBM?
- $\Delta(96)$ gives $\theta_{13} \neq 0°$ but is large
- We identified the smallest group that gives TBM!



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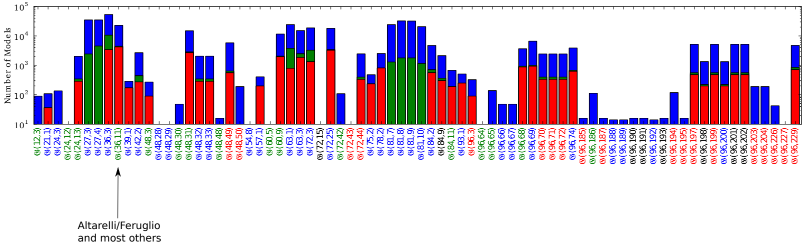

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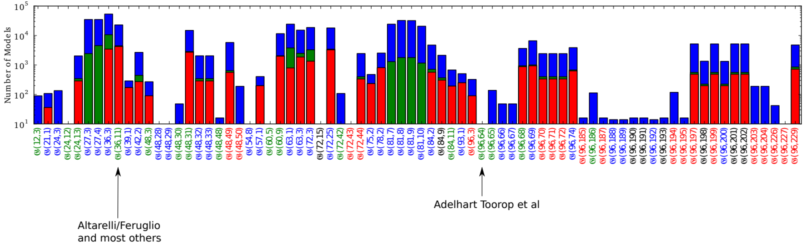

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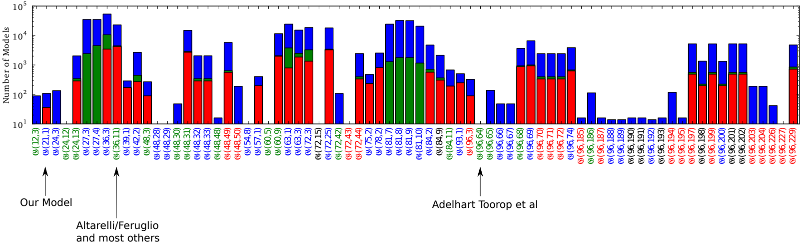

Discrete Flavor Symmetries

C. Luhn, K. M. Parattu, A. Wingerter, *work in progress*

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times T_7 \times \mathrm{U}(1)_R$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	T_7	$\mathrm{U}(1)_R$
L	$(\mathbf{2},\text{-}1)$	$\mathbf{3}$	1
e	$(\mathbf{1},\ 2)$	$\mathbf{1}$	1
μ	$(\mathbf{1},\ 2)$	$\mathbf{1'}$	1
τ	$(\mathbf{1},\ 2)$	$\mathbf{1''}$	1
h_u	$(\mathbf{2},\ 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2},\text{-}1)$	$\mathbf{1}$	0
φ	$(\mathbf{1},\ 0)$	$\mathbf{3}$	0
$\widetilde{\varphi}$	$(\mathbf{1},\ 0)$	$\mathbf{3'}$	0

❸ Breaking the family symmetry

$$\langle \varphi \rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$$

All you need to know about T_7

Tensor Products

$\mathbf{1} \times \mathbf{1} = \mathbf{1}$
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Contractions

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{3}x_1y_1 + \frac{1}{3}\sqrt{3}x_2y_3 + \frac{1}{3}\sqrt{3}x_3y_2 \\ x_1y_2\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_2y_1\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_3y_3\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) \\ x_1y_3\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_2y_2\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_3y_1\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) \end{pmatrix}$$

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Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

- Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

- Contract family indices (need to know Clebsch-Gordan coefficients):

$$y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3$$
$$+ y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2$$
$$+ y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1$$

- Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & \quad + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & \quad + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{matrix} & e & \mu & \tau \\ L_1^{(2)} & y_e & 0 & 0 \\ L_2^{(2)} & 0 & y_\mu & 0 \\ L_3^{(2)} & 0 & 0 & y_\tau \end{matrix}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{matrix} & L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \\ L_1^{(1)} & \sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi \\ L_2^{(1)} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} \\ L_3^{(1)} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} & \sqrt{2}y_2 v_\varphi \end{matrix}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form-diagonalizable! Mixing does not depend on A, B, masses do!

➢ Mixing angles: $\theta_{12} = 35.26°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$ Tribimaximal ✔

Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\widetilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{matrix} e & \mu & \tau \end{matrix} \\ \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{matrix} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{matrix} \\ \begin{pmatrix} \sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} & \sqrt{2}y_2 v_\varphi \end{pmatrix} \end{array}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\rm PMNS} = D_{\rm L} U_{\rm L}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \overset{\begin{array}{ccc} e & \mu & \tau \end{array}}{\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \overset{\begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array}}{\begin{pmatrix} \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B & \sqrt{2}A \end{pmatrix}}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

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Our T_7 Model at Leading Order

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B & \sqrt{2}A \end{pmatrix} \end{array}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

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➢ Mixing angles: $\theta_{12} = 35.26^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ Tribimaximal ✔

Next-to-Leading-Order Corrections

- Remember leading-order superpotential:

> - Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:
>
> $$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$
>
> $$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

- Superpotential (now mass dimension $\leq$6 or 7)

$$\begin{aligned}
&C_9\, L\, e\, h_d\, \widetilde{\varphi} + C_{14}\, L\, \mu\, h_d\, \widetilde{\varphi} + C_{19}\, L\, \tau\, h_d\, \widetilde{\varphi} + \\
&C_{10}\, L\, e\, h_d\, \varphi\, \varphi + C_{11}\, L\, e\, h_d\, \varphi\, \widetilde{\varphi} + C_{12}\, L\, e\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{13}\, L\, e\, h_u\, h_d\, h_d\, \widetilde{\varphi} + \\
&C_{15}\, L\, \mu\, h_d\, \varphi\, \varphi + C_{16}\, L\, \mu\, h_d\, \varphi\, \widetilde{\varphi} + C_{17}\, L\, \mu\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{18}\, L\, \mu\, h_u\, h_d\, h_d\, \widetilde{\varphi} + \\
&C_{20}\, L\, \tau\, h_d\, \varphi\, \varphi + C_{21}\, L\, \tau\, h_d\, \varphi\, \widetilde{\varphi} + C_{22}\, L\, \tau\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{23}\, L\, \tau\, h_u\, h_d\, h_d\, \widetilde{\varphi} + \\
&C_1\, L\, L\, h_u\, h_u\, \varphi + C_2\, L\, L\, h_u\, h_u\, \widetilde{\varphi} + \\
&C_3\, (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \varphi\, \varphi + C_4\, (L\, L)_{\mathbf{3}'}\, h_u\, h_u\, \varphi\, \varphi + \\
&C_7\, (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \widetilde{\varphi}\, \widetilde{\varphi} + C_8\, (L\, L)_{\mathbf{3}'}\, h_u\, h_u\, \widetilde{\varphi}\, \widetilde{\varphi} + \\
&C_5\, (L\, L)_{\mathbf{3}}\, h_u\, h_u\, \varphi\, \widetilde{\varphi} + C_6\, (L\, L)_{\mathbf{3}'}\, h_u\, h_u\, \varphi\, \widetilde{\varphi}
\end{aligned}$$

- Leading-order terms, neglected terms, NLO terms

Next-to-Leading-Order Corrections

➢ The Charged Lepton Sector

$$\Delta M_\ell = -\frac{1}{3\sqrt{2}} v_d \Big[C_{10} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{11} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C_{12} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+ C_{15} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{16} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} + C_{17} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+ C_{20} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{21} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} + C_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Big]$$

➢ The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{24\sqrt{3}} \Big[C_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 3\sqrt{2} C_4 \begin{pmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{pmatrix} + 4 C_7 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
$$+ C_8 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} + 4 C_5 \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \sqrt{2} C_6 \begin{pmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{pmatrix} \Big]$$

Switching the constants on and off

	$\exp(2\pi i)$			$\exp(2\pi i/3)$			$\exp(2\pi i/5)$		
	θ_{12}	θ_{23}	θ_{13}	θ_{12}	θ_{23}	θ_{13}	θ_{12}	θ_{23}	θ_{13}
C_3	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_4	28.52	45.00	3.47	37.73	56.24	8.90	32.28	57.91	8.50
C_5	35.34	45.00	3.48	35.42	41.51	5.04	35.41	41.80	4.83
C_6	31.22	45.00	2.18	36.92	37.73	5.57	33.64	36.76	5.60
C_7	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_8	33.57	45.00	0.00	36.13	45.00	0.00	34.72	45.00	0.00
C_{10}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{11}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{12}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{15}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{16}	31.68	44.95	3.58	37.08	44.83	4.10	33.85	44.90	3.79
C_{17}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{20}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00
C_{21}	31.03	51.15	4.22	37.40	42.38	4.97	33.52	47.78	4.52
C_{22}	35.26	45.00	0.00	35.26	45.00	0.00	35.26	45.00	0.00

Switching the constants on and off

➢ The Charged Lepton Sector

$$\Delta M_\ell = -\frac{1}{3\sqrt{2}} v_d \Big[C_{10} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{11} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C_{12} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ C_{15} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{16} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} + C_{17} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ C_{20} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_{21} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} + C_{22} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Big]$$

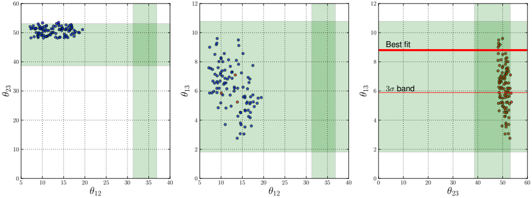
➢ The Neutrino Sector

$$\Delta M_\nu = \frac{v_u^2}{24\sqrt{3}} \Big[C_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 3\sqrt{2} C_4 \begin{pmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{pmatrix} + 4 C_7 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

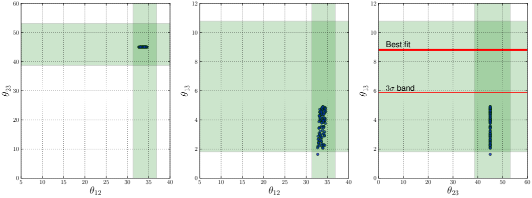
$$+ C_8 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} + 4 C_5 \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \sqrt{2} C_6 \begin{pmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{pmatrix} \Big]$$

Varying the Constants

- No constraints on the C_i

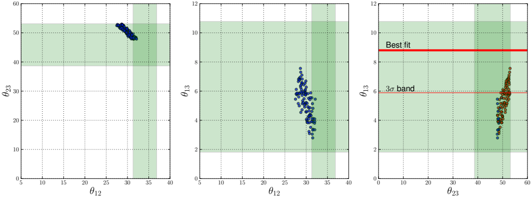


- $C_4 = C_6 = C_{16} = C_{21} = 0$

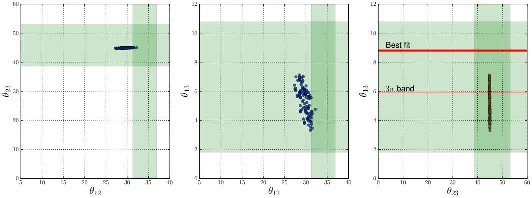


Varying the Constants

- $C_4 = C_6 = C_{16} = 0, \; C_{21} \neq 0$



- $C_4 = C_6 = C_{21} = 0, \; C_{16} \neq 0$



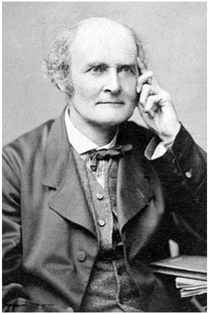
Which Flavor Symmetry?

When would we claim that a given flavor symmetry is specifically well-suited to describe neutrino mixing?

- First of all, it should reproduce the data!
  TBM is excluded at 5σ. What does this mean for A_4 (or T_7)?
- It should convincingly reproduce the data!
  - One can (probably) always tweak TBM models to give $\theta_{13} \neq 0$.
  - θ_{12}, θ_{23} in agreement w/TBM. Is that enough?
  - Angles should be insensitive to $C_i \to$ Form-diagonalizable?
  - Or: Correlation between neutrino masses and angles
- Ideally, flavor symmetry arises from some more fundamental theory
- The choice of group should be guided by above principles
- "Intuition", geometric imagination and "easiness" may be misleading $\to$ Consider the case of A_4

Which Flavor Symmetry?

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and 6 . . .



The Small Groups library

All groups (423,164,062) of order $\leq$ 2000 except 1024

Hans Ulrich Besche, Bettina Eick and Eamonn O'Brien

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[1, 1]	1	✗	---	---	---	✗
[2, 1]	$\mathbb{Z}_2$	✗	---	---	---	✗
[3, 1]	$\mathbb{Z}_3$	✗	---	---	---	✗
[4, 1]	$\mathbb{Z}_4$	✗	---	---	---	✗
[4, 2]	$\mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[5, 1]	$\mathbb{Z}_5$	✗	---	---	---	✗
[6, 1]	S_3	✗	✔	✔	✔	✗
[6, 2]	$\mathbb{Z}_6$	✗	---	---	---	✗
[7, 1]	$\mathbb{Z}_7$	✗	---	---	---	✗
[8, 1]	$\mathbb{Z}_8$	✗	---	---	---	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[8, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_2$	✗	---	---	---	✗
[8, 3]	D_4	✗	✔	✔	✔	✗
[8, 4]	Q_8	✗	✔	✔	✔	✗
[8, 5]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[9, 1]	$\mathbb{Z}_9$	✗	---	---	---	✗
[9, 2]	$\mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
[10, 1]	D_5	✗	✔	✔	✔	✗
[10, 2]	$\mathbb{Z}_{10}$	✗	---	---	---	✗
[11, 1]	$\mathbb{Z}_{11}$	✗	---	---	---	✗
[12, 1]	$\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[12, 2]	$\mathbb{Z}_{12}$	✗	---	---	---	✗
[12, 3]	A_4	✔	✔	✗	✗	✔
[12, 4]	D_6	✗	✔	✔	✔	✗
[12, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_2$	✗	---	---	---	✗
[13, 1]	$\mathbb{Z}_{13}$	✗	---	---	---	✗
[14, 1]	D_7	✗	✔	✔	✔	✗
[14, 2]	$\mathbb{Z}_{14}$	✗	---	---	---	✗
[15, 1]	$\mathbb{Z}_{15}$	✗	---	---	---	✗
[16, 1]	$\mathbb{Z}_{16}$	✗	---	---	---	✗
[16, 2]	$\mathbb{Z}_4 \times \mathbb{Z}_4$	✗	---	---	---	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[16, 3]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_{\varphi} \mathbb{Z}_2$	✗	✔	✗	✔	✗
[16, 4]	$\mathbb{Z}_4 \rtimes_{\varphi} \mathbb{Z}_4$	✗	✔	✗	✔	✗
[16, 5]	$\mathbb{Z}_8 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 6]	$\mathbb{Z}_8 \rtimes_{\varphi} \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 7]	D_8	✗	✔	✔	✔	✗
[16, 8]	QD_8	✗	✔	✔	✔	✗
[16, 9]	Q_{16}	✗	✔	✔	✔	✗
[16, 10]	$\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[16, 11]	$\mathbb{Z}_2 \times D_4$	✗	✔	✗	✔	✗
[16, 12]	$\mathbb{Z}_2 \times Q_8$	✗	✔	✗	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[16, 13]	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[16, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[17, 1]	$\mathbb{Z}_{17}$	✗	---	---	---	✗
[18, 1]	D_9	✗	✔	✔	✔	✗
[18, 2]	$\mathbb{Z}_{18}$	✗	---	---	---	✗
[18, 3]	$\mathbb{Z}_3 \times S_3$	✗	✔	✔	✔	✗
[18, 4]	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_2$	✗	✗	✗	✗	✗
[18, 5]	$\mathbb{Z}_6 \times \mathbb{Z}_3$	✗	---	---	---	✗
[19, 1]	$\mathbb{Z}_{19}$	✗	---	---	---	✗
[20, 1]	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[20, 2]	$\mathbb{Z}_{20}$	✗	---	---	---	✗
[20, 3]	$\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$	✗	✗	✗	✗	✗
[20, 4]	D_{10}	✗	✔	✔	✔	✗
[20, 5]	$\mathbb{Z}_{10} \times \mathbb{Z}_2$	✗	---	---	---	✗
[21, 1]	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[21, 2]	$\mathbb{Z}_{21}$	✗	---	---	---	✗
[22, 1]	D_{11}	✗	✔	✔	✔	✗
[22, 2]	$\mathbb{Z}_{22}$	✗	---	---	---	✗
[23, 1]	$\mathbb{Z}_{23}$	✗	---	---	---	✗
[24, 1]	$\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[24, 2]	$\mathbb{Z}_{24}$	✗	---	---	---	✗
[24, 3]	SL(2, 3)	✔	✔	✔	✔	✗
[24, 4]	$\mathbb{Z}_3 \rtimes_\varphi Q_8$	✗	✔	✔	✔	✗
[24, 5]	$\mathbb{Z}_4 \times S_3$	✗	✔	✔	✔	✗
[24, 6]	D_{12}	✗	✔	✔	✔	✗
[24, 7]	$\mathbb{Z}_2 \times (\mathbb{Z}_3 \rtimes_\varphi \mathbb{Z}_4)$	✗	✔	✗	✔	✗
[24, 8]	$(\mathbb{Z}_6 \times \mathbb{Z}_2) \rtimes_\varphi \mathbb{Z}_2$	✗	✔	✔	✔	✗
[24, 9]	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	✗	---	---	---	✗
[24, 10]	$\mathbb{Z}_3 \times D_4$	✗	✔	✔	✔	✗
[24, 11]	$\mathbb{Z}_3 \times Q_8$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[24, 12]	S_4	✔	✔	✗	✗	✔
[24, 13]	$\mathbb{Z}_2 \times A_4$	✔	✔	✗	✗	✔
[24, 14]	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times S_3$	✗	✔	✗	✔	✗
[24, 15]	$\mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	✗	---	---	---	✗
[25, 1]	$\mathbb{Z}_{25}$	✗	---	---	---	✗
[25, 2]	$\mathbb{Z}_5 \times \mathbb{Z}_5$	✗	---	---	---	✗
[26, 1]	D_{13}	✗	✔	✔	✔	✗
[26, 2]	$\mathbb{Z}_{26}$	✗	---	---	---	✗
[27, 1]	$\mathbb{Z}_{27}$	✗	---	---	---	✗
[27, 2]	$\mathbb{Z}_9 \times \mathbb{Z}_3$	✗	---	---	---	✗

Which Flavor Symmetry?

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GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[27, 3]	$(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[27, 4]	$\mathbb{Z}_9 \rtimes_\varphi \mathbb{Z}_3$	✔	✔	✗	✗	✗
[27, 5]	$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	✗	---	---	---	✗
[28, 1]	$\mathbb{Z}_7 \rtimes_\varphi \mathbb{Z}_4$	✗	✔	✔	✔	✗
[28, 2]	$\mathbb{Z}_{28}$	✗	---	---	---	✗
[28, 3]	D_{14}	✗	✔	✔	✔	✗
[28, 4]	$\mathbb{Z}_{14} \times \mathbb{Z}_2$	✗	---	---	---	✗
[29, 1]	$\mathbb{Z}_{29}$	✗	---	---	---	✗
[30, 1]	$\mathbb{Z}_5 \times S_3$	✗	✔	✔	✔	✗
[30, 2]	$\mathbb{Z}_3 \times D_5$	✗	✔	✔	✔	✗

Which Flavor Symmetry?

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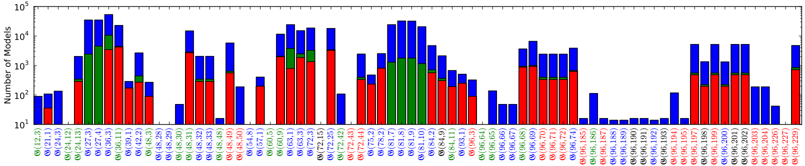

- 50% of groups have tribimaximal models
- Smallest group that can produce TBM: $\mathfrak{G}(21,1) = T_7$
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$
- A_4 has nice geometric interpretation, but what does that mean? That humans like to think in terms of geometry?
- There is probably no special connection between A_4 and TBM!

Conclusions

- We started from TBM and NLO corrections give $\theta_{13} \neq 0^\circ$
- Model is very economical:
  - Family symmetry T_7 is second-smallest group w/3-dim irreps
  - No extra (shaping) symmetries, i.e. no extra $\mathrm{U}(1)$ or $\mathbb{Z}_N$
  - Only 2 flavon fields
- Vacuum stabilization kind of a headache
  $\longrightarrow$ Either more symmetry or more flavons
- Which flavor symmetry to use?
  - Daya Bay & Reno have shattered the TBM paradigm
  - Probably there is no special connection between A_4 and TBM
  - Need to look beyond smallest groups and also critically review motivation for family symmetries