Relating neutrino mixing angles to neutrino masses

FLASY12

June 30, 2012, Dortmund

Morimitsu Tanimoto (Niigata University)
with Y. Shimizu and A. Watanabe

Plan of my talk

1. Introduction

   Tri-bimaximal mixing Paradigm
2. Breaking with Tri-bimaximal mixing
3. Large θ_{13} and the neutrino masses
4. Summary

1 Introduction
Tri-bimaximal mixing Paradigm

Before Daya Bay, RENO, DChooz and T2K,
Neutrino Data suggested
Tri-bimaximal Mixing of Neutrinos

$$\sin^2\theta_{12} = 1/3, \ \sin^2\theta_{23} = 1/2, \ \sin^2\theta_{13} = 0,$$

$$U_{\rm tri-bimaximal} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

Tri-bi maximal mixing gives us beautiful flavor structure of Neutrino Mass Matrix:

$$M_\nu^{\rm exp} \simeq V^*_{\rm tri-bi} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V^\dagger_{\rm tri-bi}$$

$$\boxed{= \frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}}$$

- integer (inter-family related) matrix elements

  $\Longleftrightarrow$ non-abelian discrete flavor sym

Mixing angles are independent of mass eigenvalues

$$\left(\theta_{ij} \cancel{\sim} \sqrt{\frac{m_i}{m_j}}\right)$$

Different from quark mixing angles

$$M_\nu^{\exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}$$

The third matrix is A_4 symmetric !

The first and second matrices are well known to be of S_3 symmetric.

In order to get the first and second matrix in A_4 model, non-trivial flavons are required.

E. Ma and G. Rajasekaran, PRD64(2001)113012

A_4 Flavor Group

Four irreducible representations in A_4 symmetry

1 1' 1" 3

Consider A_4 triplet of leptons **3** $(l_e, l_\mu, l_\tau)_L$

Tensor Product of $\mathbf{3}$ (a_1, a_2, a_3) and $\mathbf{3}$ (b_1, b_2, b_3)

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$l\, l\, h\, h/\Lambda$ $(3 \times 3 \times 1 \times 1)$ gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$(l_e l_e + l_\mu l_\tau + l_\mu l_\tau)\, v^2/\Lambda$

A_4 symmetric

In order to get Tri-bimaximal Mixing,
one needs

◎ **Non-trivial Flavons 3**

◎ **Additional U(1) or Zn**

◎ **VEV Alignment of flavon 3: (1,0,0) , (1,1,1) ......**

Typical Non-Abelian Discrete Groups

S_3 S_4 A_4 T' D_4 $\Delta(27)$ $\Delta(54)$ ……

○ : includes both 2 and 3

Singlet 1 1' 1'' …..

Doublet 2 ….. 2 families

Triplets 3 …. 3 families

Remark: S_3 and A_4 are sub-groups of S_4 .

H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1-163. [arXiv:1003.3552 [hep-th]].

Text book to be published at Springer for Physicist (2012)

Remark:

Tri-bimaximal Mixing realized in

$$M_{\rm TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

Additional Matrices break Tri-bimaximal mixing

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}, \quad \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} - \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

which could appear in A_4, S_4, $\Delta(27)$ flavor symmetries.

3 Breaking with tri-bimaximal mixing

Daya Bay suggests us

the breaking with tri-bimaximal paradigm !

Let us show
how to go beyond the tri-bi maximal mixing.

Consider Modified A_4 Model to get large θ_{13}

Modify G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

$$\mathbf{3} \times \mathbf{3} = \mathbf{3} + \mathbf{3} + \mathbf{1} + \mathbf{1}' + \mathbf{1}''$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}' = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}'' = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad$ ξ' $\qquad$ remark ξ''

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1}, \qquad \mathbf{1}'' \times \mathbf{1}' \Rightarrow \mathbf{1} \qquad 1' \times 1'' \rightarrow 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

There is one relation $a = -3b$

$$m_1 = a + \sqrt{c^2 + d^2 - cd}$$
$$m_2 = c + d$$
$$m_3 = -a + \sqrt{c^2 + d^2 - cd}$$

Neglecting phases,
3 parameters

For normal hierarchical limit m1<< m2<< m3, we have

$$m_1 \simeq 0 \quad \rightarrow \quad \Delta m^2_{\rm atm} \simeq 4(c^2 + d^2 - cd), \quad \Delta m^2_{\rm sol} \simeq (c+d)^2$$

After rotating by $V_{\text{tri-bi}}$, we get

$$M_\nu = V_{\text{tri-bi}} \begin{pmatrix} a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a+3b+c+d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2} \end{pmatrix} V_{\text{tri-bi}}^T$$

$$a = -3b$$

$$U_{\text{MNS}} = V_{\text{tri-bi}} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d}$$

Mixing matrix elements are given

$$|U_{e2}| = \frac{1}{\sqrt{3}} \,, \quad |U_{e3}| = \frac{2}{\sqrt{6}} |\sin\theta| \,, \quad |U_{\mu 3}| = \left| -\frac{1}{\sqrt{6}} \sin\theta - \frac{1}{\sqrt{2}} \cos\theta \right|$$

Then, we obtain

$$\sin\theta_{13} = \frac{2}{\sqrt{6}} |\sin\theta| \qquad \sin\theta_{23} = \frac{1}{\cos\theta_{13}} \left(\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{6}} \sin\theta \right)$$

where $\sin\theta$ is fixed by neutrino masses

$$\tan 2\theta = \frac{\sqrt{3} d}{-2c + d} \qquad \frac{\Delta m_{31}^2}{\Delta m_{21}^2} = \frac{4(c^2 + d^2 - cd)}{(c+d)^2}$$

for normal hierarchy limit m1 << m2 << m3

Taking account exp. with 3σ $\dfrac{\Delta m^2_{31}}{\Delta m^2_{21}} = 26 \sim 38$

we obtain $\sin\theta = 0.16 \sim 0.18$ **for normal hierarchy.**

Then, we predict

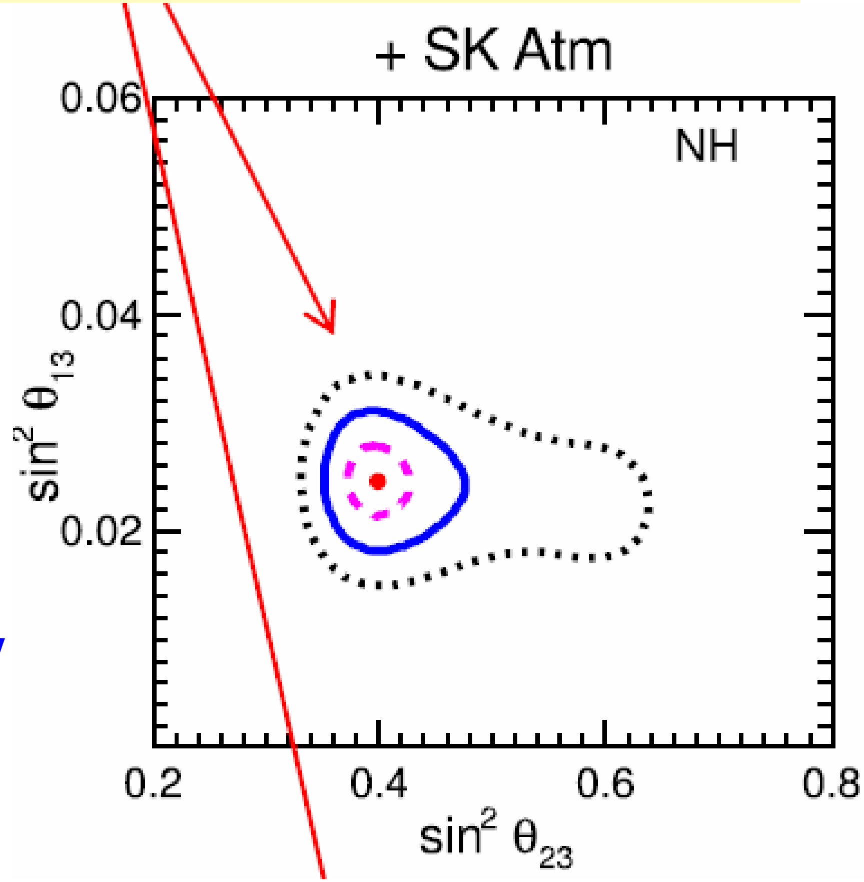
$$\sin\theta_{13} = 0.133 \sim 0.146$$

Exp. (Daya Bay 3σ) $\sin\theta_{13} = 0.12 \sim 0.18$

$$\sin\theta_{23} = 0.64 - 0.65, \quad 0.77 \sim 0.78$$

$$\sin\theta_{23}^2 = 0.40 - 0.42, \quad 0.59 \sim 0.61$$

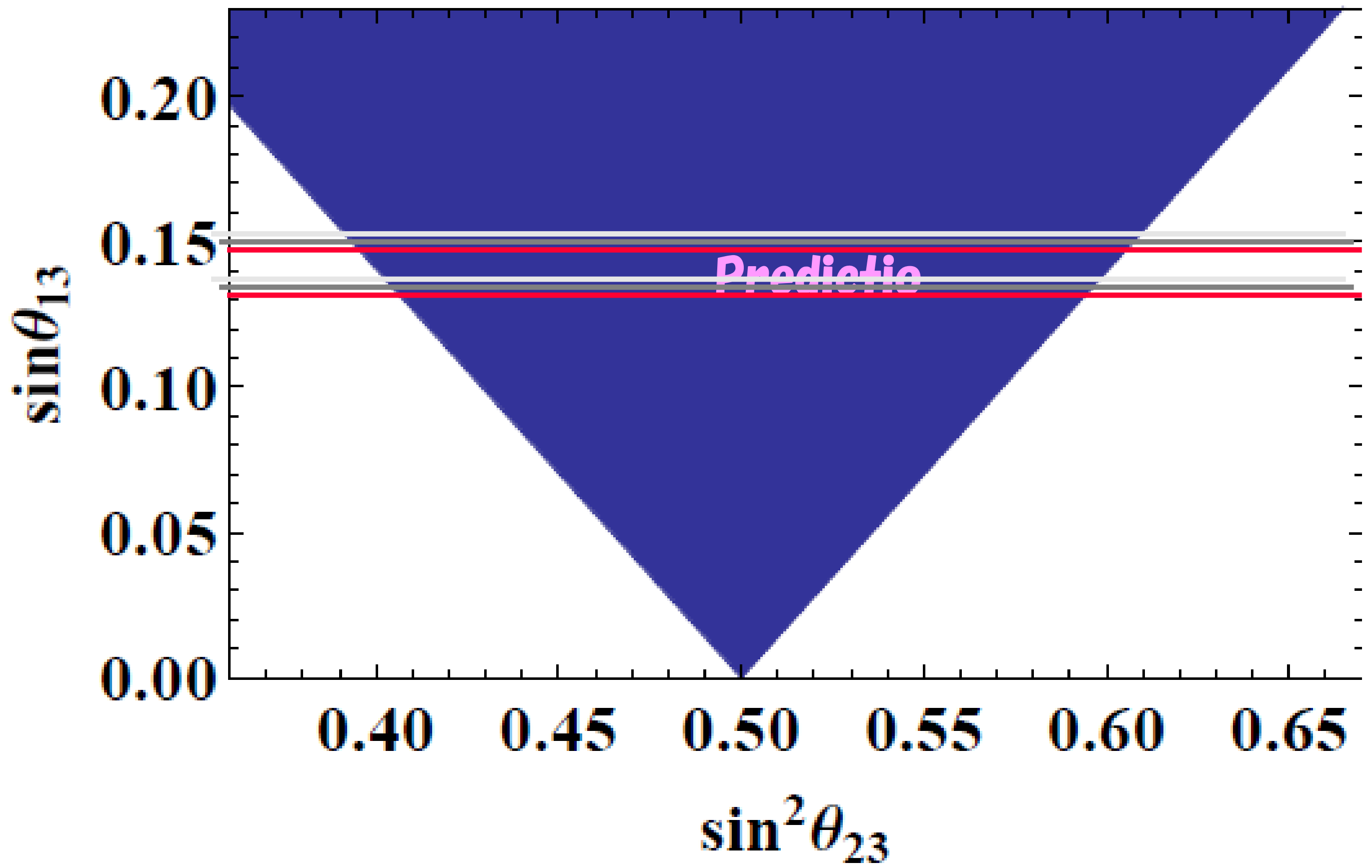
the preference for θ_{23} in the 1st octant is corroborated



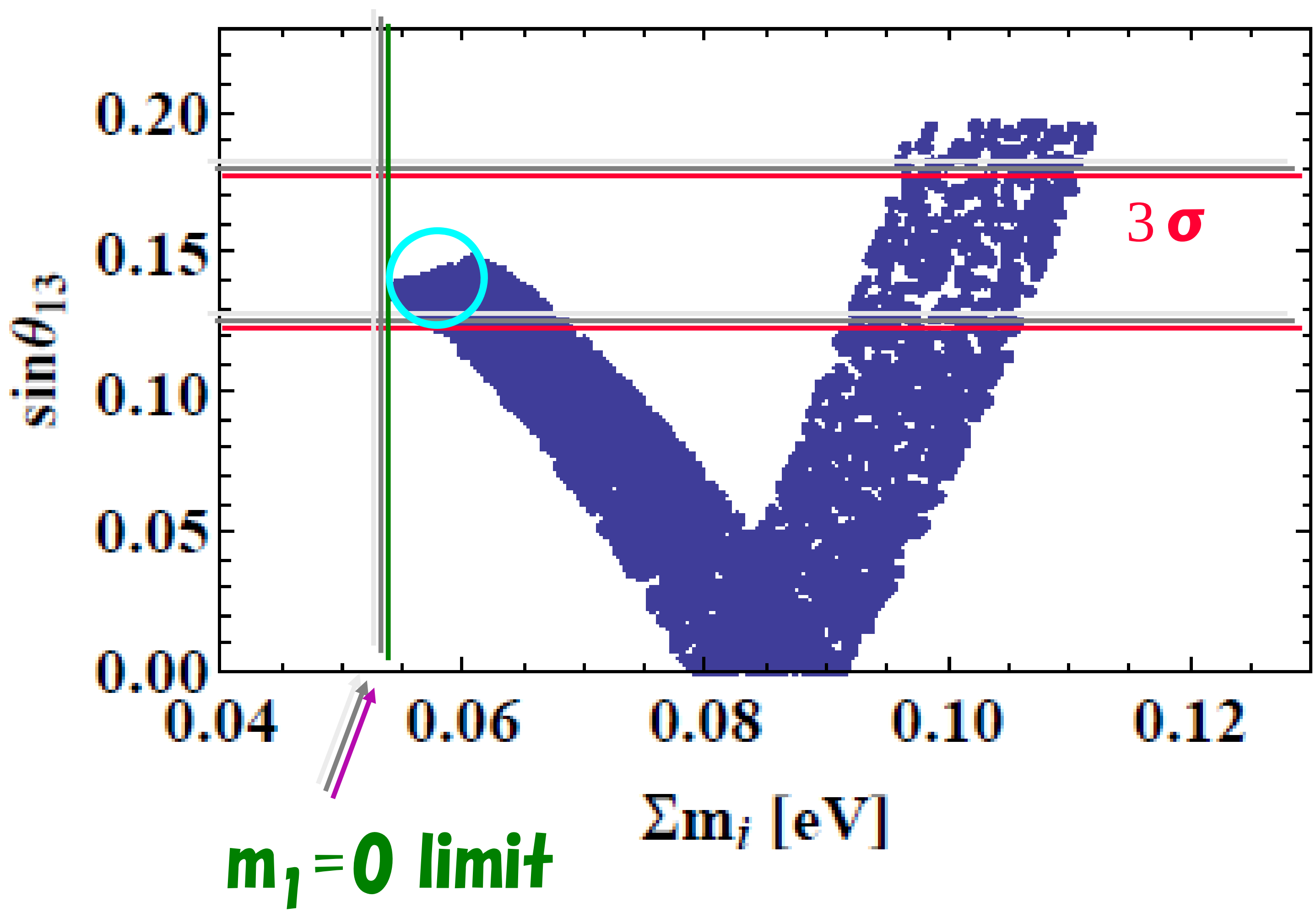
Normal Hierarchy

Taking account phase, we obtain allowed region

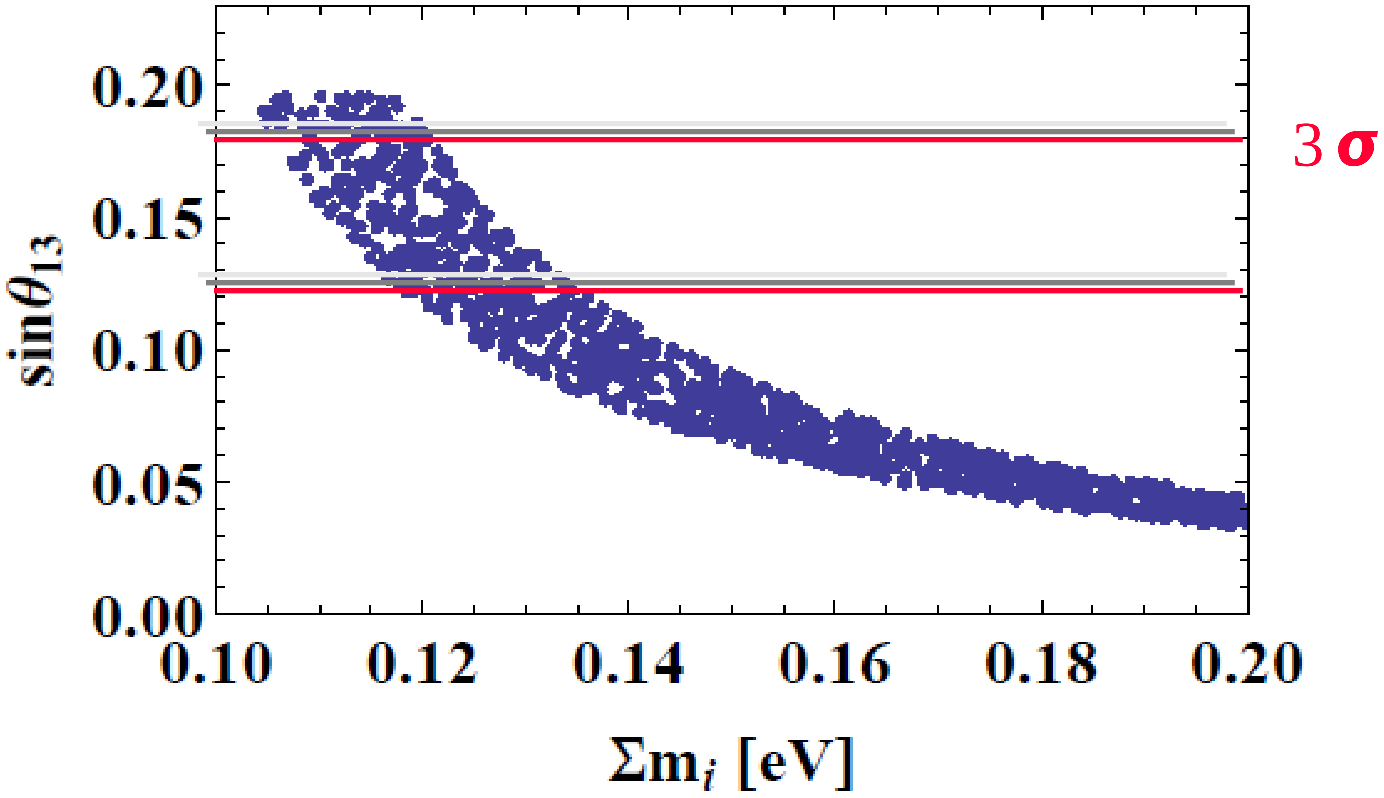
$$\sin\theta_{13} = \frac{2}{\sqrt{6}} |\sin\theta e^{i\phi}| \qquad \sin\theta_{23} = \frac{1}{\cos\theta_{13}} \left(\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{6}} \sin\theta e^{i\phi} \right)$$



Normal Hierarchy of Neutrino Masses



Inverted Hierarchy of Neutrino Masses



Remark: *Three combinations give same result.*

Present analyses

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$\boxed{\xi \text{ and } \xi'}$

1 and 1'

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{\xi \text{ and } \xi''}$

1 and 1''

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{\xi' \text{ and } \xi''}$

1' and 1''

3 Large θ_{13} and the neutrino masses

Daya Bay results

$$\sin\theta_{13} = 0.15 \pm 0.01$$ Order of Cabbibo angle !?

What does this value of θ_{13} indicate ?

$$\text{Recall} \quad \sin\theta_C \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.2$$

θ_{ij} could be related to neutrino masses explicitly.

Ratio of Neutrino Mass Squared differences

$$\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} = 0.026 \sim 0.038 = \mathcal{O}(\lambda^2)$$

$$\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.160 \sim 0.196 = \mathcal{O}(\lambda) \qquad \theta_{13}\ ?$$

$$\sqrt[4]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.40 \sim 0.44 = \mathcal{O}(\sqrt{\lambda})$$

$$\sqrt[8]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.63 \sim 0.66 = \mathcal{O}(\sqrt[4]{\lambda}) \qquad \theta_{23}\ ?$$

λ =0.2

Simple Model to get Large θ_{13} in terms of neutrino masses

Fukugita, Tanimoto, Yanagida, PLB 562(2003) 273 [arXiv:/0303177].

$$m_E = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \qquad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}$$

$$M_R = M_0 \mathbf{I}$$

$$m_i = \left(m_{\nu D}^T M_R^{-1} m_{\nu D} \right)_i$$

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix}$$

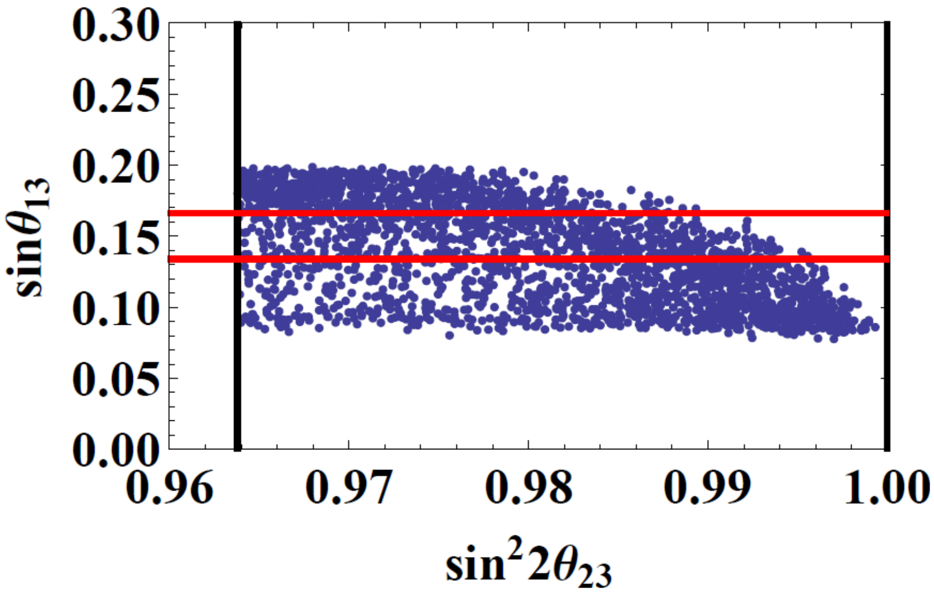
$$U = U_\ell^\dagger \, Q \, U_\nu$$

Fritzsch texture failed in Quark sector!

Lepton Mixing Matrix

Fukugita, Shimizu, Tanimoto, Yanagida (PLB 2012), arXiv:1204.2389

$$
\begin{aligned}
U_{e2} &\simeq -\left(\frac{m_1}{m_2}\right)^{1/4} + \left(\frac{m_e}{m_\mu}\right)^{1/2} e^{i\sigma} \\
U_{\mu 1} &\simeq \left(\frac{m_1}{m_2}\right)^{1/4} e^{i\sigma} - \left(\frac{m_e}{m_\mu}\right)^{1/2} \\
U_{\mu 3} &\simeq \left(\frac{m_2}{m_3}\right)^{1/4} e^{i\sigma} - \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\tau} \\
U_{\tau 2} &\simeq -\left(\frac{m_2}{m_3}\right)^{1/4} e^{i\tau} + \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\sigma} \\
U_{e3} &\simeq \left(\frac{m_e}{m_\mu}\right)^{1/2} U_{\mu 3} + \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right) \\
U_{\tau 1} &\simeq \left(\frac{m_1}{m_2}\right)^{1/4} U_{\tau 2}
\end{aligned}
$$



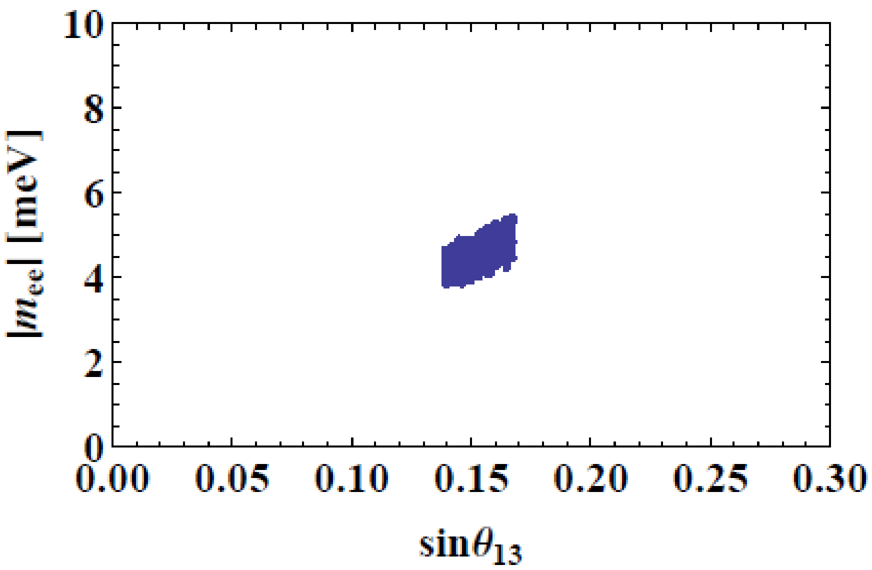
$$\boxed{\sin\theta_{23} \simeq \sqrt[8]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.63 \sim 0.66 = \mathcal{O}(\sqrt[4]{\lambda})}$$

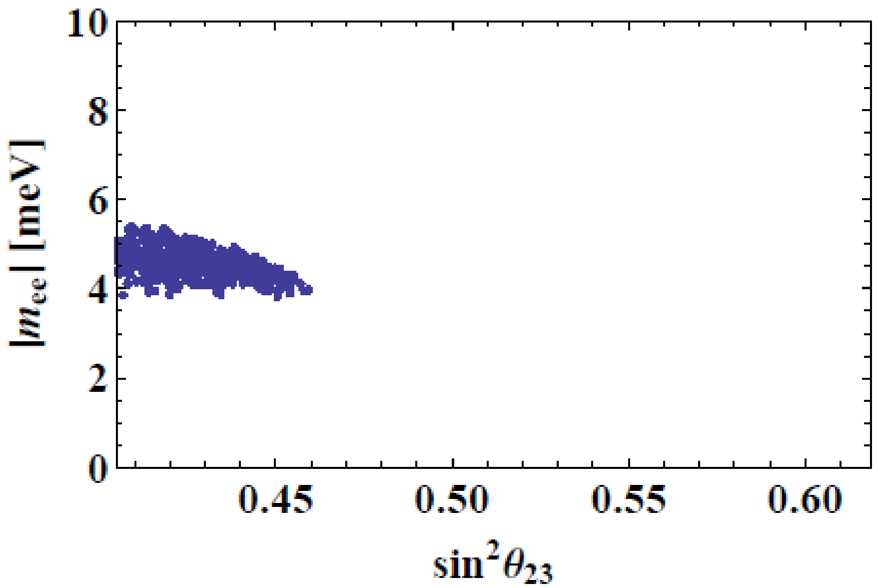
$$\boxed{\sin\theta_{13} \simeq (\sin\theta_{23})^3 \sin\theta_{12} \simeq 0.16}$$

maximal θ_{23} is excluded

Normal Mass Hierarchy

Neutrinoless Double Beta Decay





4 - 5.5 meV

4 Summary

Large $\theta_{13} \fallingdotseq 0.15$ suggests

breaking with tri-bimaximal mixing.

θ_{13} probably depends on neutrino masses !

Large θ_{13} is due to the neutrino mass ratios.

We should consider the origin of neutrino mass spectrum !

Why $\sqrt{\dfrac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.160 \sim 0.196 = \mathcal{O}(\lambda)$?

At first

let us understand how to get the tri-bimaximal mixing in the example of A_4 flavor model.

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$A_4 \times Z_3$ charge assignment

A_4 Flavor model

	(L_e, L_μ, L_τ)	R_e^c	R_μ^c	R_τ^c	$H_{u,d}$	χ_ℓ	χ_ν	χ
A_4	3	1	$1'$	$1''$	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω

χ_ℓ, χ_ν, χ are new scalars of gauge singlets.

$\omega^3 = 1$

A_4 invariant superpotential can be written by:

for charged leptons

$1' \times 1'' \rightarrow 1$

$$
\begin{aligned}
W_L &= \frac{y_e}{\Lambda}(L_e\chi_{\ell_1} + L_\mu\chi_{\ell_3} + L_\tau\chi_{\ell_2})R_e H_d && \mathbf{3_L \times 3_{flavon} \rightarrow 1} \\
&+ \frac{y_\mu}{\Lambda}(L_e\chi_{\ell_2} + L_\mu\chi_{\ell_1} + L_\tau\chi_{\ell_3})R_\mu H_d && \mathbf{3_L \times 3_{flavon} \rightarrow 1''} \\
&+ \frac{y_\tau}{\Lambda}(L_e\chi_{\ell_3} + L_\mu\chi_{\ell_2} + L_\tau\chi_{\ell_1})R_\tau H_d + h.c., && \mathbf{3_L \times 3_{flavon} \rightarrow 1'}
\end{aligned}
$$

for neutrinos

$$
\begin{aligned}
W_\nu &= \frac{y_1}{\Lambda^2}(L_eL_e + L_\mu L_\tau + L_\tau L_\mu)H_uH_u\chi && \mathbf{3_L \times 3_L \times 1_{flavon} \rightarrow 1} \\
&+ \frac{y_2}{3\Lambda^2}[(2L_eL_e - L_\mu L_\tau - L_\tau L_\mu)\chi_{\nu_1} \\
&+ (-L_eL_\tau + 2L_\mu L_\mu - L_\tau L_e)\chi_{\nu_2} && \mathbf{3_L \times 3_L \times 3_{flavon} \rightarrow 1} \\
&+ (-L_eL_\mu - L_\mu L_e + 2L_\tau L_\tau)\chi_{\nu_3}]H_uH_u + h.c.,
\end{aligned}
$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

After $A_4 \times Z_3$ symmetry is spontaneously broken by VEVs of χ_ℓ, χ_ν, and χ, mass matrices are obtained as

$$M_l = \frac{v_d}{\Lambda}\begin{pmatrix} y_e\langle\chi_{\ell_1}\rangle & y_e\langle\chi_{\ell_3}\rangle & y_e\langle\chi_{\ell_2}\rangle \\ y_\mu\langle\chi_{\ell_2}\rangle & y_\mu\langle\chi_{\ell_1}\rangle & y_\mu\langle\chi_{\ell_3}\rangle \\ y_\tau\langle\chi_{\ell_3}\rangle & y_\tau\langle\chi_{\ell_2}\rangle & y_\tau\langle\chi_{\ell_1}\rangle \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{3\Lambda}\begin{pmatrix} 3y_1\langle\chi\rangle + 2y_2\langle\chi_{\nu_1}\rangle & -y_2\langle\chi_{\nu_3}\rangle & -y_2\langle\chi_{\nu_2}\rangle \\ -y_2\langle\chi_{\nu_3}\rangle & 2y_2\langle\chi_{\nu_2}\rangle & 3y_1\langle\chi\rangle - y_2\langle\chi_{\nu_1}\rangle \\ -y_2\langle\chi_{\nu_2}\rangle & 3y_1\langle\chi\rangle - y_2\langle\chi_{\nu_1}\rangle & 2y_2\langle\chi_{\nu_3}\rangle \end{pmatrix}$$

where $v_d = \langle H_d\rangle$, $v_u = \langle H_u\rangle$.

The mass matrices do not yet predict tri-bimaximal mixing !

Can one get Desired Vacuum

in Spontaneous Symmetry Breaking ?

We need Scalar Potential Analysis.

If vacuum expectation values are aligned,
$\langle \chi_\ell \rangle = (V_\ell, 0, 0)$ and $\langle \chi_\nu \rangle = (V_\nu, V_\nu, V_\nu)$,
which are obtained by potential analysis, then

$$M_l = \frac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix} .$$

where $a = y_1 V/\Lambda$, $b = y_2 V_\nu/\Lambda$.

$\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{3_{flavon}}$ $\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}}$

$$M_\nu = \frac{v_u^2 b}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{v_u^2 b}{3\Lambda} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v_u^2 a}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore, mixing matrix is tri-bimaximal matrix, and masses are

$$m_1 = \frac{v_u^2(a+b)}{\Lambda}, \quad m_2 = \frac{v_u^2 a}{\Lambda}, \quad m_3 = -\frac{v_u^2(a-b)}{\Lambda}.$$