*FLA*vor *SY*mmetry
&
Stability of DM

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Hirsch, M, Peinado, Valle, ***PRD 82*** ***(2010)***
Meloni, M, Peinado, ***PLB 697*** *(2011)*
Boucenna,Hirsch,M,Peinado,Taoso,Valle, ***JHEP 1105*** *(2011)*
Meloni,M,Peinado ***PLB 703*** *(2011)*
Adelhart,Bazzocchi,M ***NPB 856*** *(2012)*
Boucenna,M,Peinado,Shimizu,Valle *1204.4733*
Lavoura,M,Valle *1205.3442*

DM life-time > age of the universe

stable DM | decaying DM

Z_2 : DM → − DM

SM → + SM

Z2 is typically imposed by hand

It seems there are no motivations to assume an **ad-hoc symmetry**

since in the SM there are stable particles without introducing ad hoc symmetry

photon, electron, neutrino, proton

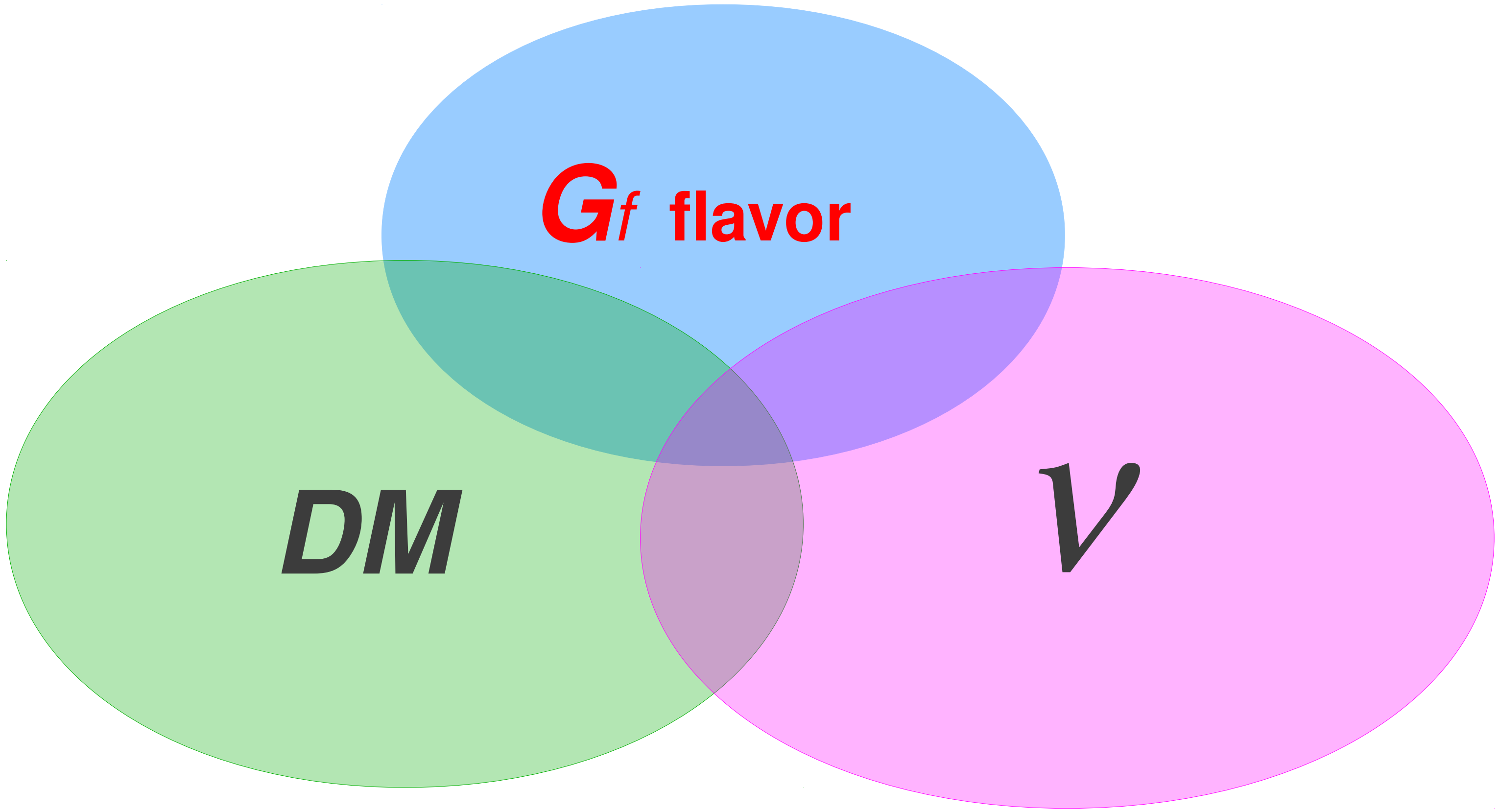
possible origin of DM stability :

- R - parity in MSSM (LSP)

- $U(1)_{B-L}$ from GUT

Hambye 1012.4587

DM stability from flavor symmetry

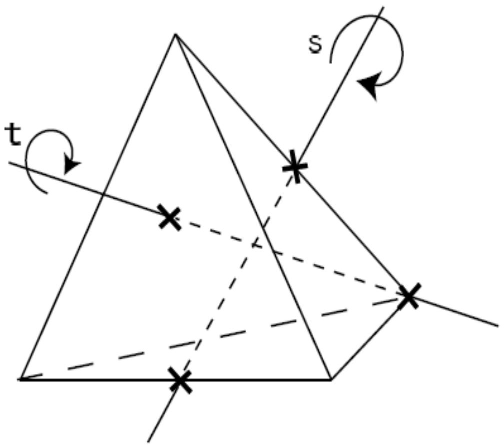


The flavor symmetry (*an example*)

Ma, Rajasekaran PRD64 (2001)

A4

Discrete group
even permutations
four objects



tetrahedron

Altarelli, Feruglio Nucl.Phys. B720 (2005)

A4 is spontaneously broken in

Z3 in the charged sector

Z2 in the neutrino sector

TBM

A4 **totally broken!**

Hirsch, M, Peinado, Valle, ***PRD 82***

A4 is spontaneously broken in

Z2 in the charged sector

Z2 in the neutrino sector

TBM X

Z2 unbroken that stabilize the DM

How is it defined the Z_2 parity from the spontaneously breaking of A_4?

Singlets *irreps*

$$S^2 = T^3 = (ST)^3 = 1$$

Z2 (from S^2) **Z3** (from T^3)

1	$S = 1$	$T = 1$
$1'$	$S = 1$	$T = e^{i2\pi/3} \equiv \omega$
$1''$	$S = 1$	$T = e^{i4\pi/3} \equiv \omega^2$

→ ***1, 1', 1''*** invariant under ***Z2***

Charged leptons in *singlets* of A4

	L_e	L_μ	L_τ	l_e^c	l_μ^c	l_τ^c
$SU(2)$	2	2	2	1	1	1
A_4	1	$1'$	$1''$	1	$1''$	$1'$

M*l* *diagonal* and *Z2* invariant

Triplet irrep

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; ; \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Z_2 :

$$\begin{pmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} +\eta_1 \\ -\eta_2 \\ -\eta_3 \end{pmatrix}$$

Triplet irrep

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$$\langle \eta \rangle \sim (1, 0, 0)$$

→ Z2 unbroken

DM candidate

$$Z_2: \begin{pmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} +\eta_1 \\ -\eta_2 \\ -\eta_3 \end{pmatrix}$$

DM

$$\langle \eta \rangle \sim (1, 0, 0)$$

DM couplings

	L_1	N_3	η_2
A_4	1	3	3
Z_2	+	–	–

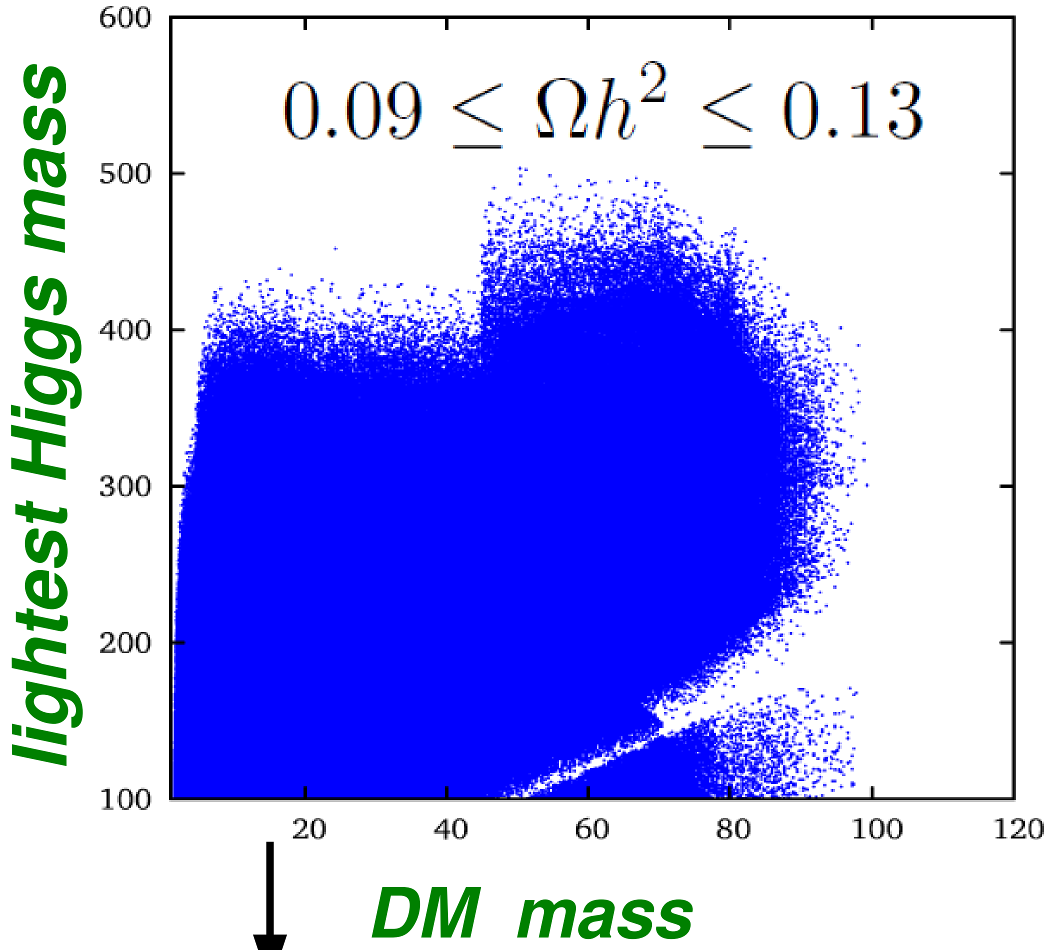
Higgs portal

$$\eta^\dagger \eta H^\dagger H \longrightarrow$$ See Boucenna's talk

Neutrino mixing & relic density

Boucenna, Hirsch, M, Peinado, Taoso, Valle, ***JHEP 1105***

Hirsch, M, Peinado, Valle, ***PRD 82***



See Boucenna's talk

IH : $m_3 = 0$

0.03 eV< 0nubb <0.05 eV

$\theta_{13} = 0$

Meloni, M, Peinado, ***PLB 697*** *(2011)*

Is it possible to assign ***L*** to ***irrep > 1*** ?

Is it possible to assign ***L*** to ***irrep > 1*** ?

	L_2	N_1	η_3
A4	***3***	***3***	***3***
Z2	−	+	−

DM can decays into light neutrinos !

We search for a group G that contains at least ***two irrep***

$$r_a \quad r_b \quad \dim(r_{a,b}) > 1$$

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all its components
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We search for a group *G* that contains at least ***two irrep***

$$r_a \quad r_b \quad \dim(r_{a,b}) > 1$$

all its components are invariant under a subgroup Z_N of *G*

at least one of its component transforms with respect to Z_N

DM candidate

$D(54)$ has a subgroup → **Z3 x Z3**

Boucenna,M,Peinado,Shimizu,Valle 1204.4733

$$a^3 = a'^3 = I$$

	$\mathbf{1}_+$	$\mathbf{1}_-$	$\mathbf{2}_1$	$\mathbf{2}_2$
a	1	1	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$
a'	1	1	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$

Stolen from Ishimori et al., Prog.Theor.Phys.Suppl. 183

$D(54)$ has a subgroup → $Z3 \times Z3$

Boucenna,M,Peinado,Shimizu,Valle *1204.4733*

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Ishimori et al., Prog.Theor.Phys.Suppl. 183

Invariant under Z3 x Z3

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Boucenna,M,Peinado,Shimizu,Valle *1204.4733*

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Ishimori et al., Prog.Theor.Phys.Suppl. 183

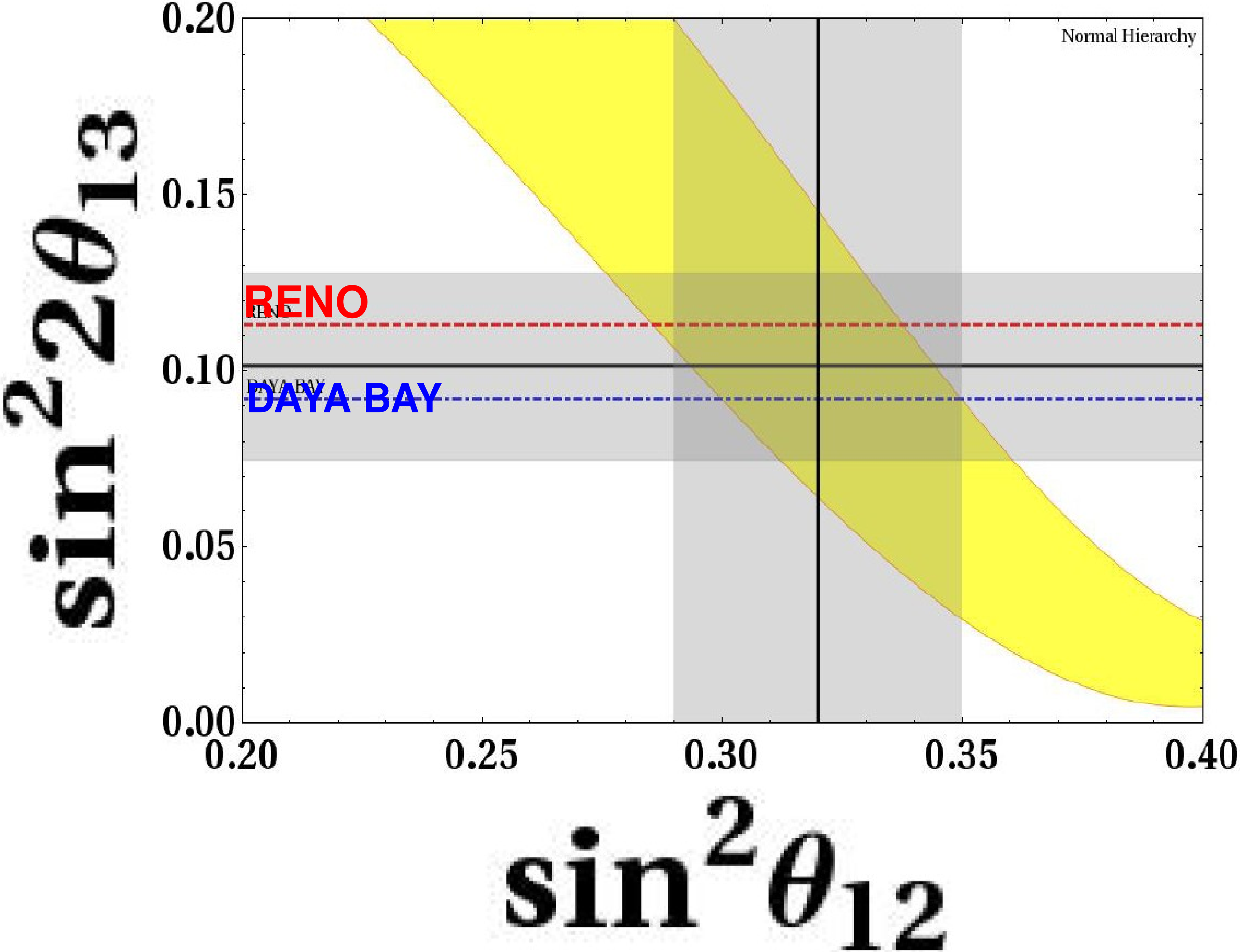
transforms under Z3 x Z3 and contains DM

	$\overline{L}_e$	$\overline{L}_D$	e_R	l_D	H	χ	η	Δ
$SU(2)$	2	2	1	1	2	2	2	3
$\Delta(54)$	$\mathbf{1}_+$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{2}_1$	$\mathbf{2}_3$	$\mathbf{2}_1$

$$\mathbf{2_k} \times \mathbf{2_k} = \mathbf{1}_+ + \mathbf{1}_- + \mathbf{2_k}$$
$$\mathbf{2_1} \times \mathbf{2_2} = \mathbf{2_3} + \mathbf{2_4}$$

Stable: does not couple to fermions and quarks

	$Q_{1,2}$	Q_3	(u_R, c_R)	t_R	d_R	s_R	b_R
$SU(2)$	2	2	1	1	1	1	1
$\Delta(54)$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{1}_-$	$\mathbf{1}_+$	$\mathbf{1}_+$

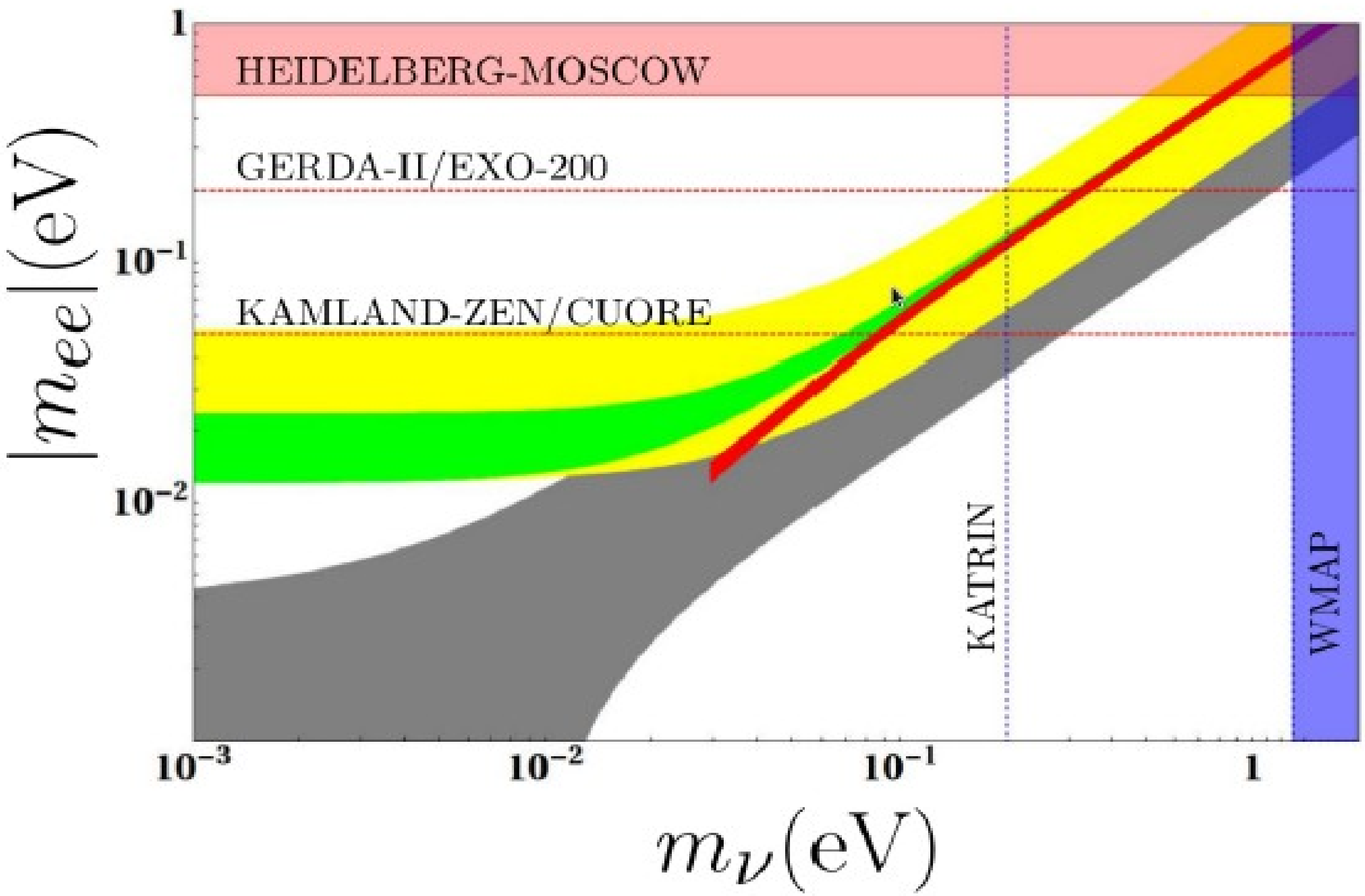


Normal Hierarchy
RENO
DAYA BAY
0.20
0.15
0.10
0.05
0.00
0.20
0.25
0.30
0.35
0.40
$\sin^2 2\theta_{13}$
$\sin^2\theta_{12}$

Sum mass rule

$$m_1^\nu + m_2^\nu = m_3^\nu$$

Barry, Rodejohann NPB 842 (2011)
Dorame,Meloni,M,Peinado,Valle NPB861(2011)



Accidental stability of DM

Lavoura,M,Valle *1205.3442*

SU(2) is the ***double cover*** of SO(3)

1, *2*, 3, *4* ,... *irrep*

1, 3, 5, ... *irrep*

Accidental stability of DM

Lavoura,M,Valle *1205.3442*

SU(2) is the ***double cover*** of SO(3)

1, *2*, 3, *4* ,... *irrep*

spinorial

We can call

1, 3, 5, ... *irrep*

vectorial

spinorial x *spinorial* → vectorial

vectorial x vectorial → vectorial

Accidental stability of DM

Lavoura,M,Valle *1205.3442*

SU(2) is the ***double cover*** of SO(3)

1, 2, 3, 4,... *irrep* **1, 3, 5, ...** *irrep*

spinorial vectorial

We can call

- *spinorial* x - *spinorial* → + vectorial

+ vectorial x + vectorial → + vectorial

As an accidental Z_2

An example: T' double cover of A_4

vectorial *spinorial*

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
$\mathbf{1}_3$			$\mathbf{1}_2$	$\mathbf{3}$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
$\mathbf{3}$				$\mathbf{3}, \mathbf{3}, \mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$
$\mathbf{2}_1$					$\mathbf{3}, \mathbf{1}_1$	$\mathbf{3}, \mathbf{1}_2$	$\mathbf{3}, \mathbf{1}_3$
$\mathbf{2}_2$						$\mathbf{3}, \mathbf{1}_3$	$\mathbf{3}, \mathbf{1}_1$
$\mathbf{2}_3$							$\mathbf{3}, \mathbf{1}_2$

A_4

An example: T'

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
$\mathbf{1}_3$			$\mathbf{1}_2$	$\mathbf{3}$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
$\mathbf{3}$				$\mathbf{3}, \mathbf{3}, \mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$
$\mathbf{2}_1$					$\mathbf{3}, \mathbf{1}_1$	$\mathbf{3}, \mathbf{1}_2$	$\mathbf{3}, \mathbf{1}_3$
$\mathbf{2}_2$						$\mathbf{3}, \mathbf{1}_3$	$\mathbf{3}, \mathbf{1}_1$
$\mathbf{2}_3$							$\mathbf{3}, \mathbf{1}_2$

vectorial x vectorial ➡ vectorial

An example: T'

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
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spinorial x *spinorial* → vectorial

An example: T'

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
$\mathbf{1}_3$			$\mathbf{1}_2$	$\mathbf{3}$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
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spinorial x vectorial → *spinorial*

An example: T'

Your favorite A4 model **Stable DM**

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
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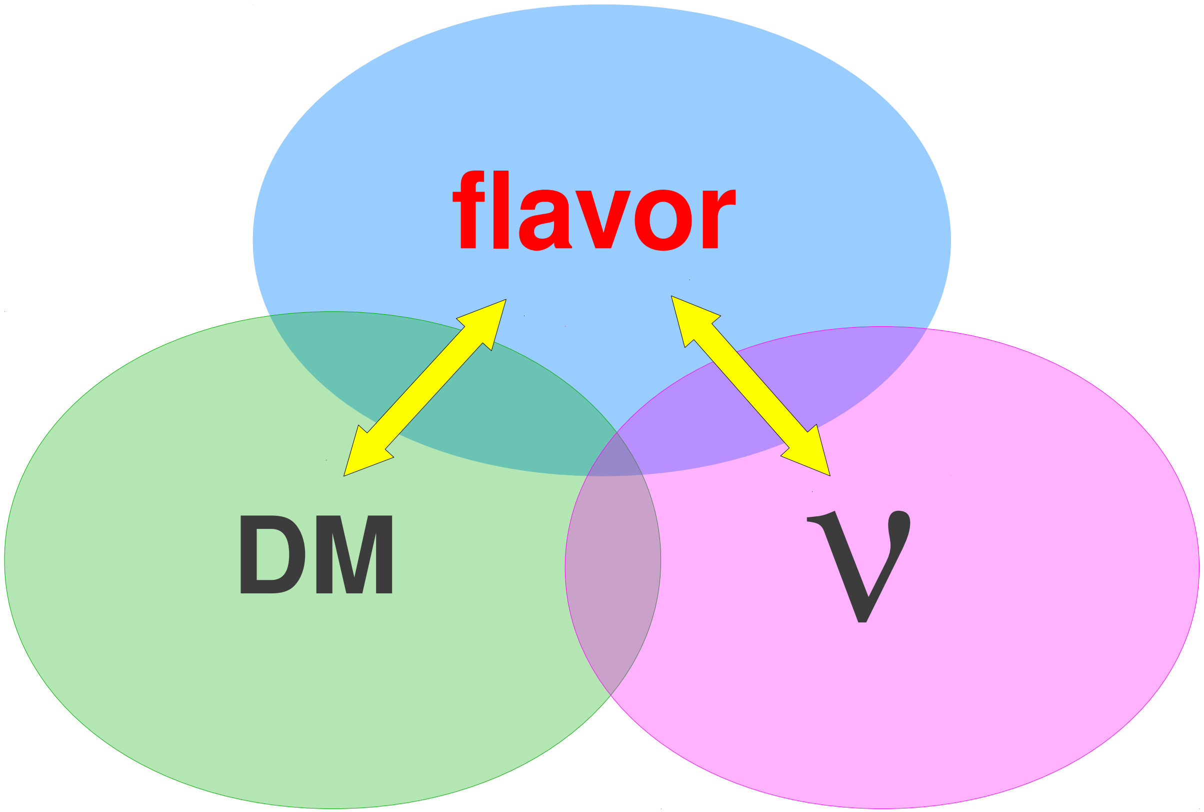
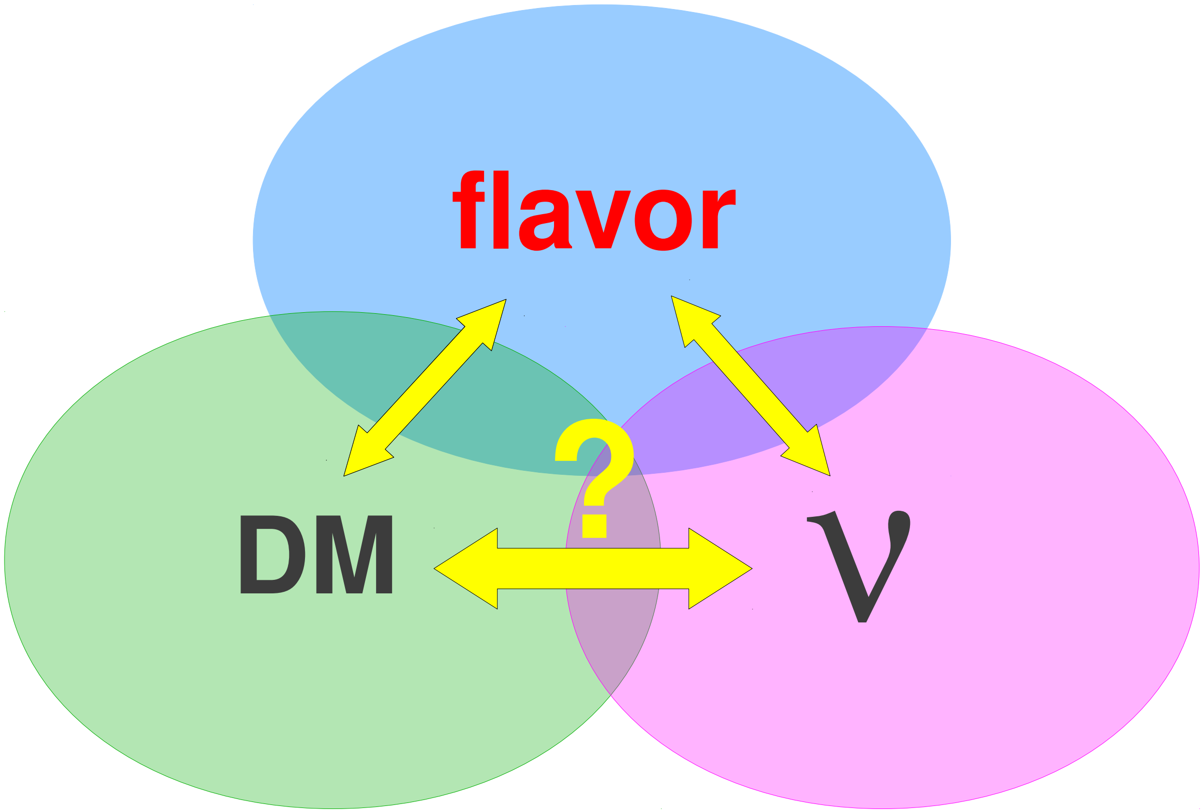
Conclusions



Conclusions



Bi-large instead of TBM (after MINOS)

Reactor angle as seed for atm and sol angles

Boucenna et al 1206.2555

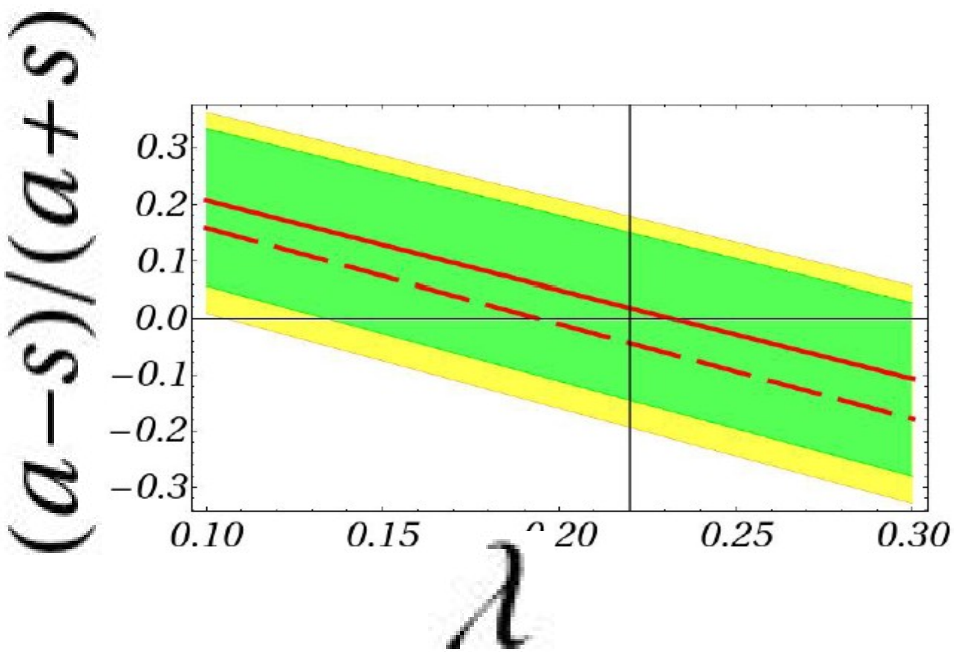
$$\sin\theta_{13} = \lambda\,; \quad \sin\theta_{12} = s\,\lambda\,; \quad \sin\theta_{23} = a\,\lambda\,,$$

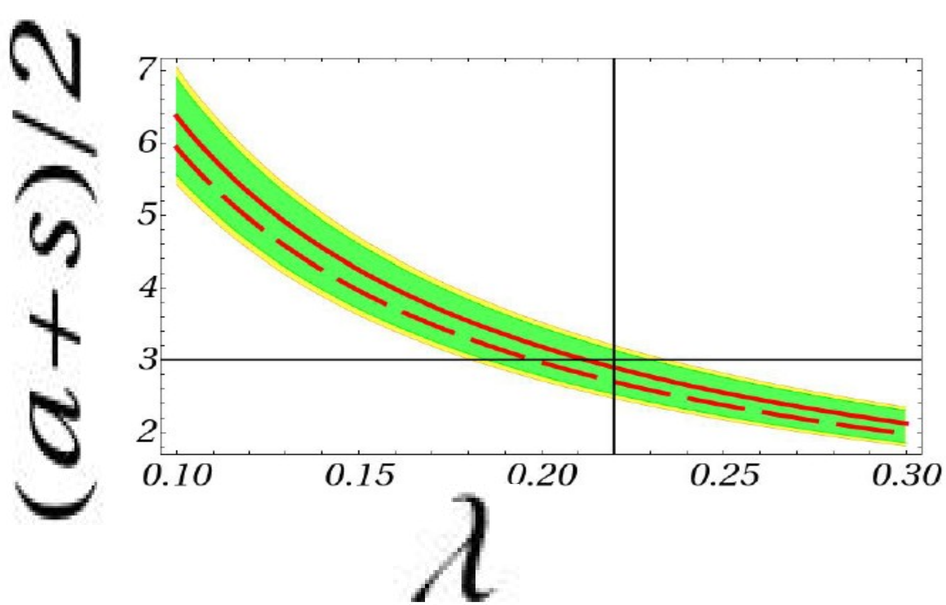
$$s = a$$

$$\sin\theta_{13} = \lambda - \epsilon\,; \quad \sin\theta_{12} = s\lambda - \epsilon\,; \quad \sin\theta_{23} = s\lambda + \epsilon\,.$$

Ref.	λ	s	ϵ
Forero *et al.* [14]	0.23 ± 0.04	$2.8^{+0.5}_{-0.4}$	$0.067^{+0.035}_{-0.025}$
Fogli *et al.* [16]	$0.19^{+0.03}_{-0.02}$	$3.0^{+0.5}_{-0.3}$	$0.038^{+0.019}_{-0.018}$

s = 3





Backup slides

Neutrino in discrete DM model

	L_e	L_μ	L_τ	l_e^c	l_μ^c	l_τ^c	N_T	N_4	H	η
$SU(2)$	2	2	2	1	1	1	1	1	2	2
A_4	1	$1'$	$1''$	1	$1''$	$1'$	3	1	1	3

$$\begin{aligned}\mathcal{L} &= y_e L_e l_e^c \hat{H} + y_\mu L_\mu l_\mu^c \hat{H} + y_\tau L_\tau l_\tau^c \hat{H} + \\ &+ y_1^\nu L_e (N_T \eta)_1 + y_2^\nu L_\mu (N_T \eta)_{1''} + y_3^\nu L_\tau (N_T \eta)_{1'} + \\ &+ y_4^\nu L_e N_4 \hat{H} + M_1 N_T N_T + M_2 N_4 N_4 + \text{h.c.}\end{aligned}$$

$$m_D = \begin{pmatrix} x_1 & 0 & 0 & x_4 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}$$

$$m_\nu = -m_{D_{3\times 4}} M_{R_{4\times 4}}^{-1} m_{D_{3\times 4}}^T \equiv \begin{pmatrix} y^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$$

D(54) isomorphic to → $(Z_3 \times Z_3) \rtimes S_3$

N=3

$\Delta(6N^2)$

$$a^N = a'^N = b^3 = c^2 = (bc)^2 = e,$$
$$aa' = a'a,$$
$$bab^{-1} = a^{-1}a'^{-1}, \qquad ba'b^{-1} = a,$$
$$cac^{-1} = a'^{-1}, \qquad ca'c^{-1} = a^{-1}.$$

Ishimori et al., Prog.Theor.Phys.Suppl. 183

	$\overline{L}_e$	$\overline{L}_D$	e_R	l_D	H	χ	η	Δ
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	$Q_{1,2}$	Q_3	(u_R, c_R)	t_R	d_R	s_R	b_R
$SU(2)$	2	2	1	1	1	1	1
$\Delta(54)$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{2}_1$	$\mathbf{1}_+$	$\mathbf{1}_-$	$\mathbf{1}_+$	$\mathbf{1}_+$

We assume real parameters

$$M_\ell = \begin{pmatrix} a & br & b \\ cr & d & e \\ c & e & dr \end{pmatrix} \qquad M_d = \begin{pmatrix} ra_d & rb_d & rd_d \\ -a_d & b_d & d_d \\ 0 & c_d & e_d \end{pmatrix}$$

r common to the two sectors

$$0.1 < r < 0.2$$

$$M_\nu \propto \begin{pmatrix} 0 & \delta & \delta \\ \delta & \alpha & 0 \\ \delta & 0 & \alpha \end{pmatrix} \qquad M_u = \begin{pmatrix} ra_u & b_u & d_u \\ b_u & a_u & rd_u \\ c_u & rc_u & e_u \end{pmatrix}$$

8 parameters

10 parameters

Accidental stability of DM

-                                -   +
<u>*spinorial*</u> x <u>*spinorial*</u> → vectorial

+   +   +
vectorial x vectorial → vectorial

As an accidental Z_2

$\eta\eta$ $\eta\eta\eta\eta$ $\eta\eta H H$

$\langle \eta \rangle_0 = 0$ No terms linear in η