The quark NNI textures rising from $SU(5) \times Z_4$ symmetry

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Based on

Phys. Rev. D 85 (2012) 016003, **D. Emmanuel-Costa and C. S.**

Phys. Lett. B 690 (2010) 62, **G. C. Branco, D. Emmanuel-Costa and C.S.**

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Outline

- Motivation
- The Model
- NNI texture from Discrete Flavour Symmetries
- Proton decay
- Gauge Couplings Unification
- Exploring the leptonic sector
- Other GUT implementations? **SO**(**10**) and Flipped **SU**(**5**)
- Conclusions

Motivation

- The simplest attempt at understanding the flavour structure encoded in the fermion mass matrices is by imposing some texture zeroes on the matrix elements

  Nearest-Neighbour-Interaction (NNI) basis $\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$

- The texture zeroes imposed through flavour symmetries have physical content
- The fact that quarks and leptons are placed together in multiplets is not enough to determine their masses and mixings.
- Flavour symmetries implemented in GUT contexts lead to a particular pattern for quark sector that affects the leptonic sector

  $$\downarrow$$

  NNI basis for quark mass matrices in the context of SU(5) GUT model realised through the implementation of a Z_4 flavour symmetry

  Consequences on the leptonic sector when introducing three right-handed neutrinos, ν_i^c

The model: SU(5) (i)

[Georgi and Glashow, 1974]

$SU(5)$ model
+
Abelian discrete flavour symmetry Z_n
+
NNI quark mass matrices @ low energy

Fermionic sector

$\mathbf{10}_i = (Q, u^c, e^c)_i$

$\mathbf{5}^*_i = (L, d^c)_i$

$\nu^c_1, \nu^c_2, \nu^c_3$

Scalar sector

$\Sigma(24)$

$H_1(5)$ and $H_2(5)$

$\mathbf{Z_n}$ symmetry:

$$\Psi_j \longrightarrow \Psi'_j = e^{i\,\frac{2\pi}{n}\,\mathcal{Q}(\Psi_j)}\,\Psi_j$$

$$\Phi_j \longrightarrow \Phi'_j = e^{i\,\frac{2\pi}{n}\,\mathcal{Q}(\Phi_j)}\,\Phi_j$$

$$\mathcal{Q}(\mathbf{5}^*_j) = (d_1, d_2, d_3)$$
$$\mathcal{Q}(\mathbf{10}_j) = (q_1, q_2, q_3)$$
$$\mathcal{Q}(\nu^c_j) = (n_1, n_2, n_3)$$

$$\mathcal{Q}(H_1) = \phi_1$$
$$\mathcal{Q}(H_2) = \phi_2$$
$$\mathcal{Q}(\Sigma) = 0$$

The model: **SU(5)** (ii)

The scalar potential

$$
\begin{aligned}
V = & -\frac{1}{2}\mu^2\,\mathrm{Tr}(\Sigma^2) + \frac{1}{3}a\,\mathrm{Tr}(\Sigma^3) + \frac{1}{2}b^2\left[\mathrm{Tr}(\Sigma^2)\right]^2 + \frac{\lambda}{4}\,\mathrm{Tr}(\Sigma^4) \\
& + H_1^\dagger\left(\frac{1}{2}\mu_1^2 + a_1\Sigma + \lambda_{11}\,\mathrm{Tr}(\Sigma^2) + \lambda_{12}\Sigma^2\right)H_1 \\
& + H_2^\dagger\left(\frac{1}{2}\mu_2^2 + a_2\Sigma + \lambda_{21}\,\mathrm{Tr}(\Sigma^2) + \lambda_{22}\Sigma^2\right)H_2 \\
& + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4\left(H_1^\dagger H_2 H_2^\dagger H_1\right)
\end{aligned}
$$

- Σ breaks SU(5) $\rightarrow$ SM gauge group through $\langle\Sigma\rangle = \sigma\,\mathrm{diag}(2,2,2,-3,-3)$
- $\langle\Sigma\rangle$ breaks $\Sigma(24)$ and $H(5)$ in theirs components

$$\sigma = \frac{a}{2\lambda}\,\frac{1+\sqrt{1+4\,\xi\,(60\eta+7)}}{60\eta+7}$$

Problem:

- Scalar potential acquires an accidental global symmetry

Solution:

- softly breaking the Z_4 symmetry $\mu_{12}^2\,H_1^\dagger\,H_2$
- introducing a singlet S

$$V_S = \left[H_1^\dagger\,(\mu'_{12} + \lambda'_{12}\Sigma)\,H_2\,S + H.c.\right] - \frac{1}{2}\,\mu_S^2\,|S|^2 + \lambda_S\,|S|^4 + \lambda'_S(S^4 + H.c.)$$

The model: SU(5) (iii)

- Each doublet H_1, H_2 get a VEV and break SM $\to SU(3)_c \times U(1)_{em}$

$$v^2 \equiv |v_1|^2 + |v_2|^2 = (246.2\,\text{GeV})^2$$

- Generate the fermion masses via Yukawa interactions.

The most general Yukawa Lagrangian

$$\begin{aligned} -\mathcal{L}_Y = & \frac{1}{4}\left(\Gamma_u^1\right)_{ij} 10_i 10_j H_1 + \frac{1}{4}\left(\Gamma_u^2\right)_{ij} 10_i 10_j H_2 \\ & + \sqrt{2}\left(\Gamma_d^1\right)_{ij} 10_i 5_j^* H_1^* + \sqrt{2}\left(\Gamma_d^2\right)_{ij} 10_i 5_j^* H_2^* \\ & + \left(\Gamma_D^1\right)_{ij} 5_i^* \nu_j^c H_1 + \left(\Gamma_D^2\right)_{ij} 5_i^* \nu_j^c H_2 + \frac{1}{2}(M_R)_{ij}\, \nu_i^c \nu_j^c + \text{H.c.} \end{aligned}$$

$$M_u = v_1\, \Gamma_u^1 + v_2\, \Gamma_u^2 \qquad\qquad M_d = v_1^*\, \Gamma_d^1 + v_2^*\, \Gamma_d^2$$

Discrete Flavour Symmetries

Mass matrices

$$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix} \longrightarrow$$

$\mathcal{Q}(10_i\, 10_j\, H_1)$

$$\begin{pmatrix} 2q_1+\phi_1 & q_1+q_2+\phi_1 & q_1+q_3+\phi_1 \\ q_2+q_1+\phi_1 & q_1+q_2+\phi_1 & q_2+q_3+\phi_1 \\ q_3+q_1+\phi_1 & q_3+q_2+\phi_1 & 2q_3+\phi_1 \end{pmatrix}$$

$\mathcal{Q}(10_i\, 10_j\, H_2)$

$$\begin{pmatrix} 2q_1+\phi_2 & q_1+q_2+\phi_2 & q_1+q_3+\phi_2 \\ q_2+q_1+\phi_2 & q_1+q_2+\phi_2 & q_2+q_3+\phi_2 \\ q_3+q_1+\phi_2 & q_3+q_2+\phi_2 & 2q_3+\phi_2 \end{pmatrix}$$

$\longrightarrow$ $\mathcal{Q}(10_i\, 5^*_j\, H^*_1)$

$$\begin{pmatrix} q_1+d_1-\phi_1 & q_1+d_2-\phi_1 & q_1+d_3-\phi_1 \\ q_2+d_1-\phi_1 & q_1+d_2-\phi_1 & q_2+d_3-\phi_1 \\ q_3+d_1-\phi_1 & q_3+d_2-\phi_1 & 2q_3+d_3-\phi_1 \end{pmatrix}$$

$\mathcal{Q}(10_i\, 5^*_j\, H^*_2)$

$$\begin{pmatrix} q_1+d_1-\phi_2 & q_1+d_2-\phi_2 & q_1+d_3-\phi_2 \\ q_2+d_1-\phi_2 & q_1+d_2-\phi_2 & q_2+d_3-\phi_2 \\ q_3+d_1-\phi_2 & q_3+d_2-\phi_2 & 2q_3+d_3-\phi_2 \end{pmatrix}$$

Zero entry $\longrightarrow$ $\mathcal{Q}(\text{entry}) \neq 0$

Non Zero entry $\longrightarrow$ $\mathcal{Q}(\text{entry}) = 0$

- Choice: $M_{u(3,3)} \neq 0 \implies \phi_2 = -2q_3$

$$\mathcal{Q}(\mathbf{10}_i) = (3q_3+\phi_1, -q_3-\phi_1, q_3)$$

$$\mathcal{Q}(\mathbf{5}^*_i) = (q_3+2\phi_1, -3q_3, -q_3+\phi_1)$$

Discrete Flavour Symmetries

$\mathcal{Q}(10_i\, 10_j)$:

$$\begin{pmatrix} 6q_3 + 2\phi_1 & 2q_3 & 4q_3 + \phi_1 \\ 2q_3 & -2\phi_1 - 2q_3 & -\phi_1 \\ 4q_3 + \phi_1 & -\phi_1 & 2q_3 \end{pmatrix}$$

$\mathcal{Q}(10_i\, 5^*_j)$:

$$\begin{pmatrix} 4q_3 + 3\phi_1 & \phi_1 & 2q_3 + 2\phi_1 \\ \phi_1 & -\phi_1 - 4q_3 & -2q_3 \\ 2q_3 + 2\phi_1 & -2q_3 & \phi_1 \end{pmatrix}$$

- $\phi_1 \neq \phi_2$
- Minimal realisation: $\mathbf{Z_4}$

$$\phi_2 = 0 \text{ or } 2 \Longrightarrow \phi_1 = 1 \text{ or } 3$$

The mass matrices:

$$M_u = v_1 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b'_u & 0 \end{pmatrix} + v_2 \begin{pmatrix} 0 & a_u & 0 \\ a'_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}$$

$$M_d = v_1^* \begin{pmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} + v_2^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}$$

The model: SU(5) $\times Z_4$

- SU(5) @ GUT scale: $M_e = M_d^T$
- Since M_d has NNI form $\Longrightarrow$ M_e has also NNI form
- Dirac mass matrix, m_D, and Majorana mass matrix, M_R with unknown shape
- $M_e = M_d^T$ not compatible with down-type quark and charged-lepton masses hierarchies @ low energy

`Two ways to solve it:`

<u>nonrenormalisable higher dimension operators</u>

- dim-5 operators

$$\sum_{n=1,2} \frac{\sqrt{2}}{\Lambda'} (\Delta_n)_{ij} \; H^*_{n\,a} \, 10_i^{ab} \, \Sigma_b^c \, 5^*_{jc}$$

$$M_d - M_e^\top = 5 \frac{\sigma}{\Lambda'} (v_1^* \, \Delta_1 + v_2^* \, \Delta_2)$$

- dim-6 operators destroy NNI form BUT are much more suppressed

<u>$H_2(5)$ substituted by 45</u>

$$M_d - M_e^\top = 8 \, \Gamma_d^2 \, v_{45}^*$$

Proton decay

Through the exchange of:

Heavy gauge bosons X and Y

$$M_V = \frac{25}{8} g_U^2 \sigma^2$$

- $M_{X,Y} >> m_p$ to suppress proton decay via X, Y channels

Proton decay width

$$\Gamma \approx \alpha_U^2 \frac{m_p^5}{M_V^4}$$

Partial proton lifetime

$$\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33} \text{ years}$$

$$M_V > (4.0 - 5.1) \times 10^{15} \text{ GeV}$$

for $\alpha_U^{-1} \approx 25 - 40$

Colour Higgs triplets, T_1 and T_2

- dim-6 operators
- $\propto$ products of Yukawa coupling
- smaller than X, Y contribution

@ tree-level

$$\sum_{n=1,2} \frac{(\Gamma_u^n)_{ij} (\Gamma_d^n)_{kl}}{M_{T_n}^2} \left[\frac{1}{2} (Q_i Q_j)(Q_k L_l) + (u_i^c e_j^c)(u_k^c d_l^c) \right]$$

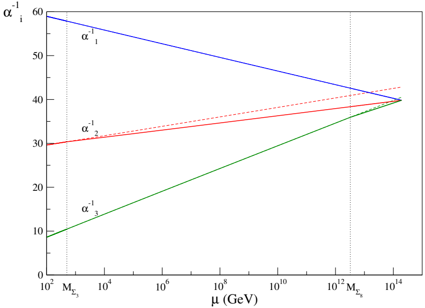
- only 3^{rd} generation contribute
- vanishes @ tree level

Unification

Unification of the gauge couplings @ 2 loop level considering the splitting between Σ_3 and Σ_8 even without threshold effects

- X, Y, T_1, T_2 @ Λ scale
- H_1, H_2 around electroweak scale

$M_{\Sigma_3} = 500$ GeV $\longrightarrow \Lambda = 1.9 \times 10^{14}$ GeV
$\longrightarrow M_{\Sigma_8} = 3.2 \times 10^{12}$ GeV



$$1.3 \times 10^{14}\ \text{GeV} \leq \Lambda \leq 2.4 \times 10^{14}\ \text{GeV}$$

$$M_Z \leq M_{\Sigma_3} \leq 1.8 \times 10^{4}\ \text{GeV}$$

$$5.4 \times 10^{11}\ \text{GeV} \leq M_{\Sigma_8} \leq 1.3 \times 10^{14}\ \text{GeV}$$

Problems

- $\Lambda < \Lambda_{X,Y}$
- spliting between M_{Σ_3} and M_{Σ_8} unnaturally large

Solution: 24 fermionic multiplet

Quark sector

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix} \quad M_d = \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix}$$

- Parameterization

$$A_{u,d} \equiv \overline{A}_{u,d}(1 - \epsilon_a^{u,d}) \qquad A'_{u,d} \equiv \overline{A}_{u,d}(1 + \epsilon_a^{u,d})$$

$$B_{u,d} \equiv \overline{B}_{u,d}(1 - \epsilon_b^{u,d}) \qquad B'_{u,d} \equiv \overline{B}_{u,d}(1 + \epsilon_b^{u,d})$$

$$\epsilon_a^{u,d} = \frac{A'_{u,d} - A_{u,d}}{A'_{u,d} + A_{u,d}} \quad \epsilon_b^{u,d} = \frac{B'_{u,d} - B_{u,d}}{B'_{u,d} + B_{u,d}}$$

Global deviation measurement:

$$\varepsilon = \frac{1}{2}\sqrt{\epsilon_a^{u\,2} + \epsilon_b^{u\,2} + \epsilon_a^{d\,2} + \epsilon_b^{d\,2}}$$

- $\epsilon_a^{u,d}, \epsilon_b^{u,d} << 1$, but not zero
- very small $\epsilon' s$ are not possible
- $\varepsilon \geq 0.19$

[G.C.Branco, Emmanuel-Costa, C.S., 2010]

Charged lepton sector

[G.C.Branco, F.Mota, 1992; G.C.Branco, Emmanuel-Costa, C.S., 2010]

$$m_\ell = \begin{pmatrix} 0 & A_\ell & 0 \\ A'_\ell & 0 & B_\ell \\ 0 & B'_\ell & C_\ell \end{pmatrix} \qquad \epsilon_a^\ell \equiv \frac{|A'_\ell|-|A_\ell|}{|A'_\ell|+|A_\ell|}\,, \qquad \epsilon_b^\ell \equiv \frac{|B'_\ell|-|B_\ell|}{|B'_\ell|+|B_\ell|} \qquad \varepsilon_\ell \equiv \sqrt{\frac{(\epsilon_a^\ell)^2+(\epsilon_b^\ell)^2}{2}}$$

- work with $h_\ell = m_\ell\, m_\ell^\dagger$ $\qquad O_\ell^\top\, h_\ell\, O_\ell = \text{diag}(m_e^2,\, m_\mu^2,\, m_\tau^2)$

For $\varepsilon_{a,b}^\ell$ small $\quad O_\ell \approx \begin{pmatrix} 1 & -\sqrt{\frac{m_e}{m_\mu}}\left(1-\epsilon_a^\ell-\frac{m_\mu}{m_\tau}\epsilon_b^\ell\right) & \sqrt{\frac{m_e\, m_\mu^2}{m_\tau^3}}\,(1+\epsilon_b^\ell-\epsilon_a^\ell) \\ \sqrt{\frac{m_e}{m_\mu}}\left(1-\epsilon_a^\ell-\frac{m_e}{m_\tau}\epsilon_b^\ell\right) & 1 & \sqrt{\frac{m_\mu}{m_\tau}}\,(1-\epsilon_b^\ell) \\ -\sqrt{\frac{m_e}{m_\tau}}\,(1-\epsilon_a^\ell-\epsilon_b^\ell) & -\sqrt{\frac{m_\mu}{m_\tau}}\left(1-\epsilon_b^\ell+\frac{m_e}{m_\mu}\epsilon_a^\ell\right) & 1 \end{pmatrix}$

Charged Lepton masses @ M_Z scale

- Running charged lepton masses from PDG'10 to M_Z in the $\overline{MS}$ scheme using RGE for QED @ 1-loop level:

[Y. Koide et al, Phys. Rev. D 57 (1998) 3986
Z. z. Xing at al, Phys. Rev. D 77 (2008)113016]

$$m_e(M_Z) = 0.486661305 \pm 0.000000056 \text{ MeV}\,,$$
$$m_\mu(M_Z) = 102.728989 \pm 0.000013 \text{ MeV}\,,$$
$$m_\tau(M_Z) = 1746.28 \pm 0.16 \text{ MeV}\,,$$

Neutrino sector (i)

Effective neutrino mass matrices: $\mathbf{m}_\nu = -\mathbf{m_D}\,\mathbf{M_R^{-1}}\,\mathbf{m_D^T}$

	q_3	$\nu = (0,1,3)$	$\nu = (1,2,3)$	$\nu_i \in \{0,2\}$
$\phi_1 = 1$	0	$\mathrm{I}_{(132)}$	$\mathrm{II}_{(12)}$	$\mathrm{III}_{(12)}$
	1	$\mathrm{I}_{(13)}$	II	III
	2	$\mathrm{II}_{(12)}$	$\mathrm{I}_{(132)}$	$\mathrm{III}_{(12)}$
	3	II	$\mathrm{I}_{(13)}$	III
$\phi_1 = 3$	0	$\mathrm{I}_{(132)}$	$\mathrm{II}_{(12)}$	$\mathrm{III}_{(12)}$
	1	II	$\mathrm{I}_{(13)}$	III
	2	$\mathrm{II}_{(12)}$	$\mathrm{I}_{(132)}$	$\mathrm{III}_{(12)}$
	3	$\mathrm{I}_{(13)}$	II	III

$$\mathrm{I}\ \begin{pmatrix}0&*&0\\ *&0&*\\ 0&*&*\end{pmatrix}\quad \mathrm{II}\ \begin{pmatrix}0&*&0\\ *&*&*\\ 0&*&*\end{pmatrix}\quad \mathrm{III}\ \begin{pmatrix}*&0&0\\ 0&*&*\\ 0&*&*\end{pmatrix}$$

$$\mathrm{I}_{(13)}\ \begin{pmatrix}*&*&0\\ *&0&*\\ 0&*&0\end{pmatrix}\quad \mathrm{II}_{(12)}\ \begin{pmatrix}*&*&*\\ *&0&0\\ *&0&*\end{pmatrix}\quad \mathrm{III}_{(12)}\ \begin{pmatrix}*&0&*\\ 0&*&0\\ *&0&*\end{pmatrix}$$

$$\mathrm{I}_{(132)}\ \begin{pmatrix}0&*&*\\ *&0&0\\ *&0&*\end{pmatrix}$$

Neutrino sector (ii)

$$m_\nu^0 = P_g^\top m_\nu^{(g)} P_g$$

$$\text{I)} \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix} \quad \text{II)} \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu e^{i\varphi} \end{pmatrix} \quad \text{III)} \begin{pmatrix} A_\nu & 0 & 0 \\ 0 & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu e^{i\varphi} \end{pmatrix}$$

$$U_\nu^\top \, m_\nu^0 \, U_\nu = \text{diag}(m_1, \, m_2, \, m_3)$$

Counting the number of parameters:

- I) 10 parameters < 12 (6 masses + 3 mixing angles + 3 phase)
- II) Compatible!
- III) Not phenomenologically viable: large mixing angles

Case II) , $\varphi = 0$:

$$|U_\nu| = \begin{pmatrix} \sqrt{\frac{m_2 m_3 (D_\nu - m_1)}{D_\nu \, (m_2 - m_1)(m_3 - m_1)}} & \sqrt{\frac{m_1 m_3 (m_2 - D_\nu)}{D_\nu \, (m_2 - m_1)(m_3 - m_2)}} & \sqrt{\frac{m_1 m_2 (D_\nu - m_3)}{D_\nu \, (m_3 - m_1)(m_3 - m_2)}} \\ \sqrt{\frac{m_1 (m_1 - D_\nu)}{(m_2 - m_1)(m_3 - m_1)}} & \sqrt{\frac{(D_\nu - m_2) m_2}{(m_2 - m_1)(m_3 - m_2)}} & \sqrt{\frac{m_3 (m_3 - D_\nu)}{(m_3 - m_1)(m_3 - m_2)}} \\ \sqrt{\frac{m_1 (D_\nu - m_2)(D_\nu - m_3)}{D_\nu \, (m_2 - m_1)(m_3 - m_1)}} & \sqrt{\frac{m_2 (D_\nu - m_1)(m_3 - D_\nu)}{D_\nu \, (m_2 - m_1)(m_3 - m_2)}} & \sqrt{\frac{m_3 (D_\nu - m_1)(D_\nu - m_2)}{D_\nu \, (m_3 - m_1)(m_3 - m_2)}} \end{pmatrix}$$

Normal Hierarchy

$$\begin{pmatrix} + & + & + \\ \text{sign}(m_1) & \text{sign}(m_2) & \text{sign}(m_3) \\ \text{sign}(m_2 m_3) & \text{sign}(m_1) & + \end{pmatrix}$$

Inverted Hierarchy

$$\begin{pmatrix} + & + & + \\ \text{sign}(m_1) & \text{sign}(m_2) & \text{sign}(m_3) \\ \text{sign}(m_3) & + & \text{sign}(m_1 m_2) \end{pmatrix}$$

[**G.C.Branco, Emmanuel-Costa, G.Felipe, H.Serôdio, 2009**]

Neutrino Oscillation data:

$sin^2 2\theta_{13} = 0.092 \pm 0.016$ [DAYA-BAY colaboration, 2012]

[Forero, Tortola, Valle, arXiv:1205.4018]

parameters	NH	IH
Δm^2_{21}	$(7.62 \pm 0.19) \times 10^{-5}$ eV^2	
$\left\|\Delta m^2_{31}\right\|$	$\left(2.53^{+0.08}_{-0.10}\right) \times 10^{-3}$ eV^2	$\left(2.40^{+0.10}_{-0.07}\right) \times 10^{-3}$ eV^2
$\sin^2 \theta_{12}$	$0.320^{+0.015}_{-0.017}$	
$\sin^2 \theta_{23}$	$0.49^{+0.08}_{-0.05}$	$0.53^{+0.05}_{-0.07}$
$\sin^2 \theta_{13}$	$0.026^{+0.003}_{-0.004}$	$0.027^{+0.003}_{-0.004}$

Other constraints

Effective Majorana mass: $m_{ee} \equiv \sum_{i=1}^{3} m_i U^{*2}_{1i}$

[Pascoli et al, 2003]

$$|m_{ee}| \lesssim 0.005 \text{ eV (NH)} \qquad 10^{-2} \text{ eV} \lesssim |m_{ee}| \lesssim 0.05 \text{ eV (IH)}$$

[Bilenky et al, 2001; Petcov et al, 2005]

Tritium β decay: $m^2_{\nu_e} \equiv \sum_{i=1}^{3} m_i^2 |U_{1i}|^2 < (2.3 \text{ eV})^2$ at 95% C.L.

[Nakamura et al, 2010(PDG)]

From cosmological and astrophysical data: $\mathcal{T} \equiv \sum_{i=1}^{3} \mathbf{m_i} < \mathbf{0.68}$ eV at 95% C.L.

Neutrino sector (iii)

Neutrino Oscillation data:

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$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$	$0.027^{+0.003}_{-0.004}$

$\Delta m^2_{ij} \equiv m^2_i - m^2_j$

The Pontecorvo-Maki-Nakagawa-Sakata matrix:

$$U_{PMNS} = O_\ell^\top \, K_\ell^\dagger \, P_g \, U_\nu$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

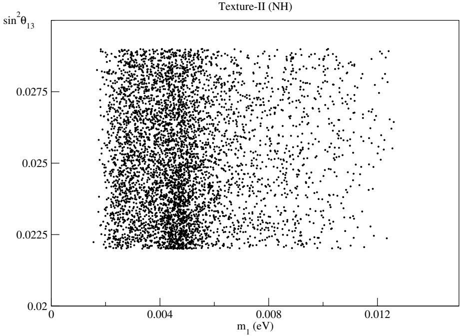
$s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$

δ: Dirac phase

α_1, α_2: Majorana phases

Results (i)

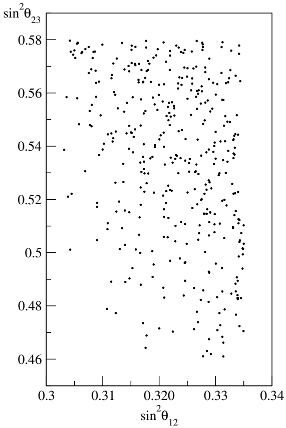
$$\mathbf{m}_\nu = \begin{pmatrix} \mathbf{0} & * & \mathbf{0} \\ * & * & * \\ \mathbf{0} & * & * \end{pmatrix}_{\mathbf{NH}}$$

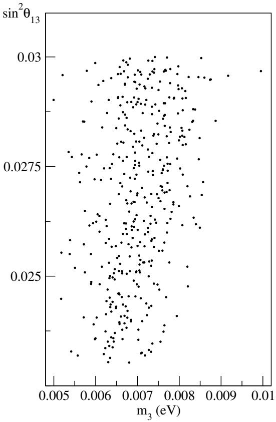


- $0.0015 \leq m_1\,[\text{eV}] \leq 0.013$
- $0.97 < |m_{ee}|\,[10^{-3}\text{eV}] < 2.1$
- $\varepsilon_\ell > 0.005$

Results (ii)

$$\mathbf{m}_\nu = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}_{\mathbf{IH}}$$

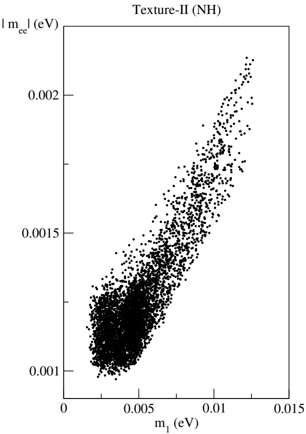


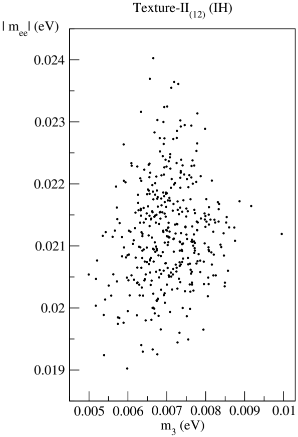


- $0.005 \leq m_3[\text{eV}] \leq 0.010$
- $0.015\,\text{eV} < |m_{ee}|\,[\text{eV}] < 0.021$
- $\varepsilon_\ell > 0.003$

Results (iii)

$$\mathbf{m}_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{\mathbf{NH}} \qquad \mathbf{m}_\nu = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}_{\mathbf{IH}}$$



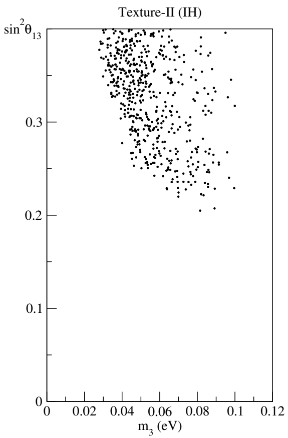


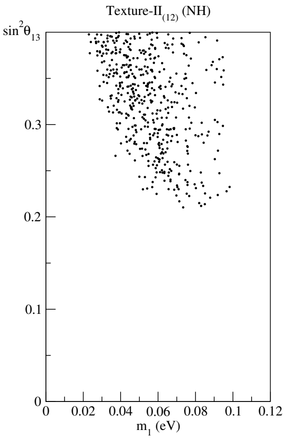
$|m_{ee}| \lesssim 0.005$ eV (NH) $\qquad 10^{-2} \lesssim |m_{ee}| \lesssim 0.05$ eV (IH)

[**Bilenky et al, 2001; Petcov et al 2005**]

Results (iv)

$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{IH} \qquad m_\nu = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}_{NH}$$





Not viable

Other implementations: $SO(10)$

SO(10):

[H.Fritzsch,P.Minkowski, 1974; H.Georgi, 1975]

$$\mathbf{16}_i = (q_L, u^c, d^c, \ell_L, e^c, \nu^c)_i$$

$$\mathcal{Q}(\mathbf{10}_i) = \mathcal{Q}(\mathbf{5}_i^*) = \mathcal{Q}(\nu_i^c) \qquad \phi_1 = -\phi_2 = 2q_3$$

Not possible in Z_4!

1) $\mathbf{Z_5}$

+ an extra Higgs singlet

or

2) SUSY **SO(10)** model:

+ 2 Higgs doublets: Φ_1^u, Φ_2^u and Φ_1^d, Φ_2^d

- Scanning $\phi_{1,2}^u$ and $\phi_{1,2}^d$: $\quad \phi_i^u = \phi_i^d$ or $\phi_i^u = \phi_{j,\, i\neq j}^d$
- m_ν is restricted only to Texture II: $\quad m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$

Other implementations: Flipped $SU(5)$

[S.Barr, 1982; J.P.Derendinger, J.E.Kim, D.V.Nanopoulos, 1984]

Flipped $\mathbf{SU}(\mathbf{5})$:

$$\mathbf{10}_i = (q_L, d^c, \nu^c)_i\,, \qquad \mathbf{5}^*_i = (\ell_L, u^c)_i\,, \qquad \mathbf{1}_i = e^c_i\,,$$

$$\mathcal{Q}(\mathbf{10}_i) = \mathcal{Q}(q_i) = \mathcal{Q}(d^c_i) = \mathcal{Q}(\nu^c_i) \qquad \mathcal{Q}(\mathbf{5}^*_i) = \mathcal{Q}(\ell_i) = \mathcal{Q}(u^c_i)$$

$\phi_1 \;=\; 2q_3$ under $\mathbf{Z}_n$:

$$\mathcal{Q}(\mathbf{10}) = (3q_3 - \phi_2,\ -q_3 + \phi_2,\ q_3)$$

$$\mathcal{Q}(\mathbf{5}^*) = (q_3 - 2\phi_2,\ -3q_3,\ -q_3 - \phi_2)$$

- e^c has free $\mathbf{Z_n}$ charges: $\mathcal{Q}(\mathbf{1}) = (e_1, e_2, e_3)$
- m_ν is restricted to only one structure:

$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

Conclusions

- Quark mass matrices in the NNI form in the context of minimal SU(5) model plus three ν_i^c and two Higgs quintets
- Through the implementation of an Abelian discrete flavour symmetry of Z_n type where the minimal realisation is $\mathbf{Z_4}$
- Study the implications of $SU(5) \times Z_4$ model on the leptonic sector
- Only two textures for m_ν

Normal Hierarchy	**Inverted Hierarchy**
$\begin{pmatrix} 0 & \star & 0 \\ \star & \star & \star \\ 0 & \star & \star \end{pmatrix}$	$\begin{pmatrix} \star & \star & \star \\ \star & 0 & 0 \\ \star & 0 & \star \end{pmatrix}$
$0.0015\,\text{eV} \leq m_1 \leq 0.013\,\text{eV}$	$0.005\,\text{eV} \leq m_3 \leq 0.010\,\text{eV}$
$0.00097\,\text{eV} < \lvert m_{ee} \rvert < 0.0021\,\text{eV}$	$0.015\,\text{eV} < \lvert m_{ee} \rvert < 0.021\,\text{eV}$
$\varepsilon_e > 0.005$	$\varepsilon_e > 0.003$