Local Flavor Symmetries

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based on
J.H., Werner Rodejohann,
PRD **84** (2011), PRD **85** (2012);
Takeshi Araki, J.H., Jisuke Kubo,
JHEP (2012).

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

INTERNATIONAL MAX PLANCK RESEARCH SCHOOL PTFS FOR PRECISION TESTS OF FUNDAMENTAL SYMMETRIES

ν $m_\nu = m_L - m_D^T M_R^{-1} m_D$ erc

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

Outline

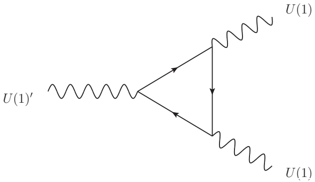
- Motivation for $U(1)$ symmetries in type-I seesaw
- Neutrino hierarchies from $U(1)$
- Texture zeros and vanishing minors
- Summary

Flavor Symmetries

- Most popular way: discrete non-abelian global symmetries ($A_4, \ldots$)
  - Drawbacks: complicated, many parameters and fields, unknown scalar sector at unreachable energies...
- Here: continuous abelian local symmetries, i.e. $U(1)$
  - Very simple, i.e. few new particles/parameters
  - Testable outside of neutrino sector, e.g. Z' at LHC
- How to choose the $U(1)$:
  - Same charge for all quarks (FCNC): $Y'(q_L) = Y'(u_R) = Y'(d_R)$.
  - Add three N_i with charges that allow m_D:

$$Y'(\ell_{Li}) = Y'(e_{Ri}) = Y'(N_{Ri})\,, \qquad Y'(\ell_{Li}) \neq Y'(\ell_{Lj})\,.$$

  - Anomaly cancellation:



Maximal Abelian Gauge Group

- Only one constraint on the charges:

$$9\, Y'(q_L) + Y'(\ell_{L1}) + Y'(\ell_{L2}) + Y'(\ell_{L3}) = 0\,.$$

- Two cases:

$$B - \sum_\alpha x_\alpha L_\alpha \text{ with } \sum_\alpha x_\alpha = 3 \text{ [E. Ma, \textit{(1998)}]}\,,$$

$$\sum_\alpha y_\alpha L_\alpha \text{ with } \sum_\alpha y_\alpha = 0\,.$$

- Famous examples: $B - L$ (GUT?), $L_\alpha - L_\beta$ (anomaly free in the SM) [R. Foot, *MPL A6 (1991)*]
- More general:

$$SU(3)_\ell \times U(1)_{B-L} \supset U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau} \times U(1)_{B-L}$$

can be added to G_{SM} without anomalies ($\ell_{Li} \sim \mathbf{3}_\ell$, $e_{Ri} \sim \mathbf{3}_\ell$, $N_{Ri} \sim \mathbf{3}_\ell$).

Now what?

- Three interesting zeroth order approximations:

$$\mathcal{M}_\nu^{L_e} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathcal{M}_\nu^{\overline{L}} \sim \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad \mathcal{M}_\nu^{L_\mu - L_\tau} \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}.$$

[S. Choubey, W. Rodejohann, *EPJC 40 (2005)*]

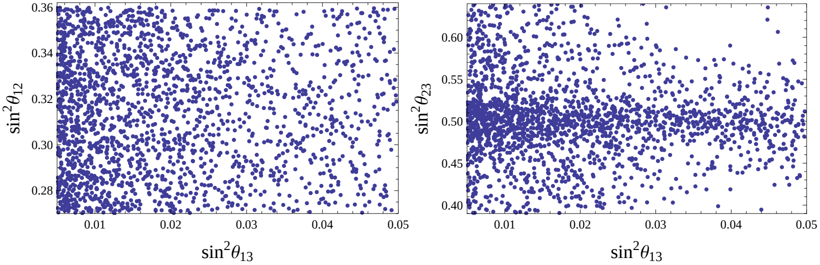
- Conserve L_e, $\overline{L} \equiv L_e - L_\mu - L_\tau$ and $L_\mu - L_\tau$, and lead to NH, IH and QD, respectively.
- $\overline{L}$ often used as global symmetry, gives bimaximal mixing.
- $L_\mu - L_\tau$ is special: $\mathcal{M}_\nu^{L_\mu - L_\tau}$ is invertible $\Rightarrow$ can be obtained from type-I seesaw.

$L_\mu - L_\tau$

- Dirac matrices diagonal due to symmetry, $\mathcal{M}_R$ of the $L_\mu - L_\tau$ symmetric form
- Seesaw gives $L_\mu - L_\tau$ symmetric $\mathcal{M}_\nu$
- Add one or two scalars S that couple to $\overline{N}_i^c N_j$ and get a VEV
- VEV fills zeros in $\mathcal{M}_R$ and $\mathcal{M}_\nu$ and gives mass to Z' boson $M_{Z'}/g' \sim \langle S \rangle$
- For mixing angles: $\langle S \rangle \sim 10^{-2} |\mathcal{M}_R|$
  $\Rightarrow$ connection of seesaw-scale and $M_{Z'}$
  $\Rightarrow$ LHC can probe low seesaw-scale

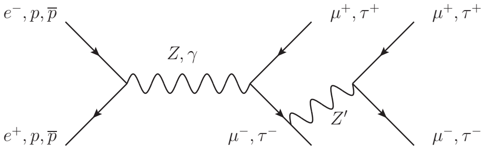
$L_\mu - L_\tau$

Two scalars, $\varepsilon = v_S/|\mathcal{M}_R| = 0.02$:

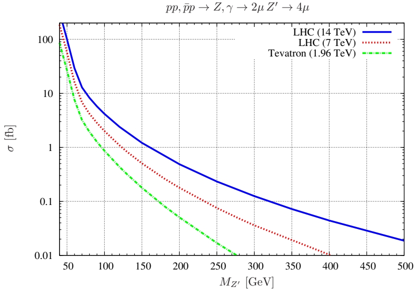


$L_\mu - L_\tau$ at LHC

- Signal at LHC [S. Baek et al, *PRD 64 (2001)*]

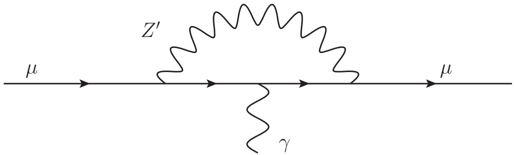


- Full run ($L = 100\,\mathrm{fb}^{-1}$) with $g' = 1$: Up to $M_{Z'} = 350\,\mathrm{GeV}$



$L_\mu - L_\tau$ at low energies

- Magnetic moment of muon:



For $M_{Z'} \gg m_\mu$: $\Delta a_\mu \simeq \frac{1}{12\pi^2}\frac{g'^2}{M_{Z'}^2} m_\mu^2$

- To explain $\sim 3.6\sigma$ deviation: $M_{Z'}/g' \simeq 220\,\mathrm{GeV}$ [E. Ma et al, *PLB 525 (2002)*]

L_e and $\overline{L}$

- Try $B - 3L_e$ and $B + 3\overline{L}$ as anomaly free symmetries with the right lepton structure.
- Same procedure as for $L_\mu - L_\tau$ does not work, $\mathcal{M}_R^{L_e}$ and $\mathcal{M}_R^{\overline{L}}$ are not invertible.
- Weird coincidence:

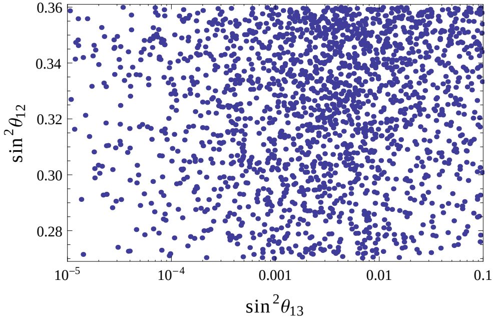
$$\mathcal{M}_R \sim \begin{pmatrix} \varepsilon & 1 & 1 \\ \cdot & \varepsilon & \varepsilon \\ \cdot & \cdot & \varepsilon \end{pmatrix} \qquad \Rightarrow \qquad \mathcal{M}_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} .$$

Low $B + 3\overline{L}$ breaking gives approximate L_e symmetry!

$\Rightarrow$ Broken $B + 3\overline{L}$ (say $\varepsilon = 0.05$) gives NH structure.

Mixing Angles and Collider

- Since μ and τ have same Y' charge, m_D not diagonal $\Rightarrow \theta_{23}$ random (i.e. large).
- One scalar S with charge $Y'(S) = 6$ and VEV 1–10 $\mathrm{TeV} < v_S \sim \varepsilon|\mathcal{M}_R|$ gives:



- LEP-II constraint: $v_S > 2.3\,\mathrm{TeV}$, LHC prospects in [H. S. Lee and E. Ma, *PLB 688 (2010)*].

How to get Inverted Hierarchy

- Problem: anomaly cancellation demands odd number of RHN, but then $\mathcal{M}_R^{\overline{L}}$ is not invertible, which destroys symmetry!
- Solution: Decouple one N_i with a $\mathbb{Z}_2$.
- Make N_3 odd:

$$\begin{aligned}\mathcal{L}_{N_3} &= i\overline{N_3}\gamma^\mu \left(\partial_\mu - i(-3)g' Z'_\mu\right) N_3 - Y_\chi S\, \overline{N_3}^c N_3 + \mathrm{h.c.}\\ &= \frac{i}{2}\chi^T \mathcal{C}\gamma^\mu\partial_\mu\chi - \frac{3}{2}g' Z'_\mu \chi^T\mathcal{C}\gamma^\mu\gamma_5\chi - Y_\chi \frac{v_S}{\sqrt{2}}\,\chi^T\mathcal{C}\chi\left(1+\frac{s}{v_S}\right)\,.\end{aligned}$$

Majorana fermion χ will be dark matter candidate.

- Neutrino mass:

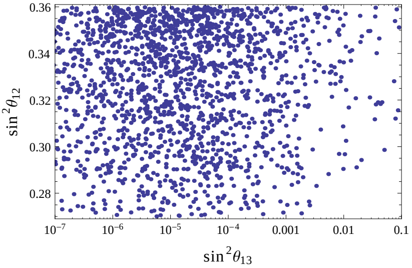
$$\mathcal{M}_\nu \simeq -\begin{pmatrix} a & 0\\ 0 & b\\ 0 & c\end{pmatrix}\begin{pmatrix} A & X\\ X & B\end{pmatrix}^{-1}\begin{pmatrix} a & 0 & 0\\ 0 & b & c\end{pmatrix} \sim \begin{pmatrix} a^2 B & -abX & -acX\\ \cdot & b^2 A & bcA\\ \cdot & \cdot & c^2 A\end{pmatrix}.$$

Mixing Angles

- Only two neutrinos massive, $\mathcal{M}_\nu$ exhibits "scaling" $\Rightarrow \theta_{13} = 0$.
- Solution: Add five N_i instead of three (still anomaly free) and decouple one of them:

$$\mathcal{M}_\nu \simeq - \begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & e & f \end{pmatrix} \begin{pmatrix} \mathcal{A} & \mathcal{X} \\ \mathcal{X}^T & \mathcal{B} \end{pmatrix}^{-1} \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ 0 & c & e \\ 0 & d & f \end{pmatrix} .$$

- Need larger breaking than before. $\varepsilon = 0.1$:



Scalar Sector

- Scalar sector identical to $U(1)_{B-L}$ models: [L. Basso et al, *PRD 80 (2009)*]

$$V(H,S) = -\mu_1^2 |H|^2 + \lambda_1 |H|^4 - \mu_2^2 |S|^2 + \lambda_2 |S|^4 + \delta |S|^2 |H|^2 \,,$$

- In unitary gauge, mixing via δ:

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}h\\ s\end{pmatrix}, \qquad \tan 2\alpha = \frac{\delta v v_S}{\lambda_2 v_S^2 - \lambda_1 v^2} \,.$$

- Masses:

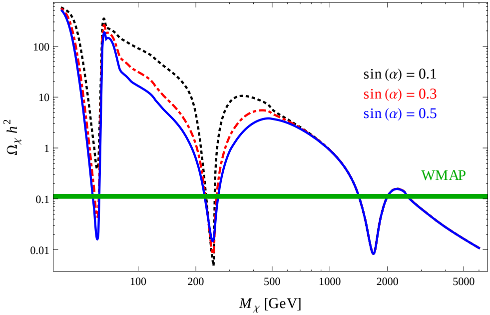
$$M_{Z'} = 6 g' v_S \,, \qquad m_2 \simeq m_s \simeq \sqrt{2\lambda_2} v_S \,, \qquad M_\chi = \sqrt{2} Y_\chi v_S \,.$$

Dark Matter

- Dark matter sector identical to $U(1)_{B-L} \times \mathbb{Z}_2$ sector: [N. Okada and O. Seto, *PRD 82 (2010)*]

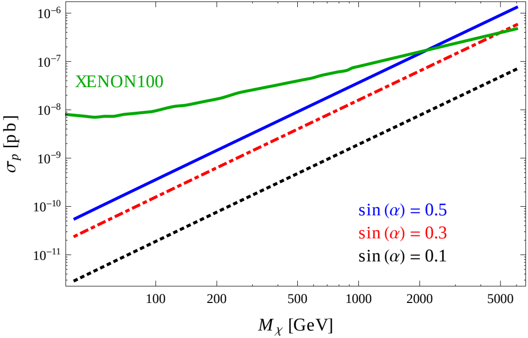
$$\mathcal{L}_\chi = \frac{i}{2}\chi^T \mathcal{C}\gamma^\mu \partial_\mu \chi - \frac{3}{2} g' Z'_\mu \chi^T \mathcal{C}\gamma^\mu \gamma_5 \chi - Y_\chi \frac{v_S}{\sqrt{2}} \chi^T \mathcal{C} \chi \left(1 + \frac{s}{v_S}\right) .$$

- Relic density can be obtained around the scalar resonances or the Z' resonance ($m_1 = 125\,\mathrm{GeV}$, $m_2 = 500\,\mathrm{GeV}$, $M_{Z'} = 3.5\,\mathrm{TeV}$):



Direct Detection

- Direct detection cross section via Z' are suppressed by Lorentz structure: $\overline{\chi}\gamma^\mu\gamma_5\chi\ \overline{f}\gamma_\mu f$.
- XENON1T only sensitive to scalar exchange:



And now for something completely different. . . Texture Zeros

- Take $\mathcal{M}_\nu$ and set two independent entries to zero $\Rightarrow$ four constraints on the nine low-energy parameters (m_1, m_2, m_3), $(\theta_{23}, \theta_{12}, \theta_{13})$ and (δ, α, β) (CP violating phases)
- 15 two-zero textures possible, only 7 allowed at 3σ: [H. Fritzsch et al, *JHEP 1109 (2011)*]

$$\mathbf{A}_1^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathbf{A}_2^\nu : \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathbf{B}_1^\nu : \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathbf{B}_2^\nu : \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},$$

$$\mathbf{B}_3^\nu : \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \qquad \mathbf{B}_4^\nu : \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}, \qquad \mathbf{C}^\nu : \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

Vanishing Minors

- Same idea, but with $\mathcal{M}_\nu^{-1}$ instead of $\mathcal{M}_\nu$.
- Seven patterns for $\mathcal{M}_\nu^{-1}$ allowed:

$$\mathbf{D}_1^R : \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & 0 \\ \cdot & \cdot & \times \end{pmatrix}, \quad \mathbf{D}_2^R : \begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & 0 \end{pmatrix}, \quad \mathbf{B}_3^R : \begin{pmatrix} \times & 0 & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}, \quad \mathbf{B}_4^R : \begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix},$$

$$\mathbf{B}_1^R : \begin{pmatrix} \times & \times & 0 \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}, \qquad \mathbf{B}_2^R : \begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}, \qquad \mathbf{C}^R : \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}.$$

- Two-zero texture in $\mathcal{M}_\nu^{-1}$ corresponds to two vanishing minors in $\mathcal{M}_\nu$. E.g.

$$\mathcal{M}^{-1} = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \quad \Rightarrow \quad \begin{vmatrix} \mathcal{M}_{11} & \mathcal{M}_{13} \\ \mathcal{M}_{31} & \mathcal{M}_{33} \end{vmatrix} = 0 = \begin{vmatrix} \mathcal{M}_{21} & \mathcal{M}_{22} \\ \mathcal{M}_{31} & \mathcal{M}_{32} \end{vmatrix}$$

Texture Zeros in $\mathcal{M}_R$

- Use lepton-nonuniversal $U(1)' \Rightarrow m_D$ and M_ℓ diagonal by symmetry.
- For diagonal m_D, texture zeros in $\mathcal{M}_\nu^{-1}$ are texture zeros of $\mathcal{M}_R$!
- Enforce zeros in $\mathcal{M}_R$ by $B - \sum_\alpha x_\alpha L_\alpha$ or $\sum_\alpha y_\alpha L_\alpha$ symmetries:

$$Y'(\overline{N}_i^c N_j) = \begin{pmatrix} -2x_e & -x_e - x_\mu & x_\mu - 3 \\ -x_e - x_\mu & -2x_\mu & x_e - 3 \\ x_\mu - 3 & x_e - 3 & 2x_e + 2x_\mu - 6 \end{pmatrix}.$$

- Symmetry can not be exact, use breaking vev to fill more entries, e.g. $L_\mu - L_\tau$ with one scalar S with $|Y'(S)| = 1$:

$$\mathcal{M}_R = M_{L_\mu - L_\tau} \begin{pmatrix} \times & 0 & 0 \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix} + \langle S \rangle \begin{pmatrix} 0 & \times & \times \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \sim \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}.$$

One Scalar

- With just one scalar, we can get five of the seven patterns:

Symmetry generator Y'	$\|Y'(S)\|$	Texture zeros in $\mathcal{M}_R$	Texture zeros in $\mathcal{M}_\nu$
$L_\mu - L_\tau$	1	$(\mathcal{M}_R)_{33}$, $(\mathcal{M}_R)_{22}$ ($\mathbf{C}^R$)	–
$B - L_e + L_\mu - 3L_\tau$	2	$(\mathcal{M}_R)_{33}$, $(\mathcal{M}_R)_{13}$ ($\mathbf{B}_4^R$)	$(\mathcal{M}_\nu)_{12}$, $(\mathcal{M}_\nu)_{22}$ ($\mathbf{B}_3^\nu$)
$B - L_e - 3L_\mu + L_\tau$	2	$(\mathcal{M}_R)_{22}$, $(\mathcal{M}_R)_{12}$ ($\mathbf{B}_3^R$)	$(\mathcal{M}_\nu)_{13}$, $(\mathcal{M}_\nu)_{33}$ ($\mathbf{B}_4^\nu$)
$B + L_e - L_\mu - 3L_\tau$	2	$(\mathcal{M}_R)_{33}$, $(\mathcal{M}_R)_{23}$ ($\mathbf{D}_2^R$)	$(\mathcal{M}_\nu)_{12}$, $(\mathcal{M}_\nu)_{11}$ ($\mathbf{A}_1^\nu$)
$B + L_e - 3L_\mu - L_\tau$	2	$(\mathcal{M}_R)_{22}$, $(\mathcal{M}_R)_{23}$ ($\mathbf{D}_1^R$)	$(\mathcal{M}_\nu)_{13}$, $(\mathcal{M}_\nu)_{11}$ ($\mathbf{A}_2^\nu$)

- Many patterns are hard to distinguish via neutrino experiments, e.g. $\mathbf{D}_1^R$ and $\mathbf{D}_2^R$, but the symmetries $B + L_e - 3L_\mu - L_\tau$ and $B + L_e - L_\mu - 3L_\tau$ are very different

$\Rightarrow$ new possibilities to disentangle texture zeros.

Two Scalars

- Remaining two patterns can be obtained with two scalars:

$\mathcal{M}_R$ pattern	Symmetry generator Y'	$\lvert Y'(S_i)\rvert$
$\mathbf{B}_1^R$	$B + 3L_\mu - 6L_\tau$	$3,\ 12$
	$B - 2L_\mu - L_\tau$	$2,\ 3$
	$B - \frac{9}{2}L_\mu + \frac{3}{2}L_\tau$	$3,\ \frac{9}{2}$
	$B - 6L_e + 3L_\mu$	$3,\ 12$
	$B + \frac{3}{2}L_e - \frac{9}{2}L_\mu$	$3,\ \frac{9}{2}$
	$B - L_e - 2L_\mu$	$2,\ 3$
	$B - L_e + 3L_\mu - 5L_\tau$	$2,\ 10$
	$B - 5L_e + 3L_\mu - L_\tau$	$2,\ 10$
$\mathbf{B}_2^R$	$\mathbf{B}_1^R$ with $L_\mu \leftrightarrow L_\tau$	

- Discrete subgroups, e.g. $\mathbb{Z}_5$, can work with just one scalar.
- With three scalars, symmetries of the type $\sum_\alpha y_\alpha L_\alpha$ can be used.

Summary

- With three right-handed neutrinos, the SM gauge group can be extended by $U(1)_{B-L} \times U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau}$.
- Can use subgroups to influence neutrino mixing by introducing just one or two scalars.
- Applications: Enforcing neutrino hierarchies, two-zero textures, two vanishing minors, maybe other stuff?
- Z' and s can be searched for outside the neutrino sector
  $\Rightarrow$ testable models!

Backup

$\mathcal{M}_R$ pattern	Symmetry generator Y'	$\lvert Y'(S_i)\rvert$
$\mathbf{D}_1^R$	$B - aL_e - 3L_\mu + aL_\tau,\ a \notin \{-9,-3,0,1,3\}$	$2\lvert a\rvert,\ \lvert 3+a\rvert$
	$B - 2L_\mu - L_\tau$	$1,\ 2$
	$B + \frac{3}{2}L_e - \frac{9}{2}L_\mu$	$3,\ \frac{3}{2}$
	$B + \frac{9}{7}L_e - \frac{27}{7}L_\mu - \frac{3}{7}L_\tau$	$\frac{18}{7},\ \frac{6}{7}$
	$B + \frac{1}{3}L_e - \frac{7}{3}L_\mu - L_\tau$	$2,\ \frac{2}{3}$
$\mathbf{D}_2^R$	$\mathbf{D}_1^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{B}_3^R$	$\mathbf{D}_1^R$ with $L_e \leftrightarrow L_\tau$	
$\mathbf{B}_4^R$	$\mathbf{B}_3^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{B}_1^R$	$B + 3L_\mu - 6L_\tau$	$3,\ 12$
	$B - 2L_\mu - L_\tau$	$2,\ 3$
	$B - \frac{9}{2}L_\mu + \frac{3}{2}L_\tau$	$3,\ \frac{9}{2}$
	$B - 6L_e + 3L_\mu$	$3,\ 12$
	$B + \frac{3}{2}L_e - \frac{9}{2}L_\mu$	$3,\ \frac{9}{2}$
	$B - L_e - 2L_\mu$	$2,\ 3$
	$B - L_e + 3L_\mu - 5L_\tau$	$2,\ 10$
	$B - 5L_e + 3L_\mu - L_\tau$	$2,\ 10$
$\mathbf{B}_2^R$	$\mathbf{B}_1^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{C}^R$	$B + 3L_e - aL_\mu - (6-a)L_\tau,\ a \notin \{-3,0,1,3,5,6,9\}$	$5,\ \lvert 3-a\rvert$
	$B - 6L_\mu + 3L_\tau$	$3,\ 6$
	$B - 3L_e \pm 9L_\mu \mp 9L_\tau$	$6,\ 12$

Backup

$\mathcal{M}_R$ pattern	Symmetry generator Y'	$\lvert Y'(S_i)\rvert$
$\mathbf{D}_1^R$	$L_e - 3L_\mu + 2L_\tau$	2, 3, 4
	$L_e + 2L_\mu - 3L_\tau$	2, 3, 6
	$3L_e - 2L_\mu - L_\tau$	1, 2, 6
$\mathbf{D}_2^R$	$\mathbf{D}_1^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{B}_3^R$	$\mathbf{D}_1^R$ with $L_e \leftrightarrow L_\tau$	
$\mathbf{B}_4^R$	$\mathbf{B}_3^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{B}_1^R$	$L_e - 3L_\mu + 2L_\tau$	1, 2, 4
	$2L_e - 3L_\mu + L_\tau$	1, 2, 4
$\mathbf{B}_2^R$	$\mathbf{B}_1^R$ with $L_\mu \leftrightarrow L_\tau$	
$\mathbf{C}^R$	$L_e - 3L_\mu + 2L_\tau$	1, 2, 3
	$L_e + 2L_\mu - 3L_\tau$	1, 2, 3

Neutrino Masses

- Neutrinos massless in Standard Model (SM)
- Simplest way out: add right-handed neutrinos N_i:

$$-\mathcal{L} \supset \overline{N}_j \left(\mathbf{Y}_\nu\right)_{j\,\alpha} L_\alpha \tilde{H}^\dagger + \frac{1}{2}\overline{N}_j^c \left(\mathcal{M}_R\right)_{jk} N_k + \mathrm{h.c.}$$

If $m_D = v\mathbf{Y}_\nu \ll \mathcal{M}_R$, Majorana mass via seesaw (type-I):

$$\mathcal{M}_\nu \simeq m_D^T \mathcal{M}_R^{-1} m_D = v^2\, \mathbf{Y}_\nu^T \mathcal{M}_R^{-1} \mathbf{Y}_\nu\,.$$

- Other ways: triplet scalar (type-II), triplet fermions (type-III), radiative neutrino masses...
- $\mathcal{M}_\nu$ symmetric complex matrix, diagonalize via unitary matrix U:

$$U^T \mathcal{M}_\nu U = \mathrm{diag}(m_1, m_2, m_3)\,.$$

Neutrino Mixing

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix $U =$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\alpha} & s_{13}e^{i(\beta-\delta_{\mathrm{CP}})} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta_{\mathrm{CP}}} & (c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta_{\mathrm{CP}}})e^{i\alpha} & s_{23}c_{13}e^{i\beta} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta_{\mathrm{CP}}} & -(s_{23}c_{12} + c_{23}s_{13}s_{12}e^{i\delta_{\mathrm{CP}}})e^{i\alpha} & c_{23}c_{13}e^{i\beta} \end{pmatrix}$$

with [D. V. Forero et al, arXiv:*1205.4018*]

$$\sin^2\theta_{13} = 0.026\,, \qquad \sin^2\theta_{12} = 0.32\,, \qquad \Delta m^2_{21} = 7.62\times 10^{-5}\,\mathrm{eV}^2,$$
$$\sin^2\theta_{23} = 0.49 \text{ (NH) or } \sin^2\theta_{23} = 0.53 \text{ (IH)},$$
$$\Delta m^2_{31} = 2.53\times 10^{-3}\,\mathrm{eV}^2 \text{ (NH) or } \Delta m^2_{31} = -2.40\times 10^{-3}\,\mathrm{eV}^2 \text{ (IH)}.$$

Why these values? Symmetries!?

Remark

Easy way to see anomaly freedom: Choose different basis in group space:

$$L_e - L_\mu = \mathrm{diag}(1,-1,0), \quad (L_e - L_\mu) + 2\,(L_\mu - L_\tau) = \mathrm{diag}(1,1,-2).$$

Cartan subalgebra of $SU(3)_\ell$ with

$$\ell_{Li} \sim \mathbf{3}_\ell\,, \qquad e_{Ri} \sim \mathbf{3}_\ell\,, \qquad N_{Ri} \sim \mathbf{3}_\ell\,.$$

Real reducible rep $\Rightarrow$ $[SU(3)_\ell]^3$ anomaly vanishes (like quarks).
Anomalies:

$$SU(3)_\ell\text{–}SU(3)_\ell\text{–}U(1)_Y: \quad \sum_{\mathbf{3}_\ell} Y = 3 \times (2\,Y(L_e) - Y(e_R)) = 0\,,$$

$$SU(3)_\ell\text{–}SU(3)_\ell\text{–}U(1)_{B-L}: \quad \sum_{\mathbf{3}_\ell} (B-L) = 3 \times (2\,(-1) + (+1) + (+1)) = 0\,.$$

$\Rightarrow G_{\mathrm{SM}} \times SU(3)_\ell \times U(1)_{B-L} \supset G_{\mathrm{SM}} \times G_{\mathrm{max}}$ is anomaly free.

One more remark

- For $U(1)$, field strength tensor $F_{\mu\nu}$ is already gauge invariant
$\Rightarrow$ Every gauge group with $U(1)_1 \times U(1)_2$ part allows for "kinetic mixing":

$$\mathcal{L}_{mix} = c_{mix} F_1^{\mu\nu} F_{2\,\mu\nu}$$

[B. Holdom, *PLB 166 (1986)*]

- Diagonalize kinetic terms: A_1 couples to j_2^μ $\Rightarrow$ new effects
- Can be extended to $U(1)_1 \times U(1)_2 \times U(1)_3$ [J.H. and W.R., *PLB 705 (2011)*]