Particle Physics on Rigid D-Branes

Gabriele Honecker

Cluster of Excellence PRISMA @ JG|U Mainz

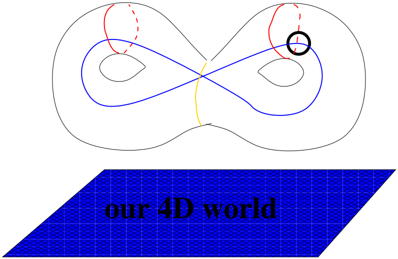
based on

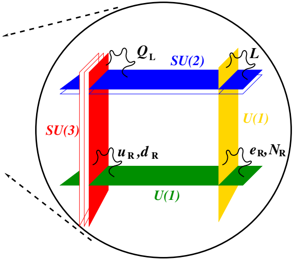
- arXiv:1209.3010 [hep-th] with **Wieland Staessens** and **Martin Ripka**
- JHEP 1101 (2011) 091 with **Stefan Förste**

Bethe Forum, Bonn, 11 October 2012

Motivation

- D6-brane model building in Type IIA/$\Omega\mathcal{R}$ string theory: use geometric intuition





- generically: non-rigid 3-cycles $\rightsquigarrow$ matter in **Adj** rep. of $U(N)$
  $\Leftrightarrow$ continuous displacements & Wilson lines
  $\Rightarrow$ continuous breaking of gauge groups & exotic matter
- choose compactification with **rigid 3-cycles**
  $\rightsquigarrow$ orbifolds with discrete torsion

Rigid D-branes & discrete torsion

- **discrete torsion** on $T^6/\mathbb{Z}_K \times \mathbb{Z}_L$ orbifolds:
  phase $\eta = e^{2\pi i m/\gcd(K,L)}$ under $\mathbb{Z}_K$ in $\mathbb{Z}_L$ twisted sector
  here: $(K,L) = (2,2M) \rightsquigarrow \boxed{\eta = \pm 1}$ without / **with** discrete torsion
- $\eta \to -\eta$ exchanges roles of $h_{11} \leftrightarrow h_{21}$
  - new 3-cycles for D6-brane model building $\boxed{\eta = -1}$
    - D6-branes stuck at $\mathbb{Z}_2$ singularities
      $\rightsquigarrow$ no open string moduli: **rigid D-branes**
  - **less** closed string **moduli** on IIA/$\Omega\mathcal{R}$ for $\boxed{\eta = -1}$
    - more h_{21} complex structures
      $\rightsquigarrow$ some fixed by SUSY conditions on D6-branes
    - less h_{11}^- Kähler moduli (but $(T^2)^3$ volumes still not fixed)
    - less h_{11}^+ vectors (e.g. dark photon)

$\rightsquigarrow$ (some) moduli projected out by construction

Hodge numbers on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with(out) discrete torsion

$T^6/$ torsion	lattice Hodge numbers	U	$\vec{w}$	$2\vec{w}$	$3\vec{w}$	$\vec{v}$	$(\vec{v}+\vec{w})$	$(\vec{v}+2\vec{w})$	$(\vec{v}+3\vec{w})$	total
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$SU(2)^6$		$(0,\frac{1}{2},-\frac{1}{2})$			$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},0,-\frac{1}{2})$			
$\eta=1$	h_{11}	3	16			16	16			51
	h_{21}	3	0			0	0			3
$\eta=-1$	h_{11}	3	0			0	0			3
	h_{21}	3	16			16	16			51
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$		$(0,\frac{1}{4},-\frac{1}{4})$	$(0,\frac{1}{2},-\frac{1}{2})$		$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{4},-\frac{1}{4})$	$(\frac{1}{2},0,-\frac{1}{2})$		
$\eta=1$	h_{11}	3	8	10		12	16	12		61
	h_{21}	1	0	0		0	0	0		1
$\eta=-1$	h_{11}	3	0	10		4	0	4		21
	h_{21}	1	8	0		0	0	0		1+8
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$SU(2)^2 \times SU(3)^2$		$(0,\frac{1}{6},-\frac{1}{6})$	$(0,\frac{1}{3},-\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{3},-\frac{1}{6})$	$(\frac{1}{2},-\frac{1}{6},-\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	8	6	8	8	8	8	51
	h_{21}	1	0	2	0	0	0	0	0	$1+2$
$\eta=-1$	h_{11}	3	0	8	0	0	4	4	0	19
	h_{21}	1	2	2	6	4	0	0	4	$15+4$
$\mathbb{Z}_2 \times \mathbb{Z}_6'$	$SU(3)^3$		$(-\frac{1}{3},\frac{1}{6},\frac{1}{6})$	$(-\frac{2}{3},\frac{1}{3},\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{6},-\frac{1}{3},\frac{1}{6})$	$(-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	9	6	6	2	2	6	36
	h_{21}	0	0	0	0	0	0	0	0	0
$\eta=-1$	h_{11}	3	1	9	0	0	1	1	0	15
	h_{21}	0	0	0	5	5	0	0	5	15

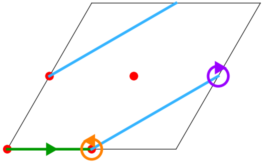
- $\mathbb{Z}_2$ sectors 'see' discrete torsion only for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with M odd $\rightsquigarrow$ potential for new SM or GUT vacua for $2M \in \{2, 6, 6'\}$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion

3 options:

- $\mathbb{Z}_2 \times \mathbb{Z}_2$: most simple case — Blumenhagen, Cvetič, Marchesano, Shiu '05
  - intensive searches by several groups
  - no global model with SM properties to date
- $\mathbb{Z}_2 \times \mathbb{Z}_6$: most complicated — Förste, G.H. '10
  - need to classify SUSY 3-cycles per complex structure
  - *a priori* SM, L-R, Pati-Salam & $SU(5)$ GUTs possible
- $\mathbb{Z}_2 \times \mathbb{Z}_6'$: intermediary — Förste, G.H. '10; G.H., Ripka, Staessens '12
  - simple classification of SUSY 3-cycles
  - $SU(5)$ GUTs *a priori* excluded

Common features: wrapping numbers $(n^i, m^i)_{i=1,2,3}$ on $(T^2)^3$ + 8 discrete param. of **rigid D6-brane**:



- 3 displacements σ
- 2 $\mathbb{Z}_2$ eigenvalues
- 3 Wilson lines τ

Orientifold projection $\Omega\mathcal{R}$

- Anti-holomorphic involution $\mathcal{R}: z^i \to \overline{z}^i$ per two-torus $T^2_{(i)}$

$+$ acts non-trivially on $\mathbb{Z}_2$ f.p. on **tilted** tori

- worldsheet duality (Klein bottle): $\eta = \eta_{\Omega\mathcal{R}} \prod_{i=1}^{3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$
  - one **exotic O6**-plane ($\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$)
  - three ordinary O6-planes ($\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = +1$)

- $\Omega\mathcal{R}$ projection on $\mathbb{Z}_2^{(i)}$ twisted sectors:

$$(-1)^{\tau^{\mathbb{Z}_2^{(i)}}} \to -\eta_{(i)} \, (-1)^{\tau^{\mathbb{Z}_2^{(i)}}} \qquad \text{with} \qquad \eta_{(i)} \equiv \eta_{\Omega\mathcal{R}} \, \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$$

- $\Omega\mathcal{R}$ inv. D6-branes $\rightsquigarrow$ enhance $U(N) \to USp(2N)$ or $SO(2N)$

Gauge enhancements to $USp(2N)$ or $SO(2N)$

- $USp(2)$ groups needed for
  - model building: $SU(2)_L = USp(2)$
  - global K-theory constraint: $USp(2)_{\text{probe}}$

$\Omega\mathcal{R}$ inv. D6-branes c:

- $\delta_i \equiv 2b_i\tau^i\sigma^i \in \{0,1\}$ non-tivial for **tilted** tori
- indep. of $(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}$

$c \mid\mid$ to	$\Omega\mathcal{R}$ invariant for $(\eta_{(1)},\eta_{(2)},\eta_{(3)}) \stackrel{!}{=}$
$\Omega\mathcal{R}$	$(-(-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2})$
$\Omega\mathcal{R}\mathbb{Z}_2^{(1)}$	$(-(-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2})$
$\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$	$((-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2})$
$\Omega\mathcal{R}\mathbb{Z}_2^{(3)}$	$((-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2})$

- **untilted tori** ($b_i \equiv 0$): $\Omega\mathcal{R}$ inv. only for $c \mid\mid$ exotic O6 & any $(\vec{\sigma},\vec{\tau}) \rightsquigarrow USp(2N)$ $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: Blumenhagen, Cvetič, Marchesano, Shiu '05
- **tilted tori** ($b_i \equiv \frac{1}{2}$): $\Omega\mathcal{R}$ invariance for G.H., Ripka, Staessens '12
  - $c \mid\mid$ exotic O6 & $\tau^i\sigma^i \equiv 0 \; \forall i \rightsquigarrow USp(2N)$
  - $c \mid\mid$ exotic O6 & $\tau^i\sigma^i \equiv 1 \; \forall i \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i\sigma^i \neq \tau^j\sigma^j = \tau^k\sigma^k = 0 \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i\sigma^i \neq \tau^j\sigma^j = \tau^k\sigma^k = 1 \rightsquigarrow USp(2N)$

$\rightsquigarrow$ less probe brane conditions (✓) & $SU(2)_L$ candidates (✓ ↯)

Gauge enhancements cont'd

- D6-branes with USp group $\longleftrightarrow$ Euclidean D2s with O group
  - minimal amount of zero modes
  - relevant for non-perturbative couplings
    $\rightsquigarrow$ light SM generations, μ-term ...
- D6-branes with SO group $\longleftrightarrow$ Euclidean D2s with Sp group

... not further discussed here

Massless spectra of $U(N)$ & completely rigid D-branes

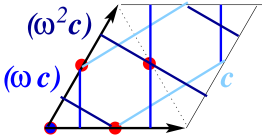
Chiral spectrum	
rep.	multiplicity
$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$	$\Pi_a \circ \Pi_b$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\Pi_a \circ \Pi'_b$
$(\mathbf{Anti}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6}\right)$
$(\mathbf{Sym}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6}\right)$

$\boxed{\mathbb{Z}_2 \times \mathbb{Z}_2}$:

- topological intersection number $\Pi_a \circ \Pi_b$ of 3-cycles Π_a, Π_b $=$ net-chirality of
  $(\mathbf{N}_a, \overline{\mathbf{N}}_b)$ for $\text{sgn}(\Pi_a \circ \Pi_b) > 0$
  $(\overline{\mathbf{N}}_a, \mathbf{N}_b)$ for $\text{sgn}(\Pi_a \circ \Pi_b) < 0$
- $|\Pi_a \circ \Pi_b|$ $=$ total amount of matter

$\boxed{\mathbb{Z}_2 \times \mathbb{Z}_{2M>2}}$:

- net-chirality *not* sufficient: cancellations among $a(\omega^k b)_{k\in\{0,1,2\}}$ sectors
- *usually* $\mathbf{Adj}_c$ at $c(\omega^k c)$ self-intersections
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}_6}$: ~~$\mathbf{Adj}_c$~~ for *some* shortest 2-cycles on T^4
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}'_6}$: ~~$\mathbf{Adj}_c$~~ for *some* shortest 3-cycles on T^6



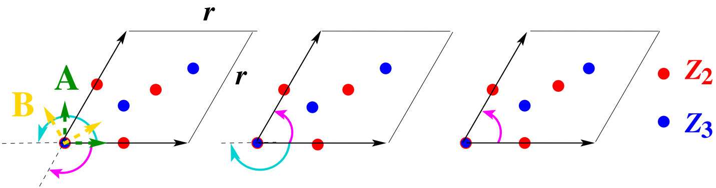
Förste, G.H. '10

G.H., Ripka, Staessens '12

IIA/$\Omega\mathcal{R}$ on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion: geometry

- $\boxed{\Pi_a^{\rm rigid} = \frac{1}{4}\left(\Pi_a^{\rm bulk} + \sum_{i=1}^3 \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$ for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ $(\eta = -1)$

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ generators: $\vec{v} = \frac{1}{2}(1,-1,0)$ $\quad \vec{w}' = \frac{1}{6}(-2,1,1)$ on $SU(3)^3$



- $\boxed{\Pi_a^{\rm bulk} = X_a\, \rho_1 + Y_a\, \rho_2}$ with

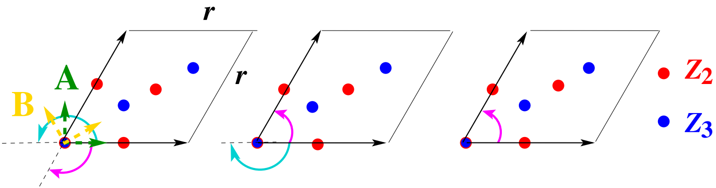
Förste, G.H. JHEP 1101 (2011) 091

$$X_a \equiv n_a^1 n_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 - \sum_{i \neq j \neq k \neq i} n_a^i m_a^j m_a^k \in \mathbb{Z}, \qquad Y_a \equiv \sum_{i \neq j \neq k \neq i} \left(n_a^i n_a^j m_a^k + n_a^i m_a^j m_a^k\right) \in \mathbb{Z}$$

$$\rho_1 \equiv 2 \sum_{k=0}^{5} \omega^k(\pi_{135}), \qquad \rho_1 \equiv 2 \sum_{k=0}^{5} \omega^k(\pi_{136}) \qquad \text{with} \quad \rho_1 \circ \rho_2 = 4$$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ geometry cont'd

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ generators: $\vec{v} = \frac{1}{2}(1,-1,0)$ $\quad \vec{w}' = \frac{1}{6}(-2,1,1)$ $\quad$ on $SU(3)^3$



- $\boxed{\Pi^{\mathbb{Z}_2^{(i)}} = \sum_{\alpha=1}^{5} \left(x^i_{\alpha,a}\, \varepsilon^{(i)}_\alpha + y^i_{\alpha,a}\, \tilde{\varepsilon}^{(i)}_\alpha \right)}$ with
  - 3 equivalent $\mathbb{Z}_2^{(i)}$ twisted sectors:
    $\varepsilon^{(i)}_{\alpha=1} = 2\sum_{k=0}^{2} \omega^k (e_{41}^{(i)} \otimes \pi_{2i-1})$ and $\tilde{\varepsilon}^{(i)}_{\alpha=1} = 2\sum_{k=0}^{2} \omega^k (e_{41}^{(i)} \otimes \pi_{2i})$
    with $\varepsilon^{(i)}_\alpha \circ \tilde{\varepsilon}^{(j)}_{\beta=1} = -4\,\delta^{ij}\,\delta_{\alpha\beta}$
  - exceptional wrappings $(x^i_{\alpha,a}, y^i_{\alpha,a}) \sim (n^i_a, m^i_a)$
    $\rightsquigarrow$ short bulk 3-cycles $(X_a, Y_a) \Leftrightarrow$ small exceptional $(x^i_{\alpha,a}, y^i_{\alpha,a})$
    important for global consistency/RR tadpole cancellation

Wrapping numbers in $\mathbb{Z}_2$ twisted sectors

G.H., Ripka, Staessens '12

Exceptional wrappings $(x^{(i)}_{\alpha,a}, y^{(i)}_{\alpha,a})$	
I	II
$(z^{(i)}_{\alpha,a}\, n^i_a \,,\, z^{(i)}_{\alpha,a}\, m^i_a)$	$\left(-z^{(i)}_{0,a}\, n^i_a + (z^{(i)}_{\alpha,a} - z^{(i)}_{0,a})\, m^i_a \,,\, (z^{(i)}_{0,a} - z^{(i)}_{\alpha,a})\, n^i - z^{(i)}_{\alpha,a}\, m^i_a\right)$
$(z^{(i)}_{\alpha,a}\, m^i_a \,,\, -z^{(i)}_{\alpha,a}\,(n^i_a + m^i_a))$	$\left((z^{(i)}_{0,a} - z^{(i)}_{\alpha,a})\, n^i_a - z^{(i)}_{\alpha,a}\, m^i \,,\, z^{(i)}_{0,a}\, m^i_a + z^{(i)}_{\alpha,a}\, n^i_a\right)$
$(-z^{(i)}_{\alpha,a}\,(n^i_a + m^i_a) \,,\, z^{(i)}_{\alpha,a}\, n^i_a)$	$\left(z^{(i)}_{\alpha,a}\, n^i_a + z^{(i)}_{0,a}\, m^i_a \,,\, -z^{(i)}_{0,a}\, n^i_a + (z^{(i)}_{\alpha,a} - z^{(i)}_{0,a})\, m^i_a\right)$

- with signs $z^{(i)}_{0,a} \equiv (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}}}$ and
  $z^{(i)}_{\alpha,a} \in \{(-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau^j_a}, (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau^k_a}, (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau^j_a + \tau^k_a}\}$
- Type I: from a single fixed point - II: sum of two fixed points
- each $\Pi^{\mathbb{Z}_2^{(i)}}$ receives contributions from only three $\alpha \in \{1 \ldots 5\}$ depending on σ^j_a, σ^k_a (at most one of Type II)
  $\rightsquigarrow$ constraints on cancellation of $\mathbb{Z}_2$ twisted tadpoles

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: RR tadpoles, SUSY & relations

Förste, G.H. '10

lattice	Bulk RR tadpole cancellation $\sum_a N_a(\Pi_a + \Pi_a') = 4\,\Pi_{O6}$	Bulk SUSY: nec. & suff. $\int_{\Pi_a} \mathrm{Im}(\Omega) = 0$	$\int_{\Pi_a} \mathrm{Re}(\Omega) > 0$
AAA	$\sum_a N_a(2\,X_a + Y_a) = 4\left(\eta_{\Omega\mathcal{R}} + 3\,\sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$Y_a = 0$	$2\,X_a + Y_a > 0$
ABB	$\sum_a N_a(X_a + 2\,Y_a) = 4\left(\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(1)}} + 3\,\sum_{i=0,2,3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$X_a = 0$	$X_a + 2\,Y_a > 0$
AAB	$\sum_a N_a(X_a + Y_a) = 4\left(3\,\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(3)}} + \sum_{i=0}^2 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$Y_a - X_a = 0$	$X_a + Y_a > 0$
BBB	$\sum_a N_a\,Y_a = 4\left(3\,\eta_{\Omega\mathcal{R}} + \sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$2\,X_a + Y_a = 0$	$Y_a > 0$

$\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = \pm 1$ for ordinary/exotic O6-planes

- pairwise relations by $(\frac{\pi}{3}, 0, 0)$ or $(0, 0, \frac{\pi}{3})$ rotation
- $(X_a\,,\; Y_a)^{\mathbf{AAA}}_{\mathbf{AAB}} = (X_a + Y_a\,,\; -X_a)^{\mathbf{ABB}}_{\mathbf{BBB}}$ and $\begin{matrix}\Omega\mathcal{R} \\ \Omega\mathcal{R}\mathbb{Z}_2^{(2)}\end{matrix} \leftrightarrow \begin{matrix}\Omega\mathcal{R}\mathbb{Z}_2^{(1/3)} \\ \Omega\mathcal{R}\mathbb{Z}_2^{(3/1)}\end{matrix}$
- pairwise relations valid for G.H., Ripka, Staessens '12
  - full massless spectra
  - field theory @ 1-loop (gauge couplings, Kähler metrics by CFT)
- maximal SUSY rank

$$\mathbf{AAA} = \begin{cases} 16 & \eta_{\Omega\mathcal{R}} = -1 \\ 8 & \text{else} \end{cases} \qquad \mathbf{BBB} = \begin{cases} 8 & \text{else} \\ 0 & \eta_{\Omega\mathcal{R}} = -1 \end{cases}$$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ relations cont'd: exceptional sectors

- rotation by $(\frac{\pi}{3}, 0, 0)$ and $(\eta_{(1)}, \eta_{(2)}, \eta_{(3)}) \to (\eta_{(1)}, -\eta_{(2)}, -\eta_{(3)})$
  - preserves relative angles $(\vec{\phi})_{a(\omega^k b)} = (\vec{\phi})_b - (\vec{\phi})_a + \frac{k\pi}{3}(-2,1,1)$
  - permutes sectors involving $\Omega\mathcal{R}$-images $(\vec{\phi})^{\mathbf{AAA}}_{a(\omega^k b')} = (\vec{\phi})^{\mathbf{ABB}}_{a(\omega^{k-1} b')}$
- $(X_a\,,\ Y_a)^{\mathbf{AAA}} = (X_a + Y_a\,,\ -X_a)^{\mathbf{ABB}}$ preserves intersection #

$$\Pi_a^{\text{rigid}} \circ \Pi_b^{\text{rigid}} = \frac{X_a Y_b - Y_a X_b}{4} - \sum_{i=1}^{3} \sum_{\alpha=1}^{5} \frac{x^{(i)}_{\alpha,a} y^{(i)}_{\alpha,b} - y^{(i)}_{\alpha,a} x^{(i)}_{\alpha,b}}{4}$$

- $\mathbb{Z}_2^{(2,3)}$ twisted RR tadpole conditions require considerations of (transformation of) fixed points

Exceptional RR tadpole cancellation conditions on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R})$			
i	α	**AAA**	**ABB**
1	1, 2, 3	$\sum_a N_a \left[x^{(1)}_{\alpha,a} - \eta_{(1)} \left(x^{(1)}_{\alpha,a} + y^{(1)}_{\alpha,a}\right)\right] = 0$ $\sum_a N_a (1+\eta_{(1)})\, y^{(1)}_{\alpha,a} = 0$	$\sum_a N_a \left[y^{(1)}_{\alpha,a} - \eta_{(1)} \left(x^{(1)}_{\alpha,a} + y^{(1)}_{\alpha,a}\right)\right] = 0$ $\sum_a N_a (1+\eta_{(1)})\, x^{(1)}_{\alpha,a} = 0$
1	4, 5	$\sum_a N_a \left[\left(2\,x^{(1)}_{4,a} + y^{(1)}_{4,a}\right) - \eta_{(1)} \left(2\,x^{(1)}_{5,a} + y^{(1)}_{5,a}\right)\right] = 0$ $\sum_a N_a \left[y^{(1)}_{4,a} + \eta_{(1)}\, y^{(1)}_{5,a}\right] = 0$	$\sum_a N_a \left[\left(x^{(1)}_{4,a} + 2\,y^{(1)}_{4,a}\right) - \eta_{(1)} \left(x^{(1)}_{5,a} + 2\,y^{(1)}_{5,a}\right)\right] = 0$ $\sum_a N_a \left[x^{(1)}_{4,a} + \eta_{(1)}\, x^{(1)}_{5,a}\right] = 0$
2, 3	1, 5	analogous to $i=1$	$\sum_a N_a \left[x^{(i)}_{\alpha,a} - \eta_{(i)}\, y^{(i)}_{\alpha,a}\right] = 0$
	3, 4		$\sum_a N_a \left[x^{(i)}_{3,a} - \eta_{(i)}\, y^{(i)}_{4,a}\right] = 0 = \sum_a N_a \left[x^{(i)}_{4,a} - \eta_{(i)}\, y^{(i)}_{3,a}\right]$
	2		$\sum_a N_a \left[x^{(i)}_{2,a} + \eta_{(i)} \left(x^{(i)}_{2,a} + y^{(i)}_{2,a}\right)\right] = 0 = \sum_a N_a (1-\eta_{(i)})\, y^{(i)}_{2,a}$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: counting of closed string states

Closed string sector on **AAA** with exotic $\Omega\mathcal{R}$-plane ($\eta_{\Omega\mathcal{R}} = -1$):

- $h_{21} = 15$ complex structures ($\mathbb{Z}_2$) $\longleftrightarrow$ D6-branes
- $h_{11}^{-} = 14$ Kähler moduli (3 bulk + 3 $\mathbb{Z}_6$ + 8 $\mathbb{Z}_3$)
- $h_{11}^{+} = 1$ vector/dark photon ($\mathbb{Z}_3$) $\rightsquigarrow$ ~~mixing~~ with open U(1)s
  - typical for D6-branes on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with $\eta = -1$: vectors only in $\mathbb{Z}_6$ and $\mathbb{Z}_3$ twisted sectors
  - for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with $\eta = +1$ and $T^6/\mathbb{Z}_{2N}$: vectors in $\mathbb{Z}_2$ (and bulk) sectors

Förste, G.H. '10

(h_{11}^{-}, h_{11}^{+}) on other lattices and/or exotic O6-planes:

- (12,3) on **AAA** with $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$
  (12=3 bulk + 1 $\mathbb{Z}_6$ + 8 $\mathbb{Z}_3$), (3=2 $\mathbb{Z}_6$ + 1 $\mathbb{Z}_3$)
- (12,3) on **BBB** with $\eta_{\Omega\mathcal{R}} = -1$ (pure $\mathcal{N} = 1$ closed spectrum)
  (12=3 bulk + 9 $\mathbb{Z}_3$), (3=3 $\mathbb{Z}_6$)
- (14,1) on **BBB** with $\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$
  (14=3 bulk + 2 $\mathbb{Z}_6$ + 9 $\mathbb{Z}_3$), (1=1 $\mathbb{Z}_6$)

Model building constraints

Open strings on **AAA**:

- ~~**Adj**$_a$~~ $\rightsquigarrow$ only possible for the 2 shortest *bulk* cycles at angles $(0,0,0)$ and $(\frac{\pi}{3},0,-\frac{\pi}{3})$ & special choices of $(\vec{\sigma},\vec{\tau})$

$$I_{a(\omega a)}+\sum_{i=1}^{3} I_{a(\omega a)}^{\mathbb{Z}_2^{(i)}}(\vec{\sigma},\vec{\tau}) \stackrel{!}{=} 0 \quad \Leftrightarrow \quad 1+\sum_{i<j}\frac{1}{p_i p_j}=0 \quad \text{with } |p_i| = \text{length of 1-cycle}$$

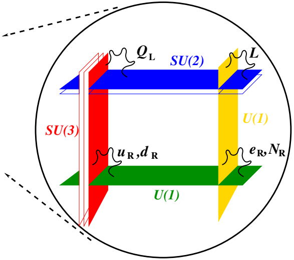
- ~~[**Sym** + $h.c.$]~~ and ~~[**Anti** + $h.c.$]~~ of **QCD** stack
  $\rightsquigarrow$ only the shortest *bulk* cycle at $(\frac{\pi}{3},0,-\frac{\pi}{3})$ and $\eta_{\Omega\mathcal{R}}=-1$
- 3 generations: case-by-case study
  $\rightsquigarrow$ same shortest *bulk* cycle for $SU(2)_L$ stack
  $\rightsquigarrow$ $SU(2)_L$ stack also completely **rigid**
  $\rightsquigarrow$ $USp(2)$ gives only 0,2 generations

Comparison with **BBB**:

- no **Adj**$_a$ ✓
- no [**Sym**$_a$ + $h.c.$] or [**Anti**$_a$ + $h.c$] ↯

A 'local' MSSM model

- 27 combinations for rigid $U(3)_a \times U(2)_b$
- no 'Spanish Quiver':
  - no 'right' & 'leptonic' stack: no intersections $(\Pi_a \circ \Pi_c, \Pi_a \circ \Pi'_c) = \mp(3,3)$
  - u_R, d_R, e_R spread over $U(1)_c \times U(1)_d$



brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X, Y)
a	(0,1;1,0,1,-1)	$(+++)$	(0,0,1)	(0,0,1)	$U(3)_a$	(1,0)
b		$(+--)$	(0,1,1)	(0,1,1)	$U(2)_b$	
c	(-1,2;2,-1;1,-1)	$(-+-)$	(1,0,0)	(1,0,0)	$U(1)_c$	(3,0)
d		$(--+)$	(0,0,1)	(1,0,1)	$U(1)_d$	

- a, b at angle $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$ and c, d at $(\frac{\pi}{2}, -\frac{\pi}{6}, -\frac{\pi}{3})$ w.r.t. $\Omega\mathcal{R}$
- a, b completely rigid $\longleftrightarrow$ $2 \times \mathbf{Adj}_c + \mathbf{Adj}_d$
- bulk RR tadpole $\rightsquigarrow$ maximal hidden rank 4

A 'local' MSSM spectrum

$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$

- Standard model particles with **one** right-handed neutrino

$$(\mathbf{3},\overline{\mathbf{2}})_{0,0} + 2\times(\mathbf{3},\mathbf{2})_{0,0} + 2\times(\overline{\mathbf{3}},\mathbf{1})_{1,0} + (\overline{\mathbf{3}},\mathbf{1})_{-1,0} + (\overline{\mathbf{3}},\mathbf{1})_{0,1} + 2\times(\overline{\mathbf{3}},\mathbf{1})_{0,-1}$$

$$Q_L^{(1)} + Q_L^{(2,3)} + \overline{d}_R^{(1,2)} + \overline{u}_R^{(1)} + \overline{d}_R^{(3)} + \overline{u}_R^{(2,3)}$$

$$+L^{(1)} + L^{(2)} + L^{(3)} + e_R^{(1,2)} + \nu_R + e_R^{(3)}$$

$$+(\mathbf{1},\mathbf{2})_{-1,0} + (\mathbf{1},\mathbf{2})_{0,-1} + (\mathbf{1},\overline{\mathbf{2}})_{0,-1} + 2\times(\mathbf{1},\mathbf{1})_{2,0} + (\mathbf{1},\mathbf{1})_{-1,1} + (\mathbf{1},\mathbf{1})_{1,1}$$

- charge selection rules allow Yukawa couplings

$$\overline{d}_R^{(3)} H_d Q_L^{(2,3)}, \qquad \overline{u}_R^{(2,3)} H_u Q_L^{(1)}, \qquad e_R^{(3)} H_d L^{(1)}$$

but only one allowed by further selection rules

- non-chiral matter with **one Higgs** (H_d, H_u)

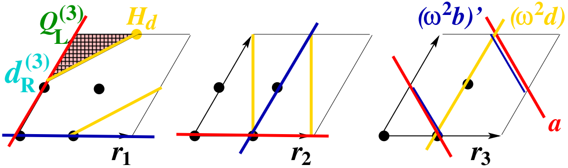
$$2\times \mathbf{Adj}_c + \mathbf{Adj}_d + [2\times(\mathbf{1},\mathbf{1})_{0,2} + (\mathbf{3},\mathbf{1})_{0,-1} + (\mathbf{1},\mathbf{1})_{1,-1} + c.c.] + (\mathbf{1},\mathbf{2})_{0,1} + (\mathbf{1},\overline{\mathbf{2}})_{0,-1}$$

A 'local' MSSM: Yukawa couplings

- charge selection rule is only *necessary*

$$\overline{d}_R^{(3)} H_d Q_L^{(2,3)}, \qquad \overline{u}_R^{(2,3)} H_u Q_L^{(1)}, \qquad e_R^{(3)} H_d L^{(1)}$$

- 'stringy' selection rule: worldsheet spanned by closed triangles: non-trivial on $T^6/\mathbb{Z}_N$ and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with $2M \neq 2$



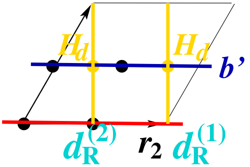
- only one massive down-type quark *by perturbative 3-pt. coupling*

$$\overline{d}_R^{(3)} H_d Q_L^{(3)} \sim \mathcal{O}(e^{-v_1/4}) \qquad \text{with} \quad v_1 \equiv \frac{\sqrt{3}\, r_1^2}{2\,\alpha'}$$

$$\rightsquigarrow m_{\text{top}} < m_{\text{bottom}} \; ↯$$

A ‘local’ MSSM: Yukawa couplings cont’d

- other *perturbative* 3-point couplings ruled out, e.g.



$\rightsquigarrow$ $\overline{d}_R^{(1,2)} H_d$ cannot form triangular worldsheet with $Q_L^{(i)}$

- additional couplings not considered here
  - non-renormalisable $(3+n)$-point couplings $\sim 1/M_{\text{string}}^n$
  - non-perturbative (instantons) $\sim e^{-S_{\text{inst}}}$

A 'local' MSSM: no global completion

RR tadpole cancellation: $\boxed{\sum_b N_b\,(\Pi_b + \Pi'_b) - 4\,\Pi_{O6} = 0}$

- implies 'generalised anomaly' condition by $\Pi_a \circ$ from left:

$$\sum_{b\neq a} N_b \left(\chi^{ab} + \chi^{ab'}\right) + (N_a - 4)\chi^{\mathbf{Sym}_a} + (N_a + 4)\chi^{\mathbf{Anti}_a} = 0 \quad \text{with } \Pi_a \circ \Pi_b \equiv \chi^{ab}$$

$\rightsquigarrow$ requires extra matter with net-charges

$$4 \times (\mathbf{1}, \overline{\mathbf{2}})_{0,0} + 4 \times (\mathbf{1}, \mathbf{1})_{-1,0} + 5 \times (\mathbf{1}, \mathbf{1})_{0,1}$$

- bulk RR tadpole cancellation $\rightsquigarrow \sum_{x \in \{\text{hidden}\}} N_x X_x = 4$
  - shortest cycles $X_x = 1 \rightsquigarrow$ hidden rank $N_x = 4$
  - one shortest + next-to-shortest cycle: $(X_{x_i}, Y_{x_i}) = (1,0)_{i=2}$ and $(3,0)_{i=1}$ with $N_{x_1} = N_{x_2} = 1$
- exceptional RR tadpoles (remember $(x_\alpha^{(i)}, y_\alpha^{(i)}) \sim (n^i, m^i)$)

$$\sum_{x\in\{a,b,c,d\}} N_x \left(\Pi_x^{\mathbb{Z}_2^{(i)}} + \Pi_{x'}^{\mathbb{Z}_2^{(i)}}\right) = \begin{cases} 3\,\varepsilon_1^{(1)} + 3\,\varepsilon_2^{(1)} + 4\,\varepsilon_3^{(1)} - \varepsilon_4^{(1)} + \tilde{\varepsilon}_4^{(1)} - \tilde{\varepsilon}_5^{(1)} \\ 3\,\varepsilon_1^{(2)} + 3\,\varepsilon_2^{(2)} + 4\,\varepsilon_3^{(2)} - 3\,\varepsilon_4^{(2)} + 2\,\varepsilon_5^{(2)} + 5\,\tilde{\varepsilon}_4^{(2)} - 5\,\tilde{\varepsilon}_5^{(2)} \\ -\varepsilon_1^{(3)} - 3\,\varepsilon_2^{(3)} - 2\,\varepsilon_3^{(3)} + \varepsilon_4^{(3)} - \tilde{\varepsilon}_4^{(3)} + \tilde{\varepsilon}_5^{(3)} \end{cases}$$

too large for cancellation by SUSY hidden branes

Recall shortest 3-cycles

- bulk wrappings

$$X_a \equiv n_a^1 n_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 - \sum_{i \neq j \neq k \neq i} n_a^i m_a^j m_a^k, \qquad Y_a \equiv \sum_{i \neq j \neq k \neq i} \left(n_a^i n_a^j m_a^k + n_a^i m_a^j m_a^k \right)$$

Exceptional wrappings $(x_{\alpha,a}^{(i)}, y_{\alpha,a}^{(i)})$	
I	II
$(z_{\alpha,a}^{(i)}\, n_a^i \,,\; z_{\alpha,a}^{(i)}\, m_a^i)$	$\left(-z_{0,a}^{(i)}\, n_a^i + (z_{\alpha,a}^{(i)} - z_{0,a}^{(i)})\, m_a^i \,,\; (z_{0,a}^{(i)} - z_{\alpha,a}^{(i)})\, n^i - z_{\alpha,a}^{(i)}\, m_a^i\right)$
$(z_{\alpha,a}^{(i)}\, m_a^i \,,\; -z_{\alpha,a}^{(i)}\, (n_a^i + m_a^i))$	$\left((z_{0,a}^{(i)} - z_{\alpha,a}^{(i)})\, n_a^i - z_{\alpha,a}^{(i)}\, m^i \,,\; z_{0,a}^{(i)}\, m_a^i + z_{\alpha,a}^{(i)}\, n_a^i\right)$
$(-z_{\alpha,a}^{(i)}\, (n_a^i + m_a^i) \,,\; z_{\alpha,a}^{(i)}\, n_a^i)$	$\left(z_{\alpha,a}^{(i)}\, n_a^i + z_{0,a}^{(i)}\, m_a^i \,,\; -z_{0,a}^{(i)}\, n_a^i + (z_{\alpha,a}^{(i)} - z_{0,a}^{(i)})\, m_a^i\right)$

- shortest $(X, Y) = (1, 0)$
  $\rightsquigarrow \pm(n^i, m^i) \in \{(1,0), (1,-1), (0,1)\}$
- next-to-shortest $(X, Y) = (3, 0)$
  $\rightsquigarrow \pm(n^i, m^i) \in \{(2,-1), (-1,2), (1,1)\}$ for two i
- only three α's per D6-brane

A ‘local’ left-right symmetric model

- $U(3)_a \times U(2)_b \times U(2)_c \times U(1)_d$
- 27 combinations for 3 generations on rigid $U(3)_a \times U(2)_b$ + ... $U(3)_a \times U(2)_c$ with c non-rigid

brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X, Y)
a	(0,1;1,0,1,-1)	$(+++)$	(0,1,0)	(0,1,0)	$U(3)_a$	(1,0)
b		$(--+)$	(1,1,0)	(1,1,0)	$U(2)_b$	
c	(1,0;2,-1;1,1)	$(--+)$	$(\vec{0})$	$(\vec{0})$	$U(2)_c$	(3,0)
d	(-1,2;2,-1;1,-1)	$(+++)$	(1,0,0)	(1,0,0)	$U(1)_d$	(3,0)

- a, b at angle $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$, c at $(0, -\frac{\pi}{6}, \frac{\pi}{6})$, d at $(\frac{\pi}{2}, \frac{\pi}{6}, -\frac{\pi}{3})$
- Standard Model matter + exotics under (B-L)
- one Higgs
- a, b completely rigid $\longleftrightarrow$ $4 \times \mathbf{Adj}_c + 2 \times \mathbf{Adj}_d$

A 'local' left-right symmetric spectrum

- Standard model particles plus **one Higgs**

$$(\mathbf{3},\overline{\mathbf{2}},\mathbf{1})_0 + 2\times(\mathbf{3},\mathbf{2},\mathbf{1})_0 + 2\times(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_0 + (\overline{\mathbf{3}},\mathbf{1},\overline{\mathbf{2}})_0 + (\mathbf{1},\overline{\mathbf{2}},\overline{\mathbf{2}})_0$$
$$Q_L^{(1)} + \quad Q_L^{(2,3)} \quad + \quad (\overline{u}_R,\overline{d}_R)^{(2,3)} + (\overline{u}_R,\overline{d}_R)^{(1)} + (H_d,H_u)$$
$$L^{(1,2,3)} + (\overline{\nu}_R,\overline{e}_R)^{(1,2,3)}$$
$$3\times(\mathbf{1},\mathbf{2},\mathbf{1})_1 + 3\times(\mathbf{1},\mathbf{1},\mathbf{2})_{-1} + 2\times(\mathbf{1},\mathbf{1},\mathbf{1})_2$$

and two exotics with B-L charge $Q_{B-L} = \frac{Q_a}{3} - Q_d = -2$

- Yukawas allowed by charge selection rules

$$Q_L^{(2,3)}\,(H_d,H_u)\,(\overline{u}_R,\overline{d}_R)^{(2,3)} \qquad L^{(1,2,3)}\,(H_d,H_u)\,(\bar{\nu}_R,\bar{e}_R)^{(1,2,3)}$$

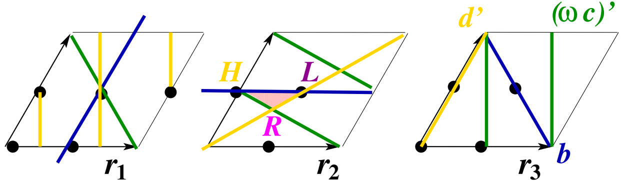
- non-chiral matter

$$4\times\mathbf{Adj}_c + 2\times\mathbf{Adj}_d +$$
$$+\left[4\times\mathbf{Anti}_c + (\mathbf{3},\mathbf{1},\overline{\mathbf{2}})_0 + (\mathbf{3},\mathbf{1},\mathbf{2})_0 + (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_1 + (\mathbf{1},\overline{\mathbf{2}},\mathbf{1})_1 + 2\times(\mathbf{1},\mathbf{1},\mathbf{2})_1 + c.c.\right]$$

A 'local' left-right symmetric model: Yukawa couplings

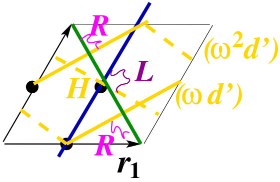
- charge selection rule is only *necessary*

$Q_L^{(2,3)}\,(H_d,H_u)\,\left(\overline{u}_R,\overline{d}_R\right)^{(2,3)}$ $\qquad$ $L^{(1,2,3)}\,(H_d,H_u)\,(\bar{\nu}_R,\bar{e}_R)^{(1,2,3)}$



$Q_L^{(3)}\,(H_d,H_u)\,\left(\overline{u}_R,\overline{d}_R\right)^{(3)} \sim \mathcal{O}(1)$ $\qquad$ $L^{(3)}(H_d,H_u)(\bar{\nu}_R,\bar{e}_R)^{(3)} \sim \mathcal{O}(e^{-v_2/24})$

$\rightsquigarrow\; m_{\text{top}}, m_{\text{bottom}} \gg m_\tau, m_{\nu_\tau} > 0$ ✓



possible further couplings

- e.g. non-renormalisable 4-point coupling $\sim 1/M_{\text{string}}$ to **$\mathbf{Adj}_d$**
- instantons

A 'local' left-right symmetric model: no global completion

- 'generalised anomaly' condition requires extra $4 \times (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1})_0 + 4 \times (\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}})_0 + 4 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_0$
- bulk RR tadpole $\rightsquigarrow$ $\rightsquigarrow$ $\sum_{x \in \{\text{hidden}\}} N_x X_x = 2$
  - shortest cycles $(X, Y) = (1, 0)$ only: $U(2)$ or $U(1)^2$
- exceptional RR tadpoles

$$\sum_{x \in \{a,b,c,d\}} N_x \left(\Pi_x^{\mathbb{Z}_2^{(i)}} + \Pi_{x'}^{\mathbb{Z}_2^{(i)}} \right) = \begin{cases} -4\, \varepsilon_1^{(1)} + 5\, \varepsilon_2^{(1)} - \varepsilon_3^{(1)} - 3\, \varepsilon_5^{(1)} - 3\, \tilde{\varepsilon}_4^{(1)} + 3\, \tilde{\varepsilon}_5^{(1)} \\ 3\, \varepsilon_2^{(2)} + 5\, \varepsilon_3^{(2)} - 4\, \varepsilon_4^{(2)} - \varepsilon_5^{(2)} + 3\, \tilde{\varepsilon}_4^{(2)} - 3\, \tilde{\varepsilon}_5^{(2)} \\ 9\, \varepsilon_1^{(3)} + 3\, \varepsilon_2^{(3)} + 8\, \varepsilon_3^{(3)} - \varepsilon_4^{(3)} + 2\, \varepsilon_5^{(3)} + 3\, \tilde{\varepsilon}_4^{(3)} - 3\, \tilde{\varepsilon}_5^{(3)} \end{cases}$$

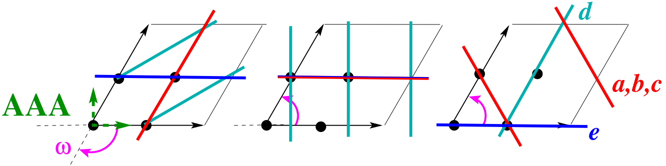
too large to cancel with SUSY hidden branes

$\rightsquigarrow$ 'local' **MSSM** and **L-R symmetric** models on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6' \times \Omega\mathcal{R})$ with exotic $\Omega\mathcal{R}$-plane and rigid $U(3)_{QCD}$ **without global completion**

A typical global Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_6$

G.H., Ripka, Staessens '12

brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X, Y)
a	(0,1;1,0,1,-1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(4)_a$	(1,0)
b		$(--+)$	$(0,1,1)$		$U(2)_b$	
c		$(-+-)$	$(1,0,1)$		$U(2)_c$	
d	(1,1;1,-2;0,1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(2)_d$	(3,0)
e	(1,0;1,0;1,0)	$(+--)$	$(1,1,1)$	$(1,1,0)$	$U(2)_e$	(1,0)



- a, b, c at $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$, d at $(\frac{\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{3})$ e at $(0,0,0)$
- all $U(1)^5$ anomalous & massive at $M_{\text{string}} \leftrightarrow h_{21} = 15(\mathbb{Z}_2)$
- $SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e$ with
  - 3 generations of quarks + leptons
  - **one Higgs** (H_d, H_u)
  - ~~**Adj**~~ on $a, b, c, e \longleftrightarrow 1 \times$ **Adj**$_d$

A typical Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}'_6$: spectrum

$SU(4)_a \times SU(2)_b \times SU(2)_c \times SU(2)_d \times SU(2)_e \times U(1)^5_{\text{massive}}$

- Standard Model particles plus **one Higgs**

$$(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + 2\,(\mathbf{4}, \mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1}) + 2\,(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}; \mathbf{1}, \mathbf{1})$$

$\rightsquigarrow$ **one massive generation** at leading order by charge selection rules

- chiral w.r.t. anomalous $U(1)^5_{\text{massive}}$

$$(\mathbf{1}, \mathbf{2}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + 3(\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}; \mathbf{1}, \overline{\mathbf{2}}) + (\mathbf{1}, \mathbf{1}, \overline{\mathbf{2}}; \mathbf{2}, \mathbf{1}) + 3(\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})$$

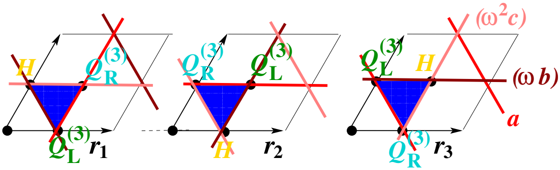
but non-chiral w.r.t. $SU(4)_a \times SU(2)_b \times SU(2)_c$

- non-chiral w.r.t. to full $U(4)_a \times U(2)^4$ with **GUT Higgses**

$$2\,\left[(\mathbf{4}, \mathbf{1}, \mathbf{1}; \overline{\mathbf{2}}, \mathbf{1}) + h.c.\right] + \left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{2}) + h.c.\right] + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{4}_{\mathbf{Adj}}, \mathbf{1})$$
$$+\, 2\,\left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{3}_{\mathbf{S}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}_{\mathbf{A}}, \mathbf{1}) + h.c.\right] + \left[(\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}_{\mathbf{S}}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}_{\mathbf{A}}) + h.c.\right]$$

Yukawa interactions for the typical Pati-Salam model

- charge selection rules not sufficient on $T^6/\mathbb{Z}_{2N}$, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ due to various sectors $a(\omega^k b)_{k\in\{0,1,2\}}$ G.H., Vanhoof '12



- Pati-Salam model: one heavy generation by G.H., Ripka, Staessens '12 $W_{Q_L^{(3)} Q_R^{(3)} H} \sim e^{-\sum_{i=1}^3 v_i/8}$ with Kähler moduli $v_i \equiv \frac{\sqrt{3}}{2}\frac{r_i^2}{\alpha'}$
- non-chiral $\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1_A},\mathbf{1}) + h.c.\right]$ massive via couplings to $(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4_{Adj}},\mathbf{1})$
- several types of $(\mathbf{1},\mathbf{2}_x,\mathbf{2}_y,\mathbf{1},\mathbf{1})_{x,y\in\{b,c,d,e\}}$ massive through 3-point couplings among each other and with SM Higgs
- other masses through higher order or non-perturbative (instanton) couplings (not computed here)

Gauge couplings @ tree-level

- **gravitational coupling** $\rightsquigarrow \frac{M^2_{\text{Planck}}}{M^2_{\text{string}}} = \frac{4\pi}{g^2_{\text{string}}} v_1 v_2 v_3$ with $v_i \equiv \frac{\sqrt{3} r_i^2}{2\alpha'}$
- **tree-level gauge coupling**

$$\frac{4\pi}{g^2_{SU(N_a),\text{tree}}} = 2\pi \Re\left(f^{\text{tree}}_{SU(N_a)}\right) = \frac{1}{4}\,\frac{1}{g_{\text{string}}}\,\frac{\prod_{i=1}^3 L_a^{(i)}}{\ell_s^3}$$

  - for the typical Pati-Salam model:

$$\frac{4\pi}{g^2_{SU(N_{a,b,c,e}),\text{tree}}} = \frac{1}{4\, g_{\text{string}}}\,\frac{r_1 r_2 r_3}{\ell_s^3} = \frac{1}{3^{3/4}\cdot 32\pi^3}\,\frac{M_{\text{Planck}}}{M_{\text{string}}} \approx 4\cdot 10^{-4}\,\frac{M_{\text{Planck}}}{M_{\text{string}}}$$

Mass scales and values of the string coupling

M_{string}	1 TeV				10^{12} GeV				10^{16} GeV			
g_{string}	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5
$v_1 v_2 v_3$	$8\cdot 10^{24}$	$8\cdot 10^{26}$	$8\cdot 10^{28}$	$2\cdot 10^{30}$	$8\cdot 10^{6}$	$8\cdot 10^{8}$	$8\cdot 10^{10}$	$2\cdot 10^{12}$	0.08	8	800	$2\cdot 10^{4}$
$4\pi/g^2_{a,\text{tree}}$	$4\cdot 10^{12}$				$4\cdot 10^{3}$				$4\cdot 10^{-1}$			

- gauge coupling unification @ tree-level for $M_{\text{string}} \sim M_{GUT}$
- $\frac{4\pi}{g^2_{SU(N_a),\text{tree}}} \gg 1$ for low M_{string}

1-loop corrections to gauge couplings

- 1-loop behaviour can change couplings drastically G.H. '11

One-loop corrections to holomorphic gauge kinetic functions on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion		
$(\phi^{(1)}_{ab}, \phi^{(2)}_{ab}, \phi^{(3)}_{ab})$	$\Re\left(\delta_b^{\text{1-loop},\mathcal{A}} \mathrm{f}_{SU(N_a)}\right)$	$\Re\left(\delta_{b=a'}^{\text{1-loop},\mathcal{M}} \mathrm{f}_{SU(N_a)}\right)$
$(0,0,0)$	$-\sum_{i=1}^3 \frac{b_{ab}^{\mathcal{A},(i)}}{4\pi^2} \ln\eta(iv_i)$ $-\sum_{i=1}^3 \frac{\tilde{b}_{ab}^{\mathcal{A},(i)}\left(1-\delta^{\sigma^i_b}_{\sigma^i_a}\,\delta^{\tau^i_b}_{\tau^i_a}\right)}{4\pi^2}\ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4}\frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2},iv_i)}{\eta(iv_i)}\right)$	$-\sum_{i=1}^3 \frac{b_{aa'}^{\mathcal{M},(i)}}{4\pi^2}\ln\eta(i\tilde{v}_i) - \frac{b_{aa'}^{\mathcal{M}}\ln(2)}{8\pi^2}$ (for $b_i=0$ or $(\sigma^i_a,\tau^i_a)=(0,0)$)
$(0^{(i)}, \phi^{(j)}_{ab}, \phi^{(k)}_{ab})$	$-\frac{b_{ab}^{\mathcal{A}}}{4\pi^2}\ln\eta(iv_i)$ $-\frac{\tilde{b}_{ab}^{\mathcal{A},(i)}\left(1-\delta^{\sigma^i_b}_{\sigma^i_a}\,\delta^{\tau^i_b}_{\tau^i_a}\right)}{4\pi^2}\ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4}\frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2},iv_i)}{\eta(iv_i)}\right)$ $+\sum_{l=j,k}\frac{N_b I_{ab}^{\mathbb{Z}_2^{(l)}}}{32\pi^2}\left(\frac{\mathrm{sgn}(\phi^{(l)}_{ab})}{2}-\phi^{(l)}_{ab}\right)$	$-\frac{b^{\mathcal{M}}_{aa'}}{4\pi^2}\ln\eta(i\tilde{v}_i)$ (for $b_i=0$ or $(\sigma^i_a,\tau^i_a)=(0,0)$) $+\frac{\ln(2)}{32\pi^2}\sum_{[m:a\perp\Omega\mathcal{R}\mathbb{Z}_2^{(m)}\text{ on }T^2_{(i)}]}\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}}\vert\tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}}\vert$
$(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$	$\sum_{l=1}^3\frac{N_b I_{ab}^{\mathbb{Z}_2^{(l)}}}{32\pi^2}\left(\frac{\mathrm{sgn}(\phi^{(l)}_{ab})+\mathrm{sgn}(I_{ab})}{2}-\phi^{(l)}_{ab}\right)$	$\frac{\ln(2)}{32\pi^2}\sum_{m=0}^3\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}}\vert\tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}}\vert$

- Annulus contributions known for all configurations of tori b_i, displacements σ^i & Wilson lines τ^i
- Möbius strip contributions only derived from first principles (and reliable for) $b_i\sigma^i\tau^i = 0$ G.H., Ripka, Staessens '12

$\rightsquigarrow$ conjecture: same prefactors $b_{aa'}^{\mathcal{M},(i)}$, $|\tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}}|$ for $b_i\sigma^i\tau^i \neq 0$

Beta function coefficients

- can be written in terms of intersection numbers G.H. ‘11

Kähler metrics and beta function coefficients on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_{2M} \times \Omega\mathcal{R})$ with discrete torsion			
$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$	$K_{\mathbf{R}_a}$	$b_{ab}^{\mathcal{A}}$	$b_{aa'}^{\mathcal{M}}$ (only for $b = a'$)
$(0,0,0)$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \sqrt{2\pi}^3 \frac{L_a^{(i)}}{\ell_s}$	$-\frac{N_b}{4} \sum_{i=1}^3 \delta_{\sigma_a^i}^{\sigma_b^i} \delta_{\tau_a^i}^{\tau_b^i} I_{ab}^{\mathbb{Z}_2^{(i)},(j\cdot k)}$	$-\frac{1}{2} \sum_{j<k} \sum_{m=0}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} (-1)^{2b_i \sigma_a^i \tau_a^i} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)},(j\cdot k)} \rvert$
$(0^{(i)}, \phi, -\phi)$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \sqrt{2\pi}^3 \frac{L_a^{(i)}}{\ell_s}$	$\frac{N_b \, \delta_{\sigma_a^i}^{\sigma_b^i} \delta_{\tau_a^i}^{\tau_b^i}}{4} \left(\lvert I_{ab}^{(j\cdot k)} \rvert - I_{ab}^{\mathbb{Z}_2^{(i)},(j\cdot k)} \right)$	$-\frac{1}{2} \sum_{\substack{m \in \{0 \ldots 3\} \text{ with} \\ a \uparrow\uparrow \Omega\mathcal{R}\mathbb{Z}_2^{(m)} \text{on } T^2_{(i)}}} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} (-1)^{2b_i \sigma_a^i \tau_a^i} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)},(j\cdot k)} \rvert$
$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$ $\sum_i \phi_{ab}^{(i)} = 0$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \prod_{i=1}^3 \frac{\Gamma(\lvert\phi_{ab}^{(i)}\rvert)}{\Gamma(1-\lvert\phi_{ab}^{(i)}\rvert)}^{-\frac{\text{sgn}(\phi_{ab}^{(i)})}{2\,\text{sgn}(I_{ab})}}$	$\frac{N_b}{8} \left(\lvert I_{ab} \rvert + \text{sgn}(I_{ab}) \sum_{i=1}^3 I_{ab}^{\mathbb{Z}_2^{(i)}} \right)$	$\frac{1}{4} \sum_{m=0}^3 c_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \rvert$

- beta function coefficients needed
  - for derivation of non-chiral matter spectrum
  - as prefactors in 1-loop correction to hol. gauge kinetic function
- factor $(-1)^{2b_i \sigma_a^i \tau_a^i}$ in $\Omega\mathcal{R}$-invariant configurations required for consistency with counting of Chan-Paton labels

G.H., Ripka, Staessens ‘12

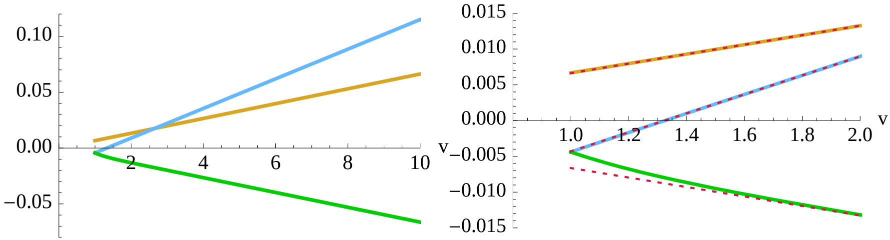
- Kähler metrics enter physical Yukawa couplings $Y_{ijk} = (K_{xy} K_{yz} K_{zx})^{-1/2} \, e^{\kappa_4^2 \mathcal{K}/2} \, W_{ijk}$

Asymptotics of 1-loop gauge corrections

1-loop corrections contain beta function coefficients $\times$

- $-\frac{1}{4\pi^2}\ln(\eta(iv)) \stackrel{v\to\infty}{\longrightarrow} \frac{v}{48\pi}$
- $-\frac{1}{4\pi^2}\ln\left(\frac{\vartheta_1(\frac{1}{2},iv)}{\eta(iv)}\right) \stackrel{v\to\infty}{\longrightarrow} \frac{v}{24\pi} - \frac{\ln 2}{4\pi^2}$
- $-\frac{1}{4\pi^2}\ln\left(e^{-\pi v/4}\frac{|\vartheta_1(-i\frac{v}{2},iv)|}{\eta(iv)}\right) \stackrel{v\to\infty}{\longrightarrow} -\frac{v}{48\pi}$

$\rightsquigarrow$ linear approximation very good in geometric regime $v > 1$:



$$\eta(iv) \equiv q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n), \quad \frac{\vartheta_1(\nu,iv)}{\eta(iv)} = 2q^{\frac{1}{12}}\sin(\pi\nu)\prod_{n=1}^{\infty}(1-2\cos(2\pi\nu)q^n+q^{2n}), \quad \begin{array}{l} q \equiv e^{-2\pi v} \\ \nu \equiv \frac{\tau-i\sigma v}{2}\end{array}$$

Gauge coupling unification or low M_{string}

- recall $\frac{4\pi}{g^2_{SU(N_a),\text{tree}}} \approx 4 \cdot 10^{-4} \frac{M_{\text{Planck}}}{M_{\text{string}}}$ with $\frac{M_{\text{Planck}}}{M_{\text{string}}} = \frac{\sqrt{4\pi}}{g_{\text{string}}} \sqrt{v_1 v_2 v_3}$

Mass scales and values of the string coupling												
M_{string}	1 TeV				10^{12} GeV				10^{16} GeV			
g_{string}	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5
$v_1 v_2 v_3$	$8 \cdot 10^{24}$	$8 \cdot 10^{26}$	$8 \cdot 10^{28}$	$2 \cdot 10^{30}$	$8 \cdot 10^{6}$	$8 \cdot 10^{8}$	$8 \cdot 10^{10}$	$2 \cdot 10^{12}$	0.08	8	800	$2 \cdot 10^{4}$
$4\pi/g^2_{a,\text{tree}}$	$4 \cdot 10^{12}$				$4 \cdot 10^{3}$				$4 \cdot 10^{-1}$			

- **one-loop** corrections contain

$$\Re\left(\delta_b^{\text{1-loop}} \mathfrak{f}_{SU(N_a)}\right) \supset -\frac{\tilde{b}_{ab}^{\mathcal{A},(i)}}{4\pi^2} \ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4} \frac{\vartheta_1\left(\frac{\tau^i_{ab} - i\sigma^i_{ab} v_i}{2}, i v_i\right)}{\eta(i v_i)}\right) \overset{v_i \to \infty}{\sim} v_i$$

  - for the typical Pati-Salam model:

$$2\pi \Re\left(\delta^{\text{1-loop}} \mathfrak{f}_{SU(N_x)}\right) \sim \begin{cases} \frac{10(v_1+v_2)-7v_3}{48} - \frac{4 \ln 2}{\pi} & x = a \\ \frac{8\, v_{2/1} - 3\, v_3}{24} - \frac{41 \ln 2}{12\pi} & b/c \end{cases}$$

with negative contribution from v_3

- **unification @ 1-loop** for $v_{1/2} = \frac{v_3}{4} + \frac{7 \ln(2)}{\pi}$ $@ M_{\text{string}} \sim M_{GUT}$
- **or** $M_{\text{string}} \sim$ **TeV** for $v_{1/2} \sim 10^6$, $v_3 \sim 10^{13}$, $g_{\text{string}} \sim 10^{-3}$

Conclusions

- **Rigid D6-branes** on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with **discrete torsion**
  - reduction of # closed & open string moduli
  - $2M \neq 2$: bulk cycles have $\mathbb{Z}_{2M}$ images
    $\rightsquigarrow$ selection rules on Yukawa interactions ($\neq T^6$)
    charge $\neq$ closed triangle G.H., Vanhoof JHEP 1204 (2012) 085
  - $2M = 6'$: ~~$SU(5)$ GUTs~~, only local MSSM and L-R models
  - $2M = 6$: *a priori* less constrained ...to be worked out
  - new maps among lattice orientations $\rightsquigarrow$ economise SM search

G.H., Ripka, Staessens arXiv:1209.3010 [hep-th]

- Example: global **Pati-Salam** model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$
  - ~~**Adj**~~ moduli
  - some vector-like states
  - perturbative Yukawa couplings for one particle generation only
  - gauge coupling unification at $M_{\text{string}} \sim M_{GUT}$ possible @ 1-loop
  - or $M_{\text{string}} \sim 1$ TeV for LARGE unisotropic volumes

G.H., Ripka, Staessens arXiv:1209.3010 [hep-th]

Outlook

- include more **couplings**:
  - higher order (non-renormalisable)
    - CFT techniques only worked out for T^6, not orbifolds
  - instantons
- details of (GUT) Higgsing & other **phenomenology**
- **dark sector**
  - kinetic mixing of open U(1)s
  - closed U(1)s completely decoupled?
  - axions of massive U(1)s
- **model building** on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$ expected to be fertile
- complex structure **deformations** to $CY_3 \stackrel{?}{\leftrightarrow}$ **SUSY** D6-branes
  - M-theory duality with heterotic models?