From Rigid D-Branes to Particle Physics

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based on

- arXiv:1209.3010 [hep-th] with **Wieland Staessens** and **Martin Ripka**
- JHEP 1101 (2011) 091 with **Stefan Förste**

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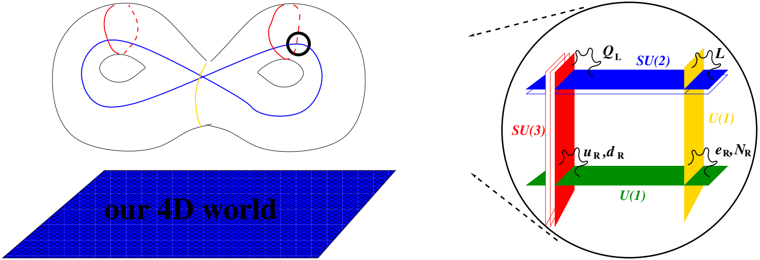
Motivation

- D6-brane model building in Type IIA/$\Omega\mathcal{R}$ string theory: geometric intuition



- generically: non-rigid 3-cycles $\rightsquigarrow$ matter in **Adj** rep. of $U(N)$
  $\Leftrightarrow$ continuous displacements & Wilson lines
  $\Rightarrow$ continuous breaking of gauge groups & exotic matter
- choose compactification with **rigid 3-cycles**
  $\rightsquigarrow$ orbifolds with discrete torsion

Rigid D-branes & discrete torsion

- **discrete torsion** on $T^6/\mathbb{Z}_K \times \mathbb{Z}_L$ orbifolds:
  phase $\eta = e^{2\pi i m/\gcd(K,L)}$ under $\mathbb{Z}_K$ in $\mathbb{Z}_L$ twisted sector
  here: $(K,L) = (2,2M) \rightsquigarrow \boxed{\eta = \pm 1}$ without / **with** discrete torsion
- $\eta \to -\eta$ exchanges roles of $h_{11} \leftrightarrow h_{21}$
  - new 3-cycles for D6-brane model building $\boxed{\eta = -1}$
    - D6-branes stuck at $\mathbb{Z}_2$ singularities
      $\rightsquigarrow$ no open string moduli: **rigid D-branes**
  - **less** closed string **moduli** on IIA/$\Omega\mathcal{R}$ for $\boxed{\eta = -1}$:
    - more h_{21} complex structures $\rightsquigarrow$ some fixed by SUSY conditions
    - less h_{11}^- Kähler moduli (but $(T^2)^3$ volumes still not fixed)
    - less h_{11}^+ vectors (e.g. dark photon)

$\rightsquigarrow$ (some) moduli projected out by construction

Hodge numbers on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with(out) discrete torsion

$T^6/$ torsion	lattice Hodge numbers	U	$\vec{w}$	$2\vec{w}$	$3\vec{w}$	$\vec{v}$	$(\vec{v}+\vec{w})$	$(\vec{v}+2\vec{w})$	$(\vec{v}+3\vec{w})$	total
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$SU(2)^6$		$(0,\frac{1}{2},-\frac{1}{2})$			$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},0,-\frac{1}{2})$			
$\eta=1$	h_{11}	3	16			16	16			51
	h_{21}	3	0			0	0			3
$\eta=-1$	h_{11}	3	0			0	0			3
	h_{21}	3	16			16	16			51
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$SU(2)^2 \times SO(5)^2$		$(0,\frac{1}{4},-\frac{1}{4})$	$(0,\frac{1}{2},-\frac{1}{2})$		$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{4},-\frac{1}{4})$	$(\frac{1}{2},0,-\frac{1}{2})$		
$\eta=1$	h_{11}	3	8	10		12	16	12		61
	h_{21}	1	0	0		0	0	0		1
$\eta=-1$	h_{11}	3	0	10		4	0	4		21
	h_{21}	1	8	0		0	0	0		1+8
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$SU(2)^2 \times SU(3)^2$		$(0,\frac{1}{6},-\frac{1}{6})$	$(0,\frac{1}{3},-\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{2},-\frac{1}{3},-\frac{1}{6})$	$(\frac{1}{2},-\frac{1}{6},-\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	8	6	8	8	8	8	51
	h_{21}	1	0	2	0	0	0	0	0	$1+2$
$\eta=-1$	h_{11}	3	0	8	0	0	4	4	0	19
	h_{21}	1	2	2	6	4	0	0	4	$15+4$
$\mathbb{Z}_2 \times \mathbb{Z}_6'$	$SU(3)^3$		$(-\frac{1}{3},\frac{1}{6},\frac{1}{6})$	$(-\frac{2}{3},\frac{1}{3},\frac{1}{3})$	$(0,\frac{1}{2},-\frac{1}{2})$	$(\frac{1}{2},-\frac{1}{2},0)$	$(\frac{1}{6},-\frac{1}{3},\frac{1}{6})$	$(-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$	$(\frac{1}{2},0,-\frac{1}{2})$	
$\eta=1$	h_{11}	3	2	9	6	6	2	2	6	36
	h_{21}	0	0	0	0	0	0	0	0	0
$\eta=-1$	h_{11}	3	1	9	0	0	1	1	0	15
	h_{21}	0	0	0	5	5	0	0	5	15

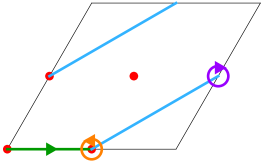
- $\mathbb{Z}_2$ sectors 'see' discrete torsion only for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with M odd $\rightsquigarrow$ potential for new SM or GUT vacua for $2M \in \{2, 6, 6'\}$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion

3 options:

- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}_2}$: most simple case — Blumenhagen, Cvetič, Marchesano, Shiu '05
  - intensive searches by several groups
  - no global model with SM properties to date
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}_6}$: most complicated — Förste, G.H. '10
  - need to classify SUSY 3-cycles per complex structure
  - *a priori* SM, L-R, Pati-Salam & $SU(5)$ GUTs possible
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}_6'}$: intermediary — Förste, G.H. '10; G.H., Ripka, Staessens '12
  - simple classification of SUSY 3-cycles
  - $SU(5)$ GUTs *a priori* excluded

Common features:



8 discrete param. of rigid D6-brane:

- 3 displacements σ
- 2 $\mathbb{Z}_2$ eigenvalues
- 3 Wilson lines τ

Orientifold projection $\Omega\mathcal{R}$

- Anti-holomorphic involution $\mathcal{R} : z^i \to \overline{z}^i$ per two-torus $T^2_{(i)}$

$+$ acts non-trivially on $\mathbb{Z}_2$ f.p. on **tilted** tori

- worldsheet duality (Klein bottle): $\eta = \eta_{\Omega\mathcal{R}} \prod_{i=1}^{3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$
  - one **exotic O6**-plane ($\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = -1$)
  - three ordinary O6-planes ($\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = +1$)
- $\Omega\mathcal{R}$ projection on $\mathbb{Z}_2^{(i)}$ twisted sectors:

$$(-1)^{\tau^{\mathbb{Z}_2^{(i)}}} \to -\eta_{(i)} (-1)^{\tau^{\mathbb{Z}_2^{(i)}}} \qquad \text{with} \qquad \eta_{(i)} \equiv \eta_{\Omega\mathcal{R}}\, \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}$$

- $\Omega\mathcal{R}$ inv. D6-branes $\rightsquigarrow$ enhance $U(N) \to USp(2N)$ or $SO(2N)$

Gauge enhancements to $USp(2N)$ or $SO(2N)$

- $USp(2)$ groups needed for
  - model building: $SU(2)_L = USp(2)$
  - global K-theory constraint: $USp(2)_{\text{probe}}$

$\Omega\mathcal{R}$ inv. D6-branes c:

- $\delta_i \equiv 2 b_i \tau^i \sigma^i \in \{0,1\}$ non-tivial for **tilted** tori
- indep. of $(-1)^{\tau^{\mathbb{Z}_2^{(i)}}}$

c || to	$\Omega\mathcal{R}$ invariant for $(\eta_{(1)},\eta_{(2)},\eta_{(3)}) \stackrel{!}{=}$
$\Omega\mathcal{R}$	$\left(-(-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(1)}$	$\left(-(-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(2)}$	$\left((-1)^{\delta_2+\delta_3}, -(-1)^{\delta_1+\delta_3}, (-1)^{\delta_1+\delta_2}\right)$
$\Omega\mathcal{R}\mathbb{Z}_2^{(3)}$	$\left((-1)^{\delta_2+\delta_3}, (-1)^{\delta_1+\delta_3}, -(-1)^{\delta_1+\delta_2}\right)$

- **untilted tori** ($b_i \equiv 0$): $\Omega\mathcal{R}$ inv. only for $c \,||$ exotic O6 & any $(\vec{\sigma},\vec{\tau}) \rightsquigarrow USp(2N)$ $\quad$ $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: Blumenhagen, Cvetič, Marchesano, Shiu '05
- **tilted tori** ($b_i \equiv \frac{1}{2}$): $\Omega\mathcal{R}$ invariance for $\quad$ G.H., Ripka, Staessens '12
  - $c \,||$ exotic O6 & $\tau^i\sigma^i \equiv 0 \;\forall i \rightsquigarrow USp(2N)$
  - $c \,||$ exotic O6 & $\tau^i\sigma^i \equiv 1 \;\forall i \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i\sigma^i \neq \tau^j\sigma^j = \tau^k\sigma^k = 0 \rightsquigarrow SO(2N)$
  - $c \perp$ exotic O6 & $\tau^i\sigma^i \neq \tau^j\sigma^j = \tau^k\sigma^k = 1 \rightsquigarrow USp(2N)$

$\rightsquigarrow$ less probe brane conditions (✓) & $SU(2)_L$ candidates (↯)

Massless spectra of $U(N)$ & completely rigid D-branes

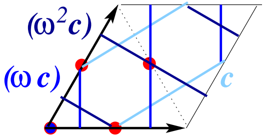
Chiral spectrum	
rep.	multiplicity
$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$	$\Pi_a \circ \Pi_b$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\Pi_a \circ \Pi'_b$
$(\mathbf{Anti}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6}\right)$
$(\mathbf{Sym}_a)$	$\frac{1}{2}\left(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6}\right)$

$\boxed{\mathbb{Z}_2 \times \mathbb{Z}_2}$:

- topological intersection number $\Pi_a \circ \Pi_b$ of 3-cycles Π_a, Π_b = net-chirality of $(\mathbf{N}_a, \overline{\mathbf{N}}_b)$ for $\text{sgn}(\Pi_a \circ \Pi_b) > 0$ $(\overline{\mathbf{N}}_a, \mathbf{N}_b)$ for $\text{sgn}(\Pi_a \circ \Pi_b) < 0$
- $|\Pi_a \circ \Pi_b|$ = total amount of matter

$\boxed{\mathbb{Z}_2 \times \mathbb{Z}_{2M>2}}$:

- net-chirality *not* sufficient: cancellations among $a(\omega^k b)_{k\in\{0,1,2\}}$ sectors
- *usually* $\mathbf{Adj}_c$ at $c(\omega^k c)$ self-intersections
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}_6}$: ~~$\mathbf{Adj}_c$~~ for *some* shortest 2-cycles on T^4
- $\boxed{\mathbb{Z}_2 \times \mathbb{Z}'_6}$: ~~$\mathbf{Adj}_c$~~ for *some* shortest 3-cycles on T^6



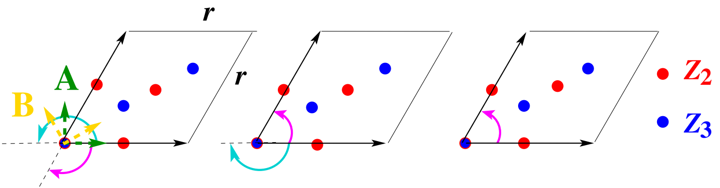
Förste, G.H. '10

G.H., Ripka, Staessens '12

IIA/$\Omega\mathcal{R}$ on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$ with discrete torsion: geometry

Förste, G.H. JHEP 1101 (2011) 091

$\mathbb{Z}_2 \times \mathbb{Z}_6'$ generators: $\vec{v} = \frac{1}{2}(1,-1,0)$ $\quad \vec{w}' = \frac{1}{6}(-2,1,1)$ $\quad$ on $SU(3)^3$



- $\boxed{\Pi_a^{\text{rigid}} = \frac{1}{4}\left(\Pi_a^{\text{bulk}} + \sum_{i=1}^{3} \Pi_a^{\mathbb{Z}_2^{(i)}}\right)}$
- $\boxed{\Pi_a^{\text{bulk}} = X_a\,\rho_1 + Y_a\,\rho_2}$ with

$$X_a \equiv n_a^1 n_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 - \sum_{i\neq j\neq k\neq i} n_a^i m_a^j m_a^k \in \mathbb{Z}, \quad Y_a \equiv \sum_{i\neq j\neq k\neq i} \left(n_a^i n_a^j m_a^k + n_a^i m_a^j m_a^k \right) \in \mathbb{Z}$$

- $\boxed{\Pi^{\mathbb{Z}_2^{(i)}} = \sum_{\alpha=1}^{5} \left(x_{\alpha,a}^i\,\varepsilon_\alpha^{(i)} + y_{\alpha,a}^i\,\tilde{\varepsilon}_\alpha^{(i)} \right)}$ with $(x_{\alpha,a}^i, y_{\alpha,a}^i) \sim (n_a^i, m_a^i)$

$T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: RR tadpoles, SUSY & relations

Förste, G.H. '10

lattice	Bulk RR tadpole cancellation $\sum_a N_a (\Pi_a + \Pi'_a) = 4\,\Pi_{O6}$	Bulk SUSY: nec. & suff. $\int_{\Pi_a} \mathrm{Im}(\Omega) = 0$	$\int_{\Pi_a} \mathrm{Re}(\Omega) > 0$
AAA	$\sum_a N_a (2\,X_a + Y_a) = 4\left(\eta_{\Omega\mathcal{R}} + 3\sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$Y_a = 0$	$2\,X_a + Y_a > 0$
ABB	$\sum_a N_a (X_a + 2\,Y_a) = 4\left(\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(1)}} + 3\sum_{i=0,2,3} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$X_a = 0$	$X_a + 2\,Y_a > 0$
AAB	$\sum_a N_a (X_a + Y_a) = 4\left(3\,\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(3)}} + \sum_{i=0}^2 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$Y_a - X_a = 0$	$X_a + Y_a > 0$
BBB	$\sum_a N_a Y_a = 4\left(3\,\eta_{\Omega\mathcal{R}} + \sum_{i=1}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}}\right)$	$2\,X_a + Y_a = 0$	$Y_a > 0$

$\eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(i)}} = \pm 1$ for ordinary/exotic O6-planes

- pairwise relations by $(\frac{\pi}{3}, 0, 0)$ or $(0, 0, \frac{\pi}{3})$ rotation
- $(X_a\,,\ Y_a)^{\mathbf{AAA}}_{\mathbf{AAB}} = (X_a + Y_a\,,\ -X_a)^{\mathbf{ABB}}_{\mathbf{BBB}}$ and $\begin{matrix}\Omega\mathcal{R} \\ \Omega\mathcal{R}\mathbb{Z}_2^{(2)}\end{matrix} \leftrightarrow \begin{matrix}\Omega\mathcal{R}\mathbb{Z}_2^{(1/3)} \\ \Omega\mathcal{R}\mathbb{Z}_2^{(3/1)}\end{matrix}$
- pairwise relations valid for G.H., Ripka, Staessens '12
  - full massless spectra
  - field theory @ 1-loop (gauge couplings, Kähler metrics by CFT)
- maximal SUSY rank

$$\mathbf{AAA} = \begin{cases} 16 & \eta_{\Omega\mathcal{R}} = -1 \\ 8 & \text{else} \end{cases} \qquad \mathbf{BBB} = \begin{cases} 8 & \text{else} \\ 0 & \eta_{\Omega\mathcal{R}} = -1 \end{cases}$$

Model building constraints

Open strings on **AAA**:

- ~~**Adj**$_a$~~ $\rightsquigarrow$ only possible for the 2 shortest *bulk* cycles at angles $(0, 0, 0)$ and $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$ & special choices of $(\vec{\sigma}, \vec{\tau})$

$$I_{a(\omega a)} + \sum_{i=1}^{3} I_{a(\omega a)}^{\mathbb{Z}_2^{(i)}}(\vec{\sigma}, \vec{\tau}) \stackrel{!}{=} 0$$

- ~~[**Sym** + *h.c.*]~~ and ~~[**Anti** + *h.c.*]~~ of **QCD stack** $\rightsquigarrow$ only the shortest *bulk* cycle at $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$ and $\eta_{\Omega\mathcal{R}} = -1$
- 3 generations $\rightsquigarrow$ same shortest *bulk* cycle for $SU(2)_L$ **stack**

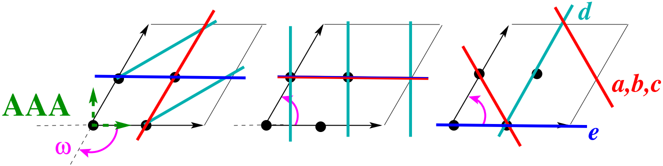
Comparison with **BBB**:

- no **Adj**$_a$ ✓
- no [**Sym**$_a$ + *h.c.*] or [**Anti**$_a$ + *h.c*] ↯

A typical Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$

G.H., Ripka, Staessens '12

brane	$(n^i, m^i)_{i=1,2,3}$	$\mathbb{Z}_2$	$(\vec{\tau})$	$(\vec{\sigma})$	group	(X, Y)
a		$(+++)$	$(0,0,1)$		$U(4)$	
b	(0,1;1,0,1,-1)	$(--+)$	$(0,1,1)$	$(\vec{1})$	$U(2)_L$	(1,0)
c		$(-+-)$	$(1,0,1)$		$U(2)_R$	
d	(1,1;1,-2;0,1)	$(+++)$	$(0,0,1)$	$(\vec{1})$	$U(2)_d$	(3,0)
e	(1,0;1,0;1,0)	$(+--)$	$(1,1,1)$	$(1,1,0)$	$U(2)_e$	(1,0)



- a, b, c at $(\frac{\pi}{3}, 0, -\frac{\pi}{3})$, d at $(\frac{\pi}{6}, -\frac{\pi}{2}, \frac{\pi}{3})$ e at $(0,0,0)$
- $U(1)^5$ anomalous & massive at M_{string}
- $SU(4) \times SU(2)_L \times SU(2)_R \times SU(2) \times SU(2)$ with
  - 3 generations of quarks + leptons
  - 1 Higgs (H_d, H_u)
  - ~~**Adj**~~ on a, b, c, e; $1 \times$ **Adj**$_d$

A typical Pati-Salam model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$: spectrum

$SU(4) \times SU(2)_L \times SU(2)_R \times SU(2) \times SU(2) \times U(1)^5_{\text{massive}}$

- Standard Model particles plus **one Higgs**

$$(\mathbf{4},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\mathbf{1})+2\,(\mathbf{4},\mathbf{2},\mathbf{1};\mathbf{1},\mathbf{1})+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})+2\,(\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{2},\overline{\mathbf{2}};\mathbf{1},\mathbf{1})$$

$\rightsquigarrow$ **one massive generation** at leading order
by charge selection rules

- chiral w.r.t. anomalous $U(1)^5_{\text{massive}}$

$$(\mathbf{1},\mathbf{2},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+3(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+(\mathbf{1},\overline{\mathbf{2}},\mathbf{1};\mathbf{1},\overline{\mathbf{2}})+(\mathbf{1},\mathbf{1},\overline{\mathbf{2}};\mathbf{2},\mathbf{1})+3(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})$$

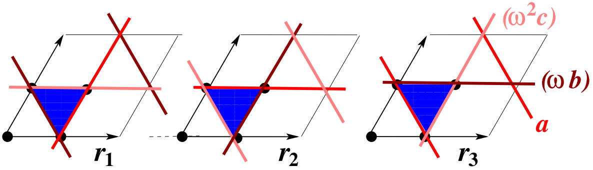
but non-chiral w.r.t. $SU(4) \times SU(2)_L \times SU(2)_R$

- non-chiral w.r.t. to full $U(4) \times U(2)^4$

$$2\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1})+h.c.\right]+\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2})+h.c.\right]+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4}_{\mathbf{Adj}},\mathbf{1})$$
$$+\,2\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{3}_{\mathbf{S}},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1}_{\mathbf{A}},\mathbf{1})+h.c.\right]+\left[(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3}_{\mathbf{S}})+(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1}_{\mathbf{A}})+h.c.\right]$$

Yukawa interactions

- charge selection rules not sufficient on $T^6/\mathbb{Z}_{2N}$, $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ due to various sectors $a(\omega^k b)_{k\in\{0,1,2\}}$ G.H., Vanhoof '12



G.H., Ripka, Staessens '12

- Pati-Salam model: one heavy generation by $W_{Q_L^{(3)} Q_R^{(3)} H} \sim e^{-\sum_{i=1}^3 v_i/8}$ with Kähler moduli $v_i \equiv \frac{\sqrt{3}}{2}\frac{r_i^2}{\alpha'}$
- non-chiral $\left[(\mathbf{4},\mathbf{1},\mathbf{1};\overline{\mathbf{2}},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1}_{\mathbf{A}},\mathbf{1}) + h.c.\right]$ massive via couplings to $(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{4}_{\mathbf{Adj}},\mathbf{1})$
- several types of $(\mathbf{1},\mathbf{2}_x,\mathbf{2}_y,\mathbf{1},\mathbf{1})$ massive through 3-point couplings among each other and with the Higgs
- other masses through higher order or non-perturbative (instanton) couplings

Gauge couplings @ 1-loop

- **gravitational coupling** $\rightsquigarrow \frac{M_{\text{Planck}}^2}{M_{\text{string}}^2} = \frac{4\pi}{g_{\text{string}}^2} v_1 v_2 v_3$
- **tree-level gauge coupling**

$$\frac{4\pi}{g^2_{SU(N_a),\text{tree}}} = 2\pi \Re\left(f^{\text{tree}}_{SU(N_a)}\right) = \frac{1}{4} \frac{1}{g_{\text{string}}} \frac{\prod_{i=1}^3 L_a^{(i)}}{\ell_s^3}$$

  - for the typical Pati-Salam model:

$$\frac{4\pi}{g^2_{SU(N_{a,b,c,e}),\text{tree}}} = \frac{1}{4\, g_{\text{string}}} \frac{r_1 r_2 r_3}{\ell_s^3} = \frac{1}{3^{3/4} \cdot 32\pi^3} \frac{M_{\text{Planck}}}{M_{\text{string}}} \approx 4 \cdot 10^{-4} \frac{M_{\text{Planck}}}{M_{\text{string}}}$$

Mass scales and values of the string coupling												
M_{string}	1 TeV				10^{12} GeV				10^{16} GeV			
g_{string}	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5
$v_1 v_2 v_3$	$8 \cdot 10^{24}$	$8 \cdot 10^{26}$	$8 \cdot 10^{28}$	$2 \cdot 10^{30}$	$8 \cdot 10^{6}$	$8 \cdot 10^{8}$	$8 \cdot 10^{10}$	$2 \cdot 10^{12}$	0.08	8	800	$2 \cdot 10^{4}$
$4\pi / g^2_{a,\text{tree}}$	$4 \cdot 10^{12}$				$4 \cdot 10^{3}$				$4 \cdot 10^{-1}$			

- gauge coupling unification @ tree-level for $M_{\text{string}} \sim M_{GUT}$

Gauge coupling unification or low M_{string}

- $\frac{4\pi}{g^2_{SU(N_a),\text{tree}}} \approx 4 \cdot 10^{-4} \frac{M_{\text{Planck}}}{M_{\text{string}}}$ with $\frac{M_{\text{Planck}}}{M_{\text{string}}} = \frac{\sqrt{4\pi}}{g_{\text{string}}} \sqrt{v_1 v_2 v_3}$

Mass scales and values of the string coupling

M_{string}	1 TeV				10^{12} GeV				10^{16} GeV			
g_{string}	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5	10^{-3}	0.01	0.1	0.5
$v_1 v_2 v_3$	$8 \cdot 10^{24}$	$8 \cdot 10^{26}$	$8 \cdot 10^{28}$	$2 \cdot 10^{30}$	$8 \cdot 10^{6}$	$8 \cdot 10^{8}$	$8 \cdot 10^{10}$	$2 \cdot 10^{12}$	0.08	8	800	$2 \cdot 10^{4}$
$4\pi/g^2_{a,\text{tree}}$	$4 \cdot 10^{12}$				$4 \cdot 10^{3}$				$4 \cdot 10^{-1}$			

- **one-loop** corrections contain

$$\Re\left(\delta_b^{\text{1-loop}} \mathfrak{f}_{SU(N_a)}\right) \supset -\frac{\tilde{b}_{ab}^{\mathcal{A},(i)}}{4\pi^2} \ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4} \frac{\vartheta_1\left(\frac{\tau^i_{ab} - i\sigma^i_{ab} v_i}{2}, i v_i\right)}{\eta(i v_i)}\right) \overset{v_i \to \infty}{\sim} v_i$$

  - for the typical Pati-Salam model:

$$2\pi\Re\left(\delta^{\text{1-loop}} \mathfrak{f}_{SU(N_x)}\right) \sim \begin{cases} \frac{10(v_1+v_2)-7v_3}{48} - \frac{4 \ln 2}{\pi} & x = a \\ \frac{8\, v_{2/1} - 3\, v_3}{24} - \frac{41 \ln 2}{12\pi} & b/c \end{cases}$$

    with negative contribution from v_3

- **unification @ 1-loop** for $v_{1/2} = \frac{v_3}{4} + \frac{7\ln(2)}{\pi}$
- **or** $M_{\text{string}} \sim$ **TeV** for $v_{1/2} \sim 10^6$, $v_3 \sim 10^{13}$, $g_{\text{string}} \sim 10^{-3}$

Conclusions

- **Rigid D6-branes** on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with **discrete torsion**
  - reduction of closed & open string moduli
  - $2M \neq 2$: bulk cycles have $\mathbb{Z}_{2M}$ images
    $\rightsquigarrow$ selection rules on Yukawa interactions ($\neq T^6$)

G.H., Vanhoof JHEP 1204 (2012) 085

  - $2M = 6'$: ~~$SU(5)$ GUTs~~, only local MSSM and L-R models
  - $2M = 6$: *a priori* less constrained ...to be worked out
  - new maps among lattice orientations $\rightsquigarrow$ economise SM search

G.H., Ripka, Staessens arXiv:1209.3010 [hep-th]

- Example: global **Pati-Salam** model on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_6'$
  - ~~**Adj**~~ moduli
  - some vector-like states
  - perturbative Yukawa couplings for one particle generation only
  - gauge coupling unification at $M_{\text{string}} \sim M_{GUT}$ possible @ 1-loop
  - or $M_{\text{string}} \sim 1$ TeV for LARGE unisotropic volumes

G.H., Ripka, Staessens arXiv:1209.3010 [hep-th]

Technical details

Counting of closed string states

Closed string sector on **AAA** with exotic $\Omega\mathcal{R}$-plane:

- $h_{21} = 15$ complex structures ($\mathbb{Z}_2$)
- $h_{11}^{-} = 14$ Kähler moduli (3 bulk + 3 $\mathbb{Z}_6$ + 8 $\mathbb{Z}_3$)
- $h_{11}^{+} = 1$ vector/dark photon ($\mathbb{Z}_3$) $\rightsquigarrow$ ~~mixing~~ with open U(1)s
  - typical for D6-branes on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with $\eta = -1$: vectors only in $\mathbb{Z}_6$ and $\mathbb{Z}_3$ twisted sectors
  - for $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with $\eta = +1$ and $T^6/\mathbb{Z}_{2N}$: vectors in $\mathbb{Z}_2$ (and bulk) sectors

Förste, G.H. '10

Wrapping numbers in $\mathbb{Z}_2$ twisted sectors

G.H., Ripka, Staessens '12

Exceptional wrappings $(x^{(i)}_{\alpha,a}, y^{(i)}_{\alpha,a})$	
I	II
$(z^{(i)}_{\alpha,a}\, n^i_a \,,\; z^{(i)}_{\alpha,a}\, m^i_a)$	$\left(-z^{(i)}_{0,a}\, n^i_a + (z^{(i)}_{\alpha,a} - z^{(i)}_{0,a})\, m^i_a \,,\; (z^{(i)}_{0,a} - z^{(i)}_{\alpha,a})\, n^i - z^{(i)}_{\alpha,a}\, m^i_a\right)$
$(z^{(i)}_{\alpha,a}\, m^i_a \,,\; -z^{(i)}_{\alpha,a}\,(n^i_a + m^i_a))$	$\left((z^{(i)}_{0,a} - z^{(i)}_{\alpha,a})\, n^i_a - z^{(i)}_{\alpha,a}\, m^i \,,\; z^{(i)}_{0,a}\, m^i_a + z^{(i)}_{\alpha,a}\, n^i_a\right)$
$(-z^{(i)}_{\alpha,a}\,(n^i_a + m^i_a) \,,\; z^{(i)}_{\alpha,a}\, n^i_a)$	$\left(z^{(i)}_{\alpha,a}\, n^i_a + z^{(i)}_{0,a}\, m^i_a \,,\; -z^{(i)}_{0,a}\, n^i_a + (z^{(i)}_{\alpha,a} - z^{(i)}_{0,a})\, m^i_a\right)$

- with signs $z^{(i)}_{0,a} \equiv (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}}}$ and
  $z^{(i)}_{\alpha,a} \in \{(-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau_a^j}, (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau_a^k}, (-1)^{\tau_a^{\mathbb{Z}_2^{(i)}} + \tau_a^j + \tau_a^k}\}$
- Type I: from a single fixed point - II: sum of two fixed points
- each $\Pi^{\mathbb{Z}_2^{(i)}}$ receives contributions from only three $\alpha \in \{1 \ldots 5\}$ depending on σ^j_a, σ^k_a
  $\rightsquigarrow$ constraints on cancellation of $\mathbb{Z}_2$ twisted tadpoles

1-loop corrections to gauge couplings

G.H. '11

One-loop corrections to holomorphic gauge kinetic functions on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ with discrete torsion		
$(\phi^{(1)}_{ab}, \phi^{(2)}_{ab}, \phi^{(3)}_{ab})$	$\Re\left(\delta_b^{\text{1-loop},\mathcal{A}} \mathrm{f}_{SU(N_a)}\right)$	$\Re\left(\delta_{b=a'}^{\text{1-loop},\mathcal{M}} \mathrm{f}_{SU(N_a)}\right)$
$(0,0,0)$	$-\sum_{i=1}^3 \frac{b_{ab}^{\mathcal{A},(i)}}{4\pi^2} \ln \eta(iv_i)$ $-\sum_{i=1}^3 \frac{\tilde{b}_{ab}^{\mathcal{A},(i)}\left(1-\delta^{\sigma^i_b}_{\sigma^i_a}\,\delta^{\tau^i_b}_{\tau^i_a}\right)}{4\pi^2} \ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4}\,\frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2}, iv_i)}{\eta(iv_i)}\right)$	$-\sum_{i=1}^3 \frac{b_{aa'}^{\mathcal{M},(i)}}{4\pi^2} \ln \eta(i\tilde{v}_i) - \frac{b_{aa'}^{\mathcal{M}} \ln(2)}{8\pi^2}$ (for $b_i = 0$ or $(\sigma^i_a, \tau^i_a) = (0,0)$)
$(0^{(i)}, \phi^{(j)}_{ab}, \phi^{(k)}_{ab})$	$-\frac{b_{ab}^{\mathcal{A}}}{4\pi^2} \ln \eta(iv_i)$ $-\frac{\tilde{b}_{ab}^{\mathcal{A},(i)}\left(1-\delta^{\sigma^i_b}_{\sigma^i_a}\,\delta^{\tau^i_b}_{\tau^i_a}\right)}{4\pi^2} \ln\left(e^{-\pi(\sigma^i_{ab})^2 v_i/4}\,\frac{\vartheta_1(\frac{\tau^i_{ab}-i\sigma^i_{ab}v_i}{2}, iv_i)}{\eta(iv_i)}\right)$ $+\sum_{l=j,k} \frac{N_b\, I_{ab}^{\mathbb{Z}_2^{(l)}}}{32\pi^2}\left(\frac{\mathrm{sgn}(\phi^{(l)}_{ab})}{2} - \phi^{(l)}_{ab}\right)$	$-\frac{b_{aa'}^{\mathcal{M}}}{4\pi^2} \ln \eta(i\tilde{v}_i)$ (for $b_i = 0$ or $(\sigma^i_a, \tau^i_a) = (0,0)$) $+\frac{\ln(2)}{32\pi^2} \sum_{[m: a \perp \Omega\mathcal{R}\mathbb{Z}_2^{(m)} \text{on } T^2_{(i)}]} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \rvert$
$(\phi^{(1)}, \phi^{(2)}, \phi^{(3)})$	$\sum_{l=1}^3 \frac{N_b\, I_{ab}^{\mathbb{Z}_2^{(l)}}}{32\pi^2}\left(\frac{\mathrm{sgn}(\phi^{(l)}_{ab})+\mathrm{sgn}(I_{ab})}{2} - \phi^{(l)}_{ab}\right)$	$\frac{\ln(2)}{32\pi^2} \sum_{m=0}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \rvert$

- Annulus contributions known for all configurations of tori b_i, displacements σ^i & Wilson lines τ^i
- Möbius strip contributions only derived from first principles (and correct for) $b_i \sigma^i \tau^i = 0$

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Beta function coefficients

G.H. '11

Kähler metrics and beta function coefficients on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_{2M} \times \Omega\mathcal{R})$ with discrete torsion

$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$	$K_{\mathbf{R}_a}$	$b_{ab}^{\mathcal{A}}$	$b_{aa'}^{\mathcal{M}}$ (only for $b = a'$)
$(0,0,0)$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \sqrt{2\pi}^3 \frac{L_a^{(i)}}{\ell_s}$	$-\frac{N_b}{4} \sum_{i=1}^3 \delta_{\sigma_a^i}^{\sigma_b^i} \delta_{\tau_a^i}^{\tau_b^i} I_{ab}^{\mathbb{Z}_2^{(i)},(j\cdot k)}$	$-\frac{1}{2} \sum_{j<k} \sum_{m=0}^3 \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} (-1)^{2b_i \sigma_a^i \tau_a^i} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)},(j\cdot k)} \rvert$
$(0^{(i)}, \phi, -\phi)$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \sqrt{2\pi}^3 \frac{L_a^{(i)}}{\ell_s}$	$\frac{N_b \, \delta_{\sigma_a^i}^{\sigma_b^i} \delta_{\tau_a^i}^{\tau_b^i}}{4} \left(\lvert I_{ab}^{(j\cdot k)} \rvert - I_{ab}^{\mathbb{Z}_2^{(i)},(j\cdot k)} \right)$	$-\frac{1}{2} \sum_{\substack{m \in \{0\dots 3\} \text{ with} \\ a \uparrow\uparrow \Omega\mathcal{R}\mathbb{Z}_2^{(m)} \text{on } T^2_{(i)}}} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} (-1)^{2b_i \sigma_a^i \tau_a^i} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)},(j\cdot k)} \rvert$
$(\phi_{ab}^{(1)}, \phi_{ab}^{(2)}, \phi_{ab}^{(3)})$ $\sum_i \phi_{ab}^{(i)} = 0$	$\frac{g_{\text{string}}}{v_1 v_2 v_3} \prod_{i=1}^3 \frac{\Gamma(\lvert\phi_{ab}^{(i)}\rvert)}{\Gamma(1-\lvert\phi_{ab}^{(i)}\rvert)}^{-\frac{\text{sgn}(\phi_{ab}^{(i)})}{2\,\text{sgn}(I_{ab})}}$	$\frac{N_b}{8} \left(\lvert I_{ab} \rvert + \text{sgn}(I_{ab}) \sum_{i=1}^3 I_{ab}^{\mathbb{Z}_2^{(i)}} \right)$	$\frac{1}{4} \sum_{m=0}^3 c_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \eta_{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \lvert \tilde{I}_a^{\Omega\mathcal{R}\mathbb{Z}_2^{(m)}} \rvert$

- Kähler metrics enter physical Yukawa couplings $Y_{ijk} = (K_{xy} K_{yz} K_{zx})^{-1/2} \, e^{\kappa_4^2 \mathcal{K}/2} \, W_{ijk}$
- beta function coefficients needed
  - for derivation of non-chiral matter spectrum
  - as prefactors in 1-loop correction to hol. gauge kinetic function
- factor $(-1)^{2b_i \sigma_a^i \tau_a^i}$ in $\Omega\mathcal{R}$-invariant configurations required for consistency with counting of Chan-Paton labels

G.H., Ripka, Staessens '12