

Nonequilibrium QFT approach to leptogenesis

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MAX PLANCK
RESEARCH SCHOOL



FOR PRECISION TESTS
OF FUNDAMENTAL
SYMMETRIES

In collaboration with M. Garny, A. Hohenegger, A. Kartavtsev, and D. Mitrouskas

Baryogenesis via Leptogenesis

- Matter-antimatter asymmetry in the Universe

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.2 \pm 0.2) 10^{-10}$$

CMB + BBN \Rightarrow



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- Dynamical generation of the baryon asymmetry:
→ 3 Sakharov conditions:
 - 1 Baryon number violation
 - 2 C and CP violation
 - 3 Out of equilibrium dynamics

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Leptogenesis

SM + 3 heavy Majorana neutrinos:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{SM} + \frac{1}{2} \bar{N}_i (i\partial^\mu - M_i) N_i \\ & - h_{\alpha i} \bar{\ell}_\alpha \tilde{\phi} P_R N_i - h_{i\alpha}^\dagger \bar{N}_i \tilde{\phi}^\dagger P_L \ell_\alpha \end{aligned}$$

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- 3 Expanding Universe + N_i decay

Canonical approach

Generation of an asymmetry:

	production	washout
N_i (inverse)decay	$\mathcal{O}(h^4)$	$\mathcal{O}(h^2)$
$\Delta L = 2$ scattering	$\mathcal{O}(h^6)$	$\mathcal{O}(h^4)$
top scattering	$\mathcal{O}(\lambda_t^2 h^4)$	$\mathcal{O}(\lambda_t^2 h^2)$
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Canonical approach

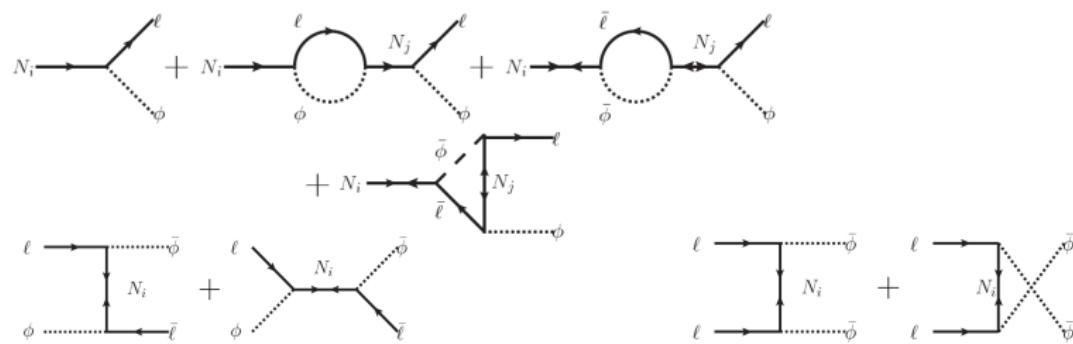
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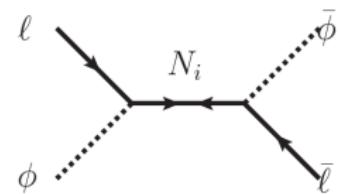


⇒ into the Boltzmann equation for the lepton number!

Canonical approach

This approach suffers from several **problems**:

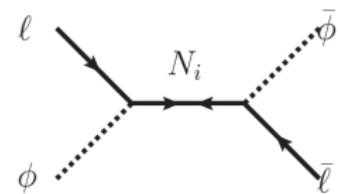
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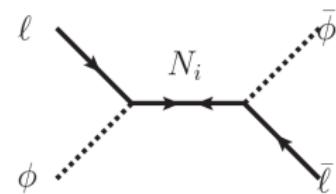
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 - **thermal decay width?**
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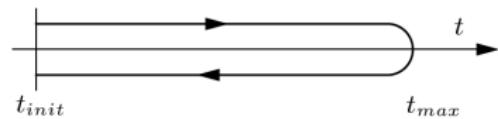


Nonequilibrium Quantum Field Theory approach is needed!!

Basics of nonequilibrium QFT

Right-handed neutrinos propagator
on CTP:

$$S_{ij}^{\alpha\beta}(x, y) = \langle T_C N_i^\alpha(x) \bar{N}_j^\beta(y) \rangle$$



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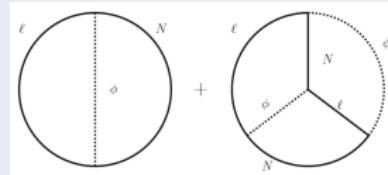
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Schwinger-Dyson equation

$$\hat{S}^{-1}(x, y) = \hat{S}_0^{-1}(x, y) - \hat{\Sigma}(x, y) , \quad \Sigma_{ij}^{\alpha\beta}(x, y) = -\frac{\delta i\Gamma_2}{\delta S_{ji}^{\beta\alpha}(y, x)}$$

$$\Gamma_2 = \text{2PI effective action} =$$



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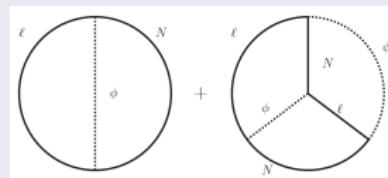
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$$\Gamma_2 = \text{2PI effective action} =$$



$$\rightarrow \text{doubling of the d.o.f.: } \hat{S}(x, y) = \underbrace{\hat{S}_F(x, y)}_{\text{statistical}} - \frac{i}{2} \text{sign}_{\mathcal{C}}(x^0 - y^0) \underbrace{\hat{S}_\rho(x, y)}_{\text{spectral}}$$

Quantum Boltzmann equation

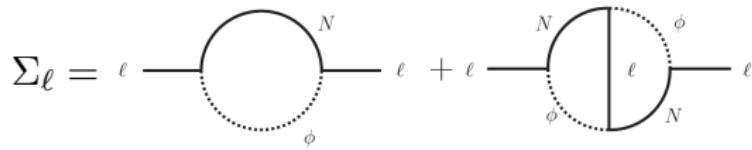
- **Wigner transform** \leftrightarrow Fourier transform wrt relative coordinates
- **Gradient expansion** \leftrightarrow Expansion in slow relative to fast time-scales (H/T)
- **Quasiparticle approximation** \leftrightarrow Narrow width approximation
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Quantum Boltzmann equation

$$\frac{d}{dt}(n_\ell(t) - n_{\bar{\ell}}(t)) = g_w \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left[\left(\underbrace{\Sigma_{\ell<} (t, p) (1 - f_\ell^p)}_{\text{gain term}} + \underbrace{\Sigma_{\ell>} (t, p) f_\ell^p}_{\text{loss term}} \right) S_{\ell\rho} (t, p) \right]$$



Lepton asymmetry

Free (thermal-) propagators into the lepton self-energy:

$$S_{\ell F} = \left(\frac{1}{2} - f_\ell^p\right) P_L \not{p} \text{sign}(p^0) (2\pi) \delta(p^2 - m_\ell^2) \quad \Delta_{\phi F} = \left(\frac{1}{2} + f_\phi^k\right) \text{sgn}(k^0) (2\pi) \delta(k^2 - m_\phi^2)$$

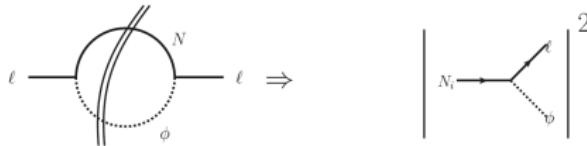
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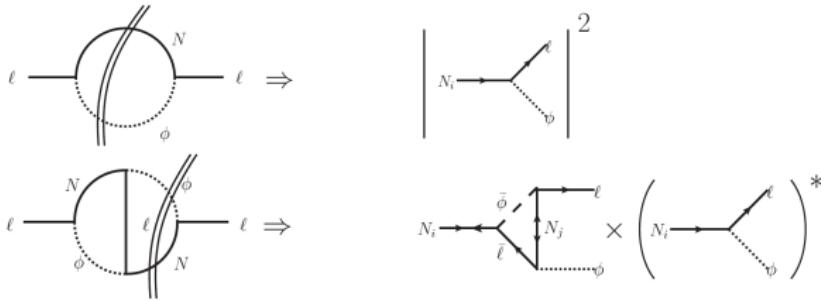


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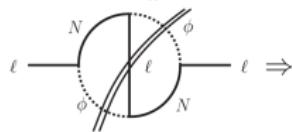
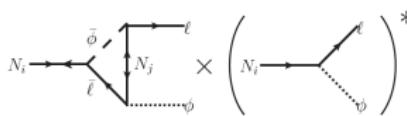
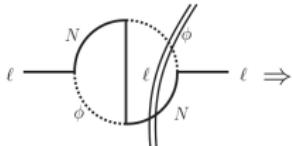
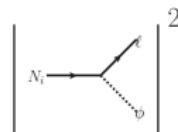
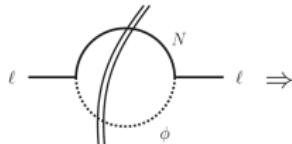


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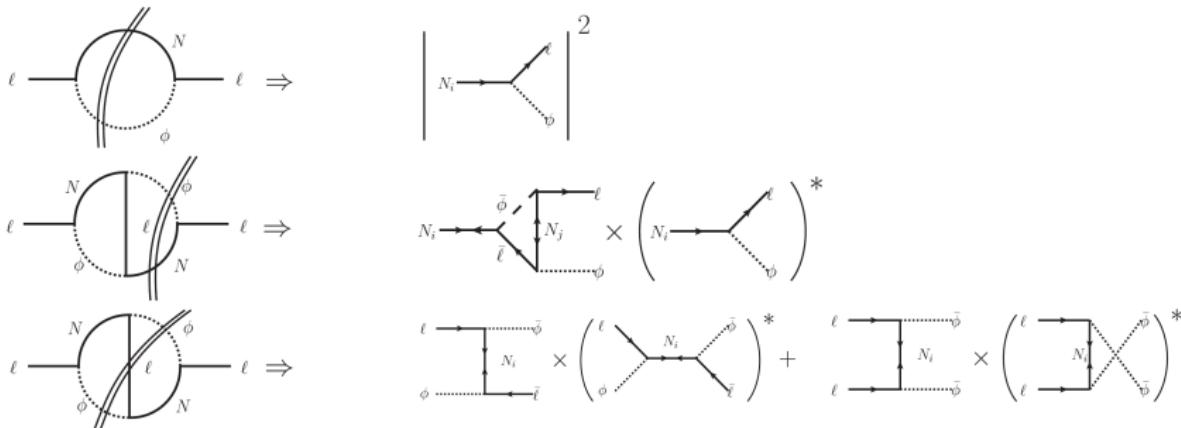


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→ $s \times s$ and $t \times t$ channels of the scattering processes are **missing**!

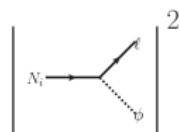
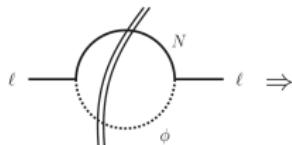
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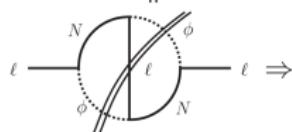
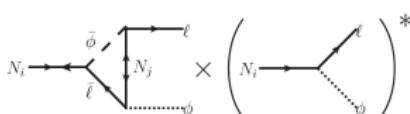
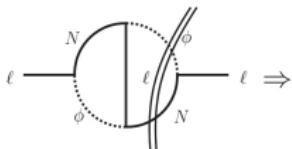
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↖ neglect the off-diagonal components



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Resummation and extended QP approximation

In **equilibrium** the *full* statistical propagator \hat{S}_F can be expressed in terms of the flavor **diagonal** propagators $\hat{\mathcal{S}}$ and flavor **off-diagonal** self-energy $\hat{\Sigma}^{od}$:

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→ expand \hat{S}_F around its poles \leftrightarrow extended QP approximation:

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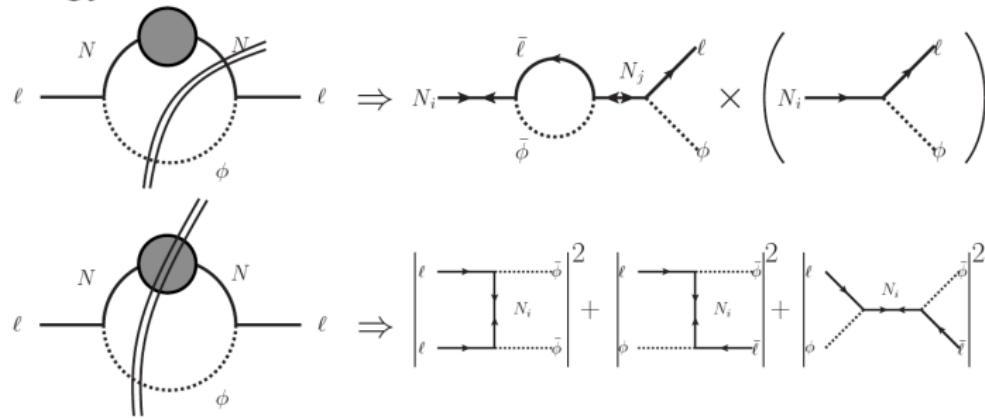
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→ assume to be valid **out of equilibrium**

Resummation and extended QP approximation

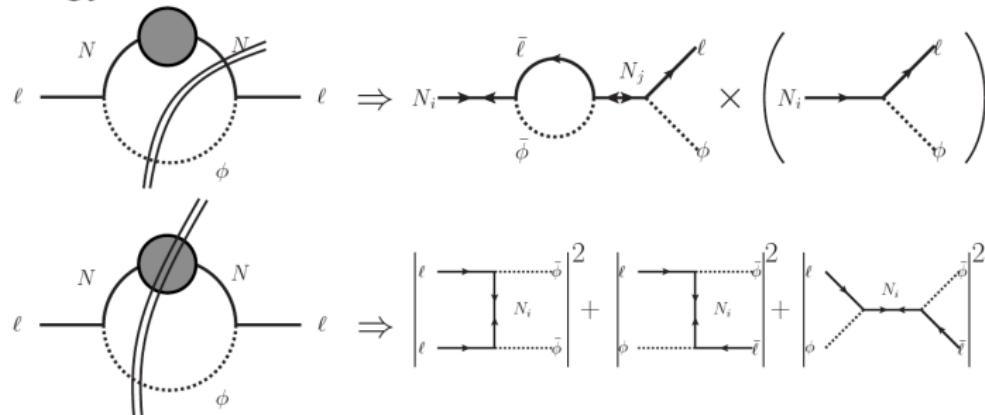
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→ The insertion of the resummed propagator reproduces the missing terms!

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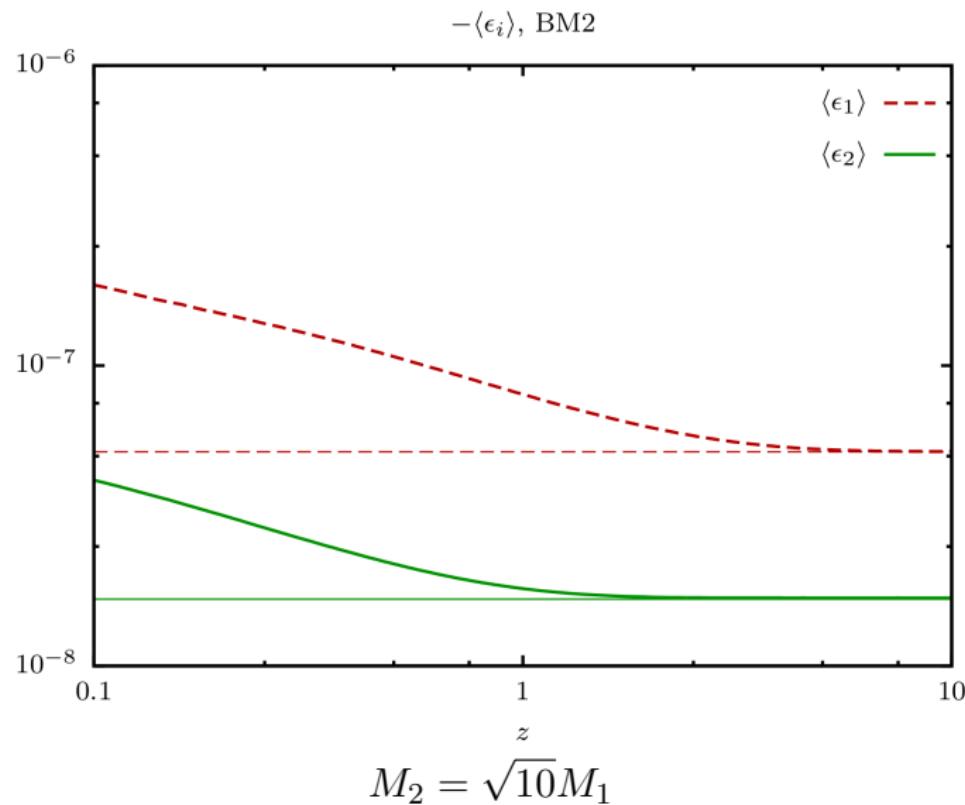
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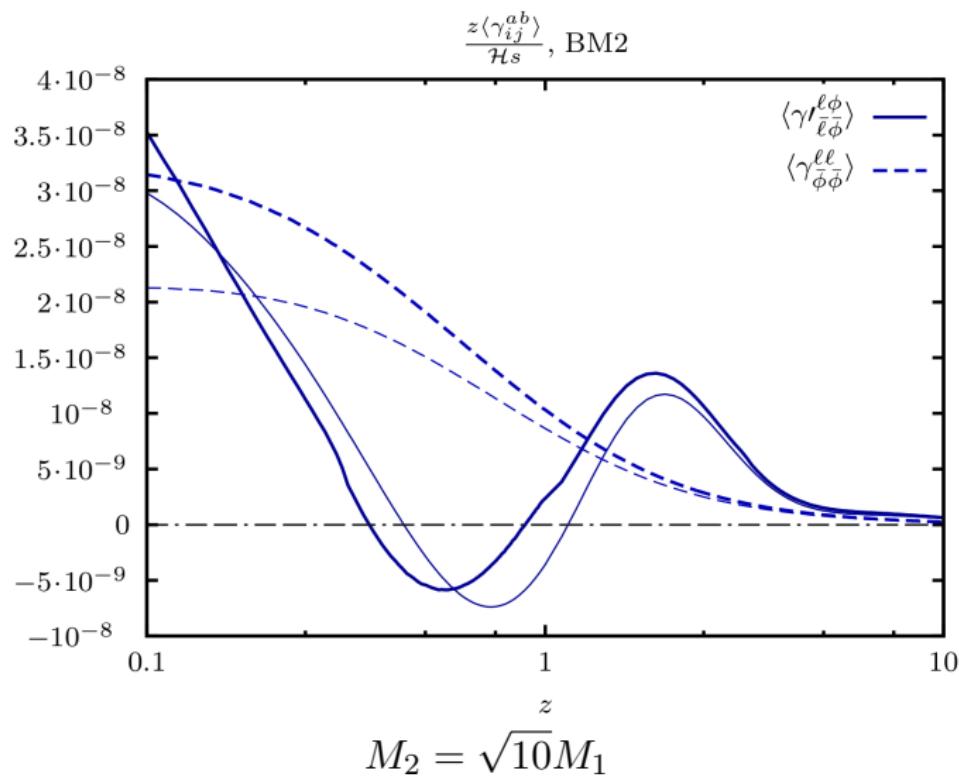
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The internal Majorana propagators are **automatically** RIS-subtracted
 ⇒ don't need to neglect the quantum statistical factors!

Numerical comparison with canonical approach



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Conclusion

Nonequilibrium quantum field theory approach to leptogenesis:

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Thank you!