DESY Theory Workshop
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Spin Correlations in Double-Parton Scattering

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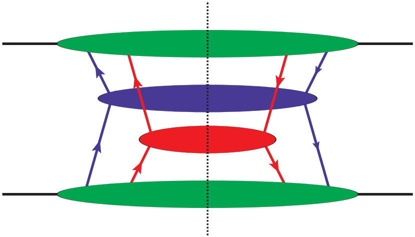
Work in collaboration with Markus Diehl

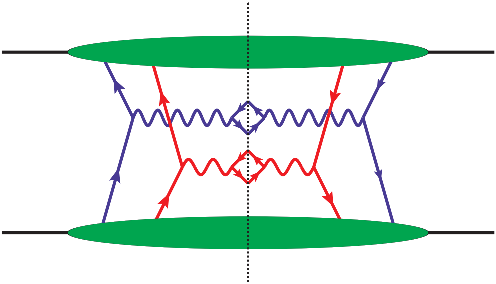


What is Double-Parton Scattering?

- Collisions with two partons (in each proton) interacting in two separate hard interactions

Why study Double-Parton Scattering?

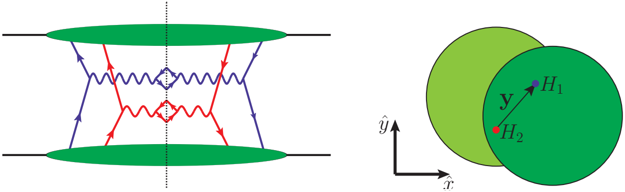
- Proton structure
- Background to other signals
- Multi-Parton Interactions and Monte Carlo
- Measured at: ISR, SPS, Tevatron and LHC
- $\sigma_{DPS}/\sigma_{SPS}$ expected to increase with energy
- No systematic theory



- Interferences in color, flavor, fermion number and spin correlations
- Study correlations between the hard interactions
- Double Drell-Yan type of process ($W^{\pm}$, Z, γ^{*})

(J. R. Gaunt, C.H. Kom, A. Kulesza, W.J. Stirling, 2010, 2011; M. Myska, 2011)

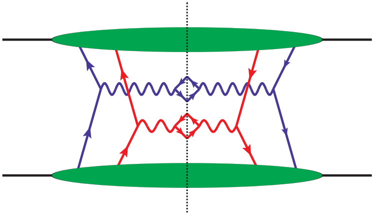
Double Parton Distributions



- Double Parton Distributions $F_{q_1 q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$
- $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ - transverse position arguments
  Fourier conjugate to (average) transverse parton momentum
- $|\boldsymbol{y}|$ - essentially distance between hard interactions

Double-Parton Cross Section

- Double-parton cross section (unpolarized, no interference)
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



$$\frac{d\sigma}{dx_1 dx_2 d\bar{x}_1 d\bar{x}_2 d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \hat{\sigma}_{q_1\bar{q}_1}(Q_1^2)\hat{\sigma}_{q_2\bar{q}_2}(Q_2^2)\int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$
$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2}\int d^2\boldsymbol{y} F_{q_1q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) F_{\bar{q}_1\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

(M. Diehl, D. Ostermeier, A. Schäfer, 2011)

DDY & Spin - Parton Distributions

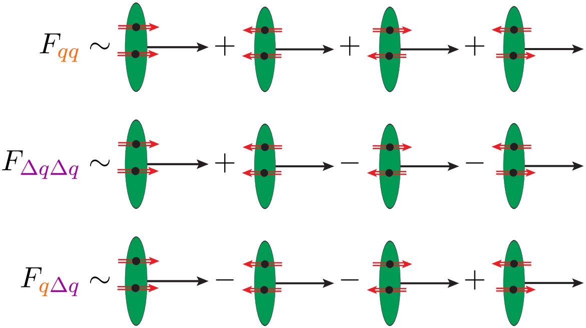
- The two hard interactions can be correlated via the spin of the quarks
- Correlation described by parton distributions

$$F_{q_1 q_2} \sim \langle p|(\bar{q}_1 \Gamma_1 q_1)(\bar{q}_2 \Gamma_2 q_2)|p\rangle$$

- $\Gamma_{1/2} = \Gamma_q, \ \Gamma_{\Delta q}, \ \Gamma_{\delta q}^j$ projection operators
  - Unpolarized (q), longitudinally- (Δq) and transversely-polarized (δq) quarks
  - Index $j = 1, 2$ corresponds to the transverse spin-vector

Double Parton Distributions as helicity eigenstates

- Unpolarized and longitudinally polarized quarks



- Transverse polarization ($F^{i}_{q\delta q}, F^{i}_{\Delta q\delta q}, F^{ij}_{\delta q\delta q}$) - helicity interference

- Unpolarized and longitudinally polarized quarks (f's scalar g's pseudo-scalar functions)

$$F_{qq} = f_{qq}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$
$$F_{\Delta q \Delta q} = f_{\Delta q \Delta q}$$
$$F_{q \Delta q} = g_{q \Delta q}$$

- Singly transversely polarized quarks ($\tilde{y}^i = y^j \epsilon^{ij}$)

$$F^i_{\Delta q \delta q} = M \left(y^i f_{\Delta q \delta q} + \tilde{y}^i g_{\Delta q \delta q} \right)$$
$$F^i_{q \delta q} = M \left(\tilde{y}^i f_{q \delta q} + y^i g_{q \delta q} \right)$$

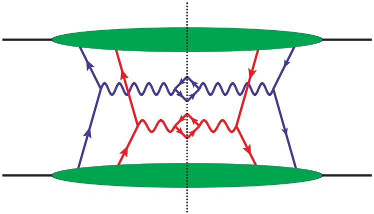
- Doubly transversely polarized quarks

$$F^{ij}_{\delta q \delta q} = \delta^{ij} f_{\delta q \delta q} + \left(2 y^i y^j - y^2 \delta^{ij} \right) M^2 f^t_{\delta q \delta q} + \left(y^i \tilde{y}^j + \tilde{y}^i y^j \right) M^2 g^s_{\delta q \delta q} + \left(y^i \tilde{y}^j - \tilde{y}^i y^j \right) M^2 g^a_{\delta q \delta q}$$

(M. Diehl, D. Ostermeier, A. Schäfer, 2011)

Double-parton Cross section

- Double-parton cross section without spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)

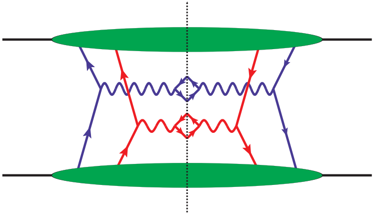


$$d\sigma \sim \hat{\sigma}_{q_1\bar{q}_1}(Q_1^2)\hat{\sigma}_{q_2\bar{q}_2}(Q_2^2) \int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$

$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2} \int d^2\boldsymbol{y} F_{q_1q_2}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) F_{\bar{q}_1\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

Double-parton Cross section

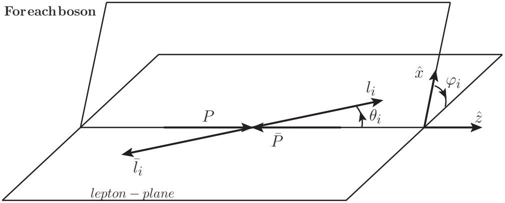
- Double-parton cross section with spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



$$d\sigma \sim \sum_{\substack{a_1,a_2=\{q,\Delta q,\delta q\} \\ \bar{a}_1,\bar{a}_2=\{\bar{q},\Delta\bar{q},\delta\bar{q}\}}} \hat{\sigma}^{ik}_{a_1\bar{a}_1}(Q_1^2)\hat{\sigma}^{jl}_{a_2\bar{a}_2}(Q_2^2)\int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$

$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2}\int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})$$

Coordinate system & angles (at $\boldsymbol{q}_i = 0$ for simplicity)



- Angles defined in rest frames of bosons
- $\hat{z}$ parallel with proton momenta, for $\boldsymbol{q}_i = 0$
- $\hat{x}$ arbitrary transverse reference axis (e.g. towards center of LHC ring)
- θ_1 and θ_2 polar angles of leptons
- φ_1 and φ_2 azimuthal angles of leptons
- φ_y angle to direction between the two collisions

- Double Drell-Yan cross section, unpolarized and longitudinally polarized quarks

($2 \times Z$ boson, one quark flavor, no interference)

$$d\sigma^{(0)} \sim \prod_{i=1}^{2} \int d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_i\boldsymbol{q}_i} \int d^2\boldsymbol{y} \sum_{\substack{a_1\bar{a}_1=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\} \\ a_2\bar{a}_2=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\}}}$$

$$\times \Big\{ \Big[K_{a_1\bar{a}_1}(1+\cos^2\theta_1) - K'_{a_1\bar{a}_1}\cos\theta_1\Big]$$

$$\times \Big[K_{a_2\bar{a}_2}(1+\cos^2\theta_2) - K'_{a_2\bar{a}_2}\cos\theta_2\Big] \Big[F_{a_1a_2}\bar{F}_{\bar{a}_1\bar{a}_2} + \text{perm.}\Big] \Big\}$$

- K, K' constants (depend of boson virtualities)
- Longitudinal polarization ⇒ change rate and angular distribution

- Double transverse polarization

$$
\begin{aligned}
d\sigma^{(2)} \sim & \prod_{i=1}^{2} \int d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_i\boldsymbol{q}_i} \int d^2\boldsymbol{y} \sin^2\theta_1 \sin^2\theta_2 \\
\times \Big\{ & \Big[A\cos 2(\varphi_1-\varphi_2) - B\sin 2(\varphi_1-\varphi_2)\Big](f\bar{f} - \boldsymbol{y}^4 M^4 g^a \bar{g}^a) \\
& + \Big[C\cos 2(\varphi_1+\varphi_2-2\varphi_y) - D\sin 2(\varphi_1+\varphi_2-2\varphi_y)\Big]\boldsymbol{y}^4 M^4 (f^t \bar{f}^t - g^s \bar{g}^s) \\
& + \Big[A\sin 2(\varphi_1-\varphi_2) + B\cos 2(\varphi_1-\varphi_2)\Big]\boldsymbol{y}^2 M^2 (f\bar{g}^a + g^a \bar{f}) \\
& - \Big[C\sin 2(\varphi_1+\varphi_2-2\varphi_y) + D\cos 2(\varphi_1+\varphi_2-2\varphi_y)\Big]\boldsymbol{y}^4 M^4 (f^t \bar{g}^s + g^s \bar{f}^t)\Big\}
\end{aligned}
$$

- Transverse dependence:
  - Azimuthal angle between lepton planes
  - Azimuthal angles between lepton planes and $\boldsymbol{y}$
- $\int d^2\boldsymbol{y}$: $\varphi_y \to$ angle to linear combination of boson momenta

- Single transverse polarization

$$d\sigma^{(1)} \sim \prod_{i=1}^{2} \int d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_i\boldsymbol{q}_i} \int d^2\boldsymbol{y} \sum_{a_1\bar{a}_1=\{q\bar{q},\Delta q\Delta\bar{q},q\Delta\bar{q},\Delta q\bar{q}\}}$$

$$\times\Big\{ \left[K_{a_1\bar{a}_1}(1+\cos^2\theta_1) - K'_{a_1\bar{a}_1}\cos\theta_1\right]$$

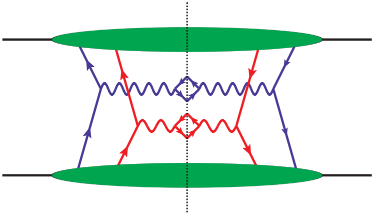
$$\times\sin^2\theta_2 \left[B_{\delta q\delta\bar{q}}\cos(2(\varphi_2-\varphi_y)) + B'_{\delta q\delta\bar{q}}\sin(2(\varphi_2-\varphi_y))\right]$$

$$\times \left[g_{q\delta q}\bar{f}_{\bar{q}\delta\bar{q}} + f_{q\delta q}\bar{g}_{\bar{q}\delta\bar{q}} + f_{\Delta q\delta q}\bar{g}_{\Delta\bar{q}\delta\bar{q}} + g_{\Delta q\delta q}\bar{f}_{\Delta\bar{q}\delta\bar{q}} + \text{perm.}\right] \Big\}$$

- Spin vector breaks z-axis rotation symmetry
- Transverse correlation between lepton plane and vector between hard interactions

Double-parton Cross section

- Double-parton cross section with spin correlations
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



$$d\sigma \sim \sum_{\substack{a_1,a_2=\{q,\Delta q,\delta q\} \\ \bar{a}_1,\bar{a}_2=\{\bar{q},\Delta\bar{q},\delta\bar{q}\}}} \hat{\sigma}^{ik}_{a_1\bar{a}_1}(Q_1^2)\hat{\sigma}^{jl}_{a_2\bar{a}_2}(Q_2^2)\int d^2\boldsymbol{z}_1 e^{-i\boldsymbol{z}_1\boldsymbol{q}_1}$$

$$\times \int d^2\boldsymbol{z}_2 e^{-i\boldsymbol{z}_2\boldsymbol{q}_2}\int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})$$

Double-parton Cross section

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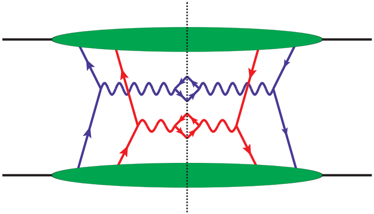
- Assume ('k_T-dependent') factorization of hard scatters (require $\boldsymbol{q}_i << $ boson virtuality)



- Integrated over transverse boson momenta:

$$d\sigma \sim \sum_{\substack{a_1,a_2=\{q,\Delta q,\delta q\} \\ \bar{a}_1,\bar{a}_2=\{\bar{q},\Delta\bar{q},\delta\bar{q}\}}} \hat{\sigma}^{ik}_{a_1\bar{a}_1}(Q_1^2)\hat{\sigma}^{jl}_{a_2\bar{a}_2}(Q_2^2)$$

$$\times \int d^2\boldsymbol{y} F^{ij}_{a_1a_2}(x_1,x_2,\boldsymbol{y}) F^{kl}_{\bar{a}_1\bar{a}_2}(\bar{x}_1,\bar{x}_2,\boldsymbol{y})$$

- Reduces number of double parton distributions
  - Unpolarized and longitudinally polarized quarks

$$F_{qq} = f_{qq}(x_1, x_2, \boldsymbol{y})$$
$$F_{\Delta q \Delta q} = f_{\Delta q \Delta q}$$
$$F_{q \Delta q} = g_{q \Delta q}$$

  - Singly transversely polarized quarks ($\tilde{y}^i = y^j \epsilon^{ij}$)

$$F^i_{\Delta q \delta q} = M \left(y^i f_{\Delta q \delta q} + \tilde{y}^i g_{\Delta q \delta q} \right)$$
$$F^i_{q \delta q} = M \left(\tilde{y}^i f_{q \delta q} + y^i g_{q \delta q} \right)$$

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$$F^{ij}_{\delta q \delta q} = \delta^{ij} f_{\delta q \delta q} + \left(2 y^i y^j - y^2 \delta^{ij}\right) M^2 f^t_{\delta q \delta q} + \left(y^i \tilde{y}^j + \tilde{y}^i y^j\right) M^2 g^s_{\delta q \delta q} + \left(y^i \tilde{y}^j - \tilde{y}^i y^j\right) M^2 g^a_{\delta q \delta q}$$

- Azimuthal correlations remain

$$d\sigma^{(2)} \sim \sin^2\theta_1 \sin^2\theta_2 \Big[A\cos 2(\varphi_1-\varphi_2) - B\sin 2(\varphi_1-\varphi_2)\Big] \times \int d^2\boldsymbol{y}\, f_{\delta q_1 \delta q_2}(x_1, x_2, \boldsymbol{y}) \bar{f}_{\delta\bar{q}_1 \delta\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

(transversely polarized quarks in both interactions)

- Still six different distributions for each combination of quark flavors
- Many distributions, poorly known
- Find constraints from general principles

Positivity bounds

- Project out helicity eigenstates
- Probability interpretation
- Can derive positivity bounds similar single parton distributions

(A. Bacchetta, M. Boglione, P.J. Mulders, 1999; M. Diehl, Ph. Hägler, 2005)

- Helicity Matrix positive semi-definite

$$\begin{pmatrix} f_{qq} + f_{\Delta q\Delta q} & -i|y|M f_{\delta q q} & -i|y|M f_{q\delta q} & 2y^2M^2 f^t_{\delta q\delta q} \\ i|y|M f_{\delta q q} & f_{qq} - f_{\Delta q\Delta q} & 2f_{\delta q\delta q} & -i|y|M f_{q\delta q} \\ i|y|M f_{q\delta q} & 2f_{\delta q\delta q} & f_{qq} - f_{\Delta q\Delta q} & -i|y|M f_{\delta q q} \\ 2y^2M^2 f^t_{\delta q\delta q} & i|y|M f_{q\delta q} & i|y|M f_{\delta q q} & f_{qq} + f_{\Delta q\Delta q} \end{pmatrix}$$

- Positivity bounds

$$f_{qq} \geq |f_{\delta q\delta q} - y^2 M^2 f^t_{\delta q\delta q}|$$

$$\Big(f_{qq} \pm (f_{\delta q\delta q} - y^2 M^2 f^t_{\delta q\delta q})\Big)^2 - \Big(f_{\Delta q\Delta q} \mp (f_{\delta q\delta q} + y^2 M^2 f^t_{\delta q\delta q})\Big)^2 \geq y^2 M^2 \Big(f_{\delta q q} \pm f_{q\delta q}\Big)^2$$

- Implying the weaker conditions

$$f_{qq} + f_{\Delta q\Delta q} \geq 2y^2 M^2 f^t_{\delta q\delta q}$$
$$f_{qq} - f_{\Delta q\Delta q} \geq 2 f_{\delta q\delta q}$$

Summary

- Many unexplored effects in double parton scatterings
- Longitudinal polarization changes magnitude and angular distribution
- Transverse polarization induce transverse correlations between:
  - decay planes of vector bosons
  - decay planes and direction between the two collisions (i.e. momentum of vector bosons)
- Azimuthal correlations also in collinear cross section
- Should hold also for quark initiated jets
- We have derived positivity bounds on distributions
- Work in progress:
  - Examine how evolution affects correlations