$\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$ from Grand Unified Theories

Vinzenz Maurer

UNI BASEL

Universität Basel

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Done in collaboration with Stefan Antusch, Christian Gross & Constantin Sluka
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The Reactor Angle $\theta_{13}^{\text{PMNS}}$ and Model Building

until 2011 One of Flavour Model Builders' Favourites:
Tri-Bimaximal Lepton Mixing

$$\sin^2\theta_{23}^{\text{PMNS}} = \frac{1}{2},\ \sin^2\theta_{12}^{\text{PMNS}} = \frac{1}{3},\ \theta_{13}^{\text{PMNS}} \approx 0^\circ$$

June 2011 Hints on $\theta_{13}^{\text{MNS}} \sim \theta_C$ from T2K

March 2012 Daya Bay: $\theta_{13}^{\text{MNS}} = 8.8^\circ \pm 1.0^\circ$
Striking resemblence with

$$\frac{\theta_C}{\sqrt{2}} = 9.2^\circ$$

with the Cabibbo angle θ_C

The Reactor Angle $\theta_{13}^{\text{PMNS}}$ and Model Building

Question for Model Building

What can we do with the circumstance

$$\theta_{13}^{\text{PMNS}} = 8.8^\circ \approx \frac{\theta_C}{\sqrt{2}} = 9.2^\circ?$$

Do we

(a) have to fit this relation

or

(b) get this naturally out as some model prediction?

Remarks: Related Works

Cabibbo Haze

Use of Wolfenstein parameter λ_C as expansion parameter also in U_{PMNS} leads to

$$\theta_{13}^{\text{PMNS}} = \mathcal{O}(1)\theta_C.$$

[Datta, Everett, Ramond '05]

Quark-Lepton Complementarity

Assume $U_{\text{CKM}} U_{\text{PMNS}} = U_{\text{BM}}$, so e.g.

$$\theta_{12}^{\text{PMNS}} + \theta_C = 45^\circ$$

can also have a similar effect on $\theta_{13}^{\text{PMNS}}$.

[Minakata, Smirnov '04]

This talk: Simple conditions for GUT flavour models that lead to

$$\theta_{13}^{\text{PMNS}} \approx \frac{\theta_C}{\sqrt{2}}$$

as a direct consequence.

Strategy to make $\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$ rigid

$$
\begin{tikzcd}
Y_u \arrow[r, leftrightarrow, "U_{\text{CKM}} = U_u^\dagger U_d"] & Y_d \arrow[d, leftrightarrow, "\text{GUTs}"] \\
m_\nu \arrow[r, leftrightarrow, "U_{\text{PMNS}} = U_e^\dagger U_\nu"] & Y_e
\end{tikzcd}
$$

Quick Rundown of Strategy:

- $\theta_{13}^{\text{PMNS}}$ dominantly coming from charged lepton contributions.
- θ_C mainly from mixing in Y_d.
- Y_e and Y_d are related by some minimal set of GUT field operators.

Condition I or „How to get $\sqrt{2}$?“

$$\begin{tikzcd}
Y_u \arrow[rr, leftrightarrow, "U_{\text{CKM}} = U_u^\dagger U_d"] & & Y_d \arrow[dd, leftrightarrow, "\text{GUTs}"] \\
& & \\
m_\nu \arrow[rr, leftrightarrow, "U_{\text{PMNS}} = U_e^\dagger U_\nu"] & & Y_e
\end{tikzcd}$$

- U_{PMNS} product of two unitary matrices
- Leading order in small angles [e.g. Antusch, King '05]

$$\begin{aligned}
s_{23}^{\text{PMNS}} e^{-i\delta_{23}^{\text{PMNS}}} &= s_{23}^\nu e^{-i\delta_{23}^\nu} - \theta_{23}^e c_{23}^\nu e^{-i\delta_{23}^e} \\
\theta_{13}^{\text{PMNS}} e^{-i\delta_{13}^{\text{PMNS}}} &= -\,\theta_{12}^e e^{-i\delta_{12}^e} (s_{23}^\nu e^{-i\delta_{23}^\nu} - \theta_{23}^e c_{23}^\nu e^{-i\delta_{23}^e}) \\
&\quad + \theta_{13}^\nu e^{-i\delta_{13}^\nu} - \theta_{13}^e c_{23}^\nu e^{-i\delta_{13}^e}
\end{aligned}$$

- $\theta_{13}^\nu, \theta_{13}^e \approx 0 \Rightarrow$ $\boxed{\theta_{13}^{\text{PMNS}} = \theta_{12}^e s_{23}^{\text{PMNS}}}$

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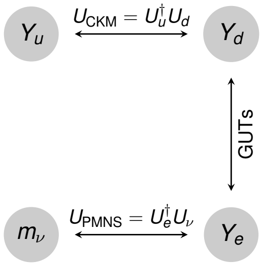
\end{aligned}$$

- $\theta_{13}^{\nu}, \theta_{13}^{e} \approx 0 \Rightarrow$ $\boxed{\theta_{13}^{\text{PMNS}} = \theta_{12}^{e} s_{23}^{\text{PMNS}}}$ with $s_{23}^{\text{PMNS}} \approx \frac{1}{\sqrt{2}}$

Condition I or „How to get $\sqrt{2}$?“

$\boxed{\theta_{13}^{\nu}, \theta_{13}^{e} \approx 0}$



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- $\theta_{13}^{\nu}, \theta_{13}^{e} \approx 0 \Rightarrow$ $\boxed{\theta_{13}^{\text{PMNS}} = \theta_{12}^{e} s_{23}^{\text{PMNS}}}$ with $s_{23}^{\text{PMNS}} \approx \frac{1}{\sqrt{2}}$

Condition II or „From where θ_C?“

$$\begin{tikzcd}[column sep=huge, row sep=huge]
Y_u \arrow[r, leftrightarrow, "U_{\text{CKM}} = U_u^\dagger U_d"] & Y_d \arrow[d, leftrightarrow, "\text{GUTs}"] \\
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\end{tikzcd}$$

GUT relations between Y_e and $Y_d \Rightarrow \theta_C$ in Y_d

- Simplest possibility: Require

$$\theta_{12}^d = \theta_C = 13^\circ, \text{ e.g. by requiring } \theta_{ij}^u \ll \theta_{12}^d.$$

- Quark mixing sum rule (with $\theta_{13}^u, \theta_{13}^d \approx 0$):

$$\theta_{12}^d \approx \left| \theta_{12}^{\text{CKM}} - \frac{\theta_{13}^{\text{CKM}}}{\theta_{23}^{\text{CKM}}} e^{-i\delta^{\text{CKM}}} \right| \approx 12.0^\circ$$

[Antusch, King, Malinsky, Spinrath '09]

Condition II or „From where θ_C?“

$$\boxed{\theta^d_{12} \approx \theta_C}$$

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$$\theta^d_{12} = \theta_C = 13^\circ, \text{ e.g. by requiring } \theta^u_{ij} \ll \theta^d_{12}.$$

- Quark mixing sum rule (with $\theta^u_{13}, \theta^d_{13} \approx 0$):

$$\theta^d_{12} \approx \left| \theta^{\mathrm{CKM}}_{12} - \frac{\theta^{\mathrm{CKM}}_{13}}{\theta^{\mathrm{CKM}}_{23}} e^{-i\delta^{\mathrm{CKM}}} \right| \approx 12.0^\circ$$

[Antusch, King, Malinsky, Spinrath ’09]

Example: $SU(5)$ with $\bar{\mathbf{5}} = (d_R^\dagger, L)$, $\mathbf{10} = (Q, u_R^\dagger, e_R^\dagger)$, $\bar{\mathbf{5}}_H = (H_d, \dots)$

- renormalizable:

$$\mathcal{L}_{\text{Yukawa}} \supset \quad y \quad \bar{\mathbf{5}}_2 \mathbf{10}_2 \bar{\mathbf{5}}_H \quad \Longrightarrow \quad y_\mu = y_s$$

- non-renormalizable:

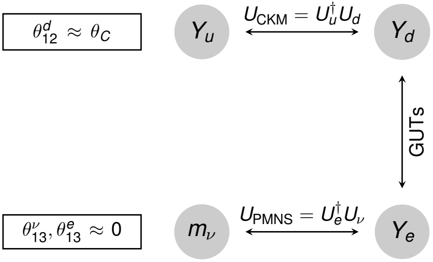
$$\mathcal{L}_{\text{Yukawa}} \supset \quad \frac{H_{\text{GUT}}}{M} \quad \bar{\mathbf{5}}_2 \mathbf{10}_2 \bar{\mathbf{5}}_H \quad \Longrightarrow \quad y_\mu = \left\{1, \tfrac{3}{2}, 6, \dots\right\} y_s$$

[Georgi, Glashow '74] [Antusch, Spinrath '09]

$\Rightarrow$ To stay predictive:

$$\boxed{\text{Only one such operator per matrix element of } Y_e/Y_d}$$

Condition IV: GUT Scale Structure of Yukawa Matrices



- Condition I: $\theta_{13}^{\text{PMNS}} = \theta_{12}^e s_{23}^{\text{PMNS}} \approx \dfrac{\theta_{12}^e}{\sqrt{2}}$
- Condition II: $\theta_{12}^d = \theta_C$
- Condition III: Predictive relation between Y_e and Y_d

$$\Rightarrow \boxed{\text{Condition IV: } \theta_{12}^d = \theta_{12}^e}$$

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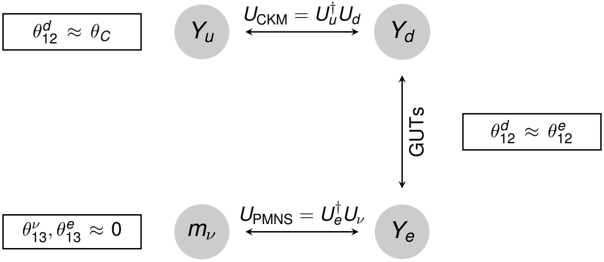

- Condition I: $\theta_{13}^{\text{PMNS}} = \theta^e_{12} s_{23}^{\text{PMNS}} \approx \dfrac{\theta^e_{12}}{\sqrt{2}}$
- Condition II: $\theta^d_{12} = \theta_C$
- Condition III: Predictive relation between Y_e and Y_d

$$\Rightarrow \boxed{\text{Condition IV: } \theta^d_{12} = \theta^e_{12}}$$

Condition IV in Pati-Salam

- Concentrate on 1-2 submatrices, neglect third generation mixing
- In Pati-Salam (PS):

$$Y_d = \begin{pmatrix} d & b \\ a & c \end{pmatrix} \Rightarrow \quad Y_e = \begin{pmatrix} c_d\, d & c_b\, b \\ c_a\, a & c_c\, c \end{pmatrix}$$

  with group-theoretical Clebsch factors c_a, c_b, c_c, c_d.
- Mixing angles at leading order:

$$\theta^d_{12} = \left|\frac{b}{c}\right|, \; \theta^e_{12} = \left|\frac{c_b}{c_c}\right| \theta^d_{12}$$

[Antusch, Maurer '11]

$$\Rightarrow \boxed{\text{Condition IV in PS: } |c_b| = |c_c|}$$

Condition IV in $SU(5)$

- In SU(5):

$$Y_d = \begin{pmatrix} d & b \\ a & c \end{pmatrix} \Rightarrow \quad Y_e = \begin{pmatrix} c_d\, d & c_a\, a \\ c_b\, b & c_c\, c \end{pmatrix}$$

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$$\theta_{12}^d = \left|\frac{b}{c}\right|, \quad \theta_{12}^e = \left|\frac{c_a}{c_c}\frac{a}{c}\right|$$

$\Rightarrow$ Have to relate a and b somehow

- Straightforward choice: Y_d, Y_e symmetric

$$\Rightarrow |a| = |b|, \ |c_a| = \boxed{|c_b| = |c_c|}$$

Blemish: Need $d \neq 0$ to get realistic mass ratios

- Second possibility is a bit less direct ...

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Condition IV in $SU(5)$, Part 2

- In SU(5):

$$Y_d = \begin{pmatrix} 0 & b \\ a & c \end{pmatrix} \Rightarrow \quad Y_e = \begin{pmatrix} 0 & c_a\, a \\ c_b\, b & c_c\, c \end{pmatrix}$$

- Mixing angles and eigenvalues at leading order:

$$\theta_{12}^d = \left|\frac{b}{c}\right|, \; \theta_{12}^e = \left|\frac{c_a}{c_c}\frac{a}{c}\right|, \; y_s = |c|, \; y_d = \left|\frac{ab}{c}\right|$$

- Yields

$$\theta_C^2 \approx \frac{1}{19 \pm 1} = \frac{m_d}{m_s} \approx \frac{y_d}{y_s} = \left|\frac{ab}{c^2}\right| = \left|\frac{a}{b}\right| \theta_C^2$$

[Gatto, Sartori, Tonin '68] [Leutwyler '96]

$$\Rightarrow |a| \approx |b|, \boxed{|c_a| = |c_c|}$$

- Only one consistent choice: $c_c = c_a = 6$, $c_b = \frac{1}{2}$.

[Antusch, Maurer '11] [Marcozza, Petcov, Romanino, Spinrath '11]

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$$\theta_C^2 \approx \frac{1}{19 \pm 1} = \frac{m_d}{m_s} \approx \frac{y_d}{y_s} \overset{\rightarrow}{=} \left| \frac{ab}{c^2} \right| = \left| \frac{a}{b} \right| \theta_C^2$$

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- Only one consistent choice: $c_c = c_a = 6$, $c_b = \frac{1}{2}$.

[Antusch, Maurer '11] [Marcozza, Petcov, Romanino, Spinrath '11]

Corrections

Several corrections to $\theta_{13}^{\text{PMNS}} = \frac{\theta_C}{\sqrt{2}}$ apply:

- Deviation of $\theta_{23}^{\text{PMNS}}$ from 45°: up to $\sim 0.8°$
- Small angle approximation: $\sim 0.6°$
- RG running of m_ν (normal hierarchy): $\leq 0.2°$ for $\tan\beta < 50$

  (inverted hierarchy): ≈ 0

  (for hierarchical neutrino masses)

- More model dependent:

  Y_ν effects in RG running, canonical normalisation

Daya Bay: $\theta_{13}^{\text{MNS}} = 8.8° \pm 1.0°$

Summary

- Only a few conditions have to be met to obtain $\theta_{13}^{\text{PMNS}} = \dfrac{\theta_C}{\sqrt{2}}$

$$
\begin{tikzcd}
\boxed{\theta_{12}^d \approx \theta_C} & Y_u \arrow[rr, leftrightarrow, "U_{\text{CKM}} = U_u^\dagger U_d"] & & Y_d \arrow[dd, leftrightarrow, "\text{GUTs}"] & \\
 & & & & \boxed{\begin{array}{c}\text{GUT operators}\\ \theta_{12}^d \approx \theta_{12}^e\end{array}} \\
\boxed{\theta_{13}^\nu, \theta_{13}^e \approx 0} & m_\nu \arrow[rr, leftrightarrow, "U_{\text{PMNS}} = U_e^\dagger U_\nu"] & & Y_e &
\end{tikzcd}
$$

- Corrections beyond leading order under control

End of the Talk

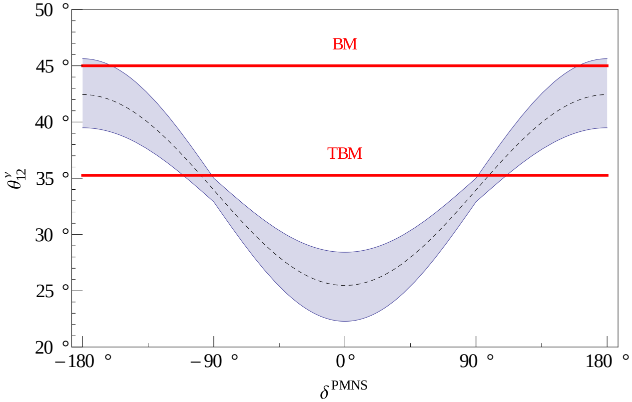
Thank you for your attention!

Using

$$\theta_{13}^{\text{PMNS}} \approx s_{23}^{\text{PMNS}} \theta_{12}^{e}$$
$$\theta_{12}^{\text{PMNS}} \approx \theta_{12}^{\nu} - s_{23}^{\text{PMNS}} \theta_{12}^{e} \cos \delta^{\text{PMNS}}$$

one can get

$$\theta_{12}^{\nu} \approx \theta_{12}^{\text{PMNS}} - \frac{\theta_C}{\sqrt{2}} \cos \delta^{\text{PMNS}}$$



Backup: Small Angle Approximation and $\theta_{12}^d \neq \theta_C$

- In SU(5):

$$Y_d = \begin{pmatrix} 0 & b \\ a & c \end{pmatrix} \Rightarrow \quad Y_e = \begin{pmatrix} 0 & 6\,a \\ \frac{1}{2}\,b & 6\,c \end{pmatrix}$$

  Fitting y_e/y_μ and θ_C to exact diagonalisation:

$$\boxed{\theta_{13}^{\text{PMNS}} = 9.8^\circ \text{ instead of } 9.2^\circ}$$

- In PS:

$$Y_d = \begin{pmatrix} d & b \\ b & c \end{pmatrix} \Rightarrow \quad Y_e = \begin{pmatrix} 9d & -3\,b \\ -3\,b & -3\,c \end{pmatrix}$$

  with Quark Mixing Sum Rule $\theta_{12}^d = \left| \theta_{12}^{\text{CKM}} - \frac{\theta_{13}^{\text{CKM}}}{\theta_{23}^{\text{CKM}}} e^{-i\delta^{\text{CKM}}} \right| = 12.0^\circ$:

$$\boxed{\theta_{13}^{\text{PMNS}} = 8.6^\circ}$$

Backup: Deviations of $\theta_{23}^{\text{PMNS}}$ from 45°

- Actual formula:

$$\theta_{13}^{\text{PMNS}} \approx \theta_C s_{23}^{\text{PMNS}}$$

Current global fit data:

$$\theta_{23}^{\text{PMNS}} = 46.1^\circ \pm 3.4^\circ$$

[Schwetz, Tortola, Valle '11]

$$\boxed{\Rightarrow \text{Introduces} \sim 6\% \text{ error in relation for } \theta_{13}^{\text{PMNS}}}$$

- Other global fits find even larger uncertainties

$$\theta_{23}^{\text{PMNS}} = 40.4^{\circ}{}^{+4.6^\circ}_{-1.6^\circ}$$

[Fogli, Lisi, Marrone, Palazzo, Rotunno '11]

$$\boxed{\Rightarrow \text{Shift for } \theta_{13}^{\text{PMNS}} \text{ of } -0.8^\circ}$$

- Reminder: $\theta_{13}^{\text{MNS}} = 8.8^\circ \pm 1.0^\circ$

Backup: Renormalisation Group Running

- Starting from

$$\theta_{13}^{\text{PMNS}} \approx \theta_C s_{23}^{\text{PMNS}}$$

at the GUT scale, the correction to the relations takes the form

$$\begin{aligned}\theta_{13}^{\text{PMNS}}|_{M_Z} &= \theta_C s_{23}^{\text{PMNS}}|_{M_{\text{GUT}}} + \Delta\theta_{13}^{\text{PMNS}} \\ &= \theta_C (s_{23}^{\text{PMNS}}|_{M_Z} - \Delta s_{23}^{\text{PMNS}}) + \Delta\theta_{13}^{\text{PMNS}}\end{aligned}$$

with $\Delta x = x|_{M_Z} - x|_{M_{GUT}}$.

- Considering strong ν-mass hierarchy $m_{\nu,\text{lightest}} = 0$, one finds

$$\text{Normal:} \quad \Delta\theta_{13}^{\text{PMNS}} \approx \theta_C \Delta s_{23}^{\text{PMNS}} + 0.2^\circ \cos(\delta^{\text{PMNS}} - \phi_2^{\text{PMNS}}) \left(\frac{\tan\beta}{50}\right)^2$$

$$\text{Inverted:} \quad \Delta\theta_{13}^{\text{PMNS}} \approx \theta_C \Delta s_{23}^{\text{PMNS}}$$

[Antusch, Kersten, Lindner, Ratz '03]

- More model dependent:

  Y_ν effects in RG running and canonical normalisation effects

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with $\Delta x = x|_{M_Z} - x|_{M_{GUT}}$.
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