Observational degeneracy between non-canonical and canonical single field inflation

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work in progress
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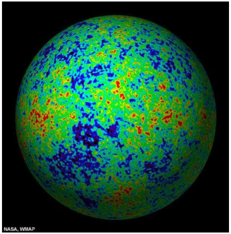
Introduction: Inflation

The ΛCDM model very successfully describes the history of the universe at least for $t > 10^{-10}\,s \Leftrightarrow E < 100\,GeV$ after the big bang.

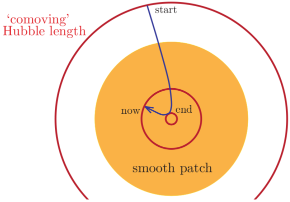
However, cosmological problems:



- Horizon problem:
  $$d_H = \int_0^a d\ln a \left(\frac{1}{aH}\right) \ll d_{\text{homogenous}}$$
- Flatness problem: $|\Omega_k| = \dfrac{1}{(aH)^2} \approx 0$

Solution: Inflation

[Guth '81], [Linde '82], [Albrecht, Steinhardt '82]



$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \;\Rightarrow\; \frac{d^2 a}{dt^2} > 0 \;\Rightarrow\; \rho + 3p < 0$$

$+$ fluctuations $\Delta T/T \sim 10^{-5}$ $+$ graceful exit.

Introduction: Generalized single field inflation

Consider scalar field ϕ coupled minimally to gravity:

$$S = \int d^4x \sqrt{g_4}\left[\mathcal{R}_4 + p(X,\phi)\right] , \quad X = -\frac{1}{2}(\partial_\mu \phi)^2 \approx \frac{1}{2}\dot{\phi}^2$$

Inflationary solution: [Garriga, Mukhanov '99]

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1,\ \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \ll 1,\ \kappa \equiv \frac{\dot{c}_s}{Hc_s} \ll 1,\ c_s^2 \equiv \left(1 + 2X\frac{\partial^2 p/\partial X^2}{\partial p/\partial X}\right)^{-1}$$

Observables at $c_s k = aH$ / $k = aH$ (horizon crossing):

$$\Delta_s^2 \sim \frac{H^2}{c_s\epsilon} , \qquad n_s - 1 = -2\epsilon - \eta - \kappa , \qquad r \sim c_s\epsilon$$

and $f_{NL}^{equilateral} \sim c_s^{-2}$ evaluated at $N = \int_{t_N}^{t_0} H(t)\,dt$

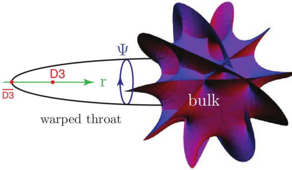
Introduction: Single field models of inflation

Canonical single field inflation:

- $p(X,\phi) = -\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)$: $\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$, $\eta = \frac{V''}{V} \ll 1$.
- $p(X,\phi) = -\frac{1}{2\phi^2}(\partial_\mu\phi)^2 - V(\phi) \Rightarrow$ Field redefinition $\psi = \ln\phi$.

Intrinsically non-canonical inflation: e.g. DBI [Alishahiha,Silverstein,Tong '04]

- $p(X,\phi) = -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X} - 1\right) - V(\phi)$
- can. limit: $f(\phi)X \ll 1$

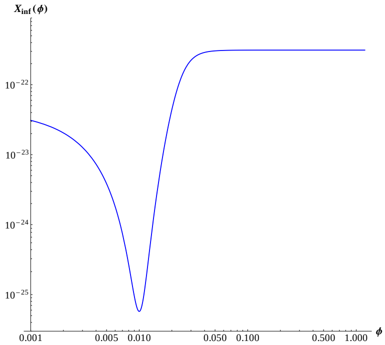


- $V_{Coul} = V_0 - T(\phi+\phi_0)^{-n}$, $V_{infl} = V_0 + \lambda(\phi-\phi_0) + \beta(\phi-\phi_0)^3$

[Baumann,Dymarsky,Kachru,Kallosh,Klebanov,Linde,Maldacena,McAllister,Murugan,Trivedi '03-'09]

From non-can. to can.: On-shell transformation

Which potential $\mathbf{V_{can}(\phi)}$ gives the same trajectory in phase space $\mathbf{X_{inf}(\phi)}$ as the non-canonical theory?



In a canonical theory:

$$-\sqrt{2X} = \dot{\phi} \simeq \frac{V'_{can}(\phi)}{3H(\phi)} \simeq \frac{V'_{can}(\phi)}{\sqrt{3V_{can}(\phi)}}$$

$$\sqrt{6X}\, d\phi = \frac{1}{\sqrt{V_{can}}}\, dV_{can}$$

$$V_{can}(\phi) = \left(\sqrt{V_{0\,can}} + \int_{\phi_0}^{\phi} \sqrt{\frac{3}{2}\, X_{inf}(\phi')}\, d\phi'\right)^2 \quad \text{with } V_{0\,can} = V_{can}(\phi_0)$$

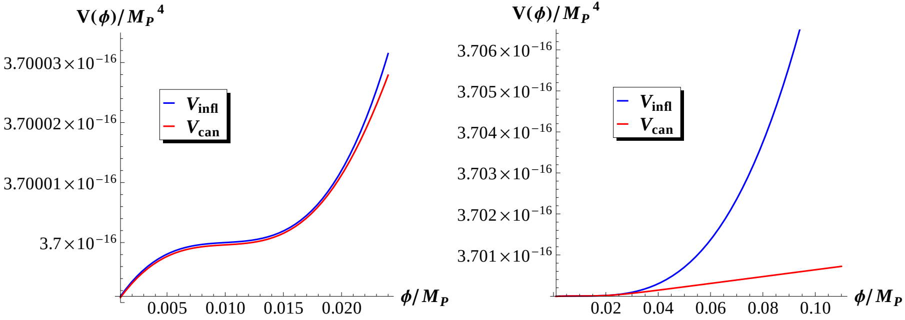
Also: Canonical transformations via generating functional $F(\phi, \tilde{\phi})$:

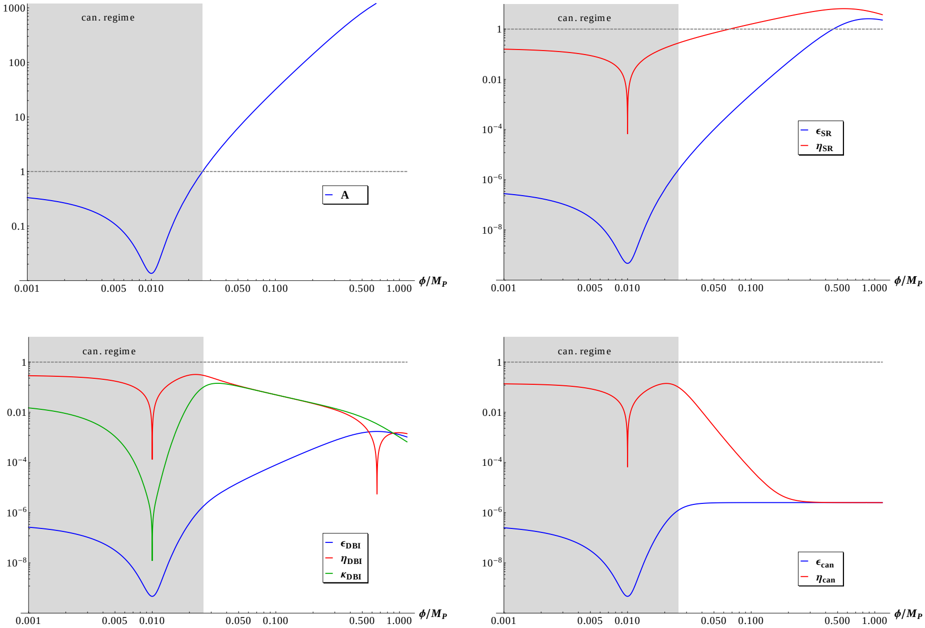
$$p = \frac{\partial F}{\partial \phi}\,, \qquad \tilde{p} = -\frac{\partial F}{\partial \tilde{\phi}} \qquad \text{(off-shell)}$$

Example: DBI + inflection point potential

Consider $V_{infl} = V_0 + \lambda(\phi - \phi_0) + \beta(\phi - \phi_0)^3$ with

$V_0 = 3.7{\cdot}10^{-16}$, $\lambda = 1.1{\cdot}10^{-20}$, $\beta = 1.1{\cdot}10^{-15}$, $\phi_0 = 0.01$, $f = (5{\cdot}10^{-6})^{-4}$




can. regime
A
ϵ_{SR}
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ϵ_{can}
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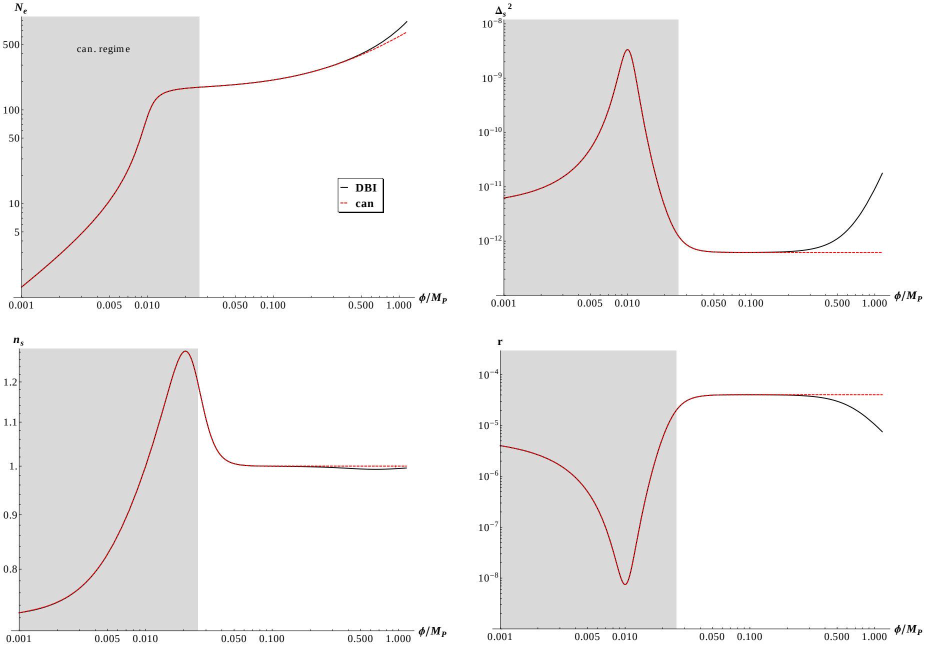


N_e
can. regime
DBI
can
ϕ/M_P
Δ_s^2
n_s
r

General understanding of degeneracy at the level of 2-point function

For $p(X,\phi) = -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X} - 1\right) - V(\phi)$ can show analytically for $A \to \infty$ + numerically for $A > 1$, e.g.:

- $\Delta^2_{s\,DBI} \simeq \Delta^2_{s\,can} \;\Leftrightarrow\; V/(c_s\epsilon_{DBI}) \simeq V_{can}/\epsilon_{can}$
- $r_{DBI} \simeq r_{can} \;\Leftrightarrow\; c_s\epsilon_{DBI} \simeq \epsilon_{can}$

if $p(X,\phi)$ has a canonical regime as long as $V(\phi) \simeq V_{can}(\phi)$.

General understanding of degeneracy at the level of 2-point function

However, consider e.g. $p(X,\phi) = \Lambda^4 \left[\left(1 + \frac{2}{3}\frac{X}{\Lambda^4} \right)^{3/2} - 1 \right] - V(\phi)$

- In this case, generically no observational degeneracy!
- $\lim_{\Lambda\to\infty} p(X,\phi) \sim \frac{1}{\Lambda^2} (\partial_\mu \phi \partial^\mu \phi)^{3/2} - V(\phi)$
- $\sqrt{1-2f\,X} \in \mathbb{R} \quad \Rightarrow f\,X \geq 1/2$

$\Rightarrow$ **When does a theory have a canonical (on-shell) description?**

Conclusions & Outlook

Conclusions:

- At the level of 2-point function observables DBI inflation can be mimicked by a canonical theory if the potential $V_{can}(\phi)$ gives the same trajectory in phase space $\{\phi, X\}$ as DBI inflation.
- This degeneracy is absent for other non-canonical theories that give rise to inflationary solutions.

Outlook:

- Non-Gaussianities: In canonical single field inflation [Maldacena '03]

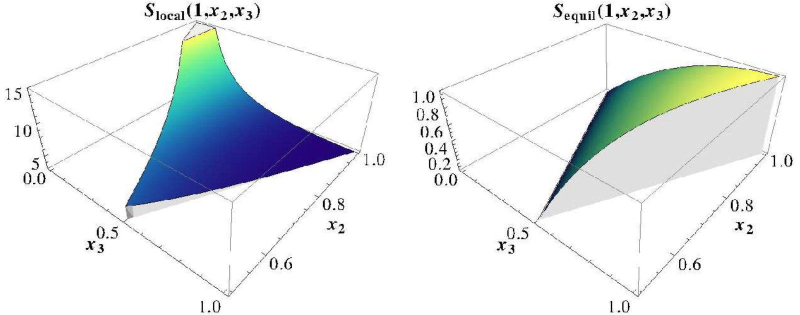
$$f_{NL} \sim \mathcal{O}(\epsilon, \eta)$$

- **Can one add features to the potential of a canonical theory that give rise to**

$$f_{NL} \sim c_s^{-2} \quad ?$$

Outlook: Non-Gaussianities

3-point function: $\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle \sim \delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\, f_{NL}\, S(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$



Use modulated potential $V(\phi) = V_0(\phi) + \sum_i A_i \cos\left(\frac{\phi + c_i}{f_i}\right)$

[Flauger,McAllister,Pajer,Westphal,Xu '09] to mimic euquilateral shape.