QCD threshold corrections for gluino pair production at NNLL

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in collaboration with Ulrich Langenfeld and Sven-Olaf Moch,
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Outline

1. Introduction: Gluino pair production at fixed order
2. Higher order partonic cross sections at the threshold
3. Hadronic cross section

1 Introduction: Gluino pair production at fixed order

2 Higher order partonic cross sections at the threshold

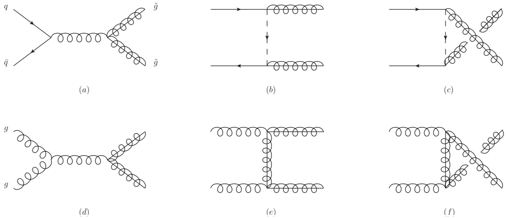
3 Hadronic cross section

Gluino production at the LHC

- The MSSM as one of the most favorite extensions of the SM provides a rich spectrum of heavy new particles.
- Hadron colliders are especially appropriated to produce color-charged particles, such as the gluino.
- The search for SUSY particles at the LHC has only produced exclusion limits so far. The limits depend on the model under consideration.
- As we hope to find evidences with an increased cms energy, a precise prediction of the production cross section near the threshold is required.
- **Known**: production cross section at NLO (fixed order) [Beenakker, Hopker, Spira, Zerwas, 1997] and the resummed threshold limit [Kulesza, Montyka, 2008], also with combined soft and Coulomb resummation at NLL [Falgari, Schwinn, Wever, 2012]
- The production of gluino-bound states has been discussed at NLO. [Kauth, Kuhn, Marquard, Steinhauser, 2011]
- The production cross section for squark antisquark is known to NNLL accuracy. [Beenakker, Brensing, Kramer, Kulesza, Laenen, 2012]

Gluino production at LO

$$
\begin{aligned}
gg &\rightarrow \tilde{g}\tilde{g}, \\
q\bar{q} &\rightarrow \tilde{g}\tilde{g}, \quad q = d, u, s, c, b \ .
\end{aligned}
$$



At NLO, also the gq channel opens. However, it is strongly suppressed near threshold.

Our assumption for the NLO calculation: All squark masses are equal.

Inclusive hadronic cross section

Recall the **standard factorization** of the **hadronic cross section**
(with the hadronic cms energy s and partonic cms energy $\hat{s}$)

$$\sigma_{pp/p\bar{p}\to\tilde{g}\tilde{g}X}(s, m_{\tilde{g}}^2, m_{\tilde{q}}^2, \mu_f^2, \mu_r^2) = \sum_{i,j=q,\bar{q},g} \int_{4m_{\tilde{g}}^2}^{s} d\hat{s}\; L_{ij}(\hat{s}, s, \mu_f^2)\; \hat{\sigma}_{ij\to\tilde{g}\tilde{g}}(\hat{s}, m_{\tilde{g}}^2, m_{\tilde{q}}^2, \mu_f^2, \mu_r^2)$$

into

- the **parton luminosity function**

$$L_{ij}(\hat{s}, s, \mu_f^2) = \frac{1}{s}\int_{\hat{s}}^{s} \frac{dz}{z} f_{i/p}\left(\mu_f^2, \frac{z}{s}\right) f_{j/p}\left(\mu_f^2, \frac{\hat{s}}{z}\right)$$

- and the **partonic cross section** assuming $\mu_r = \mu_f = \mu$ with $L_\mu = \ln\left(\mu^2/m_{\tilde{g}}^2\right)$

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_{\tilde{g}}^2}\left[f_{ij}^{(00)} + 4\pi\alpha_s\left(f_{ij}^{(10)} + f_{ij}^{(11)}L_\mu\right) + (4\pi\alpha_s)^2\left(f_{ij}^{(20)} + f_{ij}^{(21)}L_\mu + f_{ij}^{(22)}L_\mu^2\right) + \cdots\right]$$

- The scaling functions $f_{ij}^{(ab)}$ depend on the kinematic variables

$$\rho = \frac{4m_{\tilde{g}}^2}{\hat{s}}\,, \quad \beta = \sqrt{1-\rho}\,, \quad r = \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}$$

Note that for gluon fusion the functions $f_{gg}^{(00)}$ and $f_{gg}^{(11)}$ are independent of r!

1 Introduction: Gluino pair production at fixed order

2 Higher order partonic cross sections at the threshold

3 Hadronic cross section

Resummation formula

The partonic cross sections contains threshold logarithms $\ln^k(\beta)$ which stem from soft-gluon emission.

- NLO: $k \leq 2$
- NNLO: $k \leq 4$

For gluino masses of several hundred GeV, the partonic cms energy is expected to be close to the production threshold. The resulting large logarithms can be resummed to all orders in perturbation theory.

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$\longrightarrow$ classical method: **Mellin space formalism** [Sterman 1987; Catani, Trentadue, 1989]

$$\hat{\sigma}(N, m_{\tilde{g}}^2) = \int_0^1 d\rho\, \rho^{N-1}\, \hat{\sigma}(\hat{s}, m_{\tilde{g}}^2)\,, \qquad \hat{\sigma}_{ij\to\tilde{g}\tilde{g}} = \sum_{\mathrm{I}} \hat{\sigma}_{ij,\mathrm{I}}$$

- The threshold limit $\beta \to 0$ corresponds to $N \to \infty$.
- I labels the admissible $SU(3)_c$ representations of the LO scattering reactions.

$$\hat{\sigma}_{ij,\mathrm{I}}(N, m_{\tilde{g}}^2) = \hat{\sigma}^B_{ij,\mathrm{I}}(N, m_{\tilde{g}}^2)\, g^0_{ij,\mathrm{I}}(N, m_{\tilde{g}}^2)\, \exp\Big[G_{ij,\mathrm{I}}(N+1)\Big] + \mathcal{O}(N^{-1}\ln^n N)$$

Exponent of the resummation formula

$$G_{ij,\mathbf{1}}(N) = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left\{ \int_{\mu_f^2}^{4m_{\tilde{g}}^2(1-z)^2} \frac{dq^2}{q^2} \Big(A_i(\alpha_s(q^2)) + A_j(\alpha_s(q^2)) \Big) + D_{ij,\mathbf{1}}\big(\alpha_s(4m_{\tilde{g}}^2(1-z)^2)\big) \right\}.$$

where $\quad D_{ij,\mathbf{1}}(\alpha_s) = \frac{1}{2}\left(D_i(\alpha_s) + D_j(\alpha_s)\right) + D_{\tilde{g}\tilde{g},\mathbf{1}}(\alpha_s)$

Anomalous dimension functions:

- $A_{i,j}$: initial-state collinear gluon radiation (process independent)
- $D_{i,j}$: initial-state soft radiation (process independent)
- $D_{\tilde{g}\tilde{g},\mathbf{1}}$: final-state soft radiation

The process independent functions are known up to $\mathcal{O}(\alpha_s^3)$ from top-quark pair production [Kodaira, Trentadue, 1982; S. Moch, J. .A. M. Vermaseren, A. Vogt, 2004]

The functions $D_{\tilde{g}\tilde{g},\mathbf{1}}$ are known to $\mathcal{O}(\alpha_s^2)$ [Beneke, Falgari, Schwinn, 2010]

Summation of threshold logarithms

- Introduce $a_s = \alpha_s/(4\pi)$
- Defining $\lambda = a_s\,\beta_0 \ln N$, the function $G_{ij,\mathbf{I}}(N)$ can be expanded according to:

$$G_{ij,\mathbf{I}}(N) = \underbrace{\ln(N)\cdot g^1_{ij}(\lambda)}_{\text{LL}} + \underbrace{g^2_{ij,\mathbf{I}}(\lambda)}_{\text{NLL}} + \underbrace{a_s\, g^3_{ij,\mathbf{I}}(\lambda)}_{\text{NNLL}} + \dots$$

- The expansion coefficients g^k_{ij} are known up to to $k = 3$. [Moch, Uwer, 2008]

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- The explicit results involve constants γ_E, which cancel with g^0_{ij} after expansion in α_s. The appearance of γ_E can be avoided by defining

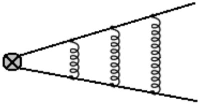
$$\widetilde{N} = N\exp(\gamma_e) \qquad \widetilde{\lambda} = a_s\,\beta_0\,\ln\widetilde{N}$$

and rearranging the sum according to

$$\boxed{G_{ij,\mathbf{I}}(N) = \ln(\widetilde{N})\cdot g^1_{ij}(\widetilde{\lambda}) + g^2_{ij,\mathbf{I}}(\widetilde{\lambda}) + a_s\,g^3_{ij,\mathbf{I}}(\widetilde{\lambda}) + \dots}$$

Coulomb corrections

• Besides threshold enhanced logarithms, one has Coulomb corrections proportional to inverse powers of β, which stem from soft gluon exchange between the final state particles (here gluinos):



• The resummation of Coulomb terms accounts for bound-state effects of the gluino pair. From NRQCD it follows that (pure) Coulomb corrections are summed to all orders by a Sommerfeld factor Δ_C: [Fadin, Khoze,Sjostrand, 1990]

$$\Delta^C = \Delta^C\left(\frac{\pi\,\alpha_s}{\beta}\,D_{\mathbf{I}}\right), \qquad \Delta^C(x) = \frac{x}{\exp(x)-1}$$

Here, $D_{\mathbf{I}} = C_{\mathbf{I}}/2 - C_A$ with $C_A = 3$ and $C_{\mathbf{I}} = \{0,3,8\}$ for $\mathbf{I} = \{\mathbf{1},\mathbf{8},\mathbf{27}\}$.

• A formal expansion in α_s reproduces the fixed order results. [Kulesza, Motyka, 2009]
For small β however, the expansion does not converge!

Hard matching constant

In Mellin space, the fixed order Coulomb terms are formally treated as part of the hard cross section, and therefore enter the hard matching constant.

$$\hat{\sigma}_{ij,\mathbf{I}}(N, m_{\tilde{g}}^2) = \hat{\sigma}^{B}_{ij,\mathbf{I}}(N, m_{\tilde{g}}^2)\; g^{0}_{ij,\mathbf{I}}(N, m_{\tilde{g}}^2)\; \exp\Big[G_{ij,\mathbf{I}}(N+1)\Big] + \mathcal{O}(N^{-1}\ln^n N)$$

The matching constant can be separated into a [Kawamura, Lo Presti, Moch, Vogt, 2012]

- hard coefficient $g^{0}_{ij,\mathbf{I}}(\alpha_s)$: Collects all constants of the fixed order result.
- Coulomb coefficient $g^{0,\,C}_{ij,\mathbf{I}}(\alpha_s, N)$: Collects all Coulomb terms and their interference with hard and soft contributions:

$$\begin{aligned} g^{0}_{ij,\mathbf{I}}(\alpha_s, N) &= g^{0}_{ij,\mathbf{I}}(\alpha_s)\, g^{0,\,C}_{ij,\mathbf{I}}(\alpha_s, N) \\ &= 1 + a_s\left(g^{0\,(1)}_{ij,\mathbf{I}} + g^{0,\,C\,(1)}_{ij,\mathbf{I}}(N)\right) \\ &\quad + a_s^2\left(g^{0\,(2)}_{ij,\mathbf{I}} + g^{0,\,C\,(2)}_{ij,\mathbf{I}}(N) + g^{0\,(1)}_{ij,\mathbf{I}}\, g^{0,\,C\,(1)}_{ij,\mathbf{I}}(N)\right) + \mathcal{O}\left(\alpha_s^3\right) \end{aligned}$$

Combined soft and Coulomb resummation

- Uses momentum-space formalism based on soft and collinear factorization in SCET. [Becher, Neubert, Pecjak, 2008]
- Threshold logarithms are summed by solving RGEs a hard function $H_{ij,\mathbf{I}}(\mu_h)$ and a soft function $W_{\mathbf{I}}(\mu_s)$, which are both evolved to the factorization scale μ_f.
- Effects from the non-relativistic potential are obtained by means of NRQCD and written in a potential function $J_{ij,\mathbf{I}}(E)$, $E = \sqrt{\hat{s}} - 2m_{\tilde{g}}$, which sums Coulomb exchange [Beneke, Falgari, Schwinn, 2010; 2011]
- For NNLL resummation, one requires the color-decomposed hard function at NLO.

$\longrightarrow$ See aslo talks of C. Schwinn and C. Wever.

NLO scaling functions near threshold

- Near the threshold ($\beta < 1$), the (color-summed) NLO functions can be factorized according to:

$$f_{ij}^{(10)} = f_{ij}^{(00)} \times \left(A \ln^2(\beta) + B \ln(\beta) + \frac{C}{\beta} + C_1^{ij} + \mathcal{O}(\beta) \right)$$

- At NLO it is also common to write $\ln(8\beta^2)$ instead of $\ln(\beta)$. Of course, there is also the freedom to pull a constant out of the parenthesis.
- Having specified the convention, the object of interest is the matching coefficient C_1^{ij}, which has to be determined from a dedicated one-loop calculation, where higher powers in β can be skipped near the threshold.
- The color-summed scaling functions of the full NLO cross section are known for a long time and have been implemented in the code `Prospino`. [Beenakker, Höpker, Spira, 1996]
- The matching constant can be extracted numerically from `Prospino` by a fit in the threshold region.

Color-decomposed NLO scaling functions

• The color decomposition for gluino pair production has not been given so far. However, there are analytic results for the decomposed NLO cross section of gluino-bound state production. [Kauth, Kuhn, Marquard, Steinhauser, 2011]

• Omitting corrections of $\mathcal{O}(\beta)$, the hard function factorizes near the threshold into the Born term and the infrared-finite parts of the UV-regularized virtual and real quantum corrections: $(z = M^2/\hat{s})$

$$\mathcal{F}_{ij\to T,\mathbf{I}} = \mathcal{F}^{\text{Born}}_{ij\to T,\mathbf{I}} \left(1 + \frac{\alpha_s(\mu_r)}{\pi}\,\overline{\mathcal{V}}_{ij,\mathbf{I}}\right)\left[\delta(1-z) + \frac{\alpha_s(\mu_r)}{\pi}\,\overline{\mathcal{R}}_{ij,\mathbf{I}}(z)\right]$$

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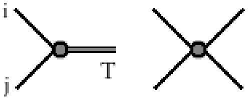
Note that the difference between the $2 \to 1$ and $2 \to 2$ kinematics (labeled by the final state T) only comes from the Born Term!



$\Rightarrow$ The hard kernel for gluino pair production is obtained by $\mathcal{F}^{\mathrm{Born}}_{ij\to T,\mathbf{I}} \to \hat{\sigma}^{B}_{ij,\mathbf{I}}$

NLO results and one-loop matching constants

- gg-channel: $\mathbf{8} \times \mathbf{8} = \mathbf{1}_s + \mathbf{8}_s + \mathbf{8}_a + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}_s$

  (Only symmetric color configurations $\mathbf{I} = \mathbf{1}_s, \mathbf{8}_s, \mathbf{27}_s$ contribute to first approx. near threshold.)

$$f^{(10)}_{gg,\,\mathbf{I}} = \frac{f^{(00)}_{gg,\,\mathbf{I}}}{4\pi^2}\left(6\ln^2\left(8\beta^2\right) - (24 + C_{\mathbf{I}})\ln\left(8\beta^2\right) - \frac{\pi^2}{2\beta}D_{\mathbf{I}} + C^{gg}_{1,\,\mathbf{I}}\right)$$

$$C^{gg\,\overline{\mathrm{MS}}}_{1,\,\mathbf{I}} = C_{\mathbf{I}}\left(4 + \ln(2) - \frac{\pi^2}{8}\right) + 36 - 6\ln^2(2) - \pi^2 + \frac{n_f}{18}A^{gg}_{\mathbf{I}}(r)$$

$$A^{gg}_{\mathbf{I}}(r) = -9\left(b_1(r) - b_4(r) + 2\,b_1'(r)\right)(1-r) + 2\left(9r - (C_{\mathbf{I}}+1)\right)c_5(r)$$

Scalar n-point integrals $b_1(r)$, etc. defined in [Kauth, Kuhn, Marquard, Steinhauser, 2011]

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- $q\bar{q}$-channel: $\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1}_s + \mathbf{8}_s + \mathbf{8}_a$ (Only the antisymmetric octet $\mathbf{I} = \mathbf{8}_a$ contributes.)

$$f^{(10)}_{q\bar{q},\,\mathbf{8}_a} = \frac{f^{(00)}_{q\bar{q},\,\mathbf{8}_a}}{4\pi^2}\left(\frac{8}{3}\ln^2\left(8\beta^2\right) - \frac{41}{3}\ln\left(8\beta^2\right) + \frac{3\pi^2}{4\beta} + C^{q\bar{q}}_{1,\,\mathbf{8}_a}\right)$$

$$C^{q\bar{q}\,\overline{\mathrm{MS}}}_{1,\,\mathbf{8}_a} = n_f\left(\ln(2) - \frac{5}{9}\right) + \frac{181}{6} - \frac{8}{3}\ln^2(2) - \frac{43}{36}\pi^2 - \frac{1}{3}\ln\left(\frac{m_t^2}{m_{\tilde{g}}^2}\right) - \frac{n_f}{6}\ln(r) + A^{q\bar{q}}_{\mathbf{8}_a}(r)$$

$$A^{q\bar{q}}_{\mathbf{8}_a}(r) = \cdots \quad \text{lengthy expression; involves more } n\text{-point integrals}$$

Cross check

- We have calculated analytic expressions for the required scalar one-, two-, and three-point integrals $a_1(r)$, $b_i(r)$, $b_i'(r)$, $c_i(r)$.

- The one-loop matching constants $C^{ij}_{1,\mathbf{I}}$ depend on the chosen renormalization scheme. Kauth. et. al use $\overline{\text{DR}}$(full MSSM). Beenakker et. al. use $\overline{\text{MS}}(n_l = 5)$, which is also implemented in `Prospino`. In order to cross check our (color-summed) analytic results with the numerical output of `Prospino`, we had to decouple the heavy particles and switch the renormalization scheme.

- Comparison with `Prospino`:
  - gg-channel: relative numerical agreement at the per mill level!
  - $q\bar{q}$-channel: deviations of a few per cent due to a different treatment of the top mass (only logarithms are kept by Kauth et. al.).

- The function $f^{(11)}_{ij}$ on the other hand directly follows from the running of α_s and the parton evolution and is computed by a Mellin convolution:

$$f^{(11)}_{ij} = \frac{1}{16\pi^2}\left(2\beta_0 f^{(00)}_{ij} - f^{(00)}_{kj,\mathbf{I}} \otimes P^{(0)}_{ki} - f^{(00)}_{ik} \otimes P^{(0)}_{kj}\right) \quad \text{where} \quad P_{ij}(x) = \alpha_s P^{(0)}_{ij}(x) + \alpha_s^2 P^{(1)}_{ij}(x) + \ldots$$

.

Approximated NNLO

- The NNLO cross section in the threshold approximation can be constructed by expanding the r.h.s. of the general resummation formula in α_s, and changing to β-space by an inverse Mellin transformation.
- The general answer for an arbitrary color-representation has already been given in the literature as a function of the one-loop matching constant, including non-relativistic kinetic energy corrections, which depend on the spin configuration. [Beneke, Czakon, Falgari, Mitov, Schwinn, 2010]
- Now we know the explicit result for gluino-pair production!
- The scale-dependent part again follows from the parton evolution:

[van Neerven, A. Vogt, 2000; Kidonakis, Laenen, Moch, R. Vogt, 2001]

$$f_{ij}^{(21)} = \frac{1}{(16\pi^2)^2}\left(2\beta_1 f_{ij}^{(00)} - f_{kj}^{(00)} \otimes P_{ki}^{(1)} - f_{ik}^{(00)} \otimes P_{kj}^{(1)}\right) + \frac{1}{16\pi^2}\left(3\beta_0 f_{ij}^{(10)} - f_{kj}^{(10)} \otimes P_{ki}^{(0)} - f_{ik}^{(10)} \otimes P_{kj}^{(0)}\right)$$

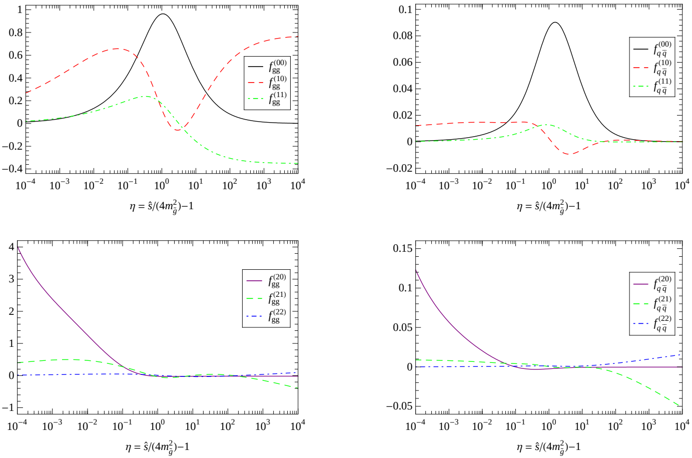
$$f_{ij}^{(22)} = \frac{1}{(16\pi^2)^2}\left(f_{kl}^{(00)} \otimes P_{ki}^{(0)} \otimes P_{lj}^{(0)} + \frac{1}{2} f_{in}^{(00)} \otimes P_{nl}^{(0)} \otimes P_{lj}^{(0)} + \frac{1}{2} f_{nj}^{(00)} \otimes P_{nk}^{(0)} \otimes P_{ki}^{(0)} + 3\beta_0^2 f_{ij}^{(00)} - \frac{5}{2}\beta_0 f_{ik}^{(00)} \otimes P_{kj}^{(0)} - \frac{5}{2}\beta_0 f_{kj}^{(00)} \otimes P_{ki}^{(0)}\right)$$

$$f^{(20)}_{gg,\mathbf{I}} = \frac{f^{(00)}_{gg,\mathbf{I}}}{(16\pi^2)^2}\Bigg[\frac{4D_{\mathbf{I}}^2\pi^4}{3\beta^2} + \frac{D_{\mathbf{I}}\pi^2}{\beta}\Big\{-192\ln^2(\beta) + \Big(44 + 16C_{\mathbf{I}} - \frac{8}{3}n_l - 192\ln(2)\Big)\ln(\beta) - 8C^{gg}_{1,\mathbf{I}}$$
$$+\frac{1090}{3} + 16C_{\mathbf{I}} + \frac{4}{3}n_l\Big(\frac{5}{3} - 2\ln(2)\Big) + 44\ln(2) + 8C_{\mathbf{I}}\ln(2) - 48\ln^2(2) - 16\pi^2\Big\} + 4608\ln^4(\beta)$$
$$+\Big\{-19840 - 768C_{\mathbf{I}} + \frac{256}{3}n_l + 27648\ln(2)\Big\}\ln^3(\beta) + \Big\{384\,C^{gg}_{1,\mathbf{I}} + 43232 + 1712C_{\mathbf{I}} + 32C_{\mathbf{I}}^2$$
$$+n_l\Big(-\frac{1088}{3} - \frac{32C_{\mathbf{I}}}{3} + 384\ln(2)\Big) - 89280\ln(2) - 3456C_{\mathbf{I}}\ln(2) + 62208\ln^2(2) - 2400\pi^2\Big\}\ln^2(\beta)$$
$$+\Big\{-\frac{262624}{3} - \frac{6584}{3}C_{\mathbf{I}} + \Big(-768 - 32C_{\mathbf{I}} + 1152\ln(2)\Big)C^{gg}_{1,\mathbf{I}}$$
$$+n_l\Big(\frac{6976}{9} + \frac{368}{9}C_{\mathbf{I}} - 1088\ln(2) - 32C_{\mathbf{I}}\ln(2) + 576\ln^2(2) - 32\pi^2\Big) + 129696\ln(2)$$
$$+5136C_{\mathbf{I}}\ln(2) + 96C_{\mathbf{I}}^2\ln(2) - 133920\ln^2(2) - 5184C_{\mathbf{I}}\ln^2(2) + 62208\ln^3(2) + 5328\pi^2$$
$$+200C_{\mathbf{I}}\pi^2 - 7200\ln(2)\pi^2 + 33264\zeta_3 - 48C_{\mathbf{I}}\zeta_3 + 16\pi^2D_{\mathbf{I}}\Big(3 - 2D_{\mathbf{I}}(1 + v_{\mathrm{spin}})\Big)\Big\}\ln(\beta) + C^{gg}_{2,\mathbf{I}}\Bigg]$$

$$f^{(20)}_{q\bar{q},\mathbf{8}_a} = \frac{f^{(00)}_{q\bar{q}}}{(16\pi^2)^2}\Bigg[\frac{3\pi^4}{\beta^2} + \frac{\pi^2}{\beta}\Big\{128\ln^2(\beta) + \Big(-138 + 4n_l + 128\ln(2)\Big)\ln(\beta) + 12\,C^{q\bar{q}}_{1,\mathbf{8}_a} - 297 + n_l\Big(-\frac{10}{3} + 4\ln(2)\Big)$$
$$-102\ln(2) + 32\ln^2(2) + \frac{32\pi^2}{3}\Big\} + \frac{8192}{9}\ln^4(\beta) + \frac{512}{27}\Big\{-279 + 288\ln(2) + 2n_l\Big\}\ln^3(\beta) + \Big\{12976 + \frac{512}{3}C^{q\bar{q}}_{1,\mathbf{8}_a}$$
$$+n_l\Big(-\frac{5216}{27} + \frac{512}{3}\ln(2)\Big) - 23808\ln(2) + 12288\ln^2(2) - \frac{4480}{9}\pi^2\Big\}\ln^2(\beta) + \Big\{\Big(-\frac{1312}{3} + 512\ln(2)\Big)C^{q\bar{q}}_{1,\mathbf{8}_a}$$
$$-\frac{667624}{27} + n_l\Big(\frac{37840}{81} - \frac{5216}{9}\ln(2) + 256\ln^2(2) - \frac{128}{9}\pi^2\Big) + 38928\ln(2) - 35712\ln^2(2) + 12288\ln^3(2)$$
$$+\frac{13592}{9}\pi^2 - \frac{4480}{3}\ln(2)\pi^2 + \frac{60080}{9}\zeta_3 + 16\pi^2D_{\mathbf{I}}\Big(3 - 2D_{\mathbf{I}}(1 + v_{\mathrm{spin}})\Big)\Big\}\ln(\beta) + C^{q\bar{q}}_{2,\mathbf{8}_a}\Bigg]$$

Exact scaling functions (except for $f_{ij}^{(20)}$)

$$\hat{\sigma}_{ij} = \frac{\alpha_s^2}{m_{\tilde{g}}^2}\left[f_{ij}^{(00)} + 4\pi\alpha_s\left(f_{ij}^{(10)} + f_{ij}^{(11)} L_\mu\right) + (4\pi\alpha_s)^2\left(f_{ij}^{(20)} + f_{ij}^{(21)} L_\mu + f_{ij}^{(22)} L_\mu^2\right) + \cdots\right], \quad m_{\tilde{g}} = 750 \text{ GeV}, \quad r = 0.64$$



Hard matching coefficients

Finally, the general resummation formula is expanded to NNLL and matched onto the approximated NNLO cross section. Working in $\overline{\text{MS}}(n_l = 5)$ we find:

$$
\begin{aligned}
g^0_{gg,\,\mathbf{1}} &= 1 + a_s\left\{4C^{gg}_{1,\,\mathbf{1}} - 192 + C_{\mathbf{1}}\left(-8 - 4\ln(2)\right) + 24\ln^2(2) + 12\pi^2\right\} \\
&+ a_s^2\left\{16C^{gg}_{1,\,\mathbf{1}}\left(-48 + 2C_{\mathbf{1}}\left(-1+\ln(2)\right) + 72\ln(2) - 48\ln^2(2) + 3\pi^2\right) + C_{\mathbf{1}}\left(\frac{8}{3}\left(-823 + 1465\ln(2)\right)\right.\right. \\
&\quad \left. - 4000\ln^2(2) + 2496\ln^3(2) + 126\pi^2 - 344\ln(2)\pi^2 + \left(1296 + 48\ln(2)\right)\zeta_3\right) + C_{\mathbf{1}}^2\left(32\ln(2) - 64\ln^2(2) + 4\pi^2\right) \\
&\quad + \frac{4}{9}n_l\left(C_{\mathbf{1}}\left(92 - 116\ln(2) + 48\ln^2(2) - 3\pi^2\right) + 1744 - 2560\ln(2) + 1776\ln^2(2) - 624\ln^3(2) - 102\pi^2\right. \\
&\quad \left. + 108\ln(2)\pi^2 - 336\zeta_3\right) + \frac{32}{3}\left(-8207 + 12260\ln(2)\right) - 101344\ln^2(2) + 66784\ln^3(2) - 23040\ln^4(2) \\
&\quad + 3292\pi^2 - 7992\ln(2)\pi^2 + 5664\ln^2(2)\pi^2 + 204\pi^4 + \left(35728 - 49392\ln(2)\right)\zeta_3 + L_\mu\left(192C^{gg}_{1,\,\mathbf{1}}\left(-1+\ln(2)\right)\right. \\
&\quad + C_{\mathbf{1}}\left(-472 + 664\ln(2) - 576\ln^2(2) + 48\pi^2\right) - \frac{8}{3}n_l\left(-68 + 2C_{\mathbf{1}}\left(-1+\ln(2)\right) + 92\ln(2) - 48\ln^2(2) + 3\pi^2\right) \\
&\quad \left. - 21616 + 31888\ln(2) - 26304\ln^2(2) + 10368\ln^3(2) + 1332\pi^2 - 1776\ln(2)\pi^2 + 8064\zeta_3\right) \\
&\quad \left. + L_\mu^2\left(-16\left(-1+\ln(2)\right)n_l - 2568 + 2568\ln(2) - 1152\ln^2(2) + 144\pi^2\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
g^{0}_{q\bar{q},\,\mathbf{8}_a} &= 1 + a_s \left\{ 4C^{q\bar{q}}_{1,\,\mathbf{8}_a} + \frac{4}{3}\left(-82 - 9\ln(2) + 8\ln^2(2) + 4\pi^2\right) + L_\mu \left(14 - \frac{4}{3}n_l\right) \right\} \\
&+ a_s^2 \Big\{ -\frac{32}{3} C^{q\bar{q}}_{1,\,\mathbf{8}_a}\left(41 - 57\ln(2) + 32\ln^2(2) - 2\pi^2\right) + \frac{4}{81}n_l \Big(9460 - 13372\ln(2) + 8400\ln^2(2) \\
&- 2496\ln^3(2) - 489\pi^2 + 432\ln(2)\pi^2 - 1344\zeta_3\Big) - 29920\ln^2(2) + \frac{8}{27}(-83453 + 127247\ln(2)) \\
&+ \frac{2}{9}\left(79424\ln^3(2) - 20480\ln^4(2) + 5167\pi^2 - 10428\ln(2)\pi^2 + 5248\ln^2(2)\pi^2 + 168\pi^4\right) \\
&+ \frac{16}{3}(1793 - 1849\ln(2))\zeta_3 + L_\mu \Big(\frac{256}{3} C^{q\bar{q}}_{1,\,\mathbf{8}_a}(-1 + \ln(2)) + \frac{16}{27} n_l \left(409 - 553\ln(2) + 288\ln^2(2) - 18\pi^2\right) \\
&+ \frac{8}{9}\left(-8283 + 11819\ln(2) - 8640\ln^2(2) + 2304\ln^3(2) + 502\pi^2 - 408\ln(2)\pi^2 + 1792\zeta_3\right)\Big) \\
&- L_\mu^2 \frac{32}{9}\left(10n_l(-1 + \ln(2)) + 245 + 64\ln^2(2) - 245\ln(2) - 8\pi^2\right)\Big\}
\end{aligned}
$$

The formula for Coulomb terms (also including non-Coulomb kinetic energy corrections) holds for both production channels:

$$
\begin{aligned}
g^{0,\,C}_{ij,\,\mathbf{I}}(N) \quad = \quad & 1 - a_s\, 4D_{\mathbf{I}}\pi^2\sqrt{\frac{N}{\pi}} + a_s^2 \Big\{ -\frac{8}{3}D_{\mathbf{I}}^2\pi^4 N + D_{\mathbf{I}}\pi^2\sqrt{\frac{N}{\pi}}\Big(\ln(\widetilde{N})\left(-44 + \frac{8}{3}n_l\right) \\
& -\frac{124}{3} + 88\ln(2) + \frac{8}{9}n_l\left(5 - 6\ln(2)\right) - L_\mu\left(44 - \frac{8}{3}n_l\right)\Big) \\
& + 16\pi^2 D_{\mathbf{I}}\left(3 - 2D_{\mathbf{I}}(1 + v_{\mathrm{spin}})\right)\left(1 - \ln(2) - \frac{1}{2}\ln(\widetilde{N})\right)\Big\}\,.
\end{aligned}
$$

Hard function in the momentum space formalism

- Result from last weak, not published yet.
- $\mu_h = \mu_f = \mu$, and $L_\mu = \left(\mu^2/m_{\tilde{g}}^2\right)$

$$H_{gg,\,\mathbf{1}}(\mu)/H^{(0)}_{gg,\,\mathbf{1}}(\mu) = 1 + a_s\Bigg(4\,C^{gg}_{1,\,\mathbf{1}} - 12\,C_{\mathbf{1}} + C_A\left(-64 + \frac{11\pi^2}{3}\right) - \Big(2\,C_{\mathbf{1}} - 8\,C_A\ln(2)\Big)L_\mu - 2\,C_A L_\mu^2\Bigg) + \mathcal{O}(a_s^2)$$

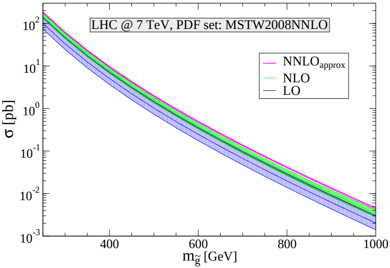
$$H_{q\bar{q},\,\mathbf{8}_a}(\mu)/H^{(0)}_{q\bar{q},\,\mathbf{8}_a}(\mu) = 1 + a_s\Bigg(4\,C^{q\bar{q}}_{1,\,\mathbf{8}_a} - 12\,C_A + C_F\left(-64 + \frac{11\pi^2}{3}\right) - \Big(2\,C_A - 8\,C_F\ln(2) - 2\beta_0 + 6\,C_F\Big)L_\mu - 2\,C_F L_\mu^2\Bigg) + \mathcal{O}(a_s^2)$$

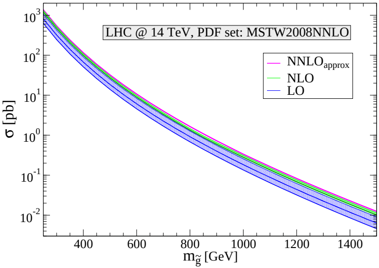
1 Introduction: Gluino pair production at fixed order

2 Higher order partonic cross sections at the threshold

3 Hadronic cross section

Fixed order cross sections

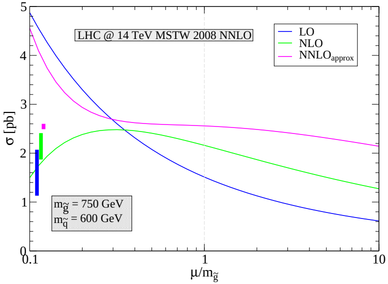


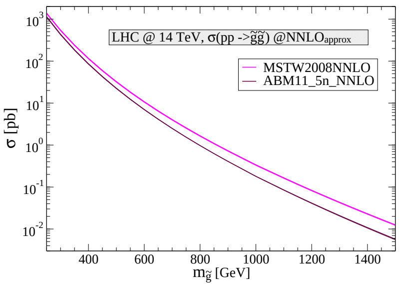


$\overline{\mathrm{MS}}(n_l = 5)$ scheme, $m_{\tilde{q}} = 4/5 m_{\tilde{g}}$, error bands correspond to a variation $m_{\tilde{g}}/2 \leq \mu \leq 2m_{\tilde{g}}$;
We have also included the threshold-suppressed gq production channel at NLO.

- We observe an increase in the predicted rates due to the approximate NNLO corrections of the order of $\mathcal{O}(15-20)\%$
  $\Rightarrow$ More stringent bounds on sparticle masses?
- As we will see below, the biggest uncertainty comes from the choice of the PDF set and the related value of $\alpha_s(M_Z)$.

Fixed order scale uncertainty and PDF comparison





- Vertical bars: total scale variation in the range $[m_{\tilde{g}}/2, 2m_{\tilde{g}}]$
- Vertical dashed gray line: cross section at the nominal scale $\mu = m_{\tilde{g}}$.

$$\sigma(\mu = m_{\tilde{g}}) = \{1.43^{+0.53}_{-0.37}\,\mathrm{pb},\ 2.16^{+0.25}_{-0.29}\,\mathrm{pb},\ 2.56^{+0.04}_{-0.07}\,\mathrm{pb}\} \quad \text{for LO, NLO, NNLO}$$

- $K_{\mathrm{NLO}} = \sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}} = 1.46, \quad K_{\mathrm{NNLO}} = \sigma_{\mathrm{NNLO}}/\sigma_{\mathrm{NLO}} = 1.13$

Gluino pair-production cross section with $\sqrt{s} = 7\,\mathrm{TeV}$

$m_{\tilde{g}}$	σ(LO)[pb]			σ(NLO)[pb]			σ(NNLO)[pb]		
[GeV]	$x = \frac{1}{2}$	$x = 1$	$x = 2$	$x = \frac{1}{2}$	$x = 1$	$x = 2$	$x = \frac{1}{2}$	$x = 1$	$x = 2$
MSTW 2008 NNLO									
300	46.839	33.335	24.383	57.703	51.178	43.701	62.941	61.264	58.908
400	7.230	5.080	3.673	8.986	7.919	6.707	9.814	9.591	9.207
500	1.475	1.026	0.735	1.855	1.624	1.365	2.029	1.990	1.906
600	0.359	0.247	0.176	0.457	0.398	0.332	0.501	0.493	0.471
700	0.098	0.067	0.047	0.127	0.110	0.091	0.140	0.138	0.131
800	0.029	0.020	0.014	0.038	0.033	0.027	0.042	0.042	0.040
ABM NNLO									
300	29.433	21.365	15.930	36.863	32.778	28.213	40.176	39.540	38.548
400	4.012	2.916	2.176	5.053	4.493	3.869	5.507	5.485	5.388
500	0.739	0.539	0.403	0.933	0.831	0.717	1.017	1.025	1.015
600	0.165	0.121	0.090	0.209	0.187	0.161	0.228	0.233	0.232
700	0.042	0.031	0.023	0.054	0.048	0.041	0.058	0.060	0.060
800	0.012	0.009	0.007	0.015	0.014	0.012	0.017	0.017	0.017

$x = \mu / m_{\tilde{g}}$

Differences between the PDF sets ABM11 and MSTW amount to the order of $\mathcal{O}(30 - 60)\%$! This issue is not treated in current experimental analysis!

Summary

- We have derived analytic results for the color-decomposed NLO and approximated NNLO cross section of gluino-pair production near the threshold.
- The resummation of threshold logarithms has been performed in Mellin space to NNLL accuracy. Alternatively, one could resum in β-space [Becher, Neubert, 2006], where general results for a combined soft- and Coulomb resummation are given in the literature. [Beneke, Falgari, Schwinn, 2010]
- We have investigated the phenomenological implications by a calculation of the approximated NNLO hadronic production cross section at fixed order in perturbation theory.
  A further phenomenology paper with results from resummation performed in both approaches is in preparation.
- At the scale $\mu = m_{\tilde{g}}$, the approximated NNLO cross section increases the NLO result of about $15 - 20\%$.
- The results are rather stable in the range $\mu \in [\frac{1}{2} m_{\tilde{g}}, 2m_{\tilde{g}}]$.
- The dominant source of uncertainty comes from the non-perturbative input. This should be kept in mind in forthcoming investigations.

Thank you for your attention!