Slow-Walking Inflation

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Slow-Roll Inflation

Standard scenario: scalar field ϕ slowly rolling down its potential

$$ds^2 \sim -dt^2 + a(t)^2 d\vec{x}^2 \quad \& \quad \phi(t,\vec{x}) \sim \phi(t)$$

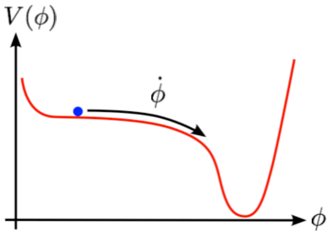
Slow-Roll Approximation

$$3\,\mathcal{H}\,\dot{\phi} \approx -V' \quad \& \quad \mathcal{H}^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \approx \frac{V(\phi)}{3M_p^2} \sim \text{const} \quad \Rightarrow a(t) \sim e^{\mathcal{H}t}$$

Justified if

$$\epsilon = \frac{M_p^2}{2}\frac{V'^2}{V^2} \ll 1$$

$$\eta = M_p^2 \frac{V''}{V} \sim \frac{m_\phi^2}{\mathcal{H}^2} \ll 1$$



η-problem

Large sensitivity to Planck-scale physics

$$\delta V \sim \mathcal{O}_4 \frac{\phi^2}{M_p^2} \quad \& \quad \langle \mathcal{O}_4 \rangle \sim \langle V \rangle \quad \Rightarrow \quad \eta \sim \mathcal{O}(1)$$

e.g. F-term inflation

$$V_F = e^{K/M_p^2} \left(K^{i\bar{\jmath}} D_i W \, D_{\bar{\jmath}} \overline{W} - 3 \, \frac{|W|^2}{M_p^2} \right) \; , \; D_i W = W_i + \frac{K_i W}{M_p^2}$$

with $K = |\phi|^2 + |X|^2 + \ldots \; \& \; F_X \neq 0$

Copeland, Liddle, Lyth, Stewart, Wands '94; Dine, Randall, Thomas '95

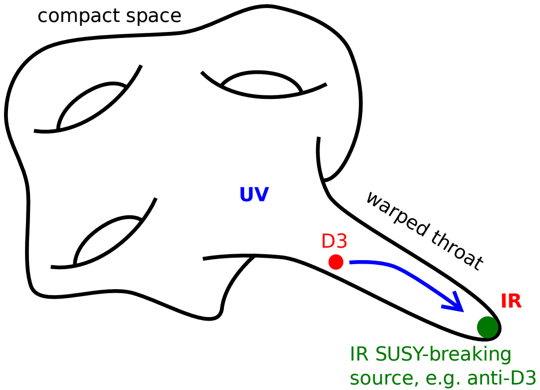
$$V_F \sim |F_X|^2 \left(1 + \frac{|\phi|^2}{M_p^2} + \ldots \right) \Rightarrow \eta \sim 1 + \ldots$$

Solution: fine-tune against dots or impose symmetry

Warped D3-brane Inflation

Probe D3-brane moving in a warped throat ($\sim AdS_5 \times X_5$)

e.g. Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi '03



IR deformation of Klebanov-Strassler dual to walking gauge theory

Warped Type IIB Backgrounds

Consider backgrounds of the form (string frame metric, $\alpha' g_s \equiv 1$)

$$
\begin{aligned}
ds^2_{st} &= h^{-\frac{1}{2}} e^{\Phi} ds^2_{1,3} + k_1 h^{\frac{1}{2}} e^{\Phi} \left(e^{2k} d\rho^2 + A^2(\rho) d\Omega_5^2 \right) \\
\mathcal{C}_4 &= h^{-1} e^{2\Phi}\, dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \\
F_5 &= (1 + \star)\, d\mathcal{C}_4 \\
F_3 &= N_c\, (\dots) \neq 0 \\
H_3 &= \dots \neq 0 \\
\Phi &= \Phi(\rho) \quad , \quad h(\rho) = k_1^{-2} \left(1 - e^{2\Phi(\rho)} \right)
\end{aligned}
$$

All functions depend on radial coordinate ρ only & we require

1. Φ bounded from above & monotonically increasing, $\Phi(\infty) = 0$
2. Approximate Klebanov-Strassler for $\rho \to \infty$ & $h(\rho_{uv}) \simeq 1$

Master Equation

Rearrange EoMs into a master equation Hoyos-Badajoz, Nunez, Papadimitriou '08; Maldacena, Martelli '09; Gaillard, Martelli, Nunzez, Papadimitriou '11; Caceres, Nunez, Pando-Zayas '11

$$P'' + P'\left(\frac{P'+Q'}{P-Q} + \frac{P'-Q'}{P+Q} - 4\coth(2\rho)\right) = 0$$

$$Q = N_c\left(2\rho\coth(2\rho) - 1\right), \quad N_c \sim \int_{S^3} F_3$$

and a set of algebraic equations

$$e^{4\Phi} = \frac{2\, e^{4\Phi_0}\sinh^2(2\rho)}{(P^2-Q^2)P'}, \quad e^{2k} = \frac{P'}{2}, \quad h = k_1^{-2}\left(1 - e^{2\Phi}\right), \quad \dots$$

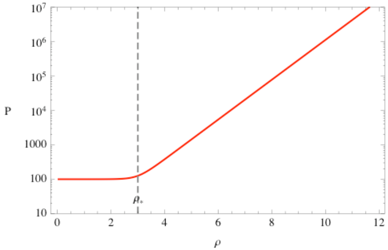
(Φ_0 & k_1 fixed by $\Phi(\infty) = 0$ & $h(\rho_{uv}) \simeq 1$)

Typical Walking Solutions

Hoyos-Badajoz, Nunez, Papadimitriou '08; Nunez, Papadimitriou, Piai '08; Elander, Gaillard, Nunez, Piai '11

$$P \sim \begin{cases} c_+ e^{4\rho/3} & \text{for } \rho \gtrsim \rho_* \\ c_0 \simeq c_+ e^{4\rho_*/3} & \text{for } \rho \lesssim \rho_* \end{cases}$$

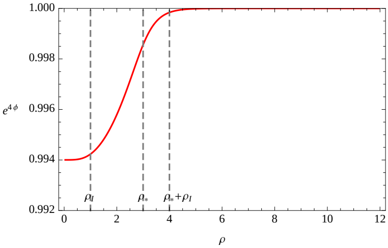
c_+, ρ_*: integration constants



VEVs of dim. 2 & 6 Operators

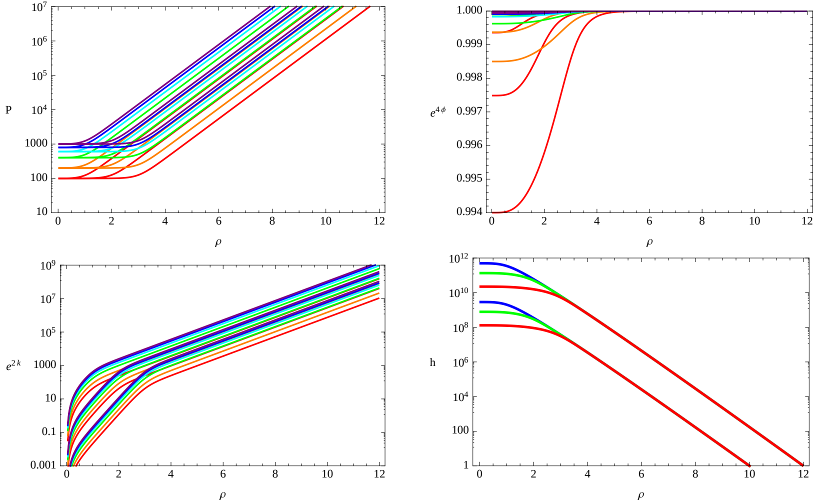
$$\langle \mathcal{U}_2 \rangle \sim \frac{N_c}{c_+}, \quad \langle \mathcal{O}_6 \rangle \sim e^{4\rho_*}$$

$\langle \mathcal{U}_2 \rangle \neq 0 \rightarrow$ baryonic branch



Typical Walking Solution

Numerical solutions for different values of c_0 & ρ_*



Approximate Analytic Solutions

Approximate analytic solution for $c_+ \gg 1$ Nunez, Papadimitriou, Piai '08

$$P = c_+ P_1 + c_+^{-1} P_{-1} + \dots$$

$$P_1 = \left(2\left(\sinh(4\rho) - 4\rho\right) + e^{4\rho_*}\right)^{1/3} \sim \begin{cases} e^{4\rho/3} & \text{for } \rho \gtrsim \rho_* \\ e^{4\rho_*/3} & \text{for } \rho \lesssim \rho_* \end{cases}$$

$$P_{-1} \sim -\frac{1}{P_1^2}\left(-P_1 Q^2 + \int P_1 Q Q' - \int\int P_1' Q Q'\right) \propto N_c^2$$

$$\dots$$

- Converges rapidly & agrees well with numerical results
- Klebanov-Strassler recovered as $c_+ \to \infty$, $\rho_* \to -\infty$

Probe D3-brane Potential

Consider probe D3-brane moving along radial direction $\rho = \rho(t)$

$$\mathcal{S}_{D3} = -T_3 \int_\Sigma d^4\xi \sqrt{-\det(g_{ind})} + T_3 \int_\Sigma \mathcal{C}_4$$

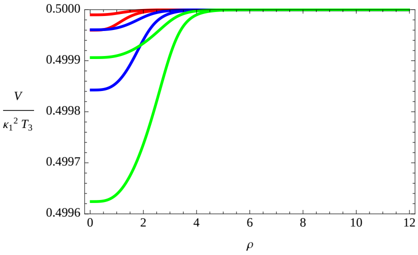
$$\simeq \int d^4x\, a^3(t) \left(\frac{1}{2} T_3 k_1 e^{2k+\Phi} \dot{\rho}^2 - \frac{k_1^2 T_3}{e^{-\Phi}+1} \right)$$

1. Non-trivial potential

$$V(\rho) = \frac{k_1^2 T_3}{e^{-\Phi}+1}$$

2. Canonical normalization

$$dr = \sqrt{T_3 k_1}\, e^{k+\frac{\Phi}{2}}\, d\rho$$



Scaling Behaviour

Using $P = c_+ P_1 + \ldots$ for $c_+ \gg 1$:

$$V \simeq \frac{k_1^2 T_3}{2} \left(1 - \frac{1}{4} \frac{N_c^2}{c_+^2} \int_\rho^\infty \frac{d\tilde{\rho}}{P_1^2} \frac{\partial \left(Q^2 \right)}{\partial \rho} \right)$$

$$dr \simeq \sqrt{k_1 T_3} \sqrt{\frac{c_+}{2} P_1'} \, d\rho$$

Scaling behaviour emerges for $c_+, e^{\rho_*} \gg 1$, *e.g.*

$$\eta(\rho) \simeq \frac{M_p^2}{k_1 T_3} \frac{N_c^2}{c_+^3} \tilde{\eta}(\rho)$$

$$\epsilon(\rho) \simeq \frac{M_p^2}{k_1 T_3} \frac{N_c^4}{c_+^5} e^{-\frac{8\rho_*}{3}} \tilde{\epsilon}(\rho) \quad (\rho \lesssim \rho_*)$$

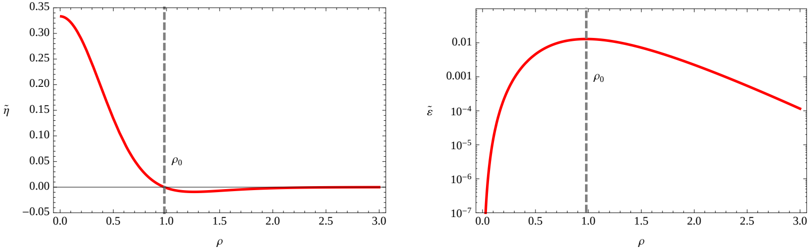
Inflection Point

Explicit expression

$$\tilde{\eta}(\rho) = \frac{3}{16}\Big(-5 + 40\rho^2 + 4(1+6\rho^2)\cosh(4\rho) + \cosh(8\rho) - \rho\,[30\sinh(4\rho) + \sinh(8\rho)]\Big)$$

vanishes at $\rho_0 \simeq 0.98$ & $\tilde{\epsilon}$ reaches its maximum

$\Rightarrow$ inflection point

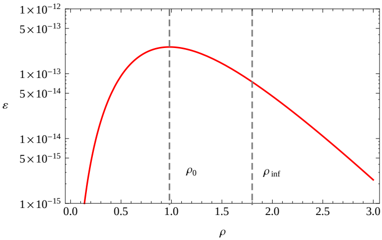


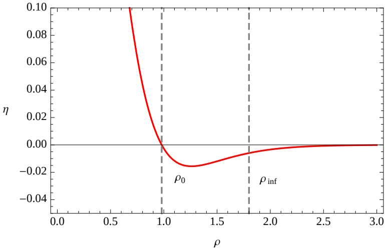
Example Trajectory

Assuming $T_3 \sim 5 \times 10^{-4}\, M_p^4$ and choosing $(k_1 \sim \frac{N_c}{c_+} e^{-4\rho_{uv}/3})$

$$\rho_* \simeq 2.9,\ \rho_{uv} = 9,\ c_+ \simeq 6.1 \times 10^3,\ N_c = 1,\ \rho_{inf} \simeq 1.8$$

1. Satisfies COBE normalization: $\frac{V}{2\epsilon M_p^4} \simeq 5.6 \times 10^{-8}$
2. $N_e \simeq 61.5$ between ρ_{inf} and ρ_0
3. $n_s \simeq 0.988,\ r \simeq 10^{-12}$





Summary & Outlook

Promising setup:

- Based on IR deformation of Klebanov-Strassler throat
- Generic inflection point: $\eta \sim 0$, $\epsilon \ll 1$
- Related to dilaton profile: $\partial_r^2 V = 0 \;\Leftrightarrow \partial_r^2 \Phi \simeq 0$

$\Rightarrow$ Potentially new way to address the η-problem

What to do next?

- Study motion in angular directions
- Add UV corrections (sourced in compact space)
- Use full DBI action & look for DBI inflation?
- Reheating?