Recent Updates on the Anomalous Magnetic Moment of the Muon in the Minimal Supersymmetric Standard Model

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in collaboration with Helvecio Fargnoli[b,c], Christoph Gnendiger[b], Dominik Stöckinger[b] and Hyejung Stöckinger-Kim[b]

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content

What is a_μ?

experimentally:

low energetic muon in an external homogeneous magnetic field:

- circular motion due to the electrical charge,
  frequency: $\omega_\mathrm{c} = -\frac{e}{m} B$
- precession due to the spin,
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measurement: $\omega_a \neq 0$

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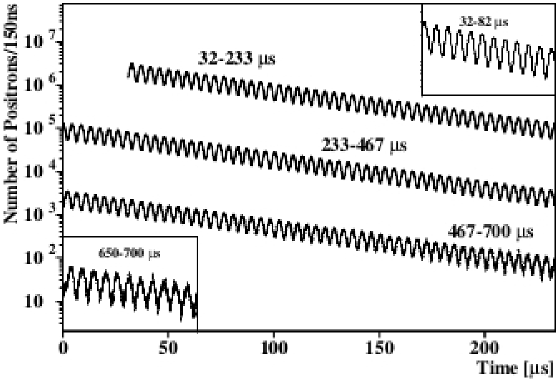
$$\Rightarrow a_\mu := \frac{g-2}{2}$$

What is a_μ?



[H. N. Brown, et. al., Physical Review Letters 86 (2001)]

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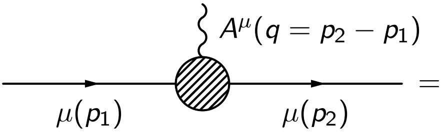

$$\mathrm{i}e\bar{u}(p_2)\left[\gamma^\mu F_\mathrm{E}(q^2) + \mathrm{i}\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_\mathrm{M}(q^2)\right]u(p_1),$$

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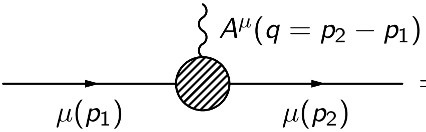

$$=$$

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$$a_\mu \equiv F_\mathrm{M}(0).$$

contributions to a_μ in the Standard Model

QED	$(11\,658\,471.896 \pm 0.008) \cdot 10^{-10}$
[T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, arXiv:1205.5370 (2012)]	
electroweak	$(15.32 \pm 0.10 \pm 0.15) \cdot 10^{-10}$
[F. Jegerlehner and A. Nyffeler, Physics Reports 477 (2009)]	
hadronic light by light	$(10.5 \pm 2.6) \cdot 10^{-10}$
[J. Prades, E. de Rafael, A. Vainshtein, Advanced Series on Directions in High Energy Physics – Vol. 20 (2009)]	
hadronic vac. pol., higher order	$(-9.98 \pm 0.04 \pm 0.09) \cdot 10^{-10}$
[F. Jegerlehner and R. Szafron (2011)]	
hadronic vac. pol., leading order	$(687.72 \pm 4.63) \cdot 10^{-10}$
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experiment: $a_\mu^{\text{BNL}} = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10}$

[G. W. Bennett et al., Physical Review D 73 (2006)],

[A. Hoecker, W. Marciano, Journal of Physics G 37 (2010)]

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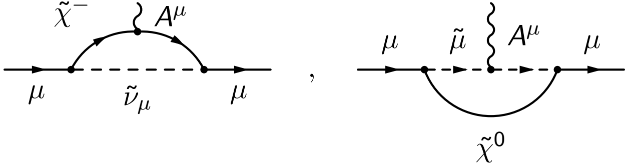
$$\Rightarrow a_\mu^{\text{BNL}} - a_\mu^{\text{SM}} = (33.4 \pm 8.2) \cdot 10^{-10} \quad (4.1\sigma)$$

MSSM contributions to a_μ: one-loop corrections

loops with Chargino $\tilde{\chi}^+$ or Neutralino $\tilde{\chi}^0$



$$a_\mu^{\tilde{\chi}_i} = \frac{e^2}{16\pi^2 \sin^2(\theta_W)} \frac{m_\mu^2}{m_{\tilde{\ell}}^2} \left[F_1^{(0)}(x_i) C_1^{\tilde{\chi}_i} + m_{\tilde{\chi}_i} F_2^{(0)}(x_i) C_2^{\tilde{\chi}_i} \right]$$

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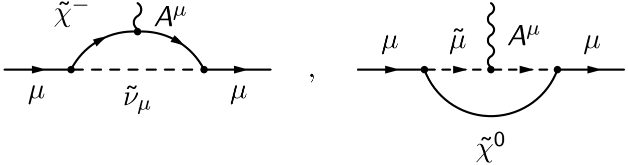

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with $C_j^{\tilde{\chi}_i}$: containing all couplings, some parts $\propto \text{sgn}(\mu) \tan\beta$

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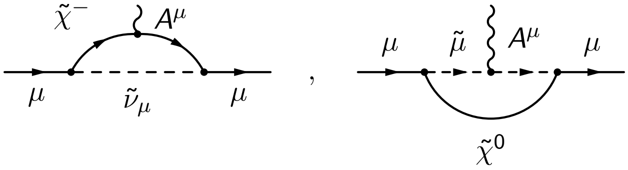

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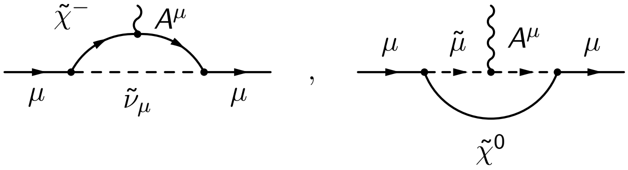

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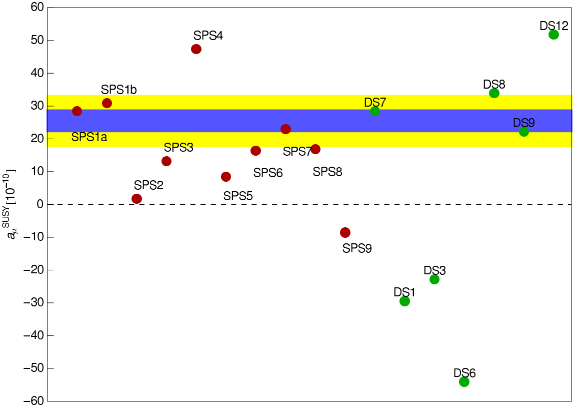
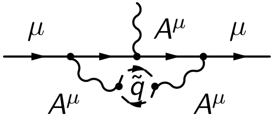
discrepancy between experiment and Standard Model prediction can be explained completely by the MSSM

a_μ useful for constraining MSSM parameters,
complementary to the LHC



plot: [D. Stöckinger (2010)],
green points: different parameter sets leading to the same LHC signature,
[Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas (2010)]

MSSM contributions to a_μ: two-loop corrections

1. MSSM corrections to Standard Model diagrams e. g. 

[S. Heinemeyer, D. Stöckinger, G. Weiglein, Nuclear Physics B 690 (2004)]

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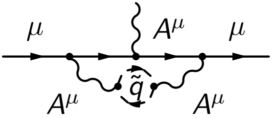
e. g. 

2. corrections to diagrams with supersymmetric particles:
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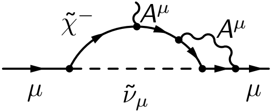
e. g. 

[G. Degrassi, G. F. Giudice, Physical Review D 58 (1998)]

[P. von Weitershausen, M. Schäfer, and H. Stöckinger-Kim, and D. Stöckinger, (2009)]

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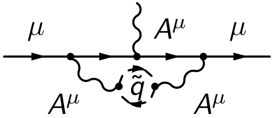

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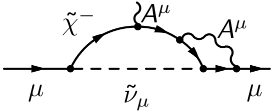

   - corrections with
     quarks, leptons
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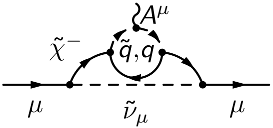

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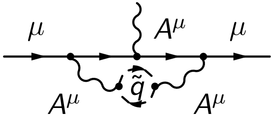

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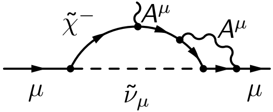

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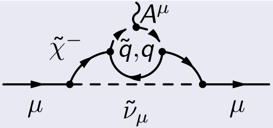
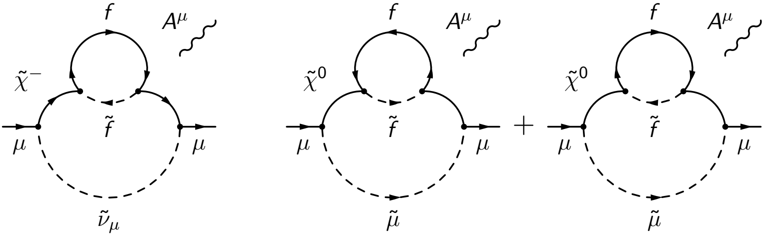


two-loop corrections with closed fermion–sfermion–loop



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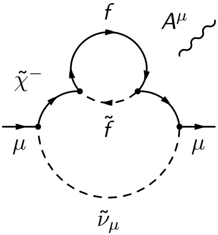
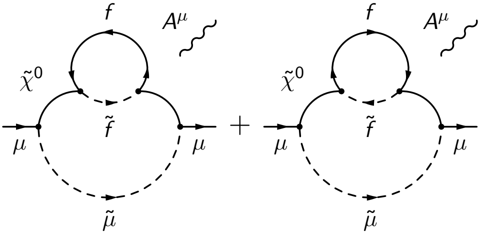
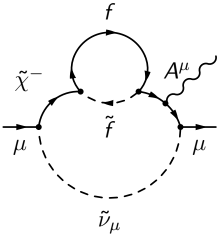
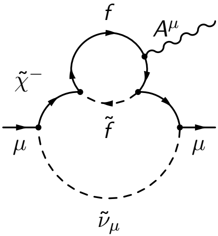




propagator contributions

vertex contributions





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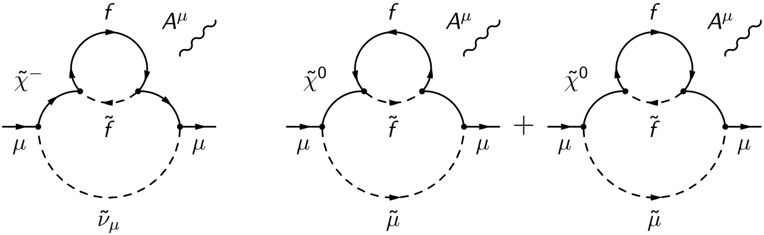

calculated by using two different methods:

- usual two-loop evaluation
  (reduction to master integrals; large mass expansion;
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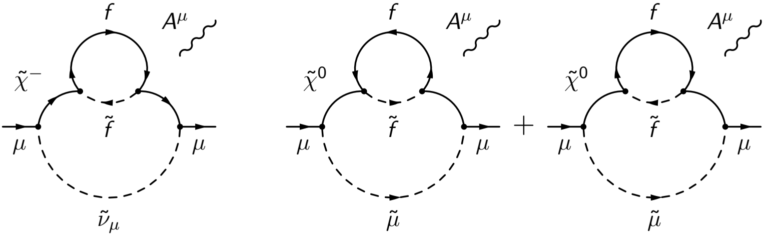

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- special method for Barr–Zee type diagrams
  (first compute inner one-loop; then insert into outer loop;
  get an integral representation)

  [S. M. Barr and A. Zee, Phys. Rev. Lett. 65 (1990)]

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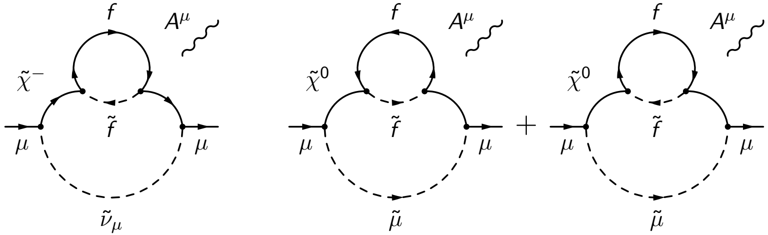


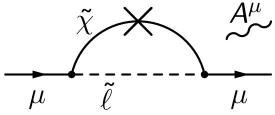
main difficulties:

- four different dimensionless mass ratios
- QED Ward identity with four different covariants
- inner loop can be ultraviolet divergent,
  evaluation of higher orders in ϵ of the outer loop necessary
  $\Big($dimensional regularisation parameter $\epsilon = \frac{1}{2}\,(4 - D)\Big)$

renormalisation and counterterm contributions to a_μ

necessary classes of counterterm corrections:

- corrections to Chargino or Neutralino propagator



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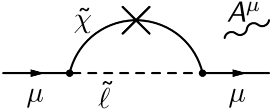
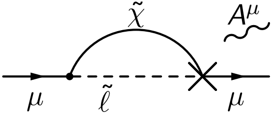


- corrections to vertices
  (no corresponding two-loop diagram $\Rightarrow$ finite!)



renormalisation constants

different renormalisation constants neccessary:

- mass corrections to W and Z bosons: δM_W^2, δM_Z^2
- mass corrections to the μ parameter and the soft SUSY breaking parameters M_1 and M_2: $\delta\mu$, δM_1, δM_2
- field corrections to the Photon and Higgs bosons and the Photon–Z–mixing: δZ_{AA}, $\delta Z_{h^0 h^0}$, $\delta Z_{H^0 H^0}$, δZ_{ZA}

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on-shell renormalisation of the MSSM as far as possible:

- SM particles on-shell
- both Charginos and the lightest Neutralino on-shell
- $\overline{\text{DR}}$ definition of the Higgs renormalisation constants (i. e. $\overline{\text{DR}}$ definition of $\tan\beta$)

numerical results

general treatment:

- all parameters considered to be real (no complex phases)
- at the moment only $t, \tilde{t}, b, \tilde{b}$ considered
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general treatment:

- all parameters considered to be real (no complex phases)
- at the moment only $t, \tilde{t}, b, \tilde{b}$ considered
- all divergencies cancel; therefore only finite parts are shown
- regularisation parameter μ_{dim} set to $\approx 500\text{GeV}$
- start from a standard set of values for the important MSSM parameters:

  $$\tan\beta = 20, \quad \mu = 500\text{GeV}, \quad m_{\tilde{t}_{\text{L,R}}} = 750\text{GeV}, \quad A_t = -1000\text{GeV},$$

  $$M_1 \approx \frac{1}{2} M_2, \quad M_2 = 400\text{GeV}, \quad m_{\tilde{\mu}_{\text{L,R}}} = 250\text{GeV}, \quad A_\mu \approx 0\text{GeV}$$

  (consistent with exclusion limits and $h \to \gamma\gamma$ signal)
- find those, which have most influence on the result (absolut value and ratio to one-loop result)

mass matrices in the MSSM

Sfermion mass matrices:

$$\mathcal{M}_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_\mathrm{L}} + m_f^2 + g_{1,f} & m_f \left(A_f^* - \mu t_{\tilde{f}}(\beta)\right) \\ m_f \left(A_f - \mu^* t_{\tilde{f}}(\beta)\right) & m^2_{\tilde{f}} + m_f^2 + g_{2,f} \end{pmatrix},$$

$$\left(f, \tilde{f}, \tilde{f}_\mathrm{L}\right) \in \left\{ \left(u, \tilde{u}, \tilde{q}\right), \left(d, \tilde{d}, \tilde{q}\right), \left(e, \tilde{e}, \tilde{\ell}\right) \right\},$$

$$t_{\tilde{f}}(\beta) = \begin{cases} \tan\beta \text{ for } \tilde{f} \in \left\{\tilde{d}, \tilde{e}\right\}, \\ \dfrac{1}{\tan\beta} \text{ for } \tilde{f} = \tilde{u}, \end{cases}$$

$$g_{1,f} = M_Z^2 \cos(2\beta) \left(T_{3,f} - Q_f s_\mathrm{W}^2\right),$$

$$g_{2,f} = M_Z^2 \cos(2\beta) Q_f s_\mathrm{W}^2.$$

mass matrices in the MSSM

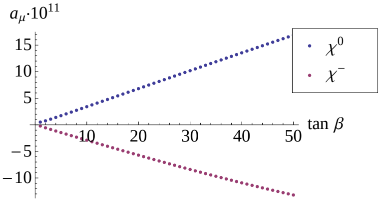
Chargino and Neutralino mass matrices:

$$\mathcal{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix},$$

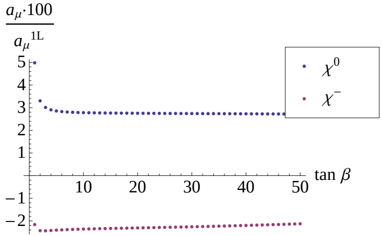
$$\mathcal{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_\mathrm{W} \cos\beta & M_Z s_\mathrm{W} \sin\beta \\ 0 & M_2 & M_Z c_\mathrm{W} \cos\beta & -M_Z c_\mathrm{W} \sin\beta \\ -M_Z s_\mathrm{W} \cos\beta & M_Z c_\mathrm{W} \cos\beta & 0 & -\mu \\ M_Z s_\mathrm{W} \sin\beta & -M_Z c_\mathrm{W} \sin\beta & -\mu & 0 \end{pmatrix}.$$

dependence on tan β

two-loop contribution

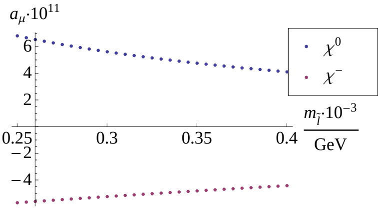


ratio to full one-loop result

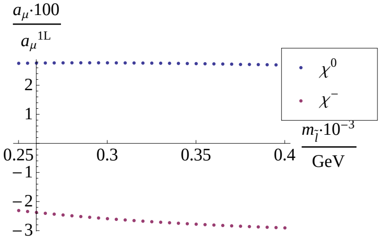


dependence on $m_{\tilde{\ell}}$

two-loop contribution

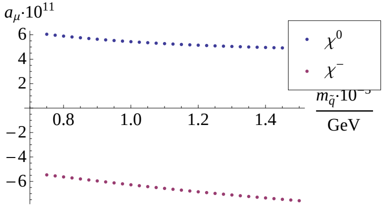


ratio to full one-loop result

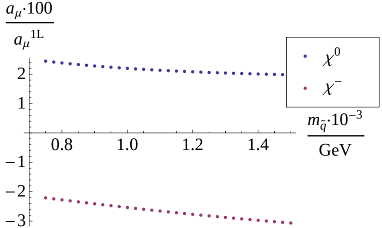


dependence on $m_{\tilde{q}}$

two-loop contribution

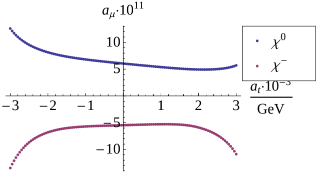


ratio to full one-loop result

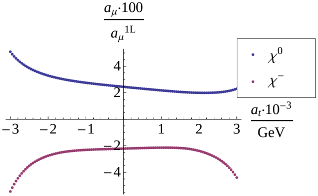


dependence on A_t

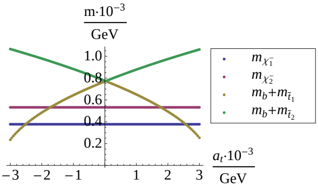
two-loop contribution

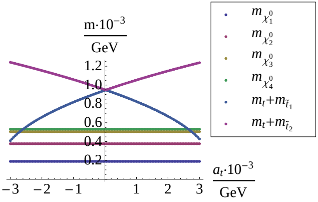


ratio to full one-loop result

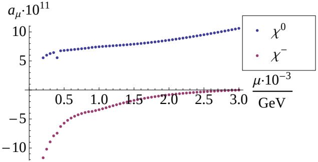


important masses (threshold effects)

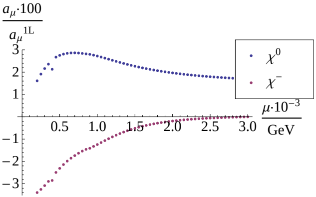




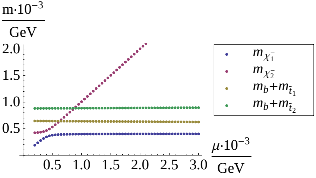
two-loop contribution

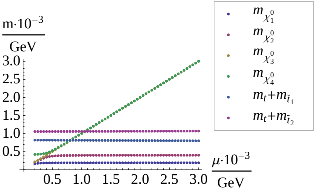


ratio to full one-loop result



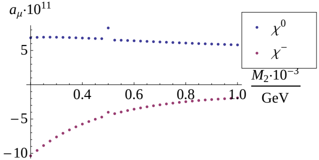
important masses (threshold effects)



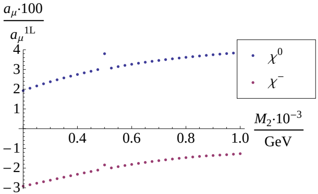


dependence on M_2

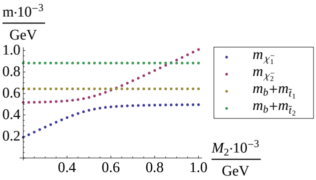
two-loop contribution

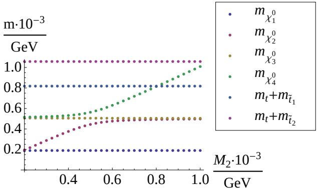


ratio to full one-loop result



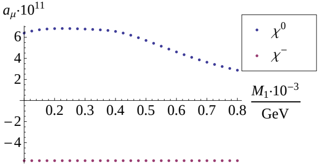
important masses (threshold effects)



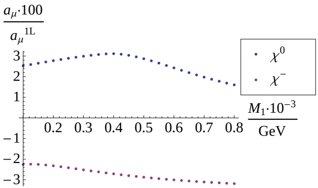


dependence on M_1

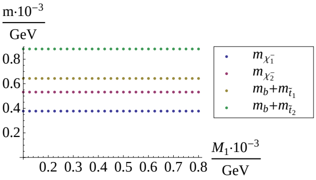
two-loop contribution

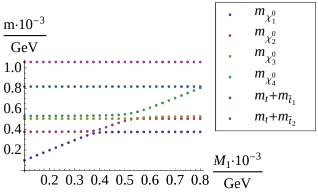


ratio to full one-loop result



important masses (threshold effects)





conclusions

- parameters of stop sector most interesting
- threshold effects generated by counterterms
- location of thresholds depending on masses of Neutralinos, Charginos and Squarks, therefore M_1, M_2 and μ become important

conclusions

- parameters of stop sector most interesting
- threshold effects generated by counterterms
- location of thresholds depending on masses of Neutralinos, Charginos and Squarks, therefore M_1, M_2 and μ become important
- for other parameters the ratio to the one-loop result is flat
- corrections by Chargino diagrams as important as corrections by Neutralino diagrams,
  but:
  for different choice of parameters
  Chargino diagrams can dominate

outlook

- two-loop corrections with a closed fermion–sfermion–loop have been evaluated analytically in the Feynman-diagrammatic approach, numerical analysis nearly finished
- full two-loop calculation in progress

$\Rightarrow$ error on a_μ^{MSSM} lower than $1 \cdot 10^{-10}$

$\Rightarrow$ new experiment / more precise Standard Model prediction become more valuable

(stronger restrictions on MSSM parameter space)

Thanks for your attention!