Theory of Heavy Quarkonium Production

Mathias Butenschön
(Hamburg University)

Based on work with Bernd Kniehl: PRL **104**, 072001 (2010); PRL **106**, 022003 (2011); PRD **84**, 051501R (2011); PRL **107**, 232001 (2011); PRL **108**, 172002 (2012).

Production and Decay Rates of Heavy Quarkonia

Heavy Quarkonia: Bound states of heavy quark and antiquark.

The classic approach: Color-singlet model

- Calculate cross section for heavy quark pair in physical **color singlet** (=color neutral) state. In case of J/ψ: $c\bar{c}[^3S_1^{[1]}]$
- Multiply by quarkonium wave function at origin
- Leftover IR singularities in case of P wave quarkonia
- Mid 90's: Strong disagreement with Tevatron data apparent

Nonrelativistic QCD (NRQCD):

- Rigorous effective field theory: Bodwin, Braaten, Lepage (1995)
- Based on **factorization of soft and hard scales** (Scale hierarchy: $Mv^2, Mv << \Lambda_{QCD} << M$)
- Could explain hadroproduction at Tevatron

Further models on the market: k_T factorization, Color Evaporation Model

J/ψ Production with NRQCD

Factorization theorem: $$\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$$

- ***n***: Every possible Fock state, including **color-octet** (CO) states.
- $\boldsymbol{\sigma_{c\bar{c}[n]}}$: Production rate of $c\bar{c}[n]$, calculated in perturbative QCD
- **$\langle O^{J/\psi}[n] \rangle$**: Long distance matrix elements (LDMEs): describe $c\bar{c}[n] \rightarrow J/\psi$, universal, extracted from experiment.

Scaling rules: LDMEs scale with definite power of v ($v^2 \approx 0.2$):

scaling	v^3	v^7 ("CO states")	v^{11}
n	${}^3S_1^{[1]}$	${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$	...

- **Double expansion** in v and α_s
- Leading term in v ($n = {}^3S_1^{[1]}$) equals **color-singlet model**.

NLO Corrections to Color Octet Contributions

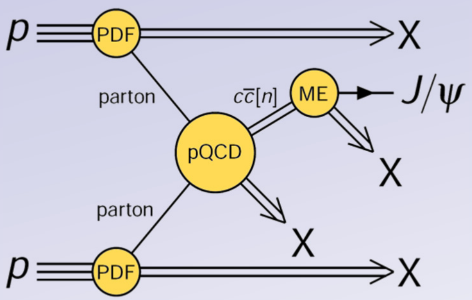
- Petrelli, Cacciari, Greco, Maltoni, Mangano (1998):
  Photo- and hadroproduction (only 2 → 1 processes)
- Klasen, Kniehl, Mihaila, Steinhauser (2005):
  γγ scattering at LEP (neglecting resolved photons)
- M.B., Kniehl (2009):
  Photoproduction at HERA (neglecting resolved photons)
- Zhang, Ma, Wang, Chao (2009):
  e^+e^- scattering at *B* factories
- Ma, Wang, Chao (2010); M.B., Kniehl (2010):
  Hadroproduction
- M.B., Kniehl (2011):
  Global fit of CO LDMEs (all above processes + resolved photons)
- M.B., Kniehl (2011):
  Polarization in Photoproduction
- M.B., Kniehl (2012); Chao, Ma, Shao, K. Wang, Y.-J. Zhang (2012); Gong, Wan, J.-X. Wang, H.-F. Zhang (2012):
  Polarization in Hadroproduction

→ Test LDME universality!

In the following

Calculate Inclusive J/ψ Production within NRQCD

Factorization formulas (here hadroproduction):



- Convolute partonic cross section with **proton PDFs:** $$\sigma_{\text{hadr}} = \sum_{i,j} \int dx\, dy\, f_{i/p}(x)\, f_{j/p}(y) \cdot \sigma_{\text{part,i,j}}$$
- **NRQCD factorization:** $$\sigma_{\text{part,i,j}} = \sum_n \sigma(ij \to c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

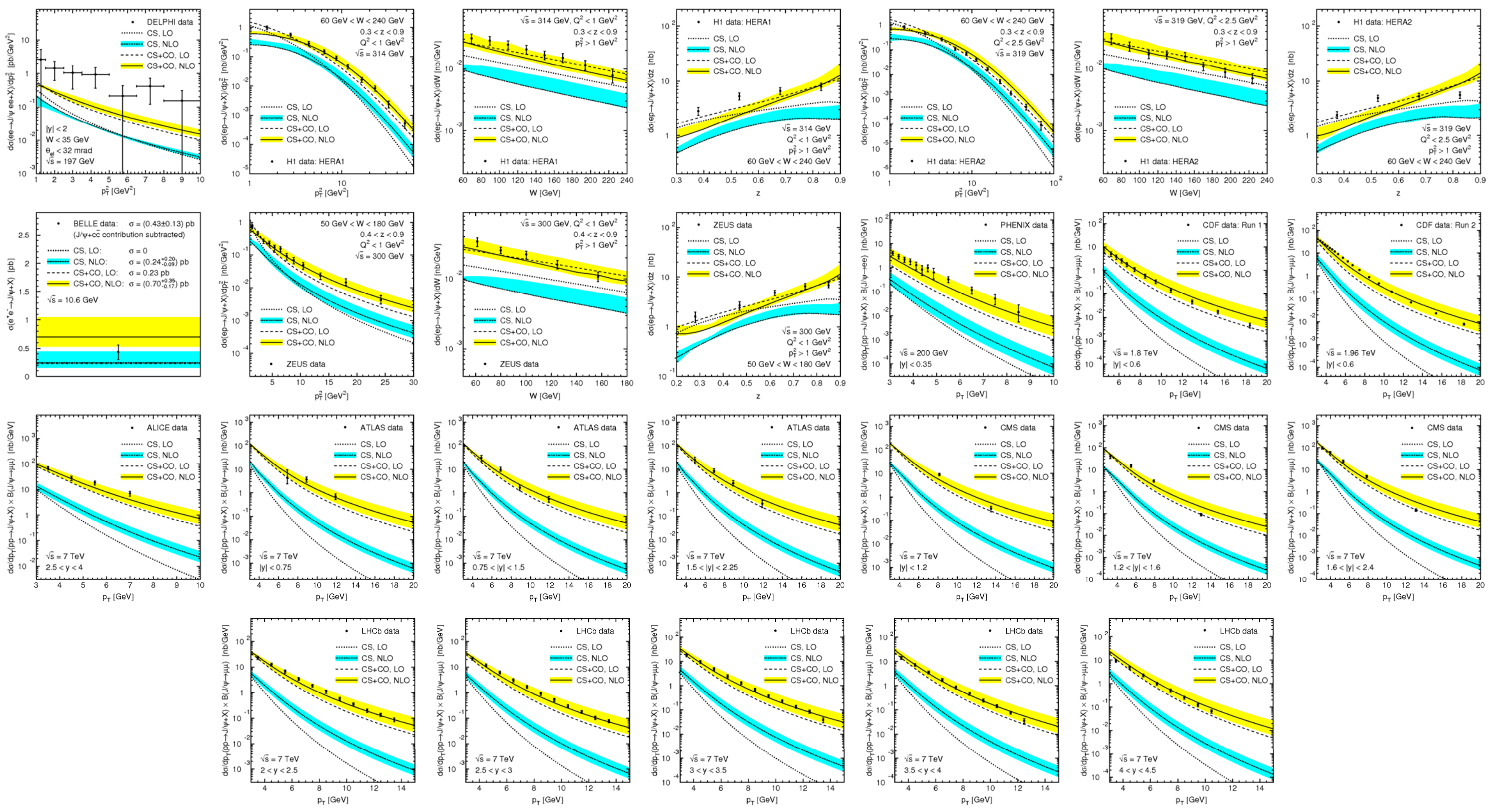
Amplitudes for $c\bar{c}[n]$ production by projector application, e.g.:

$$A_{c\bar{c}[^3S_1^{[1/8]}]} = \varepsilon_\alpha(m_S)\,\mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

$$A_{c\bar{c}[^3P_J^{[8]}]} = \varepsilon_\alpha(m_S)\,\varepsilon_\beta(m_l)\,\frac{d}{dq_\beta}\,\mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

- $A_{c\bar{c}}$: Amputated pQCD amplitude for open $c\bar{c}$ production.
- ***q***: Relative momentum between c and $\bar{c}$. ***ε***: Polarization vectors.

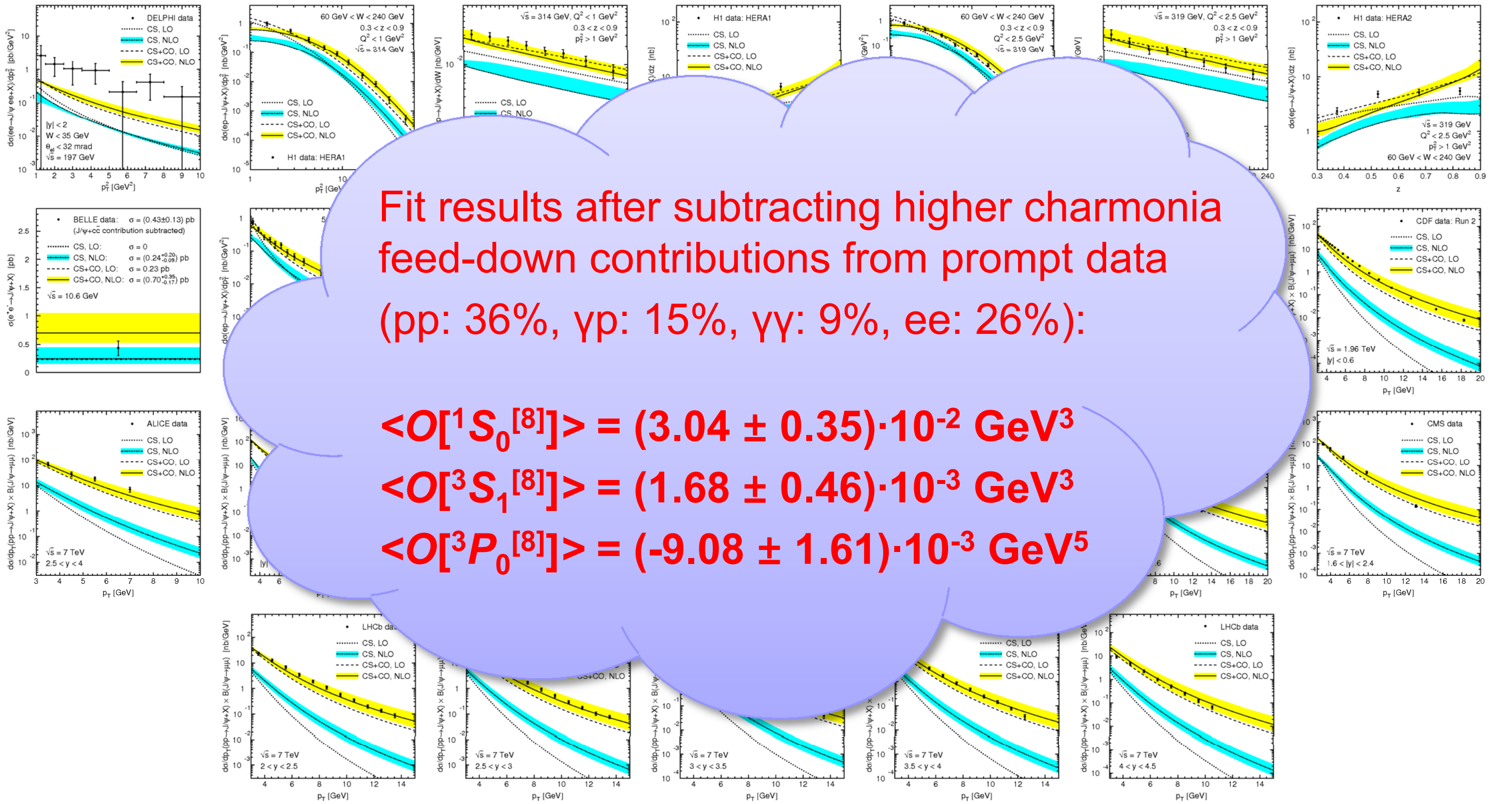
Global Fit to Unpolarized Data



$\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

Global Fit to Unpolarized Data



Fit results after subtracting higher charmonia feed-down contributions from prompt data (pp: 36%, γp: 15%, γγ: 9%, ee: 26%):

$\langle O[^1S_0^{[8]}]\rangle = (3.04 \pm 0.35)\cdot 10^{-2}$ GeV^3

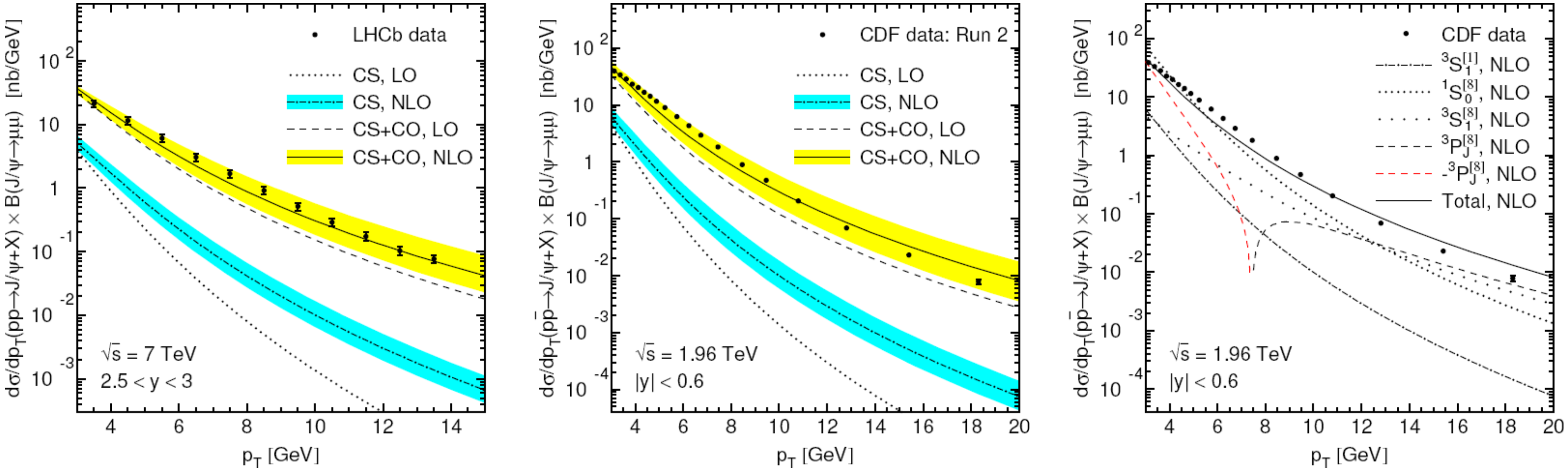
$\langle O[^3S_1^{[8]}]\rangle = (1.68 \pm 0.46)\cdot 10^{-3}$ GeV^3

$\langle O[^3P_0^{[8]}]\rangle = (-9.08 \pm 1.61)\cdot 10^{-3}$ GeV^5

$\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}$ GeV^3 $\quad$ $\langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}$ GeV^3

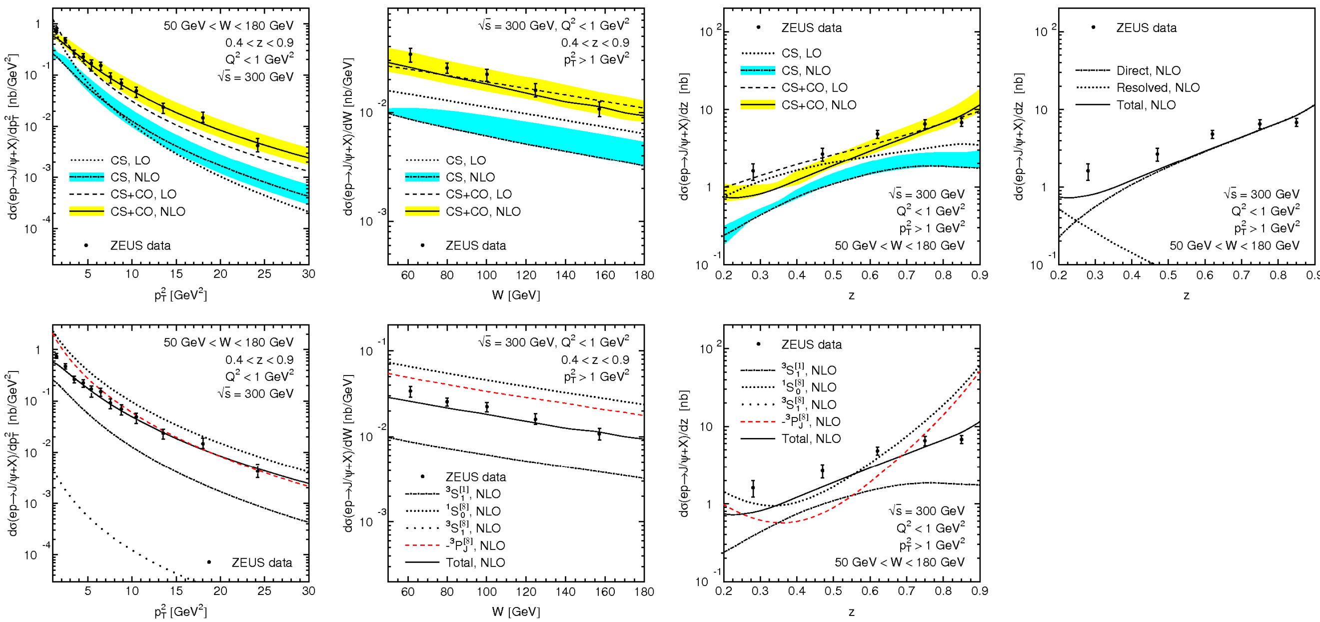
$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}$ GeV^5

In Detail: Hadroproduction (LHC, Tevatron)



- Color singlet model **far below** data. **CS+CO** describes data **well**.
- $^3P_J^{[8]}$ short distance cross section **negative** at $p_T > 7$ GeV.
- But: Short distance cross sections and LDMEs **unphysical**
  ➡ No problem!
- Hadroproduction data below $p_T = 3$ GeV excluded from our fit.
- Observation: Change s or rapidity y just **rescaling** of cross sections: CO LDMEs describing RHIC or Tevatron must also describe LHC!

In Detail: Photoproduction at HERA

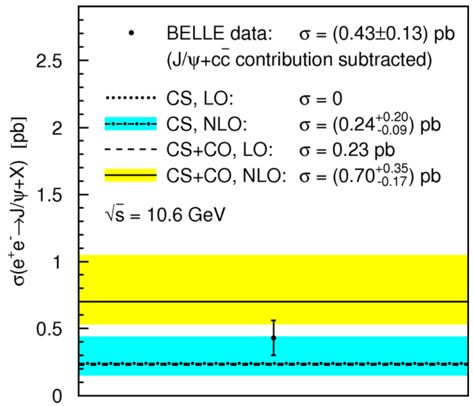


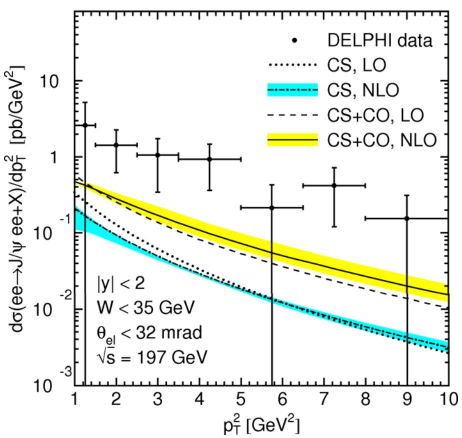
- **Distributions:** Transverse momentum (p_T), photon-proton c.m. energy (W), and z = Fraction of photon energy going to J/ψ.
- Again: Color singlet alone **below** the data, **CS+CO** describes data well.
- Calculation includes **resolved** photon contributions: Important at low z.
- **Good description at high z:** No increase like in older Born analyses!

In Detail: e^+e^- and $\gamma\gamma$ Collisions

Electron-Positron Collisions at BELLE:

- CS: Large overlap with data, CS+CO: Small overlap.
- But: Only 4+ charged track events measured.
  ➡ Actual BELLE **data larger** by unknown factor.
- For e^+e^- color singlet, NNLO terms been calculated, increasing cross section. Not part of the global fit. [Ma, Zhang, Chao (2009); Gong, Wang (2009)]





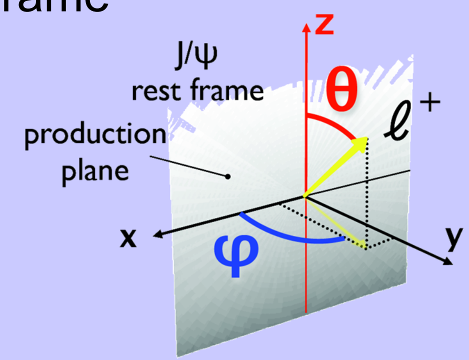
Two Photon scattering at DELPHI (LEP):

- Includes direct, single and double resolved photons.
- CS below data, but also **CS+CO** curve **too low**. Possible explanations:
  - Uncertainties in the measurement (Just 16 events involved!)
  - Hint at problems with LDME universality.

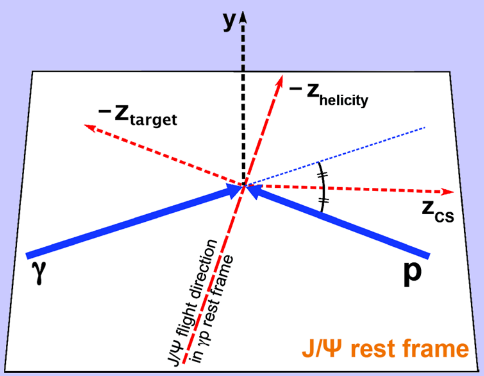
J/ψ Polarization

- **Angular distribution** of decay lepton l^+ in J/ψ rest frame
  ➡ Polarization observables ***λ***, ***μ***, ***ν:***

$$\frac{d\Gamma(J/\psi \to l^+ l^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda \cos^2\theta + \mu \sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$



- Depends on choice of **coordinate system:**
  - Helicity frame: z axis $\parallel -(\vec{p}_\gamma + \vec{p}_p)$
  - Collins-Soper frame: z axis $\parallel \vec{p}_\gamma/|\vec{p}_\gamma| - \vec{p}_p/|\vec{p}_p|$
  - Target frame: z axis $\parallel -\vec{p}_p$

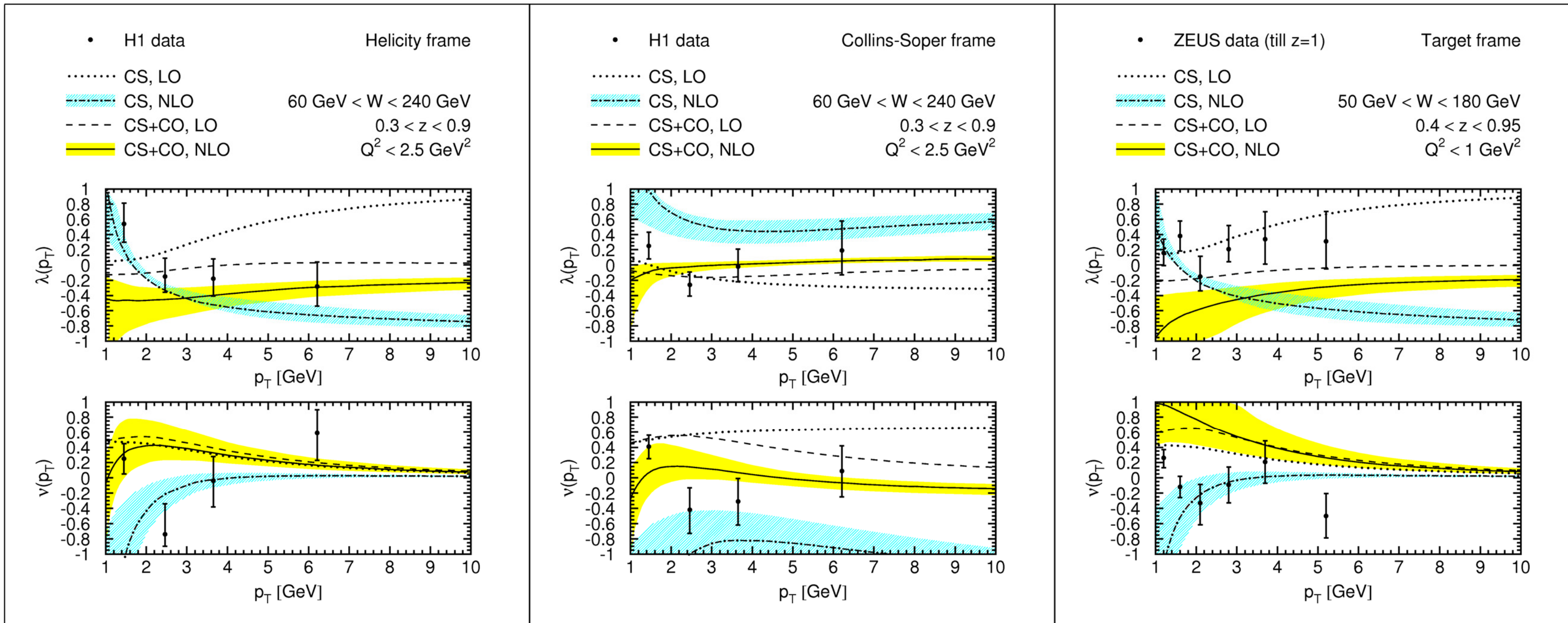


- **In Calculation:** Plug in explicit expressions for $c\bar{c}[n]$ spin polarization vectors according to

$$\lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \mu = \frac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \nu = \frac{2 d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$$

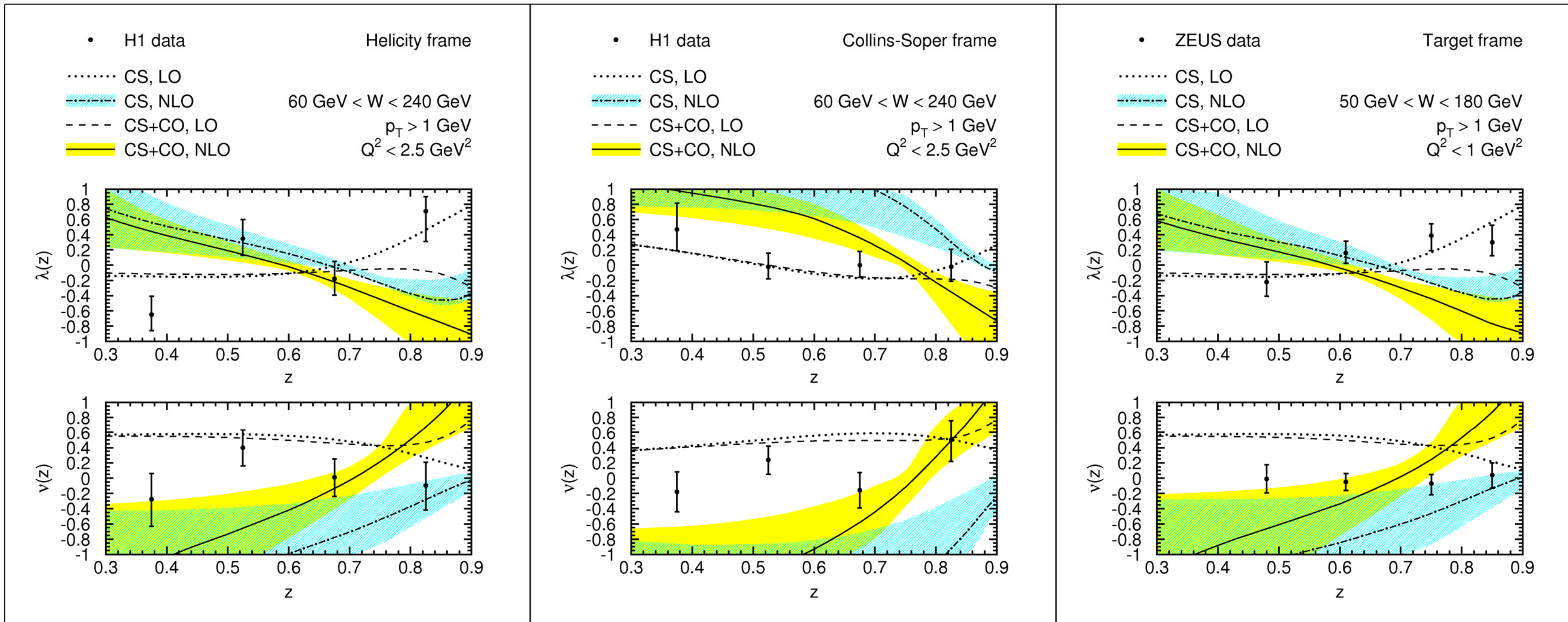
- We use the CO LDME set with feed-down contributions subtracted.

J/ψ Polarization in Photoproduction: p_T Distribution



- Bands: Uncertainties due to scale variation and CO LDMEs.
- **CSM** predicts **longitudinal** *J/ψ* at high p_T.
- **CS+CO:** largely **unpolarized** *J/ψ* at high p_T. α_s expansion converges better.
- H1 and ZEUS **data not precise** enough to discriminate CSM / NRQCD.

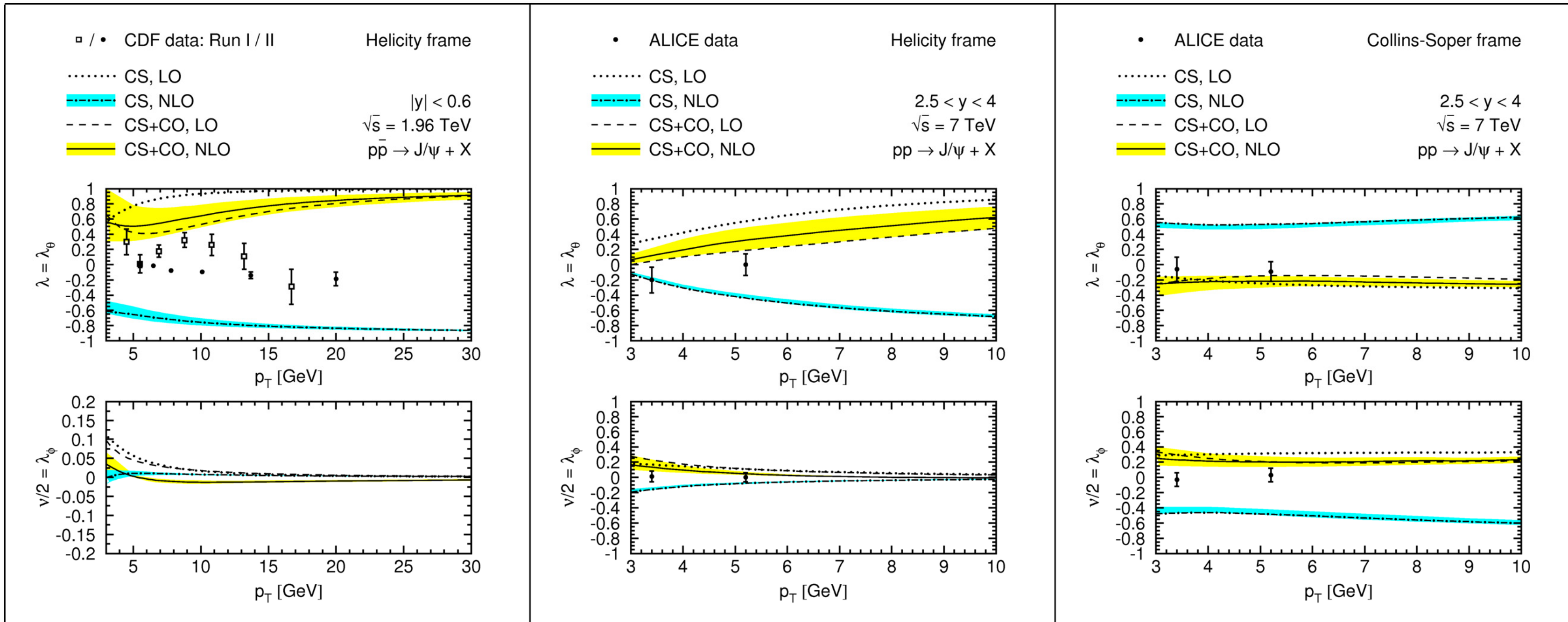
J/ψ Polarization in Photoproduction: z Distribution



- Bands: Uncertainties due to scale variation and CO LDMEs.
- **Scale** uncertainties very large.
- **Error bands** of CSM and NRQCD largely **overlap**.

➡ p_T distribution better suited to discriminate production mechanisms than z.

J/ψ Polarization in Hadroproduction



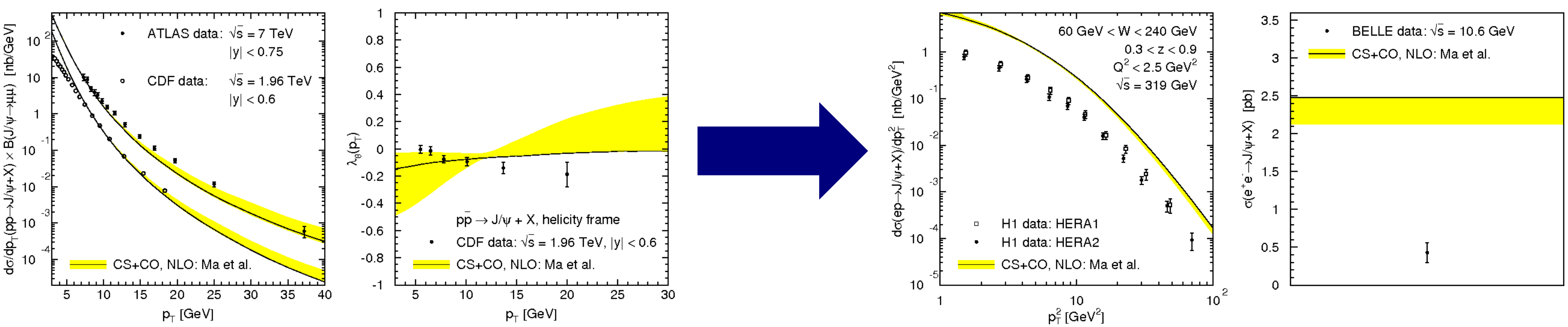
- **Helicity frame**: NRQCD predicts strong **transverse** polarization at high p_T.
- **Collins-Soper frame**: NRQCD predicts slightly longitudinal J/ψ.
- **Disagreement** with CDF Run II data, rough agreement with early ALICE data.
  ➡ Following high precision LHC data: **Confirm/rule out** LDME universality!

Polarization in Hadroproduction: Ma et al.

- Chao, Ma, Shao, Wang, Zhang (2012)
- **Fit** to CDF Tevatron *J/ψ* **yield and polarization** data with p_T>7 GeV:

$$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.089\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}(^3S_1)\rangle = 0.003\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}(^3P_0)\rangle = 0.0126\ \mathrm{GeV}^5$$

- **Describes** CDF Run II polarization data **and** *J/ψ* hadroproduction yield up to **highest measured p_T** values, not below 7 GeV.
- But: **Disagreement** with photoproduction at **HERA** and e^+e^- at **BELLE**:



- **Bands**: Two alternative LDME sets specified in Ma *et al.*:

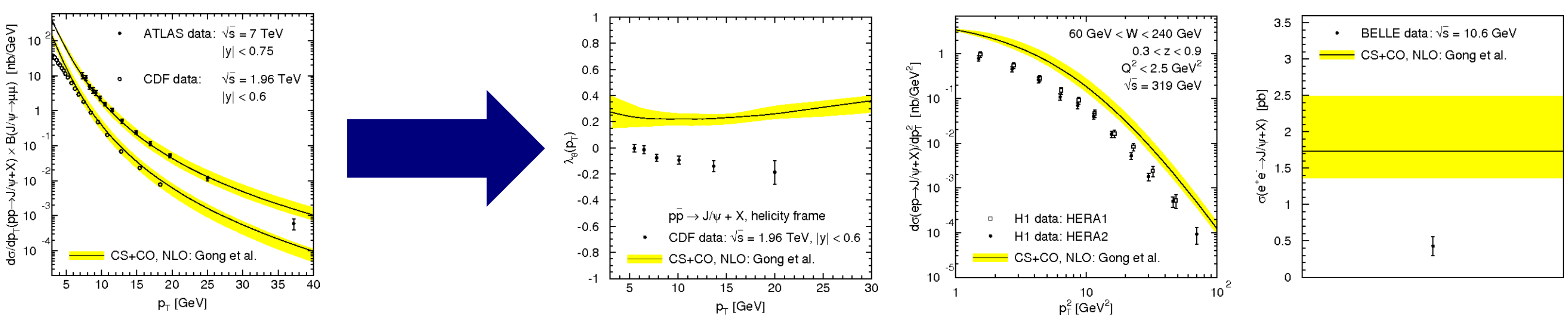
1. $\langle O_8^{J/\psi}(^1S_0)\rangle = 0 \quad \langle O_8^{J/\psi}(^3S_1)\rangle = 0.014\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}(^3P_0)\rangle = 0.054\ \mathrm{GeV}^5$
2. $\langle O_8^{J/\psi}(^1S_0)\rangle = 0.11\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}(^3S_1)\rangle = 0 \quad \langle O_8^{J/\psi}(^3P_0)\rangle = 0$

Polarization in Hadroproduction: Gong et al.

- Gong, Wan, Wang, Zhang (2012)
- **Fit only hadroproduction yield**, but consider **also ψ' and χ_{cj}** contributions:
  - Fit χ_{c0} CO LDME to LHCb data
  - Fit ψ' CO LDMEs to CDF and LHCb data (p_T>7 GeV)
  - Subtract ψ' and χ_{cj} feddowns, fit J/ψ LDMEs to CDF and LHCb data (p_T>7 GeV):

$$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0.097\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}({}^3S_1)\rangle = -0.0046\ \mathrm{GeV}^3 \quad \langle O_8^{J/\psi}({}^3P_0)\rangle = -0.0214\ \mathrm{GeV}^5$$
$$\langle O_8^{\psi'}({}^1S_0)\rangle = -0.0001\ \mathrm{GeV}^3 \quad \langle O_8^{\psi'}({}^3S_1)\rangle = 0.0034\ \mathrm{GeV}^3 \quad \langle O_8^{\psi'}({}^3P_0)\rangle = 0.0095\ \mathrm{GeV}^5$$
$$\langle O_8^{\chi_0}({}^3S_1)\rangle = 0.0022\ \mathrm{GeV}^3$$

- **Predict** J/ψ, ψ' and χ_{cj} **polarization** in prompt hadroproduction (first time!)
- Predicts **moderate** transverse J/ψ polarization, **contrary to** CDF Run II data
- Also: In **disagreement** with photoproduction at **HERA** and e^+e^- at **BELLE**:

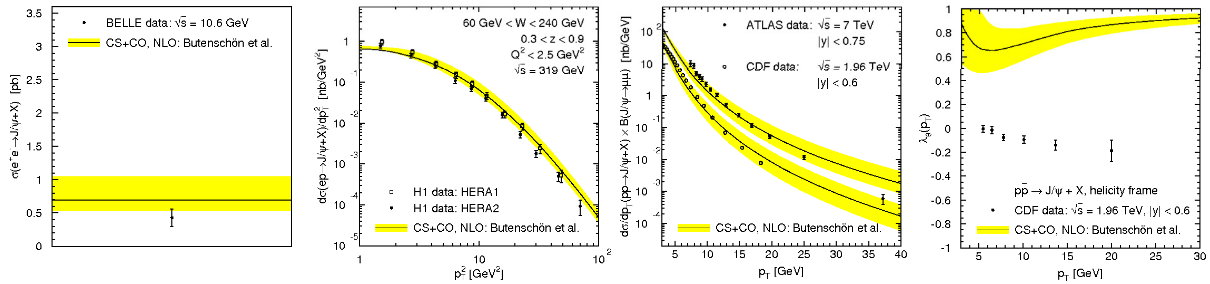


Overview: Three J/ψ Production Works

	e+e- yield:	γp yield:	pp yield:	CDF polariz.:

Butenschön, Kniehl:

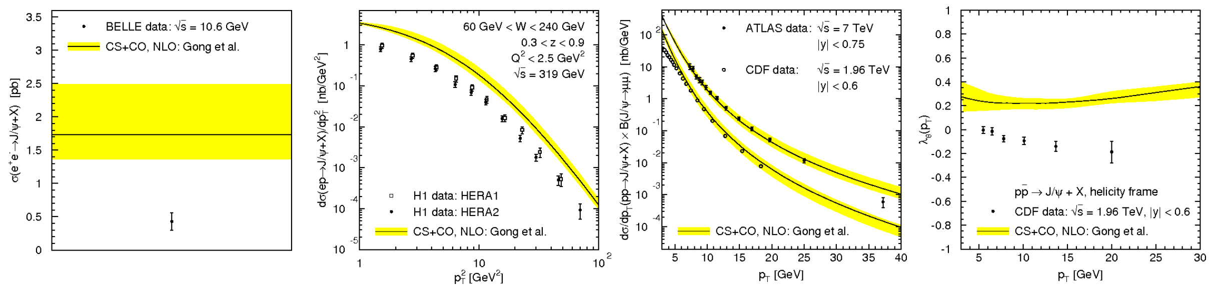
$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.0497\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.0022\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0161\ \text{GeV}^5$



Gong, Wan, J.-X. Wang, H.-F. Zhang:

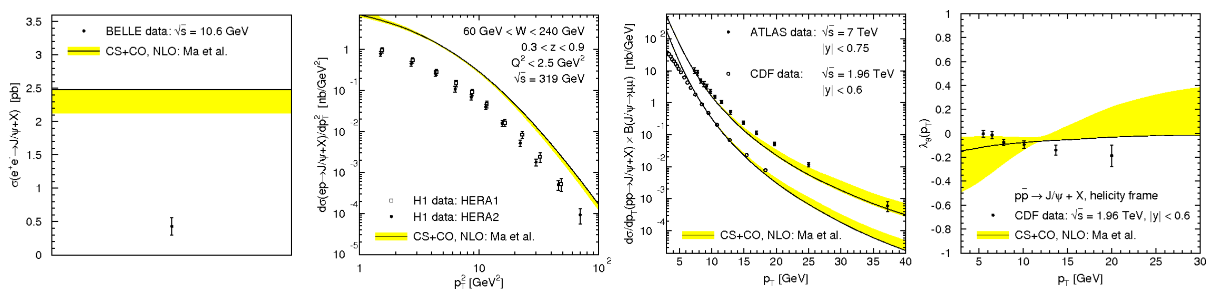
$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.097\ \text{GeV}^3$ $\langle O_8^{\psi'}(^1S_0)\rangle = -0.0001\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = -0.0046\ \text{GeV}^3$ $\langle O_8^{\psi'}(^3S_1)\rangle = 0.0034\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0214\ \text{GeV}^5$ $\langle O_8^{\psi'}(^3P_0)\rangle = 0.0095\ \text{GeV}^5$

$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.0022\ \text{GeV}^3$



Chao, Ma, Shao, K. Wang, Y.-J. Zhang:

$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.089\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.003\ \text{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = 0.0126\ \text{GeV}^5$



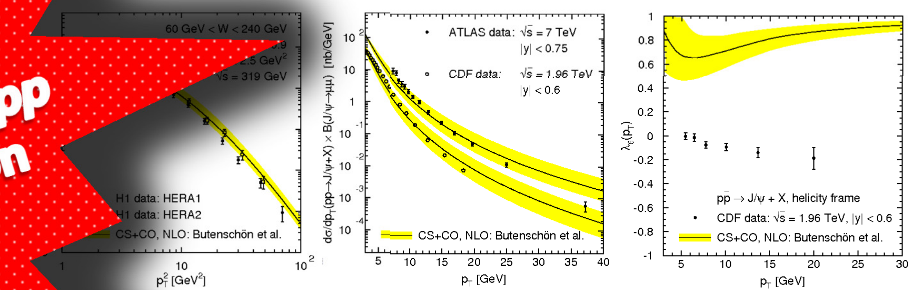
Overview: Three J/ψ Production Works

AGREEMENT:
Can NOT describe e⁺e⁻, γp, pp yield and CDF polarization with same LDMEs.

	...d:	γp yield:	pp yield:	CDF polariz.:

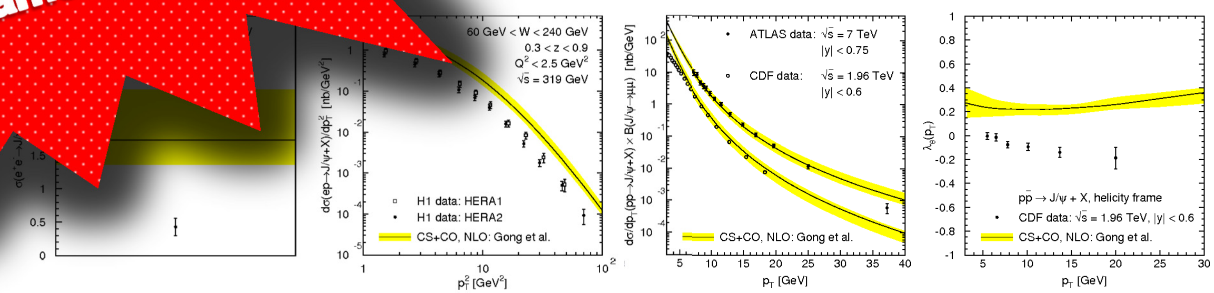
Butenschön, K...

$\langle O_8^{J/\psi}(^1S...$
$\langle O_8^{J/\psi}(^3S_1)\rangle$ ...
$\langle O_8^{J/\psi}(^3P_0)\rangle = -$ ...



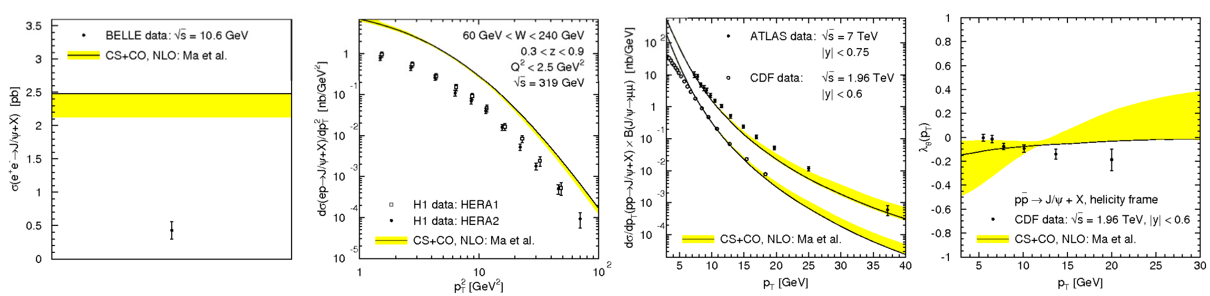
Gong, W...
H.-F. Zha...

$\langle O_8^{J/\psi}(^1S_0)\rangle$ ...
$\langle O_8^{J/\psi}(^3S_1)\rangle = -0.0046$ GeV ...
$\langle O_8^{J/\psi}(^3P_0)\rangle = -0.0214\ \mathrm{GeV}^5$ ... 0.0095 ...
$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.$ ... GeV³



Chao, Ma, Shao, K. Wang, Y.-J. Zhang:

$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.089\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3S_1)\rangle = 0.003\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}(^3P_0)\rangle = 0.0126\ \mathrm{GeV}^5$



k_T Factorization Approach

Apply k_T factorization to quarkonium production:

- **Idea:** Scales of quarkonium production much smaller than collision energy:

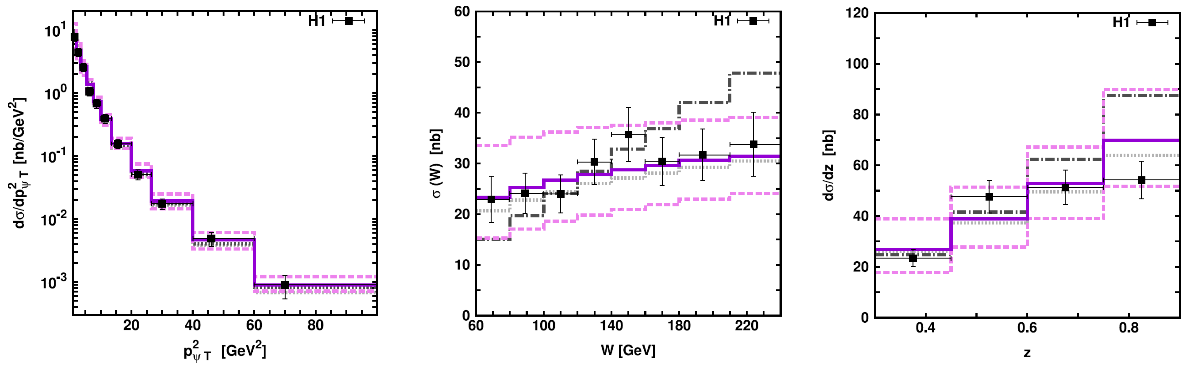
  $$p_T, m_c \ll \sqrt{s}$$

  ➡ Longitudinal parton momentum fractions x small,
  transverse parton momenta k_T should not be neglected.
- Use **off shell** matrix elements with k_T dependence entering via

  $$\varepsilon^\mu(k_T) = k_T^\mu / |\vec{k}_T| \,.$$
- Usually just LO matrix elements used.
- Fold with k_T dependent, **unintegrated PDFs**.
- **Various prescriptions** for deriving uPDFs from usual PDFs
  in DGLAP, BFKL or “CCFM” approach.
- Monte Carlo program **CASCADE** simulates initial state gluon radiation
  within k_T factorization framework [Jung, Salam (2001)].

k_T Factorization Approach: Results (1)

- Baranov, Lipatov, Zotov (2011); Baranov, Lipatov, Zotov (2012): **Color Singlet Model** predictions for various uPDFs:



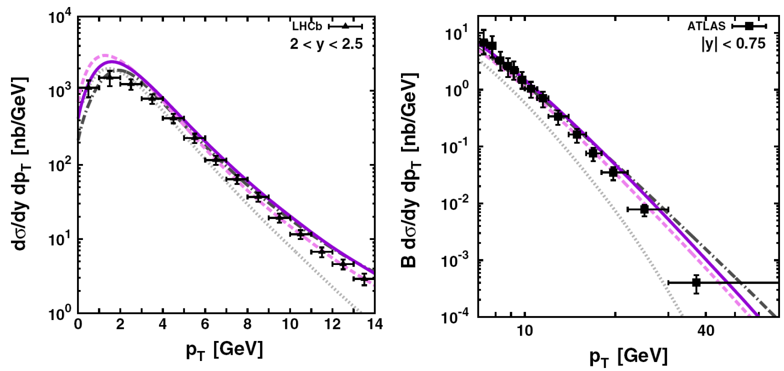


TABLE I. The polarization parameters of prompt J/ψ mesons calculated in the kinematical region of CMS and ATLAS measurements [27,28]. The CCFM A0 gluon density is used.

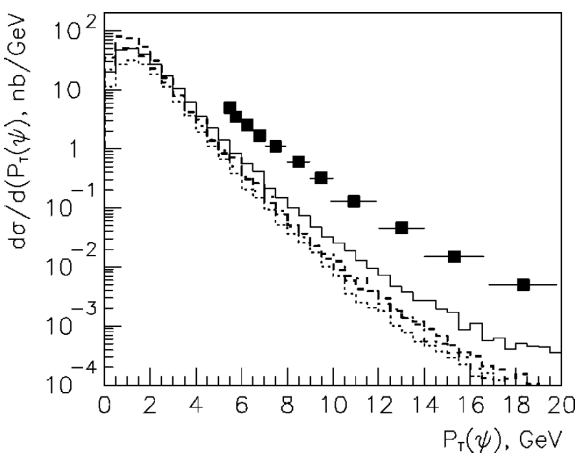
Source	λ_θ (HX)	λ_ϕ (HX)	$\lambda_{\theta\phi}$ (HX)	λ_θ (CS)	λ_ϕ (CS)	$\lambda_{\theta\phi}$ (CS)
Direct	-0.15	-0.09	0.01	0.20	-0.22	-0.01
Feed-down	0.19	0.14	0.00	0.35	0.09	0.00
Total	-0.07	-0.03	0.01	0.24	-0.14	-0.01

➡ No room and no need for color octet contributions.

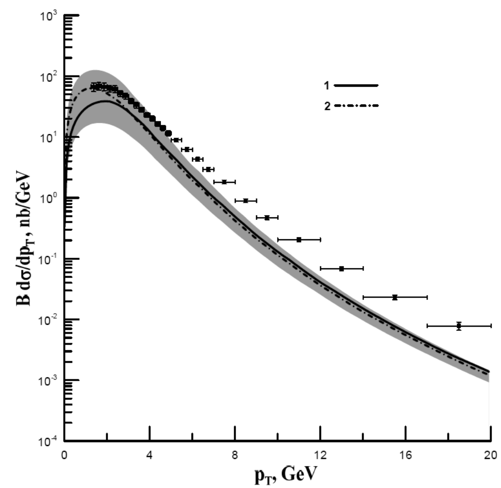
k_T Factorization Approach: Results (2)

But: Other calculations come to different conclusions:

(for hadroproduction)



[Baranov (2002)]



[Saleev, Nefedov, Shipilova (2012)]

➡ Effect of k_T **much smaller**, color singlet still **not enough!**

- In these (and other) works, also fits of CO LDMEs within k_T factorization framework been performed.

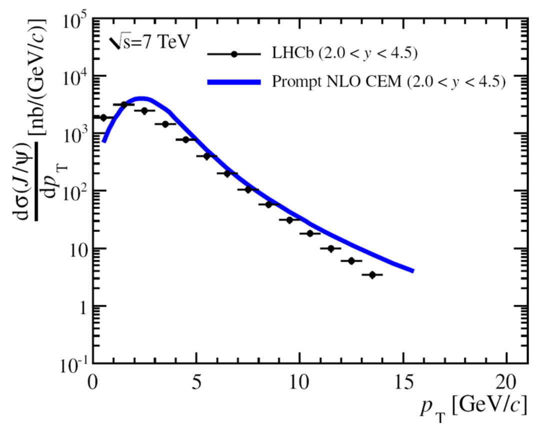
Color Evaporation Model

- Fritsch (1977); Halzen (1977); Glück, Owens, Reya (1978):

$$\sigma = F_H \int_{(2m_c)^2}^{(2m_D)^2} dm^2 \frac{d\sigma_{c\bar{c}}}{dm^2_{c\bar{c}}}$$

- Consider **open $c+\bar{c}$** production, regardless of $c+\bar{c}$ color, spin, momenta.
- Integrate over invariant $c+\bar{c}$ mass up to formation of next heavier meson pair.
- $\boldsymbol{F_H}$: Number describing formation of quarkonium H by color "evaporation".
- Qualitative picture rather than rigorous theory.

- A recent **NLO** prediction:
  [Frawley, Ullrich, Vogt (2008)]



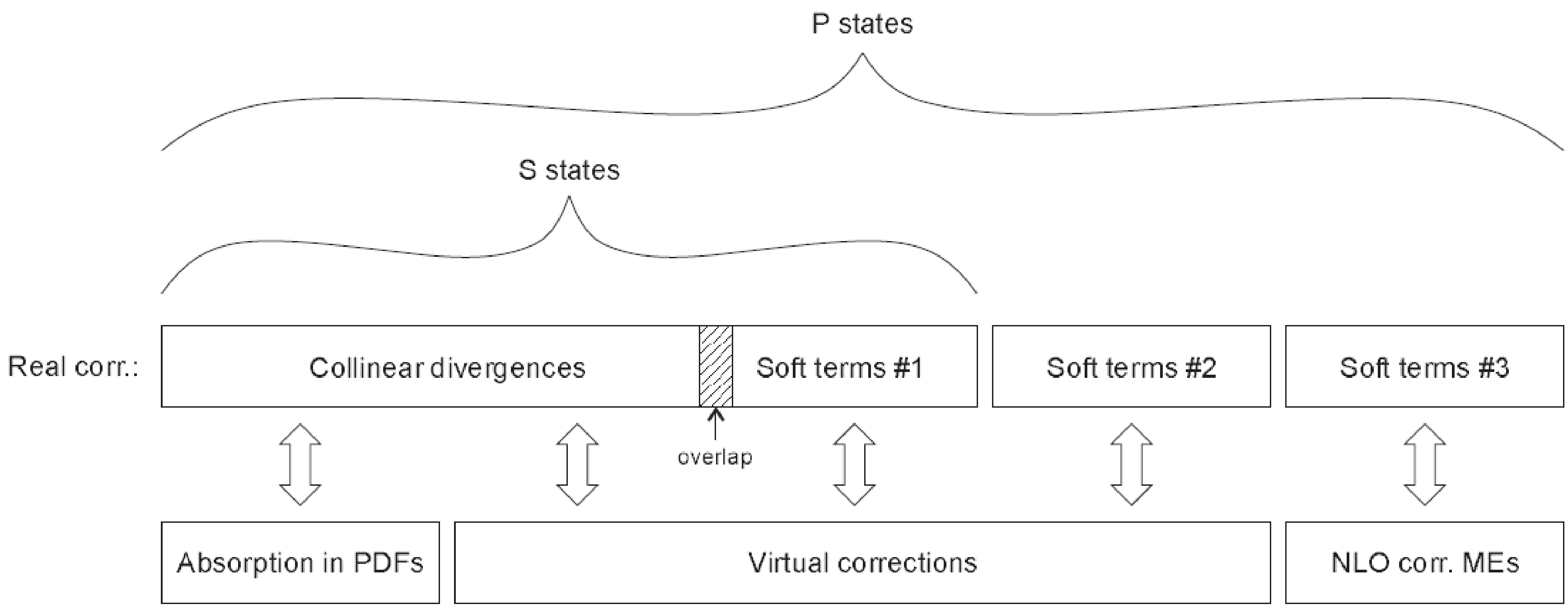
Summary

- NRQCD provides rigorous **factorization theorem** for heavy quarkonium production. But: Need to proof **LDME universality**.
- **Combined NLO fit** of NRQCD LDMEs to inclusive J/ψ production data from ALICE, ATLAS, BELLE, CDF, CMS, DELPHI, H1, LHCb, PHENIX, ZEUS.
- Good agreement for **CS+CO** with data except perhaps for $\gamma\gamma \to J/\psi+X$.
- **CSM** predictions fall **short of data** everywhere except for $e^+e^- \to J/\psi+X$.
- Fit constrained. CO LDMEs in accordance with **velocity scaling rules**.
- NLO calculations of **polarized** J/ψ cross section including CO states: Direct photoproduction at HERA and hadroproduction at Tevatron and LHC.
- **CDF Tevatron** Run II data in disagreement with our NRQCD prediction, early low-p_T **ALICE** data however still in agreement.

- **Two later** analyses **also show** that e^+e^-, γp, pp **yield** and **CDF Run II** polarization data can **not** be described with same LDME set.
  ➡ Following LHC measurements: Hopefully **clarify** LDME universality!

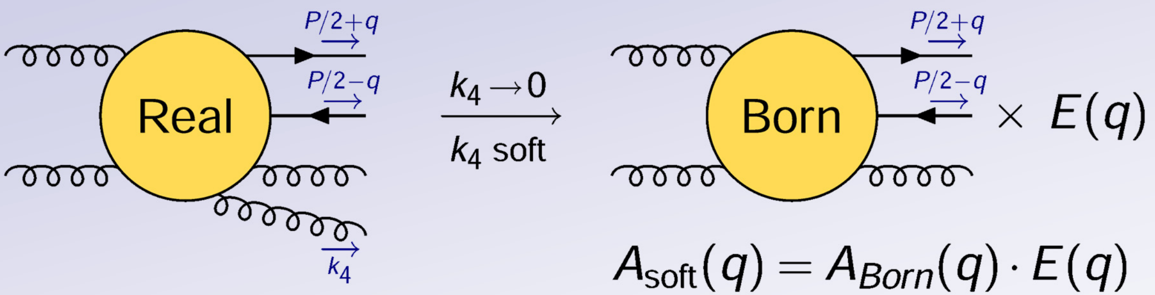
BACKUP SLIDES

Overview of IR Singularity Structure



Structure of Soft Singularities

Soft limits of the real corrections:



$$A_{\rm soft}(q) = A_{Born}(q) \cdot E(q)$$

S and P states: Soft #1 + Soft #2 + Soft #3 terms:

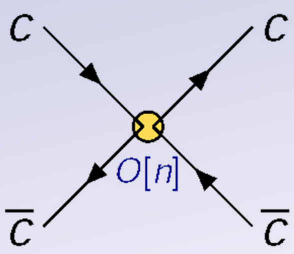
$$A_{\rm soft,s} = A_{\rm soft}(0) = A_{\rm Born,s} \cdot E(0)$$
$$A_{\rm soft,p} = A'_{\rm soft}(0) = A_{\rm Born,p} \cdot E(0) + A_{\rm Born,s} \cdot E'(0)$$

$$|A_{\rm soft,s}|^2 = |A_{\rm Born,s}|^2 \cdot E(0)^2$$
$$|A_{\rm soft,p}|^2 = |A_{\rm Born,p}|^2 \cdot E(0)^2 + 2\,\mathrm{Re}\, A^*_{\rm Born,s} A_{\rm Born,p} \cdot E(0)E'(0) + |A_{\rm Born,s}|^2 \cdot E'(0)^2$$

Radiative Corrections to LDMEs

In NRQCD: Long distance MEs = $c\bar{c}$ scattering amplitudes:

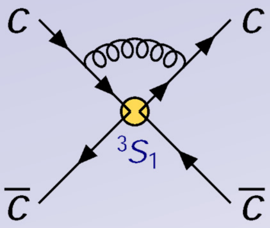
$\langle O^{J/\psi}[n] \rangle =$



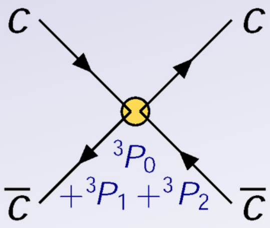
$O[n]$ = 4-fermion operators

$(n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}, \ldots)$

Corrections to $<O^{J/\psi}[{}^3S_1^{[1/8]}]>$ with NRQCD Feynman rules:

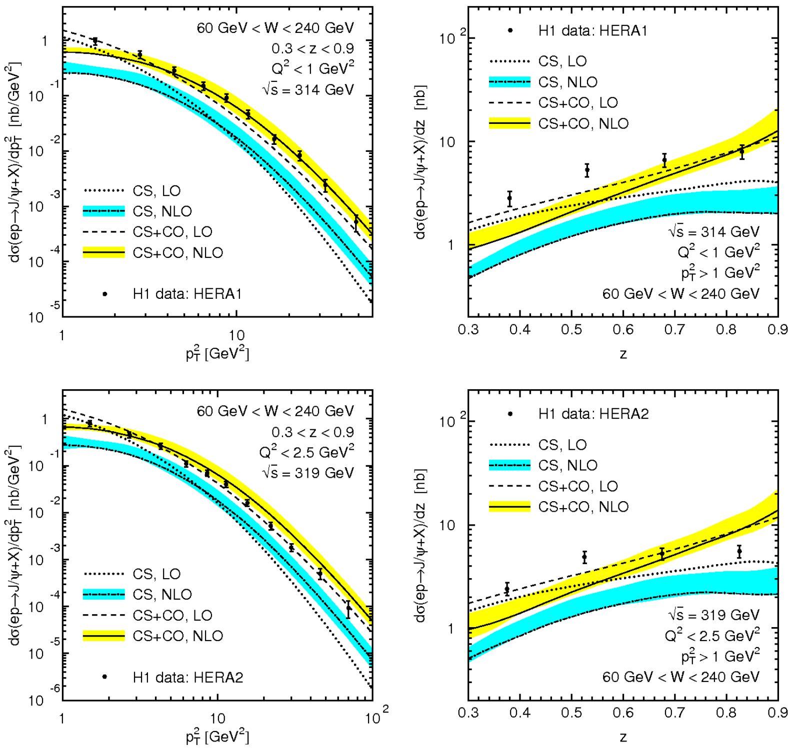


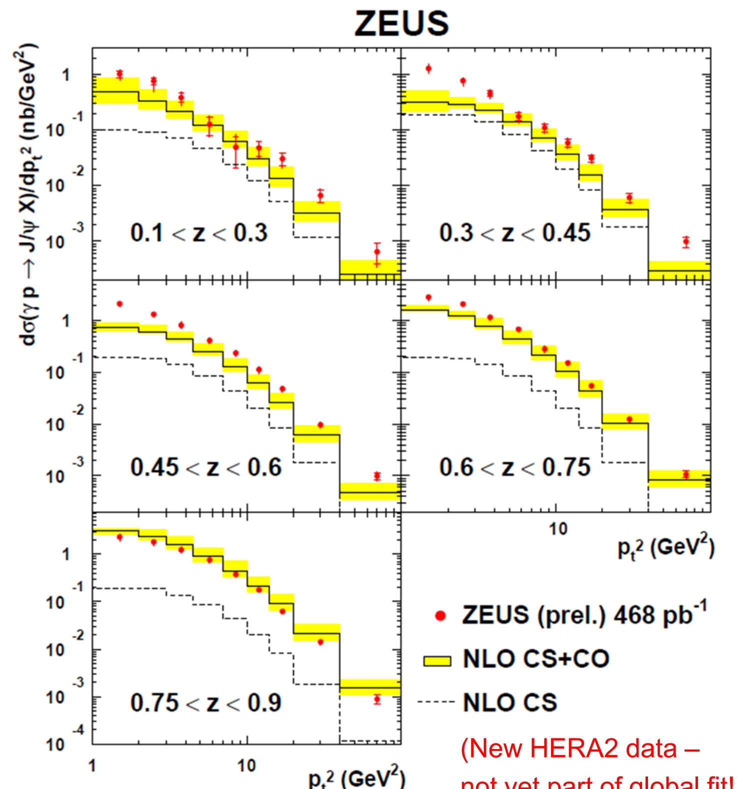
$+$ similar diagrams $\propto \frac{4\alpha_s}{3\pi m_c^2}\left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}\right) \cdot$



- **UV singularity** cancelled by renormalization of 4-fermion operator.
- **IR singularity** cancels soft #3 terms of *P* states.

In Detail: More Photoproduction



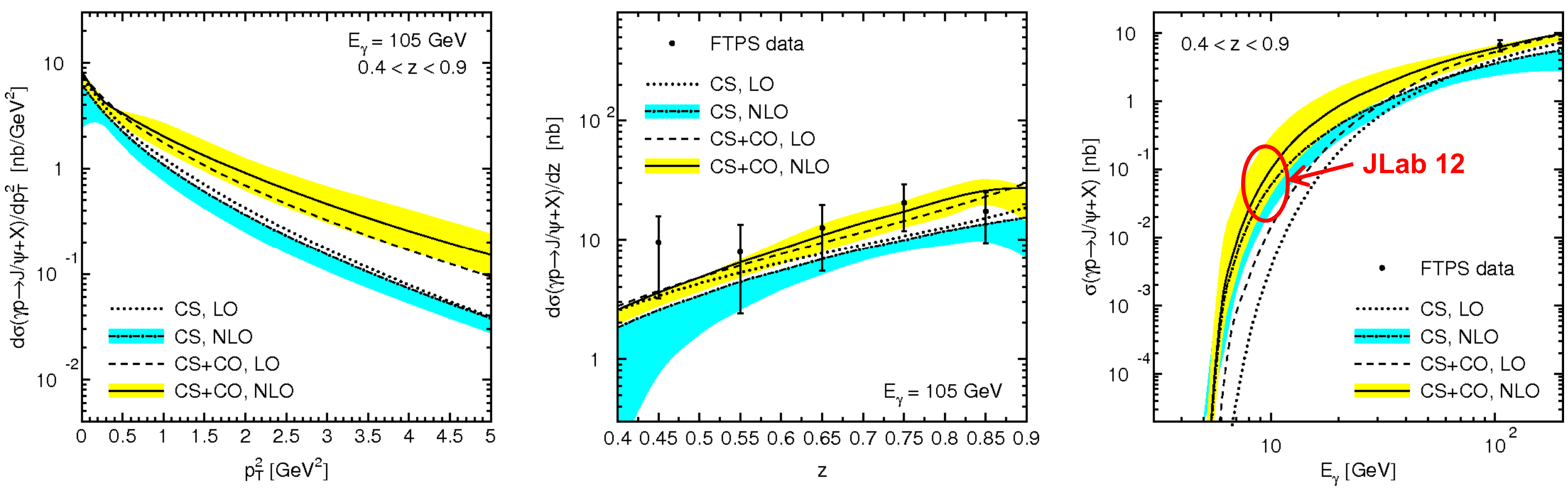


(New HERA2 data – not yet part of global fit!)

- Again: CS alone **below** data; **CS+CO** good description, especially at high *z*.
- H1 HERA2 data systematically below H1 HERA1 and ZEUS HERA1 + 2.

Low-energy inelastic J/ψ photoproduction

- **FTPS experiment** at Fermilab ('80s): 105 GeV photons on hydrogen target.
- Measured **inelastic** *J/ψ* production ($z = E_{J/\psi} / E_\gamma < 0.9$)
  NRQCD yields good description even at this low-energy range:



- Planned **JLab** near-threshold measurements: 12 GeV electrons on nuclei.
- **Total inelastic cross section**: ~ 10^{-2} nb. Measureable?
  (does of course increase with other nuclei than hydrogen)
- Close to threshold: Bad perturbative stability of parton model.

Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-p_T cut:

	p_T > 1 GeV	p_T > 2 GeV	p_T > 3 GeV	p_T > 5 GeV	p_T > 7 GeV
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	8.54 ± 0.52	16.85 ± 1.23	11.02 ± 1.67	1.68 ± 2.20	2.18 ± 2.56
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	-2.66 ± 0.69	-13.36 ± 1.60	-5.56 ± 2.19	8.75 ± 2.98	10.34 ± 3.55
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-3.63 ± 0.23	-7.70 ± 0.61	-4.46 ± 0.87	2.20 ± 1.23	3.50 ± 1.50
M_0 [10^{-2} GeV^3]	**2.25 ± 0.12**	**3.51 ± 0.19**	**3.29 ± 0.20**	**5.50 ± 0.29**	**8.24 ± 0.58**
M_1 [10^{-3} GeV^3]	**6.37 ± 0.19**	**5.80 ± 0.19**	**5.54 ± 0.20**	**3.27 ± 0.29**	**1.63 ± 0.43**

- Fit **underconstrained.** Therefore give two linear combinations of Ma *et al.*:

$$M_0 = \langle O(^1S_0^{[8]})\rangle + 3.9\,\langle O(^3P_0^{[8]})\rangle/m_c^2 \qquad M_1 = \langle O(^3S_1^{[8]})\rangle - 0.56\,\langle O(^3P_0^{[8]})\rangle/m_c^2$$

- Fit results **depend strongly** on low-p_T cut.

Agreement with Ma *et al.*'s fit to Tevatron run II data with p_T > 7 GeV:

Default: Include feed-downs, directly fit M_0 and M_1:	**M_0 = (7.4 ± 1.9) 10^{-2} GeV^3**	**M_1 = (0.5 ± 0.2) 10^{-3} GeV^3**
Ignore feed-downs, directly fit M_0 and M_1:	**M_0 = (8.92 ± 0.39) 10^{-2} GeV^3**	**M_1 = (1.26 ± 0.23) 10^{-3} GeV^3**
Ignore feed-downs, M_0 and M_1 from 3-parameter fit:	**M_0 = (8.54 ± 1.02) 10^{-2} GeV^3**	**M_1 = (1.67 ± 1.05) 10^{-3} GeV^3**
[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001		

Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-p_T cut:

	$p_T > 1$ GeV	$p_T > 2$ GeV	$p_T > 3$ GeV	$p_T > 5$ GeV	$p_T > 7$ GeV
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	8.54 ± 0.52	16.85 ± 1.23	11.02 ± 1.67	1.68 ± 2.20	2.18 ± 2.56
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	-2.66 ± 0.69	-13.36 ± 1.60	-5.56 ± 2.19	8.75 ± 2.98	10.34 ± 3.55
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-3.63 ± 0.23	-7.70 ± 0.61	-4.46 ± 0.87	2.20 ± 1.23	3.50 ± 1.50
M_0 [10^{-2} GeV^3]	**2.25 ± 0.12**	**3.51 ± 0.19**	**3.29 ± 0.20**	**5.50 ± 0.29**	**8.24 ± 0.58**
M_1 [10^{-3} GeV^3]	**6.37 ± 0.19**	**5.80 ± 0.19**	**5.54 ± 0.20**	**3.27 ± 0.29**	**1.63 ± 0.43**

- Fit **underconstrained.** Therefore give two linear combinations of Ma *et al.*:

$$M_0 = \langle O(^1S_0^{[8]})\rangle + 3.9\langle O(^3P_0^{[8]})\rangle/m_c^2 \qquad M_1 = \langle O(^3S_1^{[8]})\rangle - 0.56\langle O(^3P_0^{[8]})\rangle/m_c^2$$

- Fit results **depend strongly** on low-p_T cut.

Agreement with Ma *et al.*'s fit to Tevatron run II data with $p_T > 7$ GeV:

Default: Include feed-downs, directly fit M_0 and M_1:	**M_0 = (7.4 ± 1.9) 10^{-2} GeV^3**	**M_1 = (0.5 ± 0.2) 10^{-3} GeV^3**
Ignore feed-downs, directly fit M_0 and M_1:	**M_0 = (8.92 ± 0.39) 10^{-2} GeV^3**	**M_1 = (1.26 ± 0.23) 10^{-3} GeV^3**
Ignore feed-downs, M_0 and M_1 from 3-parameter fit:	**M_0 = (8.54 ± 1.02) 10^{-2} GeV^3**	**M_1 = (1.67 ± 1.05) 10^{-3} GeV^3**
[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001		

Global Fit: Dependence on Low-p_T Cuts (1)

Global fit: Vary low-p_T cut on hadroproduction data:

hadroproduction data left	$p_T > 1$ GeV 148 points	$p_T > 2$ GeV 134 points	$p_T > 3$ GeV 119 points	$p_T > 5$ GeV 86 points	$p_T > 7$ GeV 60 points
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	5.68 ± 0.37	4.25 ± 0.43	4.97 ± 0.44	4.92 ± 0.49	3.91 ± 0.51
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	0.90 ± 0.50	2.94 ± 0.58	2.24 ± 0.59	2.23 ± 0.62	2.96 ± 0.64
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-2.23 ± 0.17	-1.38 ± 0.20	-1.61 ± 0.20	-1.59 ± 0.22	-1.16 ± 0.23
M_0 [10^{-2} GeV^3]	**1.81 ± 0.09**	**1.85 ± 0.09**	**2.18 ± 0.10**	**2.17 ± 0.12**	**1.89 ± 0.12**
M_1 [10^{-3} GeV^3]	**6.46 ± 0.17**	**6.37 ± 0.17**	**6.25 ± 0.17**	**6.18 ± 0.17**	**5.86 ± 0.18**

↑ **Our default fit**

- **Stabilizing** influence of **photoproduction** data.
- Fit **constrained** enough: Can now extract 3 CO LDMEs.
- Fit results now **almost independent** of low-p_T cut.
- Fit less stable with low-p_T cut below 2 GeV (nonperturbative effects).

Global Fit: Dependence on Low-p_T Cuts (2)

Global fit: Vary low-p_T cut on photoproduction (including γγ-scattering):

photoproduction data left	p_T > 1 GeV 74 points	p_T > 2 GeV 30 points	p_T > 3 GeV 15 points	p_T > 5 GeV 5 points	p_T > 7 GeV 1 point
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	4.97 ± 0.44	5.10 ± 0.92	4.05 ± 1.17	5.44 ± 1.27	9.56 ± 1.59
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	2.24 ± 0.59	2.11 ± 1.22	3.52 ± 1.56	1.73 ± 1.68	-3.66 ± 2.09
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-1.61 ± 0.20	-1.58 ± 0.48	-0.97 ± 0.63	-1.63 ± 0.68	-3.73 ± 0.83
M_0 [10^{-2} GeV^3]	2.18 ± 0.10	2.36 ± 0.12	2.37 ± 0.13	2.62 ± 0.15	3.10 ± 0.19
M_1 [10^{-3} GeV^3]	6.25 ± 0.17	6.05 ± 0.18	5.94 ± 0.19	5.78 ± 0.20	5.62 ± 0.20

↑ Our default fit

- **Fit stable** against varying low-p_T cut in region 1 GeV ~ 3 GeV.
- Just 5 or 1 photoproduction against 119 hadroproduction points not enough to stabilize the fit. ➡ **Not stable** with low-p_T cut much larger than 3 GeV. (Would need more high-p_T photoproduction data.)

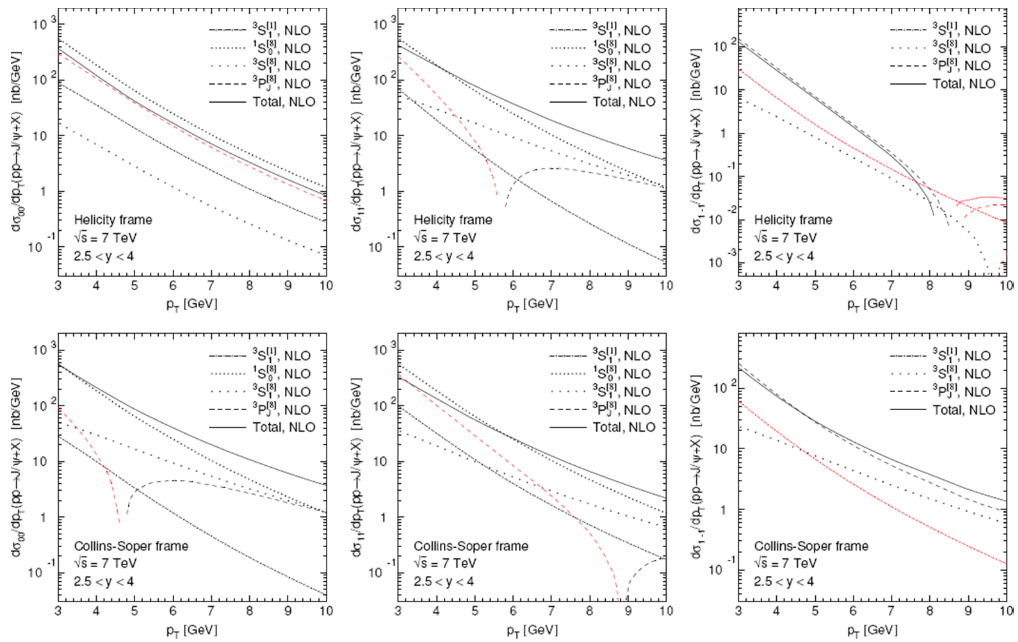
Polarization in Hadroproduction: Contributions

- **First:** Sum up contributions of intermediate states:

$d\sigma_{00}$ $d\sigma_{11}$ $d\sigma_{1\,-1}$

Helicity frame:

Collins-Soper frame:



- **Then:** $\lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_{\theta\phi} = \frac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\phi = \frac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$