Maximal R-symmetry violating amplitudes in type IIB superstring theory

(based on arXiv:1204.4208)

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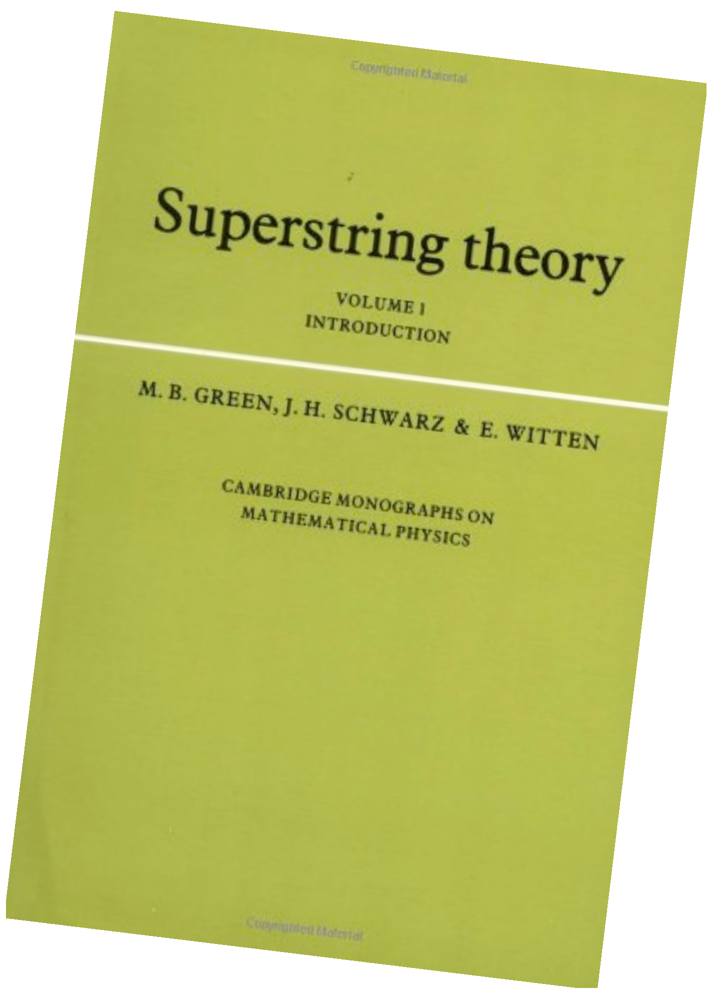

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- calculate correlation functions
- integrate over moduli

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(but go see Schlotterer's talk!)

Goal: simple answers

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- golden standard of simple scattering amplitudes: four dimensional MHV amplitudes in tree level Yang-Mills

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D=4 vs D>4?

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- every leg has (k, η_a) coordinates → on-shell superspace
- chirality constraints easy, reality conditions hard

Superamplitudes

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- component amplitudes by fermionic integration

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- 3 massless particle exception: $\frac{3}{4}\mathcal{D}$ fermionic coordinates

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- also three points, five points, on-shell recursion in paper

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• U(1)R in D=10 → part of SL(2,R)/U(1) of IIB not conserved
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Massless on-shell superspace in D=10, type IIB

D=10: 256 states in the fundamental multiplet

$$\phi(\eta) = \phi_0 + \phi^a \eta_a + \ldots + \overline{\phi}_0 (\eta)^8$$

field content:

bosonic		fermionic	
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  - only massive particle poles (n>4)
  - → no poles in field theory limit

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$$\begin{aligned}\tilde{A}_5^{\mathrm{MRV}} =& (g\alpha'^2)^3 \Big[-6\,\zeta(3)\alpha'^3 - \frac{5}{2}\,\zeta(5)\alpha'^5 \left([s_{12}^2]_5\right) \\ & +2\,\zeta(3)^2\alpha'^6 \left([s_{12}^3]_5\right) - \frac{7}{32}\zeta(7)\,\alpha'^7 \left(13[s_{12}^4]_5 + 6[s_{12}^2 s_{34}^2]_5\right) \\ & + \frac{1}{30}\zeta(3)\zeta(5)\,\alpha'^8 \left(71[s_{12}^5]_5 + 25[s_{12}^3 s_{34}^2]_5\right) + \mathcal{O}\left(\alpha'^9\right)\Big]\end{aligned}$$

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$$\begin{aligned}\tilde{A}_5^{\mathrm{MRV}} =& (g\alpha'^2)^3 \Big[-6\,\zeta(3)\alpha'^3 - \frac{5}{2}\zeta(5)\alpha'^5\left([s_{12}^2]_5\right) \\ &+2\,\zeta(3)^2\alpha'^6\left([s_{12}^3]_5\right) - \frac{7}{32}\zeta(7)\,\alpha'^7\left(13[s_{12}^4]_5 + 6[s_{12}^2 s_{34}^2]_5\right) \\ &+\frac{1}{30}\zeta(3)\zeta(5)\,\alpha'^8\left(71[s_{12}^5]_5 + 25[s_{12}^3 s_{34}^2]_5\right) + \mathcal{O}\left(\alpha'^9\right)\Big]\end{aligned}$$

- using results from: [Kawai-Lewellen-Tye, 86], [Stieberger-Taylor, 06], [Huber-Maitre, 07] + equivalence theorem

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

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- differential operator annihilates gravitational coupling

→ relates c_i for various multiplicities, up to degeneracy

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include more stringy symmetries?

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

more stringy symmetry in IIB: SL(2,Z)

[Green-Gutperle, 97], [Green et.al., 97-12]

$$\tau_b = a_b + \mathrm{i}\, e^{-\phi_b} \qquad f^k_\beta(\tau_b, \bar{\tau}_b) = \sum_{(l,m)\neq(0,0)} (l + m\tau_b)^{k-\beta}(l + m\bar{\tau}_b)^{-k-\beta}$$

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- analytic part of amplitude: the "no logs"-part
- guess for next order exists
- much work: relation to effective action, better normalization...

Summary, outlook

remarkably simple class of amplitudes in type IIB superstrings

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more examples of applications / more explicit amplitudes?

- how deep does analogy to MHV go?
- worldsheet picture? (pure spinor?)
- IIA? D=11? open strings? → `constrained superspaces'?

Your Question Here?