Towards a quantum treatment of leptogenesis

Mathias Garny (DESY, Hamburg)

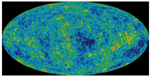
DESY Theory workshop, September 25 – 28 2012

based on work in collaboration with Andreas Hohenegger, Alexander Kartavtsev [1112.6428]

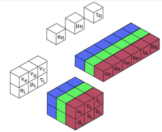
Physics beyond the Standard Model

Collider exp.

Baryon
asymmetry



⋮



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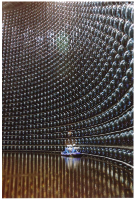
⋮



Neutrino exp.

Dark matter

Outline

Towards a quantum treatment of leptogenesis

- Leptogenesis
- Boltzmann approach
- Kadanoff-Baym approach
- Resonant enhancement on the closed time path

Leptogenesis

Standard Model (SM) extended by three heavy singlet neutrino fields $N_i = N_i^c$, $i = 1, 2, 3$ with Majorana masses $\hat{M} = \mathrm{diag}(M_i)$ in the mass eigenbasis

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \frac{1}{2}\bar{N} i \partial\!\!\!/ N - \frac{1}{2}\bar{N}\hat{M}N - \bar{\ell}\widetilde{\phi} h P_R N - \bar{N} P_L h^\dagger \widetilde{\phi}^\dagger \ell$$

Light neutrino masses via seesaw mechanism

$$m_\nu = -v_{EW}^2 h \hat{M}^{-1} h^T \quad \rightarrow \quad \text{TeV} \lesssim M_i \lesssim M_{GUT} \text{ for } m_e/v_{EW} < h_{ij} < 1$$

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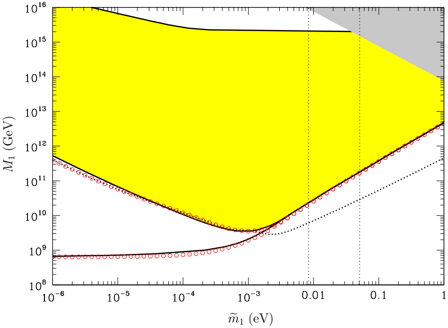
Baryogenesis via leptogenesis *Fukugita, Yanagida 86*

- B-violation via L-violating Majorana masses M_i
- CP-violation via Yukawa couplings $\mathrm{Im}[(h^\dagger h)_{ij}] \neq 0$
- Out-of-equilibrium (inverse) decay $N_i \leftrightarrow \ell\phi^\dagger$ and $N_i \leftrightarrow \ell^c\phi$

$$\begin{aligned} (\Gamma_i/H)|_{T=M_i} &\simeq \tilde{m}_i/\mathrm{meV} \sim \mathcal{O}(1) && \text{where } \tilde{m}_i = v_{EW}^2 (h^\dagger h)_{ii}/M_i \\ (\Gamma_{SM}/H)|_{T=M_i} &\sim g^4 M_{pl}/M_i \gg 1 && \text{for } M_i \ll 10^{16}\mathrm{GeV} \end{aligned}$$

Vanilla leptogenesis for hierarchical spectrum $M_1 \ll M_{2,3}$ requires large values of the reheating temperature $T_R \gtrsim \mathcal{O}(M_1) \gtrsim 10^9\,\mathrm{GeV}$

Hamaguchi, Murayama, Yanagida; Davidson, Ibarra



Buchmüller, Di Bari, Plümacher

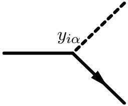
Gravitino production

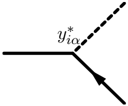
$$\Omega_{3/2}^{th} h^2 \simeq 0.27 \left(\frac{T_R}{10^9\,\mathrm{GeV}}\right)\left(\frac{10\,\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{m_{\widetilde{g}}}{1\,\mathrm{TeV}}\right)^2$$

Moroi, Murayama, Yamaguchi, 93; Bolz, Brandenburg, Buchmüller, 01;Pradler, Steffen, 06; Rychkov, Strumia, 07

Leptogenesis

L-violating decay of heavy right-handed neutrino N_i

$\mathcal{M}_{N_i \to \ell_\alpha h^\dagger} =$  $+ \ldots$

$\mathcal{M}_{N_i \to \ell^c_\alpha h} =$  $+ \ldots$

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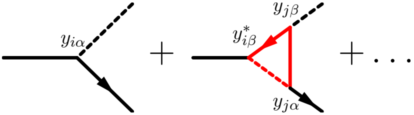

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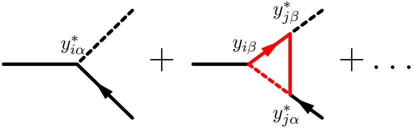
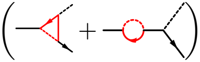


Matter-antimatter (CP) asymmetry

$\Leftrightarrow$ interference of tree and loop processes

$\Gamma(N_i \to \ell_\alpha h^\dagger) - \Gamma(N_i \to \ell^c_\alpha h) \sim \mathrm{Im}(y_{i\alpha} y_{i\beta} y^*_{j\alpha} y^*_{j\beta}) \cdot \mathrm{Im}$



Sakharov: asymmetry from out-of-equilibrium N_i decay/inverse decay

Standard Boltzmann approach

$$N_L(t) = \int d^3x \sqrt{|g|} \int \frac{d^3p}{(2\pi)^3} \sum_{\ell} [f_\ell(t, \mathbf{x}, \mathbf{p}) - f_{\bar{\ell}}(t, \mathbf{x}, \mathbf{p})]$$

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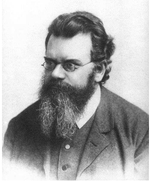

$$p^\mu \mathcal{D}_\mu f_\ell(t,\mathbf{x},\mathbf{p}) \;=\; \sum_i \int d\Pi_{N_i} d\Pi_h$$

$$\times\, (2\pi)^4 \delta(p_\ell + p_h - p_{N_i})$$

$$\times \Big[|\mathcal{M}|^2_{N_i \to \ell h^\dagger}\; f_{N_i}(1 - f_\ell)(1 + f_h) + \ldots$$

$$-\; |\mathcal{M}|^2_{\ell h^\dagger \to N_i}\; f_\ell f_h (1 - f_{N_i}) + \ldots \Big]$$

$f_\psi(t,\mathbf{x},\mathbf{p})$: distribution function of on-shell particles

$|\mathcal{M}|^2$: matrix elements computed in *vacuum*, off-shell effects

Corrections within Boltzmann framework

- Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium
  - quantum statistical factors $1 \pm f_k$
  - non-integrated Boltzmann equations

Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

- Thermal corrections via thermal QFT
  - medium correction to CP-violating parameter $\epsilon = \epsilon^{vac} + \delta\epsilon^{th}$
  - thermal masses, dispersion relation
  - production rate from a thermal plasma for $T \gg M_1$, $T \ll M_1$

Covi, Rius, Roulet, Vissani 98; Giudice, Notari, Raidal, Riotto, Strumia 04; Besak, Bödeker 10

Kiessig, Thoma, Plümacher 10; Salvio Lodone Strumia 11, Anisimov, Besak, Bödeker 11; Laine, Schroder 11; Besak, Bodeker 12...

- Flavour effects

Nardi, Nir, Roulet, Racker 06; Abada, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; de Simone, Riotto 06; Blanchet, diBari 06; ...

- Spectator processes, scatterings, N_2, ...

Double Counting Problem

Naive contribution from decay/inverse decay

$$|\mathcal{M}|^2_{N_i \to \ell h^\dagger} = |\mathcal{M}_0|^2(1+\epsilon_i) \qquad |\mathcal{M}|^2_{\ell h^\dagger \to N_i} = |\mathcal{M}_0|^2(1-\epsilon_i)$$

$$|\mathcal{M}|^2_{N_i \to \ell^c h} = |\mathcal{M}_0|^2(1-\epsilon_i) \qquad |\mathcal{M}|^2_{\ell^c h \to N_i} = |\mathcal{M}_0|^2(1+\epsilon_i)$$

$$\begin{aligned}\frac{dN_{B-L}}{dt} \quad \propto \quad & (|\mathcal{M}|^2_{N_i \to \ell h^\dagger} - |\mathcal{M}|^2_{N_i \to \ell^c h}) N_{N_i} \\ & - (|\mathcal{M}|^2_{\ell h^\dagger \to N_i} - |\mathcal{M}|^2_{\ell^c h \to N_i}) N^{eq}_{N_i}\end{aligned}$$

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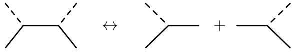
$$\begin{aligned} \frac{dN_{B-L}}{dt} &\propto (|\mathcal{M}|^2_{N_i \to \ell h^\dagger} - |\mathcal{M}|^2_{N_i \to \ell^c h}) N_{N_i} \\ &\quad - (|\mathcal{M}|^2_{\ell h^\dagger \to N_i} - |\mathcal{M}|^2_{\ell^c h \to N_i}) N^{eq}_{N_i} \\ &\propto \epsilon_i (N_{N_i} + N^{eq}_{N_i}) \end{aligned}$$

$\Rightarrow$ spurious generation of asymmetry even in equilibrium

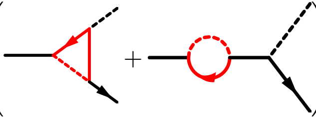
Origin: Double Counting Problem 

$\rightarrow$ need real intermediate state subtraction

... derivation from first principles / generalization ?

Efficiency of leptogenesis depends on CP-violating parameter, which is one-loop suppressed

$$\epsilon_{N_i} = \frac{\Gamma(N_i \to \ell\phi^\dagger) - \Gamma(N_i \to \ell^c\phi)}{\Gamma(N_i \to \ell\phi^\dagger) + \Gamma(N_i \to \ell^c\phi)} \propto \mathrm{Im}\left(\quad + \quad \right)$$



Self-energy (or 'wave') contribution to CP-violating parameter features a resonant enhancement for a quasi-degenerate spectrum $M_1 \simeq M_2 \ll M_3$

$$\epsilon_{N_i}^{wave} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

Flanz Paschos Sarkar 94/96; Covi Roulet Vissani 96;

Leptogenesis - resonant enhancement

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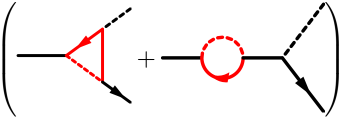

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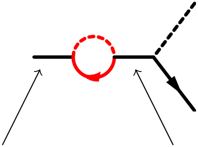


On-shell initial N_1: $p^2 = M_1^2$ Internal N_2: $\frac{i}{p^2 - M_2^2}$

Resonant leptogenesis

- *Flanz Paschos Sarkar Weiss 96;* effective Hamiltonian approach

$$\epsilon_{N_i} = -\frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{16\pi(h^\dagger h)_{ii}} \frac{M_1(M_2 - M_1)}{(M_2 - M_1)^2 + M_1^2(\mathrm{Re}(h^\dagger h)_{12}/(16\pi))^2}$$

- *Covi Roulet 96;* CP violating decay of mixing scalar fields described by effective mass matrix; formalism as in Liu Segre 93
- *Pilaftsis 97; Pilaftsis Underwood 03;* Pole mass expansion of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

- *Buchmüller Plümacher 97;* Diagonalization of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \frac{M_1 M_2(M_2^2 - M_1^2)}{(M_2^2 - M_1^2 - \frac{1}{\pi}\Gamma_i M_i \ln\frac{M_2^2}{M_1^2})^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2}$$

- *Rangarajan Mishra 99;* comparison of different approaches
- *Anisimov Broncano Plümacher 05;* Reconciliation of diagonalization approach with the pole mass expansion approach
- Invariant quantity $M_1 M_2(M_2^2 - M_1^2)\mathrm{Im}(h^\dagger h)_{12}^2$ related to CP violation appears in the enumerator

Resonant leptogenesis

The results can be summarized (neglecting log-corrections) as

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times R, \qquad R \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Different calculations correspond to different expressions for the 'regulator' A^2

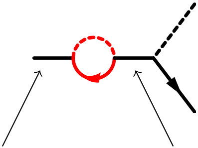
$$A^2 = \begin{cases} \frac{1}{4}(M_1 + M_2)^4 \left(\frac{\mathrm{Re}(h^\dagger h)_{12}}{16\pi}\right)^2 & \textit{Flanz Paschos Sarkar Weiss 96} \\ M_i^2 \Gamma_j^2 & \textit{Pilaftsis 97; Pilaftsis Underwood 03} \\ (M_1\Gamma_1 - M_2\Gamma_2)^2 & \textit{Buchmüller Plümacher 97; Anisimov Broncano Plümacher 05; \ldots} \\ \ldots & \end{cases}$$

The regulator is relevant for determining the maximal possible resonant enhancement, which occurs for $M_2^2 - M_1^2 = \pm A$, and is given by

$$R_{max} = \frac{M_1 M_2}{2|A|}$$

Resonant leptogenesis

The origin of the regulator is the finite width of N_1 and N_2



Off-shell initial N_1: $p^2 = M_1^2 + iM_1\Gamma_1$ Internal N_2: $\frac{i}{p^2 - M_2^2 - iM_2\Gamma_2}$

Resonant leptogenesis

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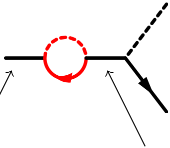

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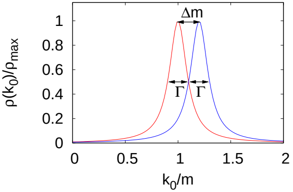
In the maximal resonant case $M_2 - M_1 = \mathcal{O}(\Gamma_i)$, the spectral functions overlap



deviation from equilibrium is essential
⇒ desirable to go beyond the quasi-particle approximation which underlies the conventional semi-classical Boltzmann approach.

CTP/Kadanoff-Baym approach to leptogenesis

Goal(s)

(1)

- derivation of kinetic equations starting from first principles
- on-/off-shell treated in a unified way (avoid double-counting)

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- coherent flavor transitions, finite-width effects, ...

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(... incomplete list) Buchmüller, Fredenhagen hep-ph/004145; De Simone, Riotto hep-ph/0703175, 0705.2183

Anisimov, Buchmüller, Drewes, Mendizabal 1001.3856, 1012.5821

MG, Kartavtsev, Hohenegger, Lindner 0909.1559,0911.4122

Beneke, Fidler, Garbrecht, Herranen, Schwaller 1002.1326,1007.4783

Garbrecht 1011.3122; Gagnon, Shaposhnikov 1012.1126

MG, Kartavtsev, Hohenegger 1002.0331, 1005.5385, 1112.6428

Drewes, Mendizabal, Weniger 1202.1301

Garbrecht, Herranen 1112.5954; Garbrecht 1202.5126;Garbrecht, Drewes 1206.5537

see also Fidler, Herranen, Kainulainen, Rahkila 1108.2309; Canetti, Drewes, Shaposhnikov 1204.4186

Going beyond the Boltzmann picture

$$
\begin{aligned}
\text{Statistical propagator} \quad S_F^{ij}(x,y) &= \langle N_i(x)\bar{N}_j(y) - \bar{N}_j(y)N_i(x)\rangle/2 \\
\text{Spectral function} \quad S_\rho^{ij}(x,y) &= i\langle N_i(x)\bar{N}_j(y) + \bar{N}_j(y)N_i(x)\rangle
\end{aligned}
$$

Going beyond the Boltzmann picture

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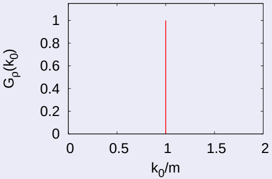
Boltzmann limit

- on-shell quasi-stable particles



$$S_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like
  fluctuation-dissipation relation

$$S_F^{ij}(t,k) = \left(\frac{1}{2} - f_k^i(t)\right) S_\rho^{ij}(k)$$

Going beyond the Boltzmann picture

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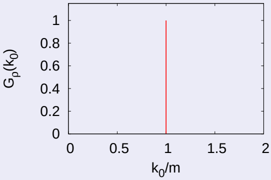

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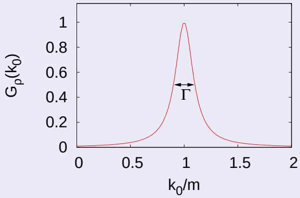
Propagation beyond Boltzmann

- spectrum with (thermal) width



$$S_\rho^{ij}(t,k) \propto \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \dots$$

- coherent $N_1 - N_2$ transitions

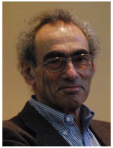
$$S_F^{ij}(t,k) = \begin{pmatrix} S_F^{11} & S_F^{12} \\ S_F^{21} & S_F^{22} \end{pmatrix}$$

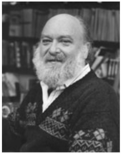
Kadanoff-Baym equations

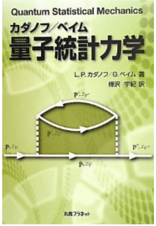
$$\left((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik}\right) S_F^{kj}(x,y) = \int_0^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{ik}(x,z) S_F^{kj}(z,y) - \int_0^{y^0} dz^0 \int d^3z\, \Sigma_{NF}^{ik}(x,z) S_\rho^{kj}(z,y)$$

$$\left((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik}\right) S_\rho^{kj}(x,y) = \int_{y^0}^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{ik}(x,z) S_\rho^{kj}(z,y)$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Memory integrals







CTP/Kadanoff-Baym approach to leptogenesis

Lepton current

$$j_L^\mu(x) = \left\langle \sum_\alpha \bar{\ell}_\alpha(x)\gamma^\mu \ell_\alpha(x) \right\rangle = -\mathrm{tr}\left[\gamma^\mu {S_\ell}^{\alpha\beta}(x,x)\right]$$

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$$n_L(t) = \frac{1}{V}\int_V d^3x\, j_L^0(t,\mathbf{x})$$

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Equation of motion

$$\frac{dn_L}{dt} = \frac{1}{V}\int_V d^3x\,\partial_\mu j_L^\mu(x) = -\frac{1}{V}\int_V d^3x\,\mathrm{tr}\left[\gamma_\mu(\partial_x^\mu + \partial_y^\mu) S_\ell{}^{\alpha\beta}(x,y)\right]_{x=y}$$

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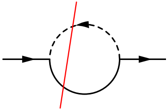
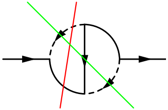
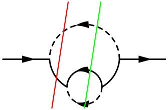
Use KB equations for leptons on the right-hand side $\Rightarrow$

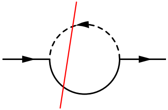
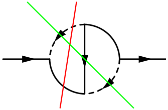
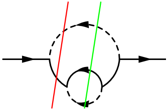
$$\begin{aligned}\frac{dn_L}{dt} \quad = \quad & i\int_0^t dt' \int \tfrac{d^3p}{(2\pi)^3}\, \mathrm{tr}\Big[\Sigma_{\ell}{}_{\rho\,\mathbf{p}}^{\alpha\gamma}(t,t') S_{\ell}{}_{F\,\mathbf{p}}^{\gamma\beta}(t',t) - \Sigma_{\ell}{}_{F\,\mathbf{p}}^{\alpha\gamma}(t,t') S_{\ell}{}_{\rho\,\mathbf{p}}^{\gamma\beta}(t',t) \\ & - S_{\ell}{}_{\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell}{}_{F\,\mathbf{p}}^{\gamma\beta}(t',t) + S_{\ell}{}_{F\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell}{}_{\rho\,\mathbf{p}}^{\gamma\beta}(t',t)\Big]\end{aligned}$$

CTP/Kadanoff-Baym approach to leptogenesis

			
$N \leftrightarrow \ell\phi^\dagger$ $N \leftrightarrow \ell^c\phi$	$\lvert tree \rvert^2$	tree $\times$ vertex-corr.	tree $\times$ wave-corr.
$\ell\phi^\dagger \leftrightarrow \ell^c\phi$		s $\times$ t	s $\times$ s, t $\times$ t

- unified description of CP-violating decay, inverse decay, scattering

CTP/Kadanoff-Baym approach to leptogenesis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- unified description of CP-violating decay, inverse decay, scattering
- dn_L/dt vanishes in equilibrium due to KMS relations

$$S_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) S_\rho^{eq} \qquad \Sigma_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) \Sigma_\rho^{eq}$$

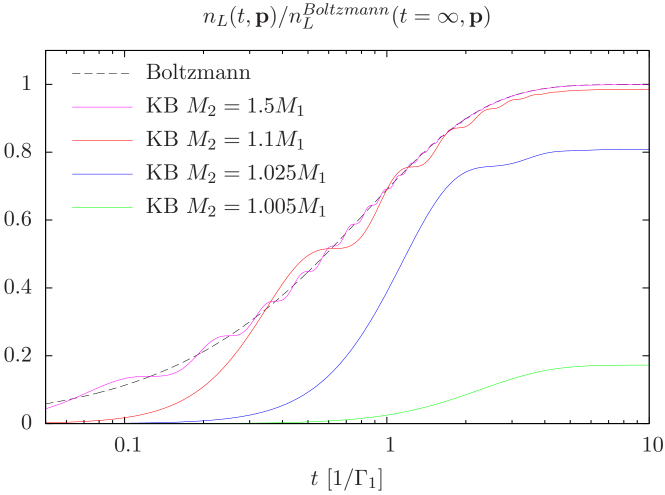
$\Rightarrow$ consistent equations free of double-counting problems

MG, Kartavtsev, Hohenegger, Lindner 09;

Beneke, Garbrecht, Herranen, Schwaller 10;

Resonant enhancement

Solve KB eqs for resummed neutrino propagator $S^{ij}(t,\mathbf{p})$ in $N_{1,2}$ flavor space, treat lepton/Higgs as thermal bath *MG, Kartavtsev, Hohenegger 1112.6428*



$\Gamma_1 = 0.01M_1, \Gamma_2 = 0.015M_1, \Gamma_{\ell\phi} \to 0$

Resonant enhancement

Resonant enhancement within the Boltzmann approach

$$R^{Boltzmann} \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

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Resonant enhancement within the Kadanoff-Baym approach, including coherent contributions ($|(h^\dagger h)_{12}| \ll (h^\dagger h)_{ii}$)

$$R^{KB}(t) = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \times (1 - f_{coherent}(t))$$

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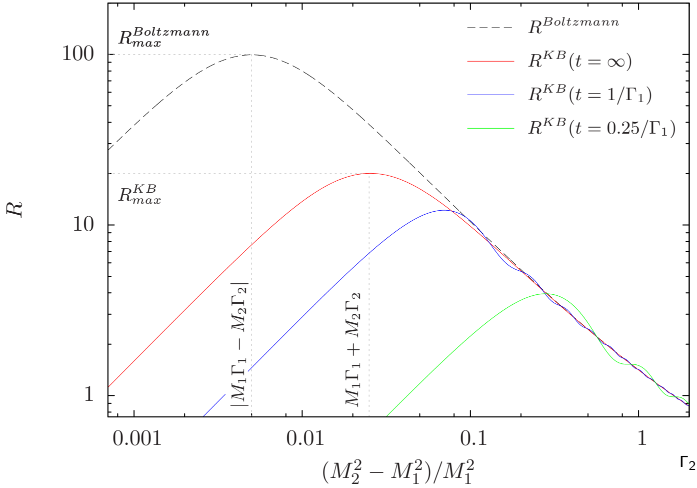
Partial cancellation of Boltzmann- and coherent contribution cuts off the enhancement in the doubly degenerate limit $M_1 \to M_2$ and $\Gamma_1 \to \Gamma_2$

$$R^{KB}(t)|_{t \gtrsim 1/\Gamma_{pl}} \quad \simeq \quad \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 + M_2\Gamma_2)^2}$$

Resonant enhancement

MG, Kartavtsev, Hohenegger 1112.6428



$\Gamma_2/\Gamma_1 = 1.5$

$$R^{Boltzmann}_{max} = M_1 M_2/(2|\Gamma_1 M_1 - \Gamma_2 M_2|),\ R^{KB}_{max} = M_1 M_2/(2(\Gamma_1 M_1 + \Gamma_2 M_2))$$

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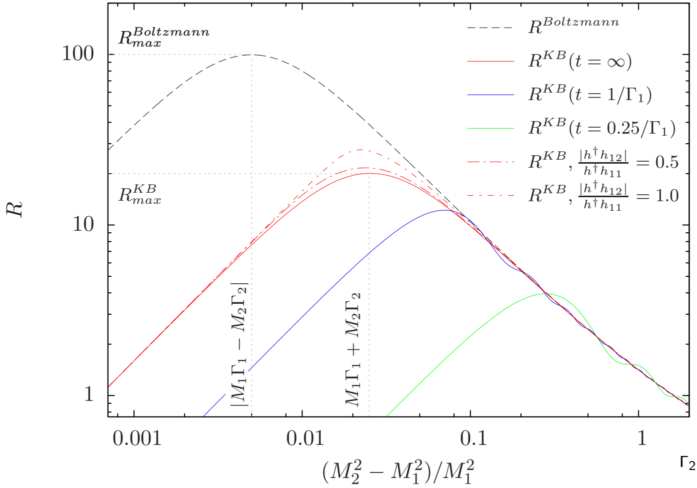

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Conclusions

Towards a quantum treatment of leptogenesis

- First-principles methods like Kadanoff-Baym equations are helpful to scrutinize classical approximations
- Resonant enhancement on the closed time path
  - valid for $\Delta M \sim \Gamma$, effective width/mass, flavor coherence
  - smaller enhancement than Boltzmann

$$
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\end{aligned}
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  - numerical solution for $|h^\dagger h|_{12} \sim h^\dagger h_{ii}$ (check BW appr.)
  - relevant *e.g.* for flavor symmetries ($\Delta M/M, \Delta\Gamma/\Gamma \ll 1$)

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thank you!

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Goal: obtain analytical result by taking essential features into account
(width, coherent flavor-mixing, memory integrals)
[later: compare with numerical treatment]

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- First step: Non-equilibrium Majorana propagator in BW appr.

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- Second step: Lepton asymmetry

$$\begin{aligned} n_L(t) \quad = \quad & i(h^\dagger h)_{ji} \int_0^t dt' \int_0^t dt'' \int \tfrac{d^3p}{(2\pi)^3} \\ & \mathrm{tr}\Big[P_R \underbrace{\Big(\Delta S^{ij}_{F\,\mathbf{p}}(t',t'') - \Delta \bar{S}^{ji}_{F\,\mathbf{p}}(t',t'') \Big)}_{\propto \text{ Deviation from equilibrium, CP-violation}} P_L S_{\ell\phi\,\rho\,\mathbf{p}}(t''-t') \Big] \end{aligned}$$

$\Delta \bar{S}^{ji}_{F\,\mathbf{p}}(t',t'') = CP \Delta S^{ij}_{F\,\mathbf{p}}(t'',t')^T (CP)^{-1}$
lepton-Higgs loop $S_{\ell\phi} = S_\ell \Delta_\phi$

Result for the lepton asymmetry

$$n_L(t) = \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, 2q\, 2k}\, (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \sum_{I,J=1,2} \sum_{\epsilon_n=\pm 1} F_{JI}^{\epsilon_n} L_{IJ}^{\epsilon_n}(t)$$

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Coefficients F depend on Yukawa couplings, thermal distributions of lepton and Higgs, resummed ret/adv propagators and initial conditions

$$F^{\epsilon_n}_{JI} = \sum_{ijkl=1,2} (h^\dagger h)_{ji} \left(\left(\tfrac{1}{2} + f_\phi(q)\right) + \epsilon_2\epsilon_3 \left(\tfrac{1}{2} - f_\ell(k)\right) \right) \mathrm{tr}\Big[P_L(|\mathbf{k}|\gamma_0 + \epsilon_2 \mathbf{k}\gamma) \times \left(S^{ik\epsilon_4}_{RI} \gamma_0 \Delta S^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 S^{lj\epsilon_1}_{AJ} - \bar{S}^{jk\epsilon_4}_{RI} \gamma_0 \Delta \bar{S}^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 \bar{S}^{li\epsilon_1}_{AJ} \right) \Big]$$

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Time-dependence: flavor diagonal and off-diagonal contributions:

$$\begin{aligned} L_{II}^{\pm}(t) \;&=\; \frac{1-e^{-\Gamma_{pI} t}}{\Gamma_{pI}} \,\mathrm{Re}\left(\frac{\Gamma_{\ell\phi}}{(\omega_{pI} - k - q + i\Gamma_{pI}/2)^2 + \Gamma_{\ell\phi}^2/4} \right) \\ L_{21}^{\pm}(t) \;&=\; \frac{1-e^{\mp i(\omega_{p1}-\omega_{p2})t} e^{-(\Gamma_{p1}+\Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} \pm 2i(\omega_{p1}-\omega_{p2})} \left(\frac{\Gamma_{\ell\phi}}{(\omega_{p1} - k - q \pm i\Gamma_{p1}/2)^2 + \Gamma_{\ell\phi}^2/4} \right. \\ & \left. + \frac{\Gamma_{\ell\phi}}{(\omega_{p2} - k - q \mp i\Gamma_{p2}/2)^2 + \Gamma_{\ell\phi}^2/4} \right) \;=\; L_{12}^{\pm}(t)^* \end{aligned}$$

Resonant enhancement

Comparison KB – Boltzmann: hierarchical limit

$$n_L(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi}\frac{M_1}{M_2}\int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_{p1}\, 2q\, 2k} 4k\cdot p_1$$
$$(2\pi)^3\delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \times \mathrm{Re}\,\frac{\Gamma_{\ell\phi}}{(\omega_{p1}-k-q+i\Gamma_{p1}/2)^2+\Gamma_{\ell\phi}^2/4}$$
$$\times (1+f_\phi(q)-f_\ell(k))\, f_{FD}(\omega_{p1})\frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}$$

$$n_L^{Boltzmann}(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi}\frac{M_1}{M_2}\int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_{p1}\, 2q\, 2k} 4k\cdot p_1$$
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$$\times (1+f_\phi(q)-f_\ell(k))\, f_{FD}(\omega_{p1})\frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}\ .$$

The thermal width of lepton and Higgs $\Gamma_{\ell\phi} = \Gamma_\ell + \Gamma_\phi$ leads to a replacement of the on-shell delta function in the Boltzmann equations by a Breit-Wigner curve, in accordance with *Anisimov, Buchmüller, Drewes, Mendizabal 10*

The coherent contributions are suppressed with Γ_{p1}/ω_{p2}

Resonant enhancement

Comparison KB – Boltzmann: degenerate case

$$n_L(t) = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k} 4k\cdot p$$
$$\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$
$$\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4\,\mathrm{Re}\left(\frac{1 - e^{-i(\omega_{p1}-\omega_{p2})t} e^{-(\Gamma_{p1}+\Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]$$

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- Regulator $M_1\Gamma_1 - M_2\Gamma_2$ is confirmed
- Additional oscillating contribution due to coherent N_1—N_2 transitions

Majorana neutrino propagator

Retarded and advanced propagators

$$\begin{aligned} S_R(x,y) &= \Theta(x^0 - y^0) S_\rho(x,y) \\ S_A(x,y) &= -\Theta(y^0 - x^0) S_\rho(x,y) \end{aligned}$$

The Kadanoff-Baym equation for the statistical propagator can be written as

$$\int_0^\infty d^4z \left[\Big(\left(i\partial\!\!\!/_x - M_i \right) \delta^{ik} - \delta\Sigma_N{}^{ik}(x) \Big) \delta(x-z) - \Sigma_N{}_R^{ik}(x,z) \right] S_F^{kj}(z,y) = \int_0^\infty d^4z \, \Sigma_N{}_F^{ik}(x,z) S_A^{kj}(z,y)$$

Special solution of the inhomogeneous equation

$$S_F^{ij}(x,y)_{inhom} = -\int_0^\infty d^4u S_R^{ik}(x,u) \int_0^\infty d^4z \, \Sigma_N{}_F^{kl}(u,z) S_A^{lj}(z,y)$$

General solution of the homogeneous equation

$$S_F^{ij}(x,y)_{hom} = -\int d^3u \, S_R^{ik}(x,(0,\mathbf{u})) \int d^3v \; A^{kl}(\mathbf{u},\mathbf{v}) S_A^{lj}((0,\mathbf{v}),y)$$

where $A^{kl}(\mathbf{u},\mathbf{v})$ is a free function

Majorana neutrino propagator

Treating lepton/Higgs as a thermal bath, the Majorana neutrino propagator is given by

$$S^{ij}_{F\,\mathbf{p}}(t,t') = S^{ij\,th}_{F\,\mathbf{p}}(t-t') + \underbrace{S^{ik}_{R\,\mathbf{p}}(t)\gamma_0 \Delta S^{kl}_{F\,\mathbf{p}}(0,0)\gamma_0 S^{lj}_{A\,\mathbf{p}}(-t')}_{\equiv \Delta S^{ij}_{F\,\mathbf{p}}(t,t')}$$

The resummed retarded and advanced functions can be obtained by solving a SD equation (we use $\overline{MS}$ at $T=0$ and $\mu = (M_1+M_2)/2$)

$$\left[\left(\not{p} - M_i\right)\delta^{ik} - \delta\Sigma_N{}^{ik}(p) - \Sigma_N{}^{ik}_{R(A)}(p)\right] S^{kj}_{R(A)}(p) = -\delta^{ij}$$

$$\Sigma_N{}^{ij}_{R(A)}(p) = -2\left[(h^\dagger h)_{ij} P_L + (h^\dagger h)_{ji} P_R\right] S_{\ell\phi\,R(A)}(p)$$

$$S_{\ell\phi}{}^{vac}_{R(A)}(p) = \frac{1}{32\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 2 - \ln\left(\frac{|p^2|}{\mu^2}\right) \pm i\pi\Theta(p^2)\mathrm{sign}(p_0)\right)\not{p}$$

$$S_{\ell\phi}{}^{th}_{R(A)}(p) \simeq -T^2\not{p}/(12p^2) \pm 2i\not{p}/(e^{|p_0|/T}-1)\Theta(p^2)\mathrm{sign}(p_0)/(32\pi)$$

Majorana neutrino propagator

Solve SD equation for $S_{R(A)}(p)$ in Breit-Wigner approximation:

$$S^{ij}_{R(A)}(p) \simeq \frac{Z^{ij}_{1R(A)}}{p^2 - x_1} + \frac{Z^{ij}_{2R(A)}}{p^2 - x_2}$$

with residua $Z^{ij}_{IR(A)}$ and complex poles x_I (basis independent)

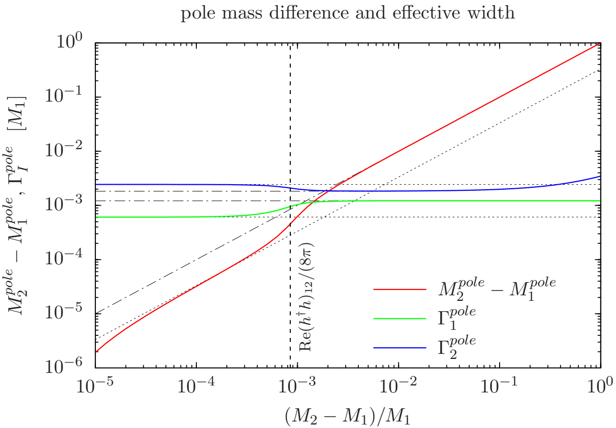
$$x_{1,2} = \frac{(V \pm W)^2}{4Q^2} \equiv \left(\omega_{pI} - i\frac{\Gamma_{pI}}{2}\right)^2 - \mathbf{p}^2$$

where

$$\begin{aligned} V &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) - \eta_2 M_2(1+i\gamma_{11}))^2 - 4\eta_1\eta_2 M_1 M_2 (\mathrm{Re}\gamma_{12})^2} \\ W &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) + \eta_2 M_2(1+i\gamma_{11}))^2 + 4\eta_1\eta_2 M_1 M_2 (\mathrm{Im}\gamma_{12})^2} \\ Q &= \det\Omega_{LR} = \det\Omega_{RL} = (1+i\gamma_{11})(1+i\gamma_{22}) + |\gamma_{12}|^2 \end{aligned}$$

$$\gamma_{ij} \simeq (h^\dagger h)_{ij}\left[\frac{\Theta(p^2)\mathrm{sign}(p_0)}{16\pi}\left(1 + \frac{2}{e^{|p_0|/T} - 1}\right) + i\left(\frac{\ln\frac{|p^2|}{\mu^2}}{16\pi^2} + \frac{T^2}{6p^2} - \frac{T^2}{6\mu^2}\right)\right]$$

Majorana neutrino propagator



Effective masses $M_I^{pole} \equiv \omega_{pI}|_{\mathbf{p}=0}$ and widths $\Gamma_I^{pole} \equiv \Gamma_{pI}|_{\mathbf{p}=0}$ of the sterile Majorana neutrinos extracted from complex poles of resummed ret/adv prop. for $(h^\dagger h)_{11} = 0.03$, $(h^\dagger h)_{22} = 0.045$, $(h^\dagger h)_{12} = 0.03 \cdot e^{i\pi/4}$ and $T = 0.25 M_1$.

Majorana neutrino propagator

- Regime $(M_2 - M_1)/M_1 \gtrsim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq M_i \,, \\ \Gamma_i^{pole} &\simeq \Gamma_i \equiv \frac{(h^\dagger h)_{ii}}{8\pi} M_i \left(1 + \frac{2}{e^{M_i/T} - 1}\right) \end{aligned}$$

- Regime $(M_2 - M_1)/M_1 \lesssim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq \frac{M_1 + M_2}{2} \pm \frac{(M_2 - M_1)((h^\dagger h)_{22} - (h^\dagger h)_{11})}{2\sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}} \,, \\ \Gamma_i^{pole} &\simeq \frac{M_i}{16\pi}\left(1 + \frac{2}{e^{M_i/T} - 1}\right)\Big((h^\dagger h)_{11} + (h^\dagger h)_{22} \\ &\qquad \pm \sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}\Big) \end{aligned}$$

The relation between the mass- and Yukawa coupling matrices at zero and finite temperature is

$$\begin{aligned} M(T) &= (P_L Z(T)^T + P_R Z(T)^\dagger) M(T=0)(P_L Z(T) + P_R Z(T)^*) \,, \\ (h^\dagger h)(T) &= Z(T)^T (h^\dagger h)(T=0) Z(T)^* \,, \end{aligned}$$

where $Z_{ij}(T) \equiv V_{ik}(T)(\delta_{kj} + (h^\dagger h)_{kj} T^2/(6\mu^2))$, $V(T)^\dagger V(T) = 1$