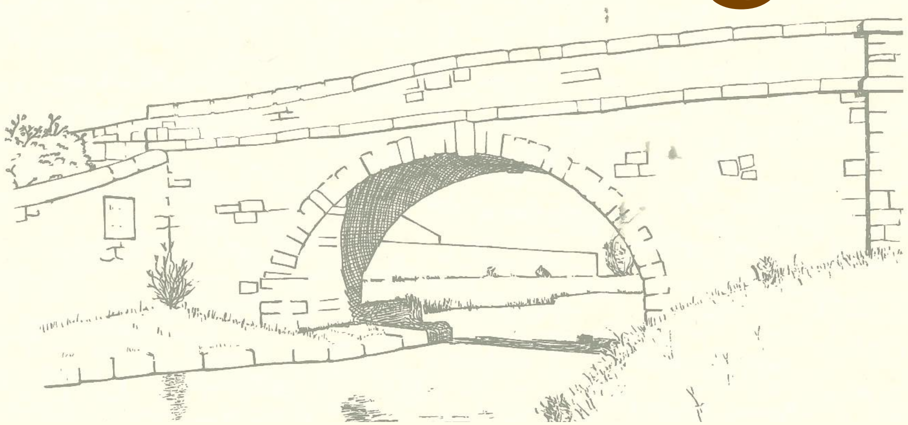
New Dualities in Three-dimensional Scattering



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Based on work with T. Bargheer, N. Beisert, S. He, F. Loebbert.

Superconformal Chern Simons matter theories

- Such three-dimensional theories (for example $\mathcal{N} = 6$ ABJM & $\mathcal{N} = 8$ BLG) describe the low energy dynamics of multiple M2-branes.
  - $\mathcal{N} = 6$ ABJM has many properties in common with 4D $\mathcal{N} = 4$ SYM. Spectrum of anomalous dimensions is integrable in the planar limit & it possesses a holographic dual string theory.
    - Scattering amplitudes also share many features with $\mathcal{N} = 4$ SYM however the $\mathcal{N} = 6$ ABJM theories are less constrained by supersymmetry and provide interesting generalisations.
      - Some evidence that on-shell they are related to three-dimensional supergravity in flat space e.g.

$$\text{“}\mathcal{N} = 8 \text{ BLG”}^2 \equiv \quad \mathcal{N} = 16 \ E_{8(8)} \text{ sugra}$$

$\mathcal{N} = 6$ ABJM theory

On-shell field content: four complex bosons and four fermions

$$\phi^{\hat{I}}(p)^{A}{}_{\bar{A}} \quad \& \quad \psi_{\hat{I}}(p)^{A}{}_{\bar{A}} \qquad \hat{I} = 1, \ldots, 4$$

transforming in bifundamental rep. of $U(N_c) \times U(\bar{N}_c)$ gauge group.   .

Useful to introduce real* spinors for onshell momenta

$$p^{\alpha\beta} = \lambda^{\alpha}\lambda^{\beta} \qquad \alpha = 1, 2$$

and Graßmann variables η^I , $I = 1, 2, 3$ for on-shell superfield

$$\Phi = \phi^4 + \eta^I \psi_I + \tfrac{1}{2}\epsilon_{IJK}\eta^I\eta^J\phi^K + \tfrac{1}{3!}\epsilon_{IJK}\eta^I\eta^J\eta^K\psi_4$$

& conjugate fermionic superfield

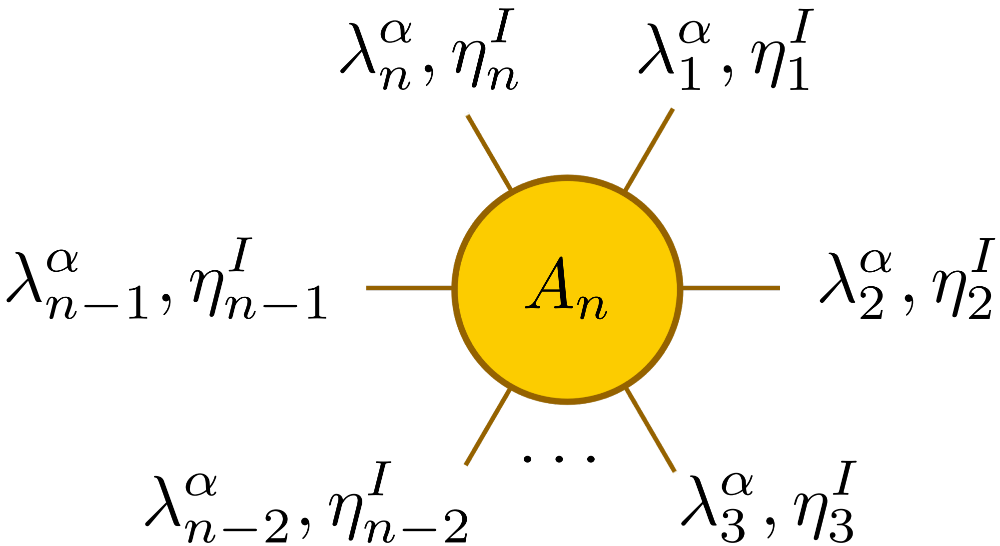
$$\bar{\Phi} = \bar{\psi}^4 + \ldots$$

* caveat emptor

We can define colour ordered, planar amplitudes

$$\mathcal{A}(\bar{\Phi}(p_1)^{\bar{A}_1}{}_{A_1}, \Phi(p_2)^{B_2}{}_{\bar{B}_2}, \dots, \Phi(p_n)^{B_n}{}_{\bar{B}_n}) = A(\bar{1}, 2, \dots, n)\delta^{B_2}{}_{A_1}\delta^{\bar{A}_3}{}_{\bar{B}_2} \dots \delta^{\bar{A}_1}{}_{\bar{B}_n} + \dots$$

sum over all permutations of even and odd sites modulo cyclic permutations by two sites.



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Ex. Four-points tree-level

$$A_4(\bar{\Phi}_1, \Phi_2, \bar{\Phi}_3, \Phi_4) = \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{\langle 12\rangle\langle 23\rangle}$$

[Bargheer, Loebbert, Meneghelli]

Lorentz invariants: $\langle ij\rangle = \lambda_i^\alpha \epsilon_{\alpha\beta}\lambda_j^\beta$

(Super)momenta: $P^{\alpha\beta} = \sum_{a=1}^{n} \lambda_a^\alpha \lambda_a^\beta \,, \quad Q^{I\alpha} = \sum_{a=1}^{n} \eta_a^I \lambda_a^\alpha$

- BCFW recursion relations generates all tree-level amplitudes (in principle) and proves there exists a Yangian symmetry for all tree-level amplitudes. [Gang, Huang, Koh, Lee, & Lipstein]

- Orthogonal Graßmannian formulation. [Lee]

- Dual to $AdS_4 \times CP^3$ type-IIA string theory
  - evidence of classical integrability
  - integrable spectrum (ABA & TBA/Y-system) [Gromov & Vieira] [Gromov, Kazakov, Vieira][Bombardelli, Fioravanti and Tateo]
  - some evidence for a duality between amplitudes and Wilson loops at four points

    e.g. [Henn, Plefka, Wiegandt], [Chen&Huang], [Bianchi et al], [Wiegandt]

Six-point one-loop amplitude

Result found using "anomalous" symmetries and unitarity [Bargheer, Beisert, Loebbert, TMcL]:

$$A_6^{(1)}(\bar{1},2,\bar{3},4,\bar{5},6) = \frac{\pi}{4} c_6(\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6}) A_6^{(0)}(\bar{6},1,\bar{2},3,\bar{4},5)$$

where

$$c_6(\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6}) = \mathrm{sgn}\langle 12\rangle \mathrm{sgn}\langle 34\rangle \mathrm{sgn}\langle 56\rangle + \mathrm{sgn}\langle 61\rangle \mathrm{sgn}\langle 23\rangle \mathrm{sgn}\langle 45\rangle .$$

- Answer is proportional to Yangian invariants, however there are additional discontinuities when two particles become collinear.
- Also found in a Feynman graph calculation. [Bianchi, Leoni, Mauri, Penati, Santambrogio] (see also [Huang, Caron-Huot], [Brandhuber, Travaglini and Wen]).
- Doesn't match bosonic Wilson loop, more akin to N=4 SYM NMHV amplitude. Optimistic conclusion:

Need a new super-Wilson loop.

SCS as a three-algebra theory

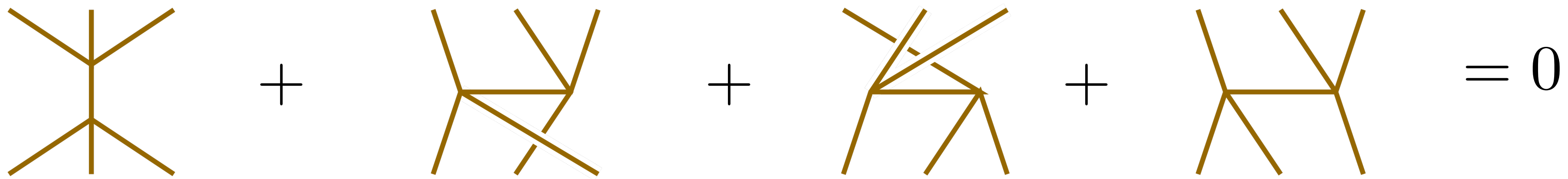
Can express the ABJM theory by rewriting the $U(N_c) \times U(N_c)$ color structure as a three-algebra [Bagger & Lambert]:

Complex vector space with basis T^a, $a = 1, \ldots, N_c^2$ and trilinear bracket

$$[T^a, T^b; \bar{T}^c] = f^{ab\bar{c}}{}_d T^d \quad \text{s.t.} \quad f^{ab\bar{c}}{}_d = -f^{ba\bar{c}}{}_d$$

A key property is the fundamental identity (c.f. Jacobi identity)

$$f^{ef\bar{g}}{}_b f^{*\bar{a}\bar{d}cb} + f^{fe\bar{a}}{}_b f^{*\bar{g}\bar{d}cb} + f^{ec\bar{d}}{}_b f^{*\bar{g}\bar{a}fb} + f^{cf\bar{d}}{}_b f^{*\bar{g}\bar{a}eb} = 0$$



SCS as a three-algebra theory

Can express the ABJM theory by rewriting the $U(N_c) \times U(N_c)$ color structure as a three-algebra [Bagger & Lambert]:

Enhanced $\mathcal{N} = 8$ supersymmetry (BLG-theory) when the vector space is real and structure constants are totally antisymmetric

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \qquad \text{s.t.} \qquad f^{abcd} = f^{[abcd]}$$

Only one finite dimensional example.

SCS as a three-algebra theory

Superfields transform as fundamental representations of three-algebra.
Four point amplitudes:

$$\mathcal{N}=6: \quad \mathcal{A}(\bar{1},2,\bar{3},4)=\frac{4\pi i}{k}\frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{\langle 12\rangle\langle 23\rangle}f^{a_2 a_4 \bar{a}_1 \bar{a}_3}$$

$$\mathcal{N}=8: \quad \mathcal{A}(1,2,3,4)=\frac{4\pi i}{k}\frac{\delta^{(3)}(P)\delta^{(8)}(Q)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}f^{a_1 a_2 a_3 a_4}$$

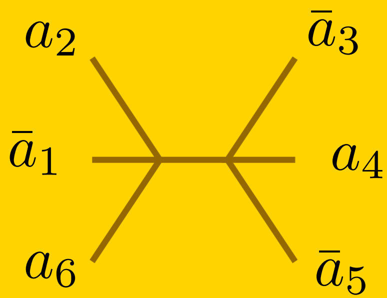
[Huang Lipstein]

A general amplitude is written as a sum of quartic graphs:

$$\mathcal{A}_n \propto \sum_{i\in \text{graphs}} \frac{n_i c_i}{\prod_{\alpha_i} \ell^2_{\alpha_i}}$$

c_i : $f^{a_n a_2 \bar{a}_1}{}_{b_1} f^{*\bar{a}_3 \bar{a}_5 b_1}{}_{\bar{b}_2} \cdots$
ℓ_i^2 : inverse propagators
n_i : kinematic numerators

e.g.



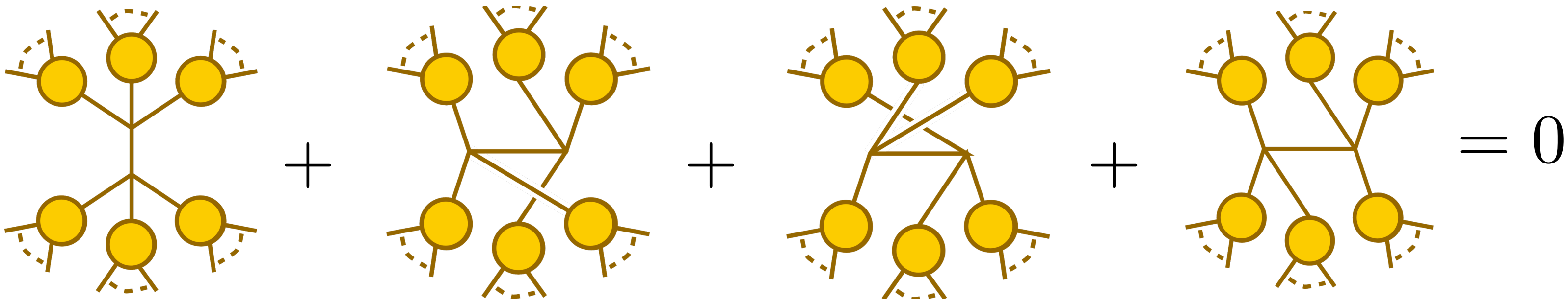
c_1 : $f^{a_6 a_2 \bar{a}_1}{}_{b_1} f^{*\bar{a}_3 \bar{a}_5 a_4 b_1}$

Color-kinematics duality

Claim: there exists a duality between color and kinematics analogous to that in YM [Bern, Johansson & Carrasco]:

$$\mathcal{A}_n \propto \sum_{i \in \text{graphs}} \frac{n_i c_i}{\prod_{\alpha_i} \ell_{\alpha_i}^2}$$

Different color structures are related by Fundamental identity:



$$c_s + c_t + c_u + c_v = 0$$

There exists numerators satisfying the same relations:

$$n_s + n_t + n_u + n_v = 0$$

[Bargheer, He, TMcL]

Evidence: four points (trivial) and six points (non-trivial)

- Implies non-trivial relations between color ordered amplitudes

Doubling to Supergravity

Given numerators satisfying the fundamental identities we can replace the color structures with another copy of the numerators:

$$\mathcal{M}_n = \sum_{i \in \text{graphs}} \frac{n_i n_i}{\prod_{\alpha_i} \ell_i^2}$$

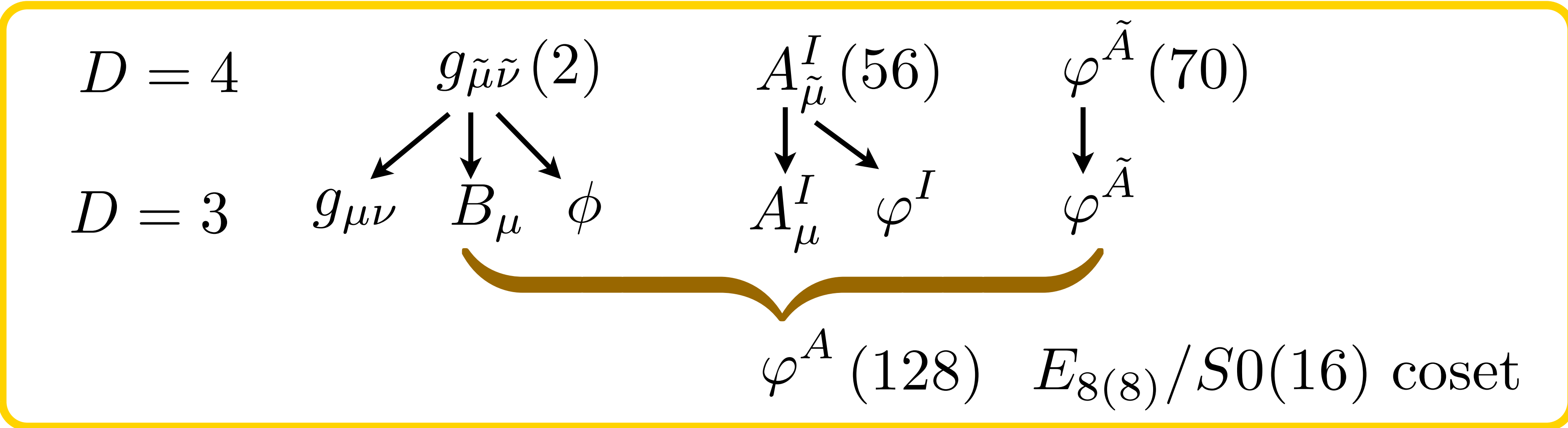
This defines the scattering amplitudes for a theory with a spectrum given by the square of gauge theory (c.f. KLT, BCJ)

- $\mathcal{N} = 8$ BLG case gives a theory with 128 bosons + 128 fermions and $\mathcal{N} = 16$ supersymmetry.
- Only has amplitudes with even numbers of external legs.
- This theory will have a hidden three-algebra structure in its kinematics!

$\mathcal{N} = 16$ Supergravity

- Maximally supersymmetric 3D supergravity constructed by Marcus and Schwarz has 128 bosons and 128 fermions transforming as SO(16) spinors $\Rightarrow$ correct spectrum and no odd-point amplitudes.

- Can be found by dimensional reduction and duality transformation of 4D $\mathcal{N} = 8$ supergravity:



using $$\triangle^2 F_{\mu\nu} = \epsilon_{\mu\nu\lambda}\partial^\lambda \varphi$$

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- Can be found by dimensional reduction and duality transformation of 4D $\mathcal{N} = 8$ supergravity.

- Four-point amplitude is the square of BLG four-point:

$$\mathcal{M}_4 = \frac{i\kappa^2}{4} \frac{\delta^{(3)}(P)\delta^{(16)}(Q)}{(\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2}$$

- Six-point is:

$$\mathcal{M}_6 = \sum_{i \in \text{graphs}} \frac{n_i n_i}{\prod_{\alpha_i} \ell_i^2}$$

where the numerators are those of the SCS theory and the sum is over the same quartic graphs.

Conclusions & Outlook

- There are interesting structures in three-dimensional supersymmetric scattering amplitudes.

- Provided evidence for tree-level color-kinematics in ABJM (and BLG) theories when written as three-algebra theories.

- Provided evidence that one can, á la BCJ, "double" BLG theory into $\mathcal{N} = 16$ E_8 supergravity and hence for a hidden three-algebra structure in 3D supergravity.

* Does this color-kinematics duality persist to higher points? Loop integrands?

* Is $\mathcal{N} = 16$ 3D supergravity finite?