One-loop approximation of Lattice HQET parameters

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26 September 2012

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Motivations

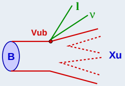
V_{ub}

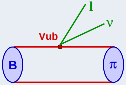
The electroweak sector of the Standard Model includes flavour-mixing transitions:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad \text{where} \quad V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Lattice needed

For processes with $b \to u$ transitions $\Gamma \sim |V_{ub}|^2$:





- inclusive semi-leptonic $B \to X_u l \nu$
- exclusive semi-leptonic $B \to \pi l \nu$ (involves $f_+(q^2)$) $\to$ lattice
- exclusive leptonic $B \to \tau \nu$ (involves f_B) $\to$ lattice

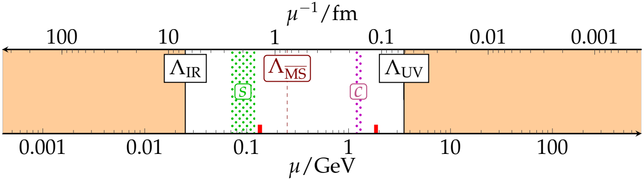
$\Rightarrow$ high precision lattice calculations needed.

Heavy Quark Effective Theory on the Lattice

Hierarchy of scales

- $L > 4/m_\pi \approx 6$ fm to suppress finite-size effects for light quarks
- $a < (2m_B)^{-1}$ to control discretization errors for the heavy quark

$\Rightarrow$ solution: HQET on the lattice



$\mathscr{L}_{\mathrm{HQET}}$

$$\mathscr{L}_{\mathrm{stat}} + \frac{1}{m_b}\left(\mathscr{L}_{\mathrm{kin}} + \mathscr{L}_{\mathrm{spin}}\right) + \ldots, \qquad \mathscr{L}_{\mathrm{stat}} = \bar{\psi}_h (m_b + D_0)\psi_h$$

At order $(\Lambda_{\mathrm{QCD}}/m_b)$

Additional contributions to the Lagrangian

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}} \sum_x \langle \mathcal{O} \mathscr{L}_{\mathrm{kin}}(x) \rangle_{\mathrm{stat}} + \omega_{\mathrm{spin}} \sum_x \langle \mathcal{O} \mathscr{L}_{\mathrm{spin}}(x) \rangle_{\mathrm{stat}}$$

and contributions to heavy-light operators

$$A_0(x) = \bar{\psi}_l(x)\gamma_0\gamma_5\psi_h(x) + \omega_{A_0,1}\delta_1 A_0(x) + \omega_{A_0,2}\delta_2 A_0(x)$$

NB. $\langle A_0(x)A_0(0) \rangle$ can be used to determine m_B and f_B

$\Rightarrow$ 3 parameters in $\mathscr{L}_{\mathrm{HQET}}$ and 2 parameters in $A_0(x)$ and ...

Matching strategy

How to determine the HQET parameters?

$$\Phi^{\mathrm{QCD}}(L, z = Lm_b, a = 0) \stackrel{!}{=} \Phi^{\mathrm{HQET}}(L, z, a)$$

Schrödinger functional correlation functions

- boundary-to-boundary

$$F_1(\theta) = \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}_l'(\mathbf{u})\gamma_5\zeta_h'(\mathbf{v})\bar{\zeta}_h(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle$$

- current insertions

$$f_{A_0}(\theta, x_0) = \sum_{\mathbf{u},\mathbf{v}} \langle \bar{\zeta}_h(\mathbf{u}\gamma_5\zeta_l(\mathbf{v})A_0(x_0)\rangle$$

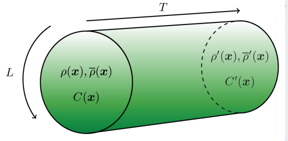


figure from D.Hesse PhD thesis

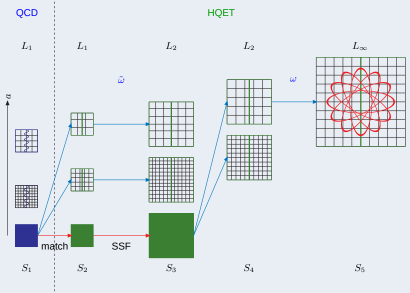
Examples of Φ observables

$$\Phi_1(\theta_1, \theta_2) = \frac{F_1(\theta_1)}{F_1(\theta_2)}, \qquad \Phi_3(\theta_1, \theta_2, x_0) = \frac{f_{A_0}(\theta_1, x_0)}{f_{A_0}(\theta_2, x_0)}$$

Matching strategy

How to determine the HQET parameters?

$$\Phi_i^{\mathrm{QCD}}(L,z,a=0) \stackrel{!}{=} \Phi_i^{\mathrm{HQET}}(L,z,a) = \varphi(L,a)_{ij}\, {\color{red}\omega(z,a)_j} + \eta(L,a)_i$$



⇒ non-perturbative definition (matching) and evolution of HQET parameters

One-loop lattice HQET parameters

Why?

Before starting non-perturbative matching one would like to investigate the sensitivity of matching conditions on the HQET paramaters.

How?

PASTOR [written by D. Hesse]: automatic tool for generation and calculation of lattice Feynman diagrams.

input: discretized action, correlation function, L/a, z, a

output: Feynman rules, Feynman diagrams, numerical contributions of each diagram

Example

Matching conditions

$$\varphi(L,a)\omega(z,a) = \Phi^{\mathrm{QCD}}(L,z,0) - \eta(L,a)$$

5x5 system

$$\begin{pmatrix} R_1^{\mathrm{kin}} & 0 & 0 & 0 & 0 \\ 0 & R_2^{\mathrm{spin}} & 0 & 0 & 0 \\ R_{A_0}^{\mathrm{kin}} & R_{A_0}^{\mathrm{spin}} & R_{\delta A} & 0 & 0 \\ \Psi_{A_0}^{\mathrm{kin}} & \Psi_{A_0}^{\mathrm{spin}} & \rho_{\delta A} & 1 & 0 \\ L\Gamma^{\mathrm{kin}} & L\Gamma^{\mathrm{spin}} & L\Gamma_{\delta A} & 0 & L \end{pmatrix} \begin{pmatrix} \omega_{\mathrm{kin}} \\ \omega_{\mathrm{spin}} \\ \omega_{A_0,1} \\ \log Z_{A_0}^{\mathrm{HQET}} \\ m_{\mathrm{bare}} \end{pmatrix} = \begin{pmatrix} R_1 - R_1^{\mathrm{stat}} \\ R_2 \\ R_{A_0} - R_{A_0}^{\mathrm{stat}} \\ R_{A_0/F} - \zeta_{A_0} \\ L\Gamma - L\Gamma^{\mathrm{stat}} \end{pmatrix}$$

$\Rightarrow$ once all diagrams are evaluated one can solve for $\omega(z,a)$

Example

Continuum extrapolations

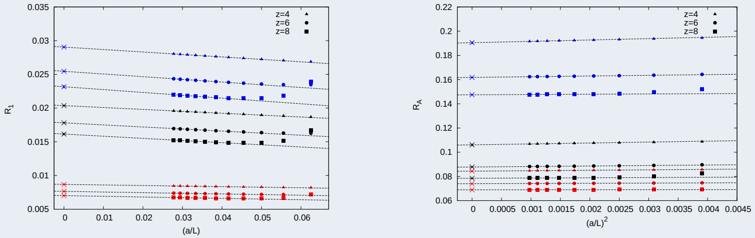


Figure: Continuum extrapolations of one-loop contribution to R_1^{QCD} and $R_{A_0}^{\mathrm{QCD}}$ for three sets of θ angles: red: $(0.5, 0.0)$, black: $(1.0, 0.0)$, blue: $(1.0, 0.5)$.

Example

z dependence

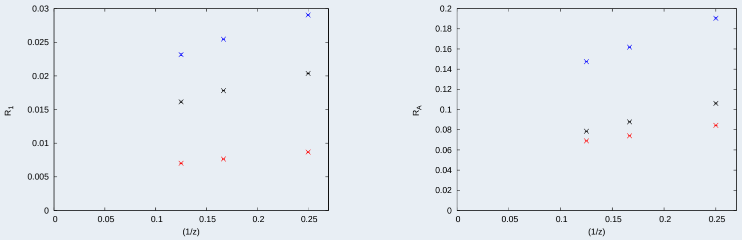


Figure: z dependence of one-loop contribution to R_1^{QCD} and $R_{A_0}^{\mathrm{QCD}}$ for three sets of θ angles: red: $(0.5, 0.0)$, black: $(1.0, 0.0)$, blue: $(1.0, 0.5)$.

Conclusions

- lattice HQET is a prototype of an effective theory where one can perform a nonperturbative matching

- we have a tool for checking the sensitivity and contamination with $1/z^2$ terms of the matching conditions
- we can further improve the tree-level optimization of the matching conditions
- necessary preparation step for the full matching at order $\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$, i.e. including parameters for $A_k(x)$, $V_0(x)$, $V_k(x)$, where 19 parameters have to be matched

- Finally: determination of V_{ub} from $B \to \tau\nu$ and $B \to \pi l \nu$.