Towards true frequentist p-values for global SUSY fits with Fittino

FITTINO SUSY

Xavier Prudent, Björn Sarrazin et al.

6th Annual Workshop of the Helmholtz Alliance "Physics at the Terascale"

PHYSICS AT THE TERA SCALE
Helmholtz Alliance

FITTINO

- C++ program for **SUSY model testing and SUSY parameter analysis**

- Currently supported SUSY models:
  CMSSM, GMSB, AMSB, MSSM24, NMSSM, NUHM1, NUHM2

- Measurements from low/high energy experiments, direct SUSY search
  LEP/SLC, Tevatron, cosmology, LHC and LC, $(g\text{-}2)_\mu$, B, K...

- Use puplic theory codes: **SPheno, SuperIso, Micromegas, FeynHiggs, HDecay**

- Parameter analysis using full correlation information:
  Auto-adaptive Markov Chain Monte Carlo (MCMC)

- Previous publications:
  arXiv:0412012 [hep-ph] (2004), arXiv:0511006 [hep-ph] (2005), arXiv:0907.2589 [hep-ph] (2009), arXiv:0909.1820 [hep-ph] (2009), arXiv:1105.5398 [hep-ph] (2011), arXiv:1102.4693 [hep-ph] (2011), arXiv:1204.4199 [hep-ph] (2012),

- General SUSY model > 120 parameters
  Current data insufficient
  Restrict to constrained model: **CMSSM**

$\tan\beta$	ratio of Higgs VEVs
A_0	common trilinear coupling parameter
$M_{1/2}$	common gaugino mass parameter
M_0	common scalar mass parameter
sign(μ)	sign of Higgsino mass parameter

Non-minimal model: additional Higgs mass parameter: **NUHM1**, **NUHM2**

M_H	universal Higgs mass $M_H=M_{Hu}=M_{Hd}$
M_{Hu}	up-Higgs mass parameter
M_{Hd}	down-Higgs mass parameter

NEW Observables

HFAG update 2012

$\mathcal{B}(b \to s\gamma)$	$(3.55 \pm 0.26)\ 10^{-4}$
$\mathcal{B}(B_s \to \mu\mu)$	$(3.2 \pm 1.5)\ 10^{-9}$
$\mathcal{B}(B \to \tau\nu)$	$(0.72 \pm 0.27)\ 10^{-4}$
Δm_{B_s}	$(17.719 \pm 0.043)\ ps^{-1}$
$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	$(28.7 \pm 8.2) \times 10^{-10}$
m_W	(80.385 ± 0.015) GeV
$\sin^2 \theta_{\mathrm{eff}}$	0.23113 ± 0.00021
$\Omega_{\mathrm{CDM}} h^2$	0.1123 ± 0.0118
m_t	(173.2 ± 1.34) GeV

1st measurement by LHCb

Belle measurement including peaking background

- **Higgs limits** via *HiggsBounds* or requiring $m(h^0) = (126 \pm 2\,(\mathrm{exp}) \pm 3(\mathrm{theo}))$ GeV

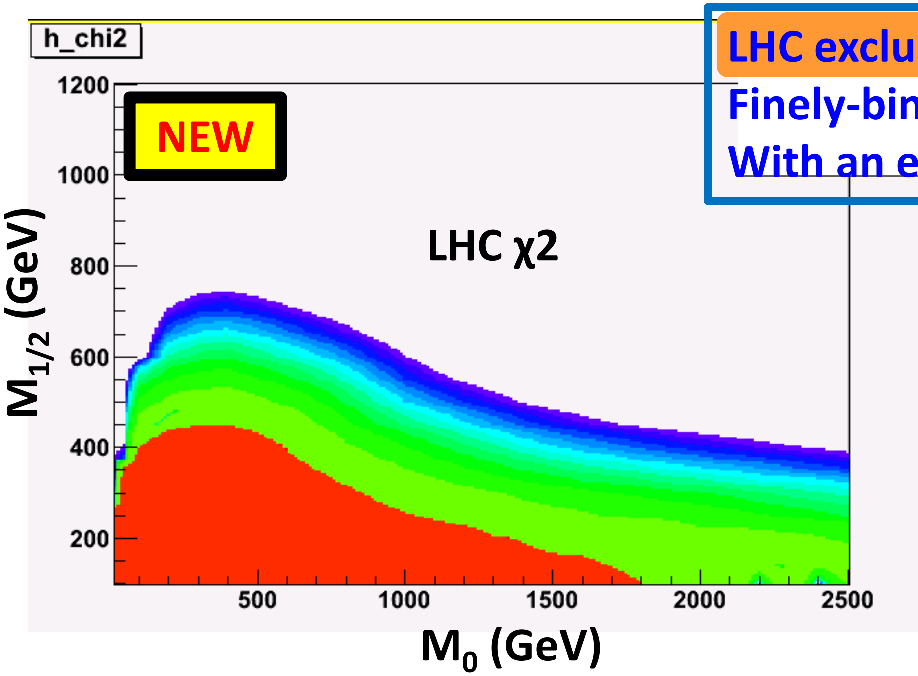
  Higgs discovery via HiggsSignals (fit of $m(h^0)$ and $\mu(h^0)$)

- **LEP chargino limit** (implications for neutralino limits)

- **LHC exclusion** from 'jets+MET' analysis L=4.7/fb

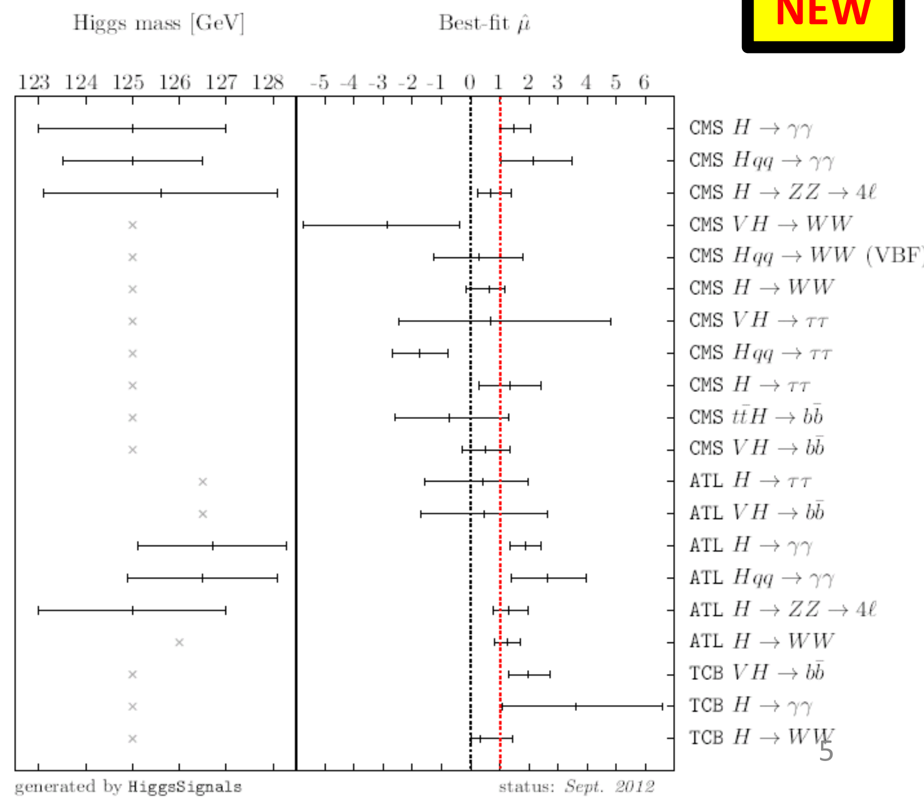
  Fine-tuning of the simulation to match the ATLAS exclusion

- **Direct detection of DM** via *AstroFit*



LHC exclusion:
Finely-binned M0-M12 grid of χ2
With an extension beyond the grid boundaries

Herwig++, Delphes, Prospino

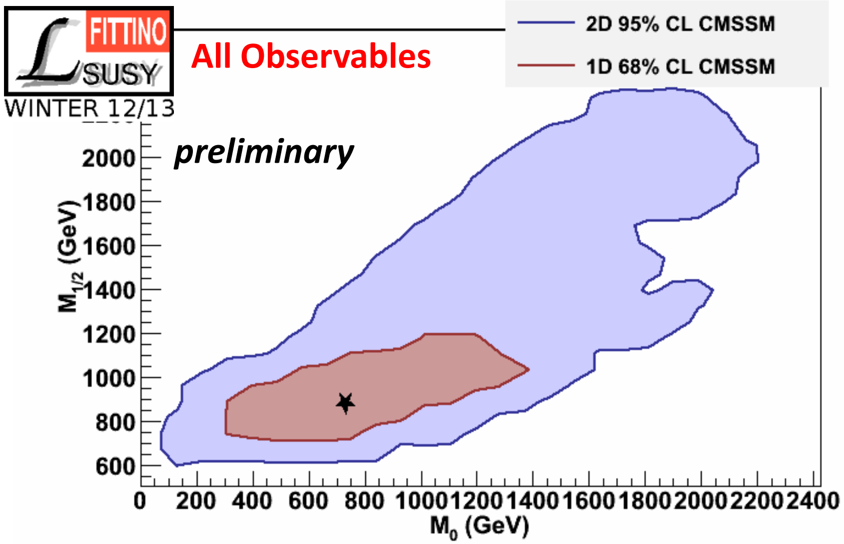


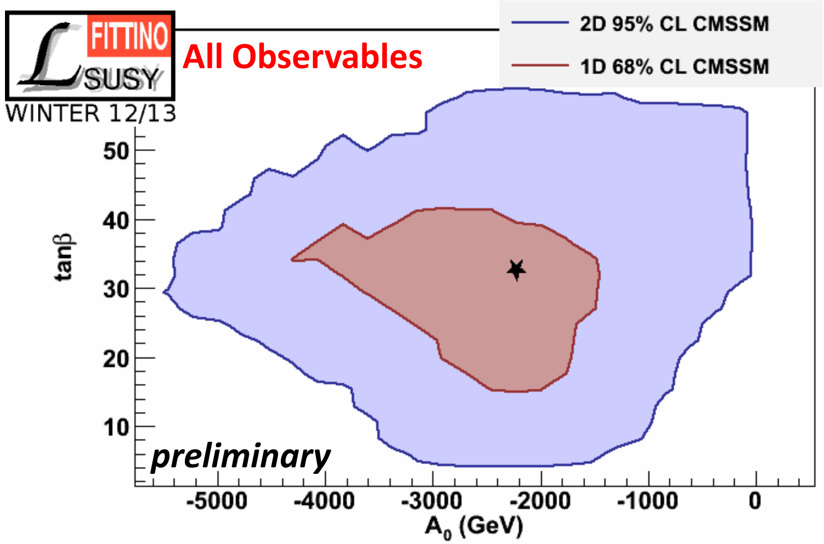
Higgs measurements HiggsSignals-1.0.0.
χ2 from Higgs mass and signal rate
χ2 from mass only with clearly observed peak

Thanks to the HiggsSignals authors : S. Heinemeyer, O. Stal, T. Stefaniak and G. Weiglein, P. Bechtle

See the talks of Tim Stefaniak and Lisa Zeune in the Higgs session: Tuesday 11:00 in SR 4a

Full 5D fit (M_0, M_{12}, tanβ, A_0, m(top)) including all observables: low energy, astrophysics, direct searches, Higgs combined constraints ($m(h^0)$, μ)





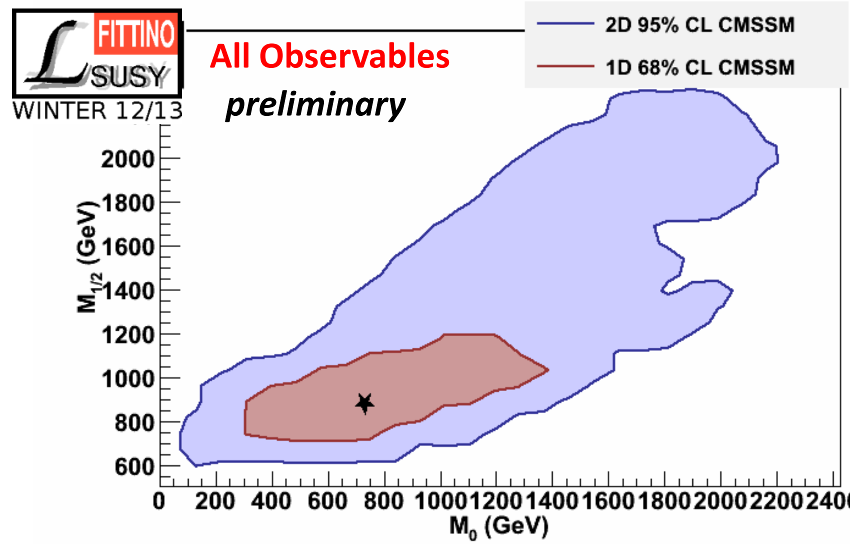
Tanβ	= 32.7	[-18.0, +9.1]
M_0	= 730.7	[-396.1, +700.5] GeV
M_{12}	= 880.5	[-158.0, +395.0] GeV
A_0	= -2225.8	[-1975.2, +768.4] GeV

minimal Chi2/ndf = 34.0/33

Pretty good fit, with tendency to higher masses and tanβ (coupling throug $(g-2)_\mu$)

Tanβ	= 32.7	[-18.0, +9.1]
M_0	= 730.7	[-396.1, +700.5] GeV
M_{12}	= 880.5	[-158.0, +395.0] GeV
A_0	= -2225.8	[-1975.2, +768.4] GeV

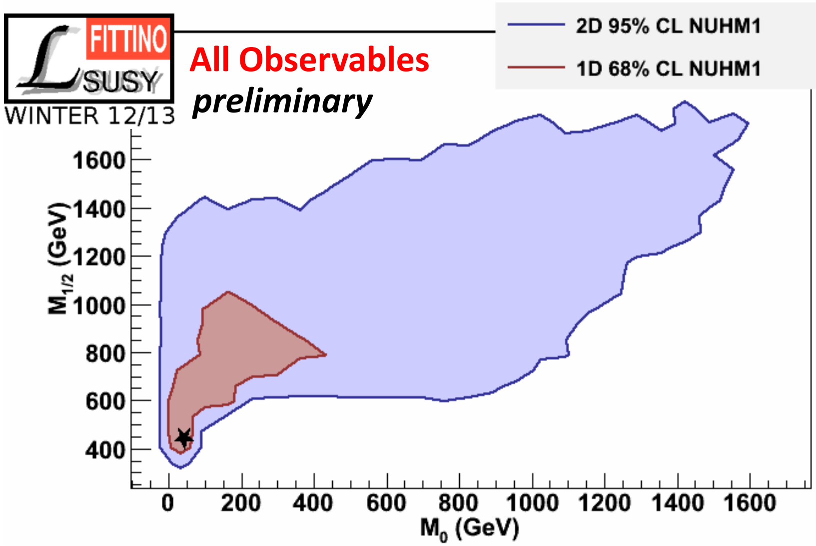
mininal Chi2/ndf = 34.0/33

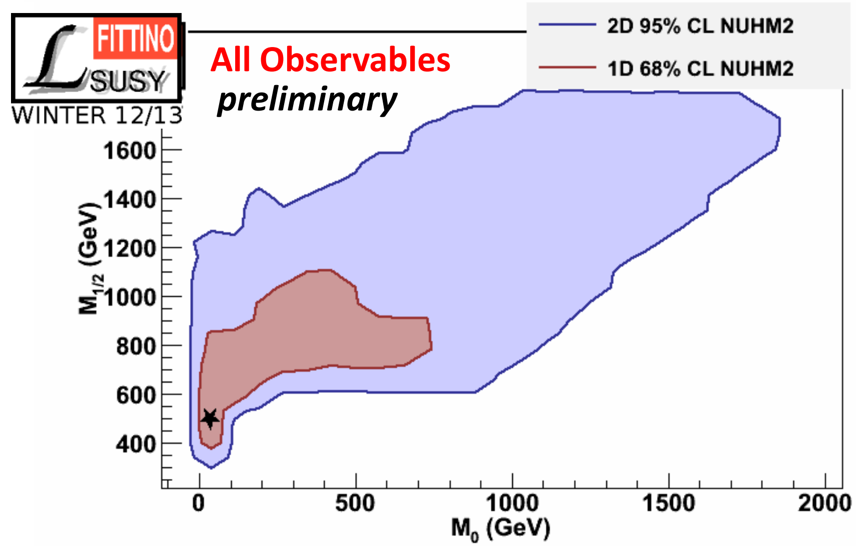


Fit quality does not necessarily improve with the non-universal models

Lower masses, tanβ

NEW





Tanβ	= 9.3	[-3.4, +19.9]
M_0	= 45.2	[-25.2, +411.2] GeV
M_H^2	= -0.47	[-8.81, +1.22] 10^6 GeV^2
M_{12}	= 442.2	[-37.7, +581.5] GeV
A_0	= -1620.8	[-2475.4, +351.7] GeV

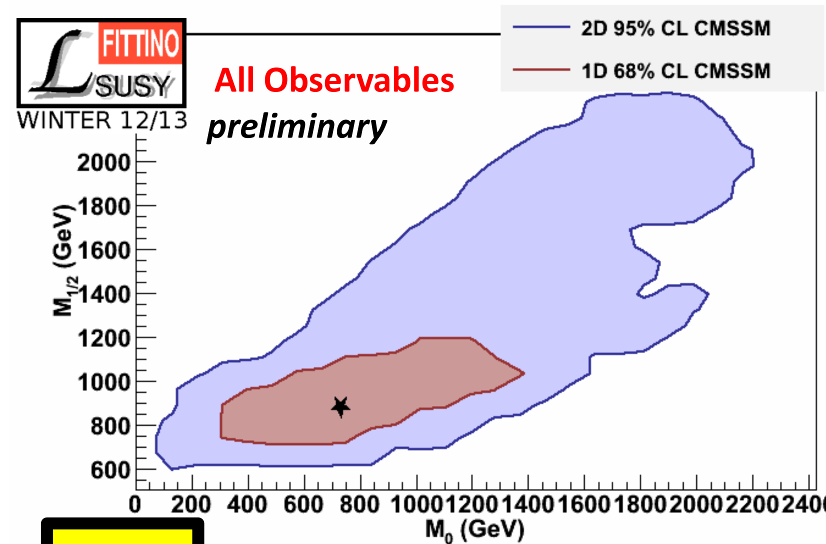
mininal Chi2/ndf = 33.1/32

Tanβ	= 10.3	[-2.6, +20.1]
M_0	= 34.34	[-14.3, +731.6] GeV
M_{Hu}^2	= -0.04	[-9.5, +1.8] 10^6 GeV^2
M_{Hd}^2	= 0.12	[-1.78, +7.38] 10^6 GeV^2
M_{12}	= 502.1	[-97.5, +570.0] GeV
A_0	= -1551.2	[-3404. 5, +246.7] GeV

mininal Chi2/ndf = 33.5/31

Tanβ	= 32.7	[-18.0, +9.1]
M_0	= 730.7	[-396.1, +700.5] GeV
M_{12}	= 880.5	[-158.0, +395.0] GeV
A_0	= -2225.8	[-1975.2, +768.4] GeV

mininal Chi2/ndf = 34.0/33

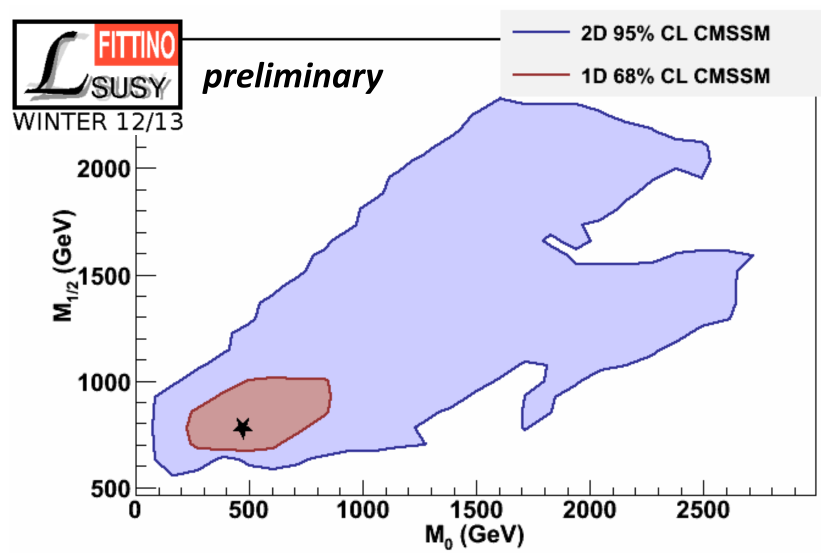


Accommodating a Higgs mass >125GeV is difficult for the CMSSM (summer2012 result)
But the agreement of all Higgs measurements makes the fit quality improving

NEW

Tanβ	= 31.7	[-9.7, +8.9]
M_0	= 475.5	[-202.4, +372.5] GeV
M_{12}	= 777.6	[-57.5, +242.2] GeV
A_0	= -1253.1	[-1085.7, +1509.1] GeV

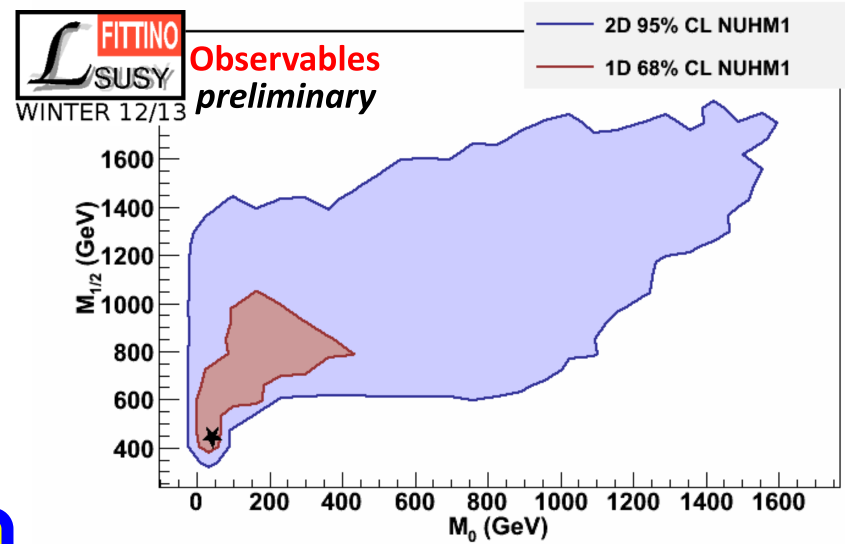
mininal Chi2/ndf = 12.0/8



All Observables, no Higgs rate constraints,
Only asking for $m(h^0)$= (125 ± 2 ± 3) GeV

Tanβ	= 9.3	[-3.4, +19.9]
M_0	= 45.2	[-25.2, +411.2] GeV
M_H^2	= -0.47	[-8.81, +1.22] 10^6 GeV^2
M_{12}	= 442.2	[-37.7, +581.5] GeV
A_0	= -1620.8	[-2475.4, +351.7] GeV

mininal Chi2/ndf = 33.1/32

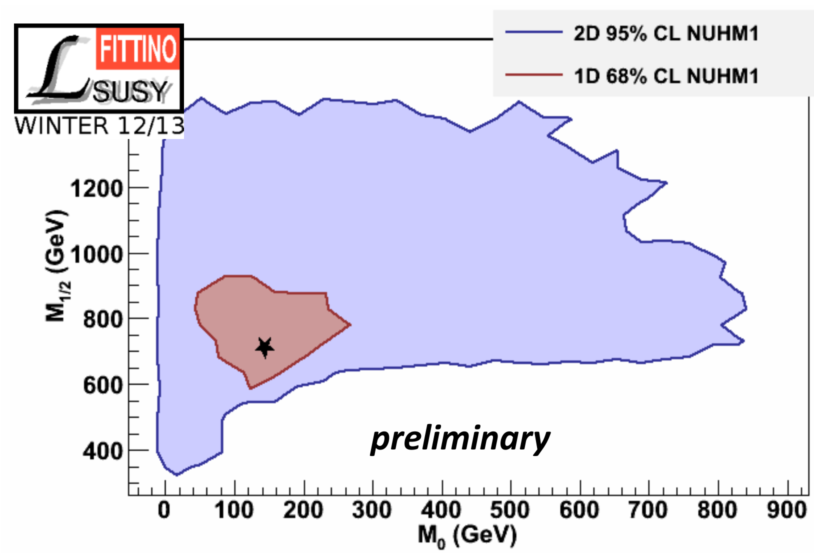


Tension relieved in the non-universal model→difference smaller

NEW

Tanβ	= 23.1	[-6.2, +8.6]
M_0	= 145.0	[-81.5, +106.5] GeV
M_H^2	= -4.06	[-2.32, +1.91] 10^6 GeV^2
M_{12}	= 713.0	[-105.5, +204.9] GeV
A_0	= -1337.1	[-565.7, +522.4] GeV

mininal Chi2/ndf = 8.0/7

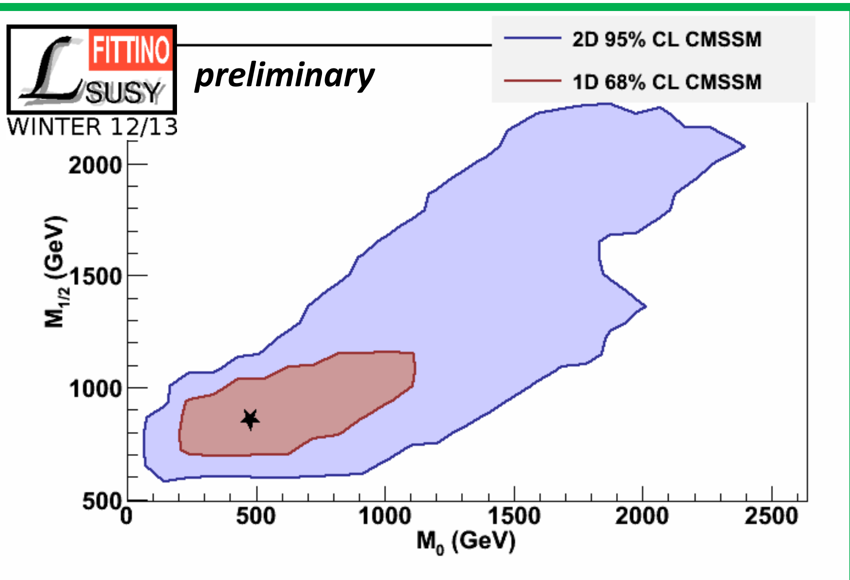


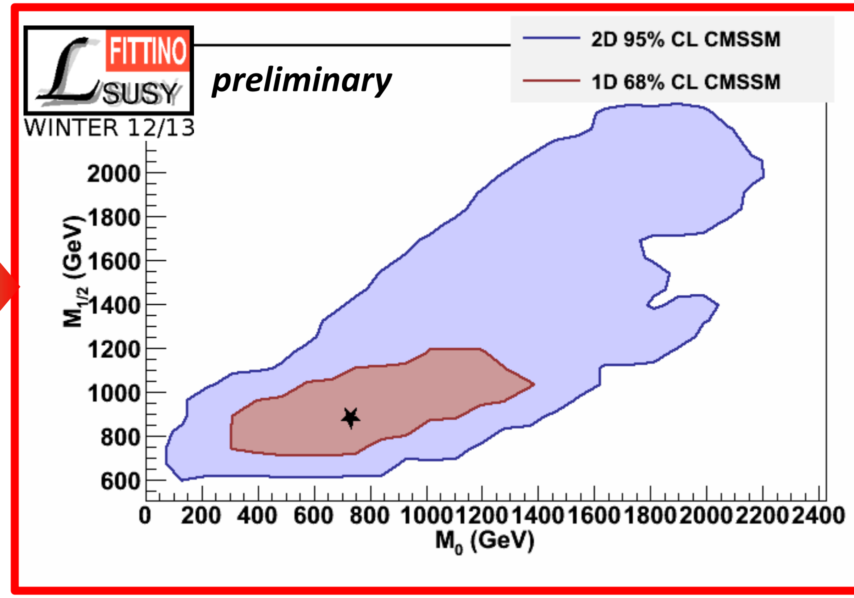
All Observables, no combined Higgs constraints,
Only asking for $m(h^0)$= (125 ± 2 ± 3) GeV

First measurement of $B_s \to \mu^+\mu^-$ by LHCb !

arxiv:1211.2674 [hep-ex]

With the upper limit — **NEW** — **With the measurement**





M_0:	477.1 (730.7)	[-396.1, +700.5] GeV
M_{12}:	861.3 (880.5)	[-158.0, +395.0] GeV
A_0:	-2332.6 (-2225.83)	[-1975.2, +768.4] GeV
Tanβ:	20.5 (32.7)	[-18.0, +9.1]
Chi2:	33.4 (34.0)	

Minor impact on the best fit point, increase of M_0 and tanβ

What about the p-values of these fits?

Tanβ	= 23.1	[-6.2, +8.6]
M_0	= 145.0	[-81.5, +106.5] GeV
M_H^2	= -4.06	[-2.32, +1.91] 10^6 GeV^2
M_{12}	= 713.0	[-105.5, +204.9] GeV
A_0	= -1337.1	[-565.7, +522.4] GeV

minimal Chi2/ndf = 8.0/7

Well-spread thought:

"The fit is good if this number is roughly one"

➔ Because the mean value of a χ2 distribution is the number of degrees of freedom

"our" χ2 : $$\chi^2 = (M - O(P))^T \text{cov}_M^{-1} (M - O(P))$$

M: measurements
O(P): predictions at point P
covM: covariance matrix

This assumption that our χ2 is distributed according to a χ2 distribution is correct only:

- if the O^i are **Gaussian-distributed and if the dependency O(P) is linear.**
- or if a **large number of observables** is used

Therefore TMath::Prob may not give the right value !!

➔Risk of drawing a wrong conclusion on the validity of a model

What is the p-value?

Assuming the best fit point found is the real one, if measurements are repeated, what is the probability to get an agreement (i.e. χ2) at least as worse as the one observed?

Computation of the p-value of the best fit point (lower χ2) with toys:

- take the observables values at this point
- smear the observables values
- calculate the χ2 for these new pseudo-measurements
- spot the new best fit point
- repeat that procedure many times
- integrate the distribution for χ2 ≥ χ2(real fit)

NEW

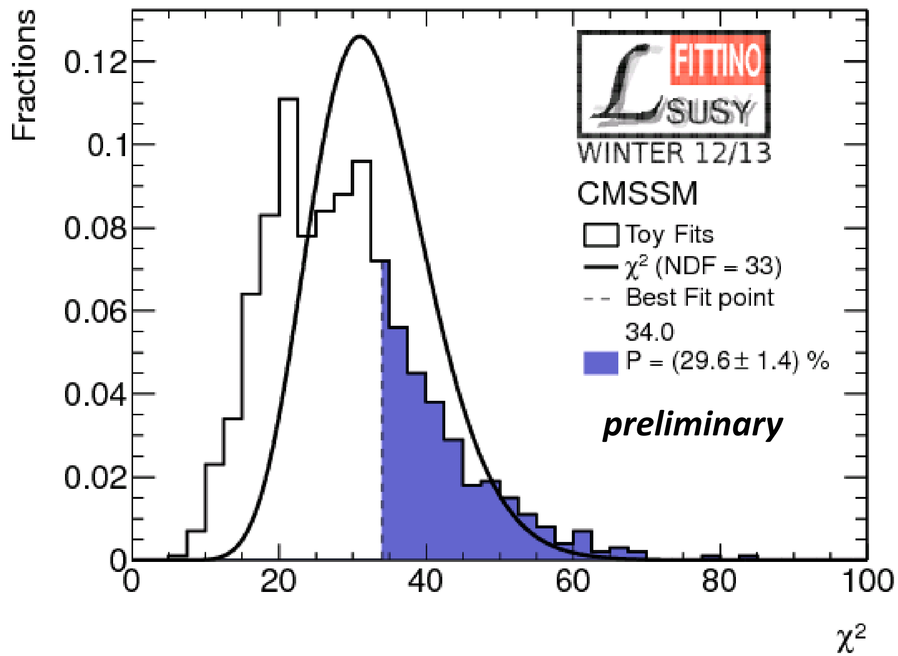
Statistical uncertainty (binomial rule:
N_p = #(χ2> χ2(real fit)), N_g = #(total)

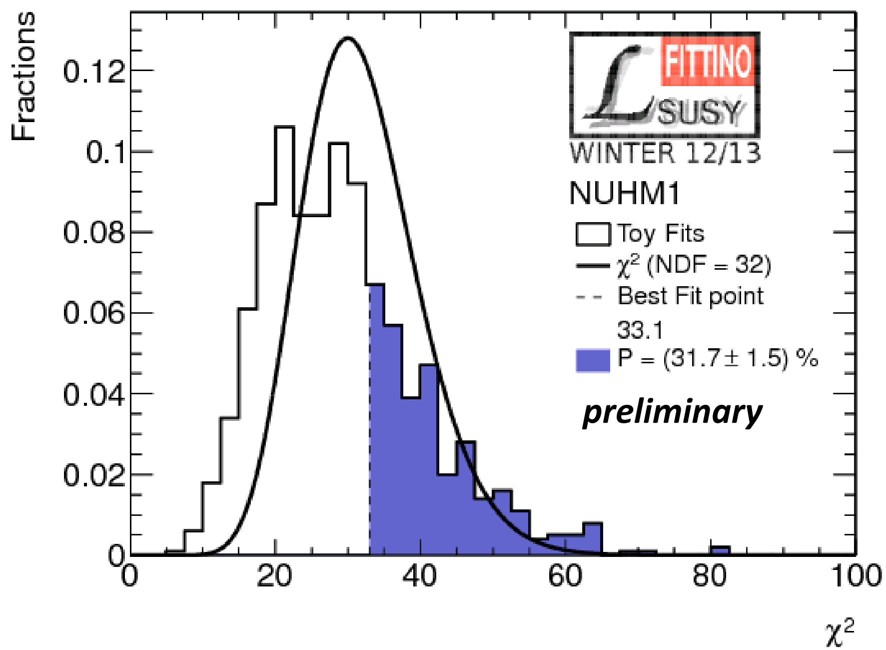
$$\frac{\sigma(X)}{\langle X \rangle} = \sqrt{\frac{1}{N_p} - \frac{1}{N_g}}$$

Results of the toys: 1st frequentist p-values for the CMSSM !

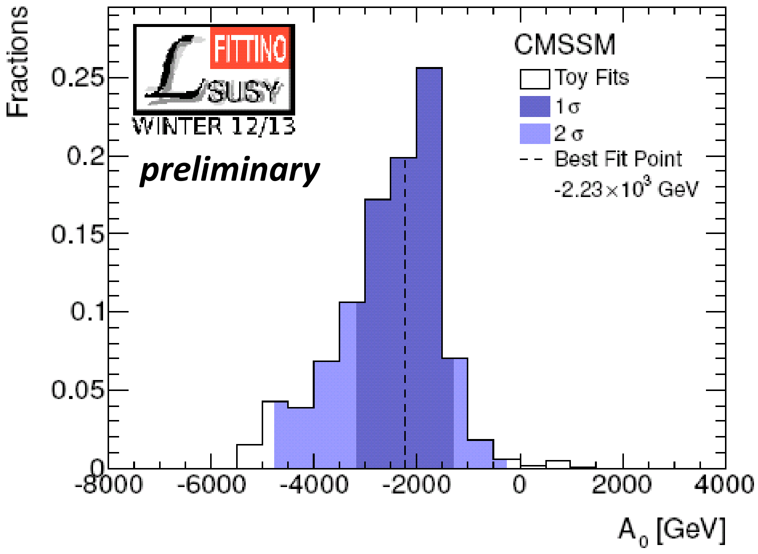
Total toy statistics per model: 1000

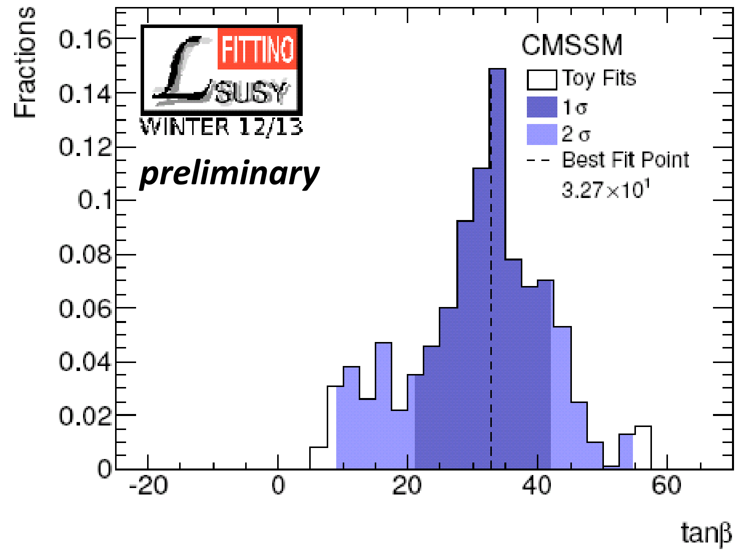
Model	Toys	χ2 approx.
CMSSM	**29.6 ± 1.4**	41.7
NUHM1	**31.7 ± 1.5**	41.4
NUHM2	**31.2 ± 1.5**	34.7

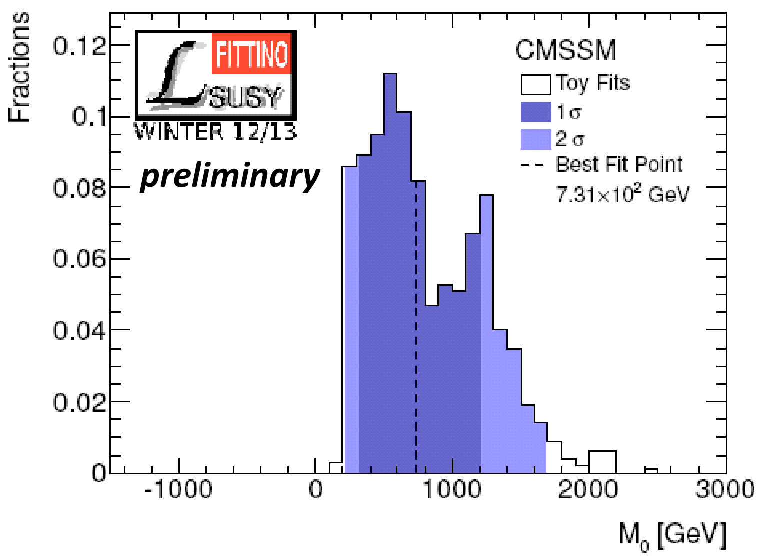


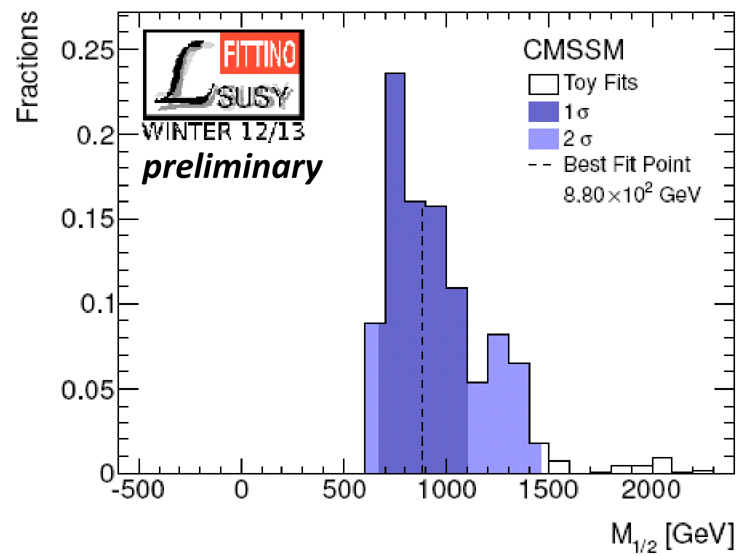


Parameters distributions for the CMSSM (more in backup)

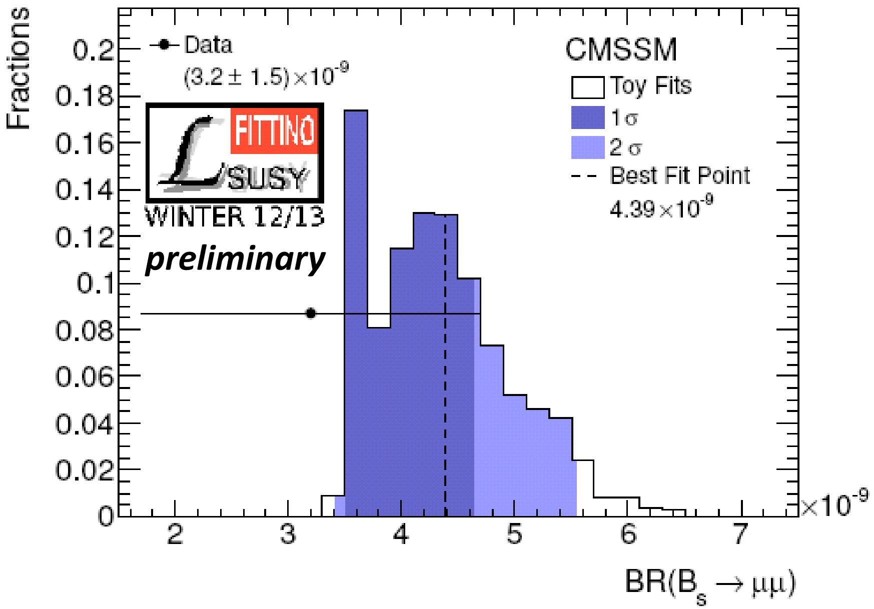


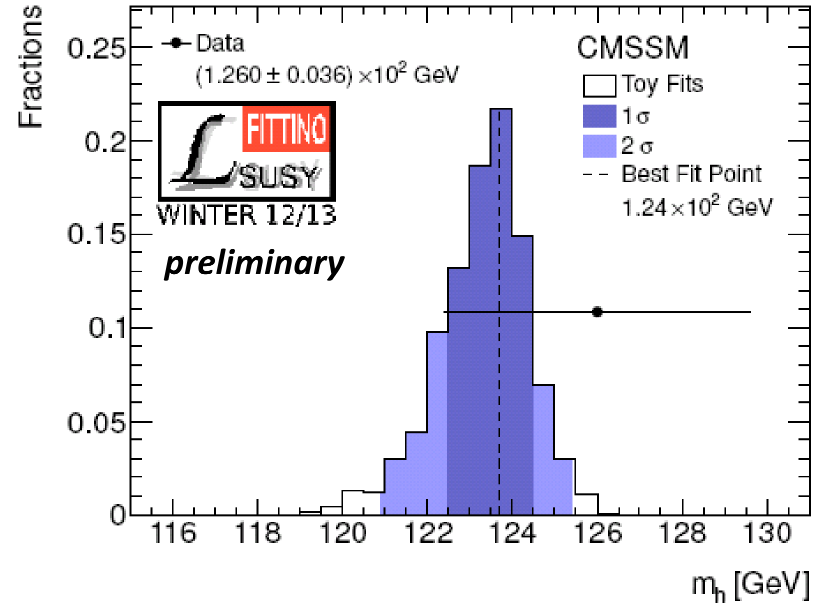


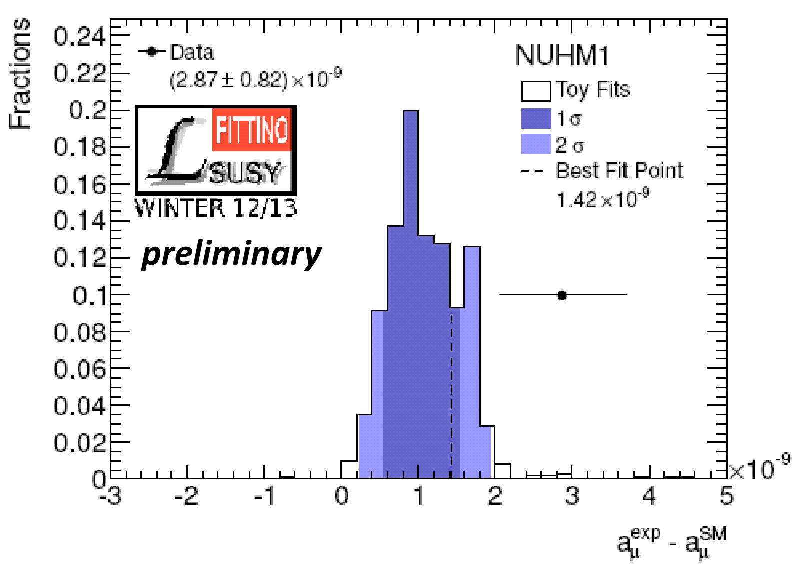


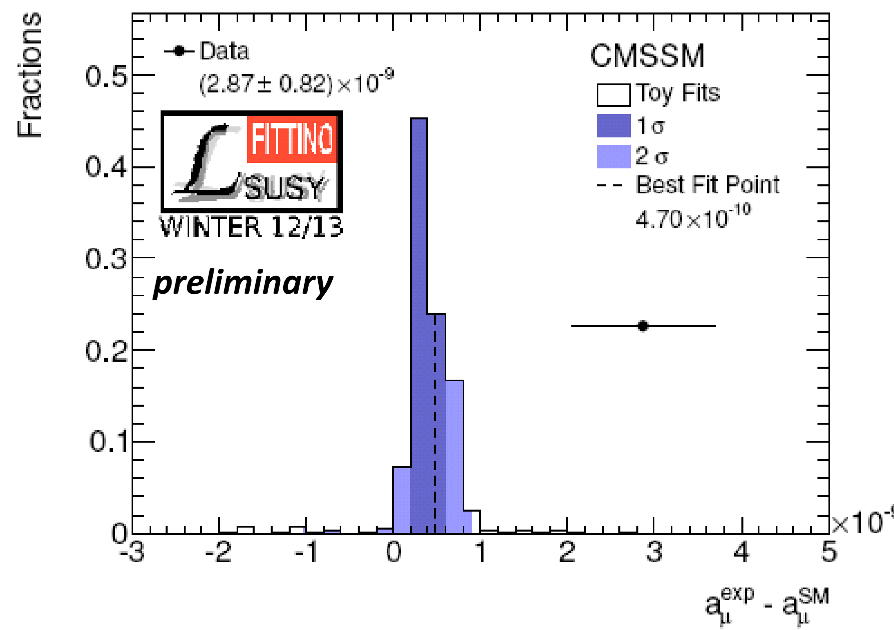


A few observables distributions for the CMSSM (more in backup)









Conclusion & Plans

- **The first real toy-based frequentist p-values for the CMSSM!**

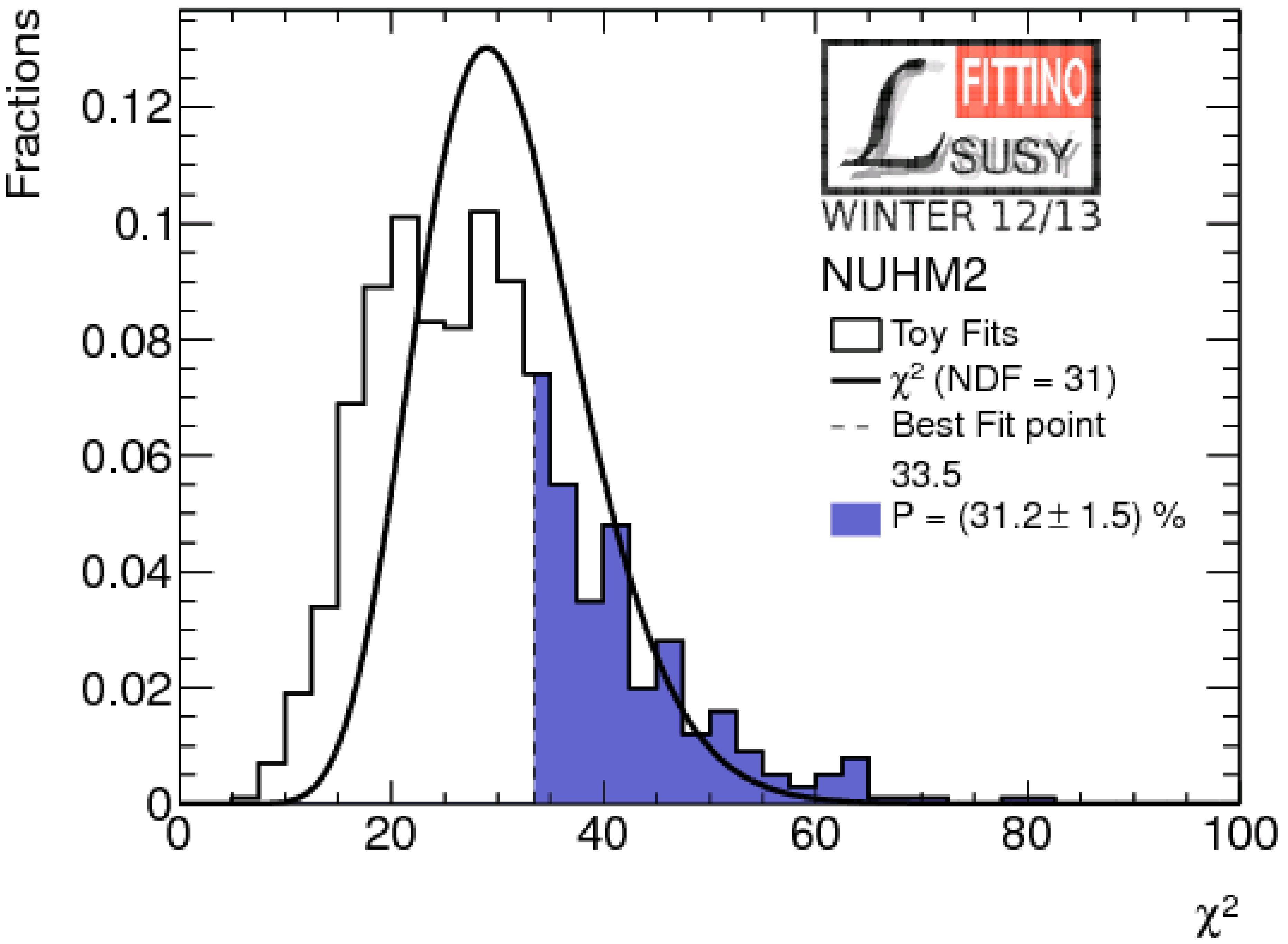
- **SM-like Higgs sector easily achievable within the CMSSM, NUHM**
  - ➔ good agreement of the fit with the SM-like Higgs rate (Tevatron, LHC)
  - ➔ improvement in Fit Quality (45%)

- First measurement of $B_s \rightarrow \mu^+\mu^-$ in agreement with the SM
  - ➔ **perfectly expected within constraint SUSY**

- **Still room for M_{12}, M_0 < 1 TeV**

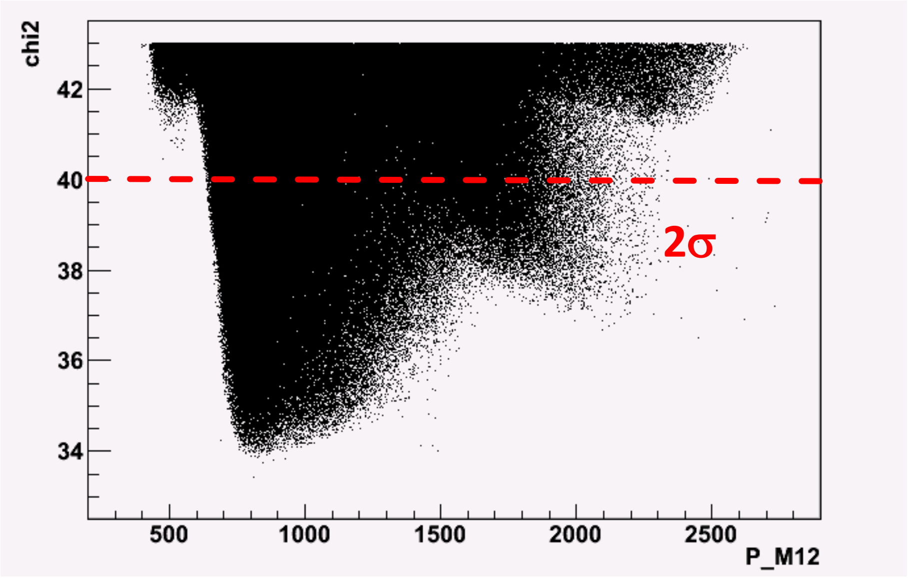
- **Higgs mass and rate measurements** push the CMSSM into a parameter space where the **differences to the SM in all observables (apart Ω_{DM}) is small**
  - ➔negligible improvement in $(g-2)_\mu$ wrt the SM

BACKUP

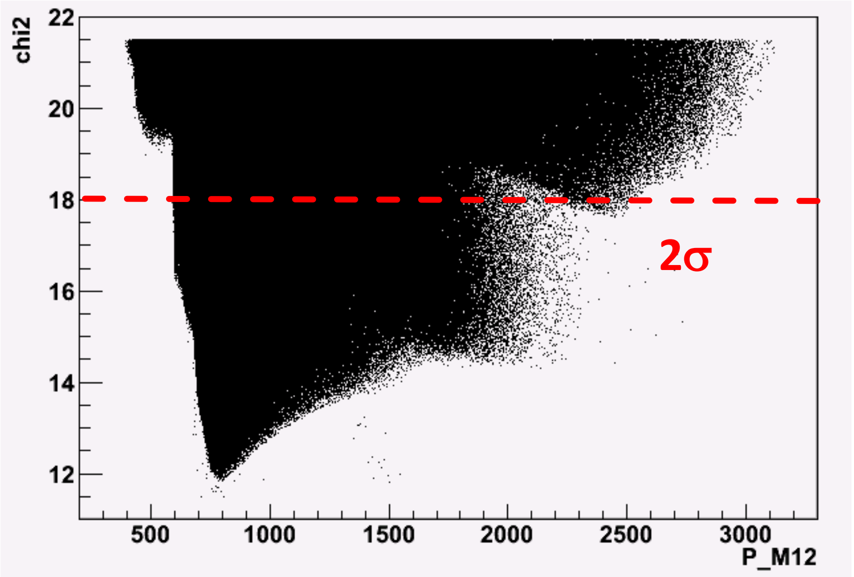

Fractions
0.12
0.1
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
χ^2
FITTINO
SUSY
WINTER 12/13
NUHM2
Toy Fits
χ^2 (NDF = 31)
Best Fit point
33.5
P = (31.2 ± 1.5) %

Where is the focus point gone?

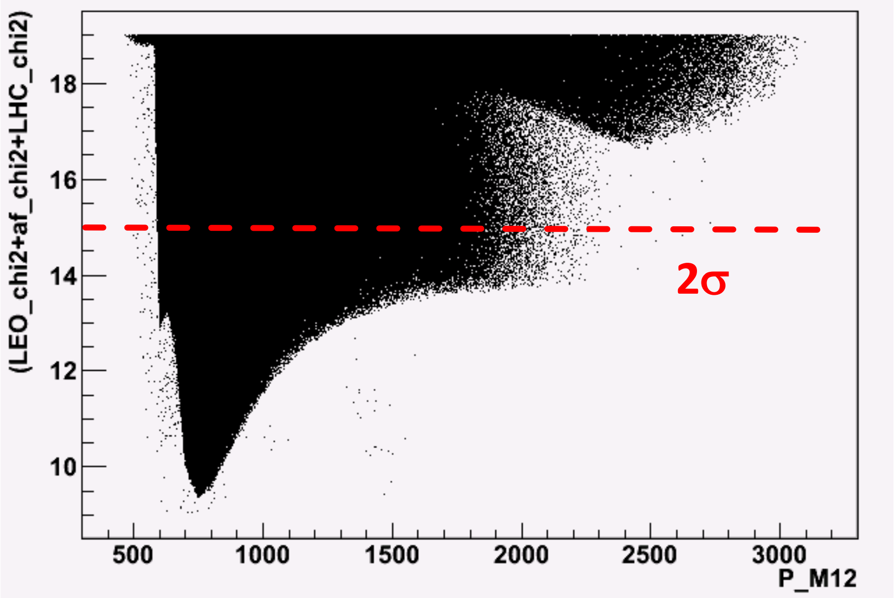
All Combined h^0 measurements



Only $m(h^0)$



No h^0 constraint



Searching for SUSY at the TeV scale ... and ?

2011: long LHC run, center-of-mass energy 7 TeV, luminosity ~5/fb.

- Direct step into Terascale
- No significant excess seen

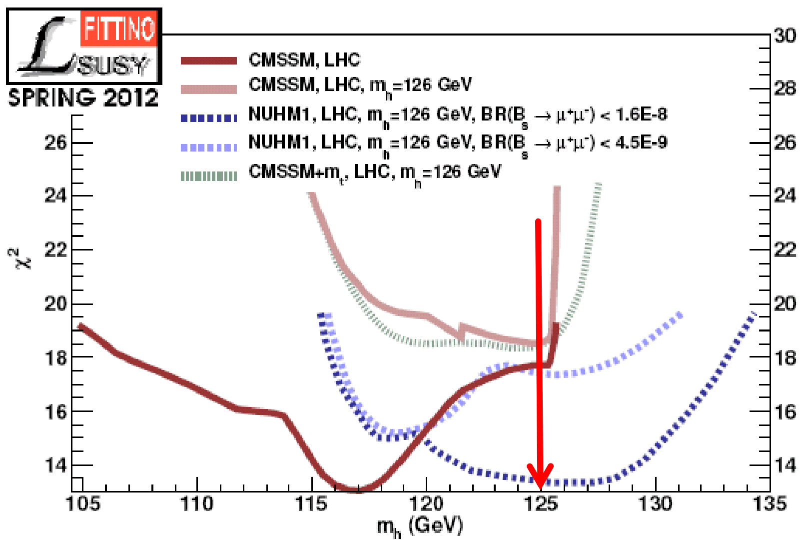
https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/SUSY-2011-19/

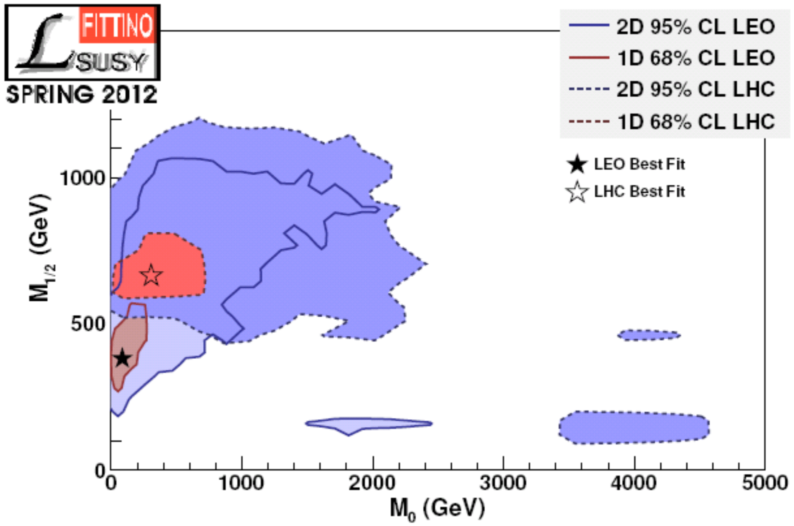
- **CMSSM/NUHM1 parameter space still allowed ?**
- **Tension between LHC and pre-LHC ?**
- **Impact of light SUSY Higgs at 126 GeV ?**
- **Impact of direct & indirect search for dark matter ?**
- **Interpretation in (non-)minimal models?**

Short reminder of the previous Fittino results

Main Conclusions

- Current LHC exclusion leads to tension within CMSSM
- Accommodate Higgs mass ≥ 125 GeV very hard in mSUGRA
- Measurement of Higgs branching ratios can discriminate SM/MSSM





Published JHEP 06 (2012) 098
Presented at ICHEP 2012 – Melbourne

And now what …?

Improvements

Observables

$\mathcal{B}(b \to s\gamma)$	$(3.55 \pm 0.34) \times 10^{-4}$
$\mathcal{B}(B_s \to \mu\mu)$	$< 4.5 \times 10^{-9}$
$\mathcal{B}(B \to \tau\nu)$	$(1.67 \pm 0.39) \times 10^{-4}$
Δm_{B_s}	$17.78 \pm 5.2\,\mathrm{ps}^{-1}$
$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	$(28.7 \pm 8.2) \times 10^{-10}$
m_W	(80.385 ± 0.015) GeV
$\sin^2\theta_{\mathrm{eff}}$	0.23113 ± 0.00021
$\Omega_{\mathrm{CDM}}h^2$	0.1123 ± 0.0118
m_t	(173.2 ± 1.34) GeV

- **Higgs limits** via *HiggsBounds* or requiring $m(h^0)$ = (126 ± 2 (exp) ± 3(theo)) GeV

- **LEP chargino limit** (implications for neutralino limits)

- **LHC exclusion** from ‘jets+MET‘ analysis L=4.7/fb

- **Direct and indirect detection of DM** via *AstroFit*

Comparison of models performed using p-values

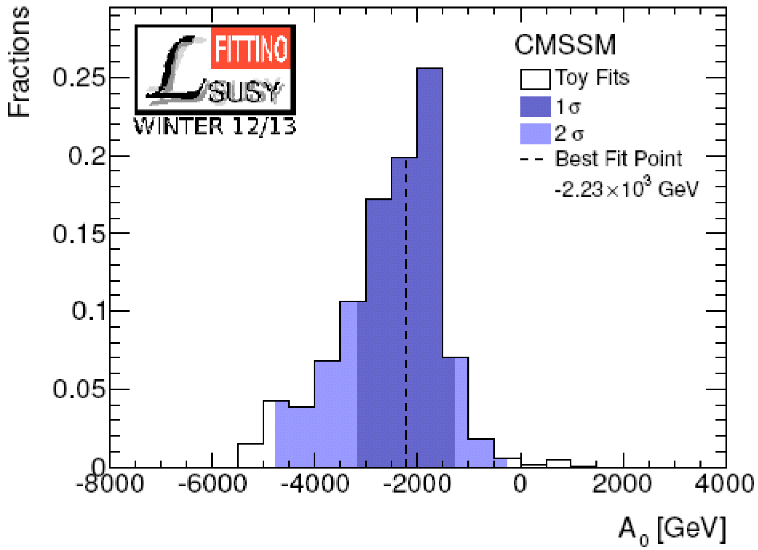
→ Computed assuming the observables likelihood to be Gaussian

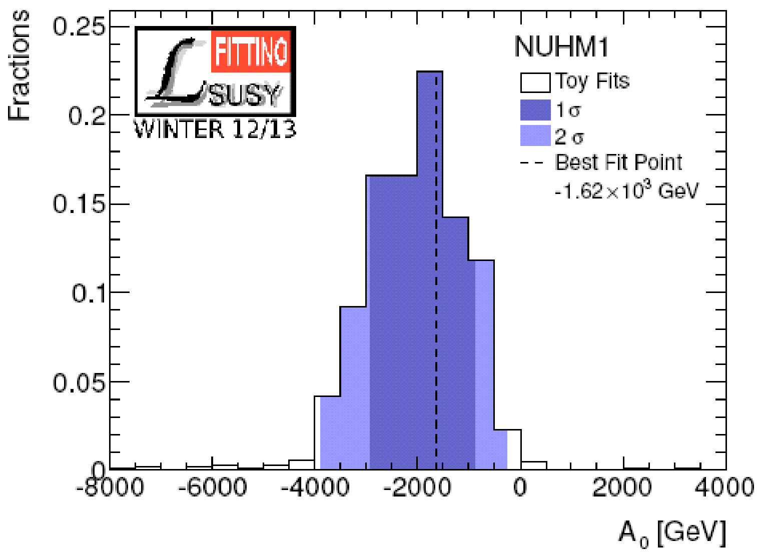
→ integration of a χ2 distribution function

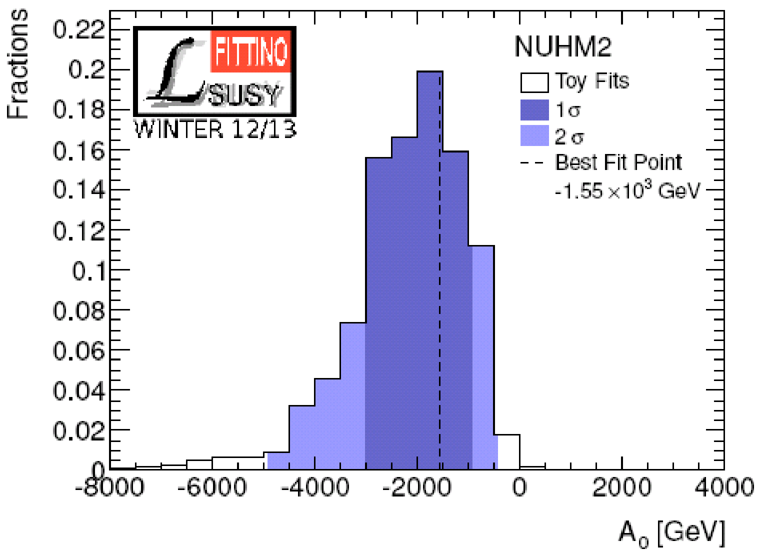
<u>What is the p-value?</u>

Assuming the best fit point found is the real one, if measurements are repeated, what is the probability to get an agreement (i.e. χ2) at least as worse as the one observed?

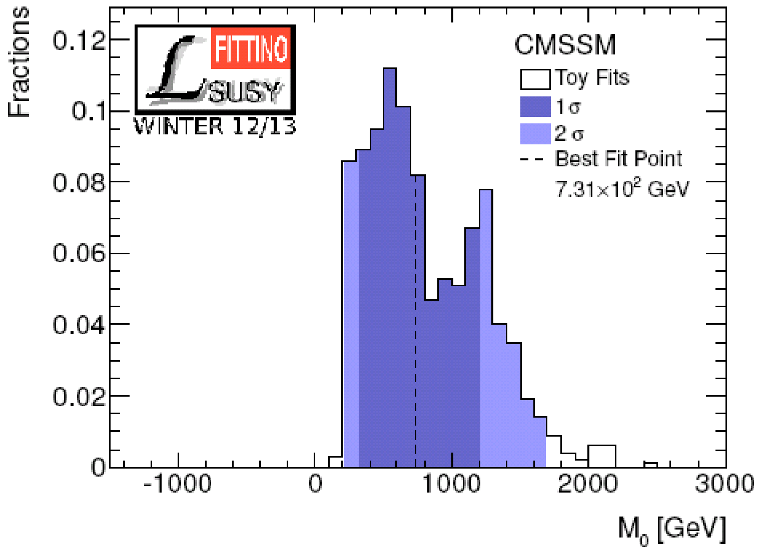
Toy fits (parameters)

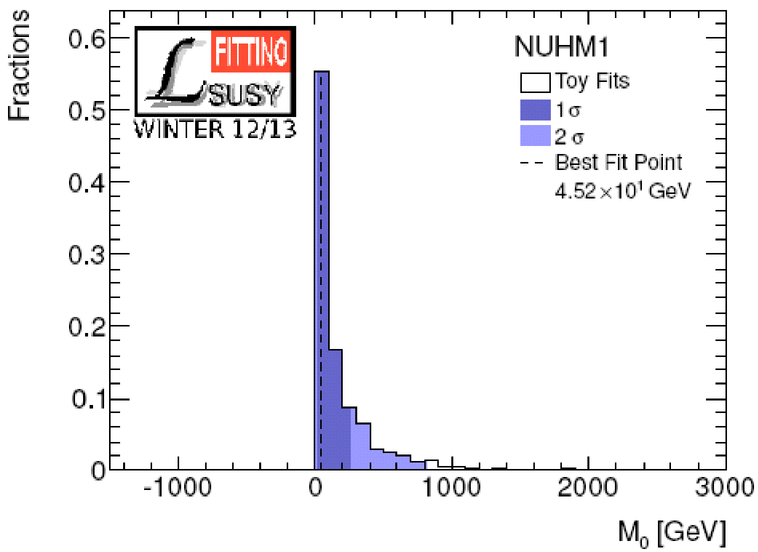


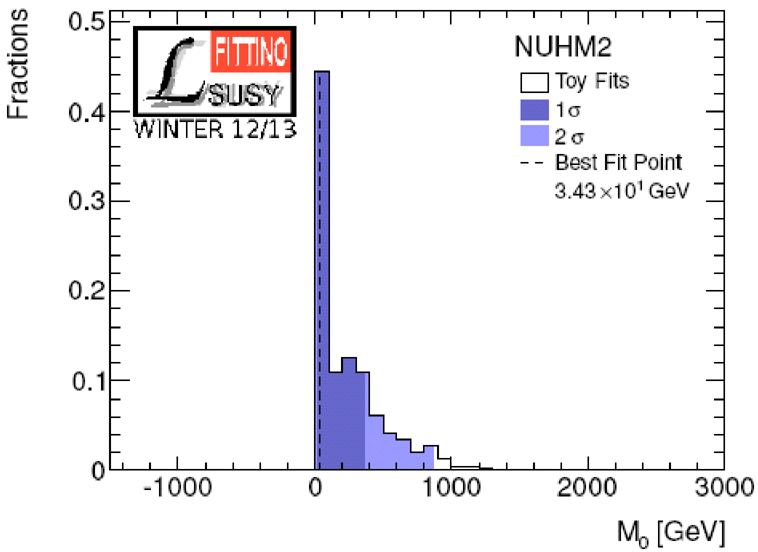




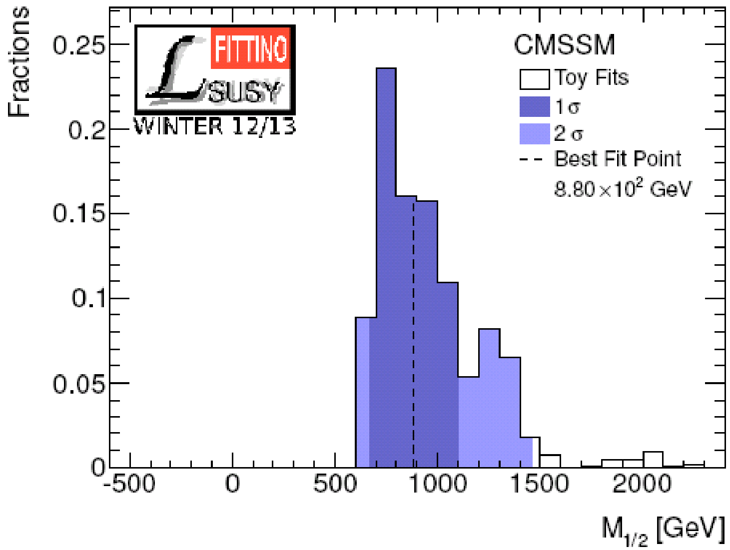
Toy fits (parameters)

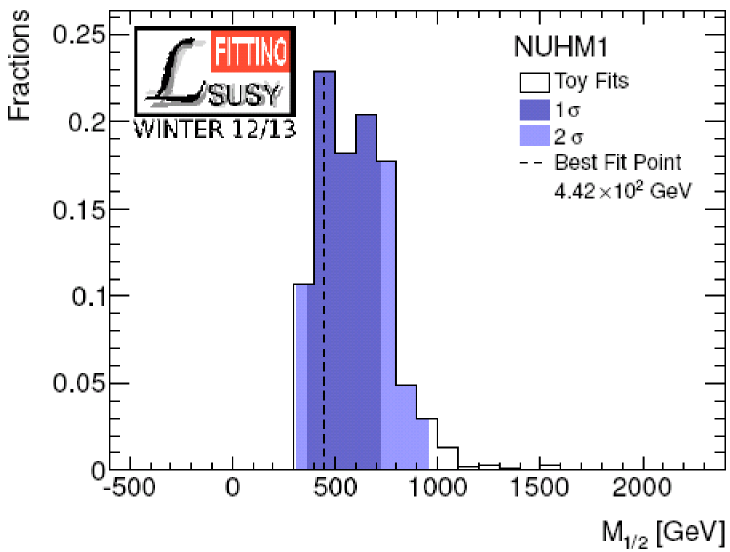


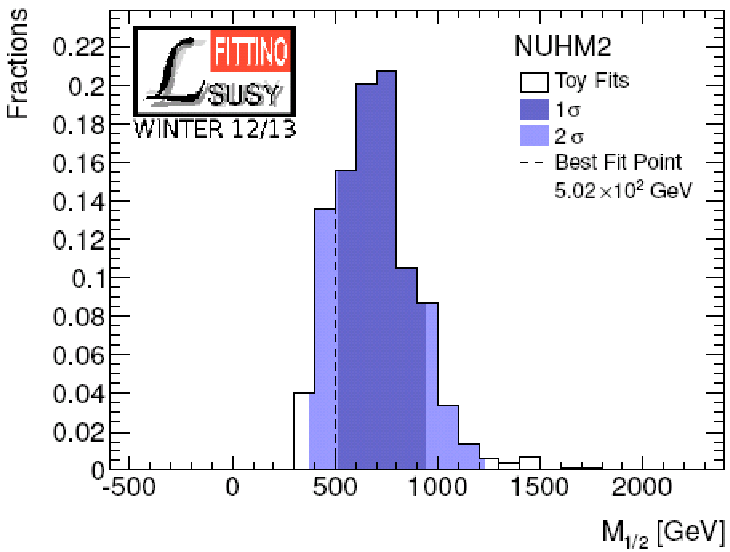




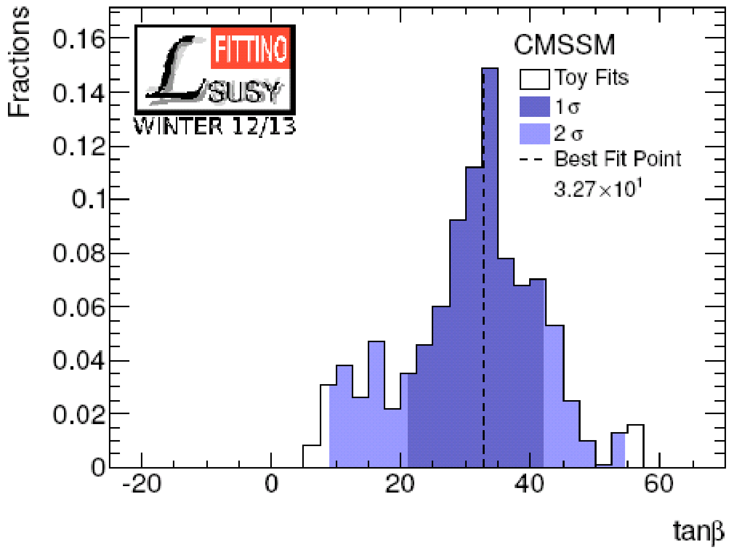
Toy fits (parameters)

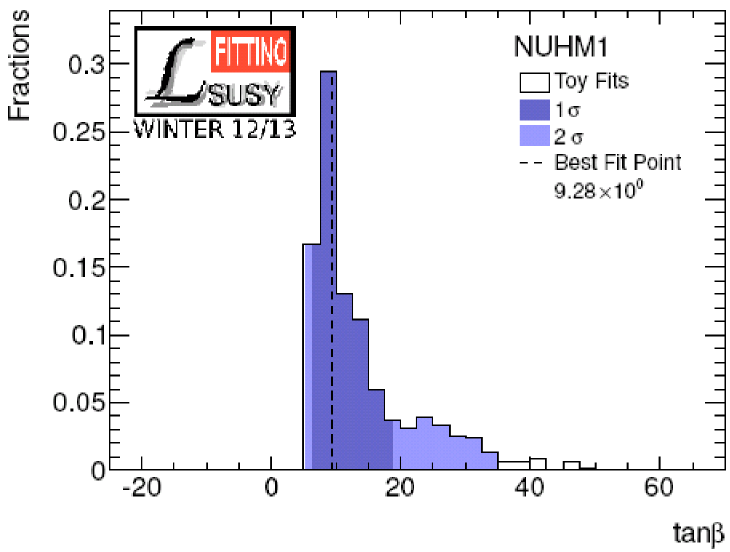


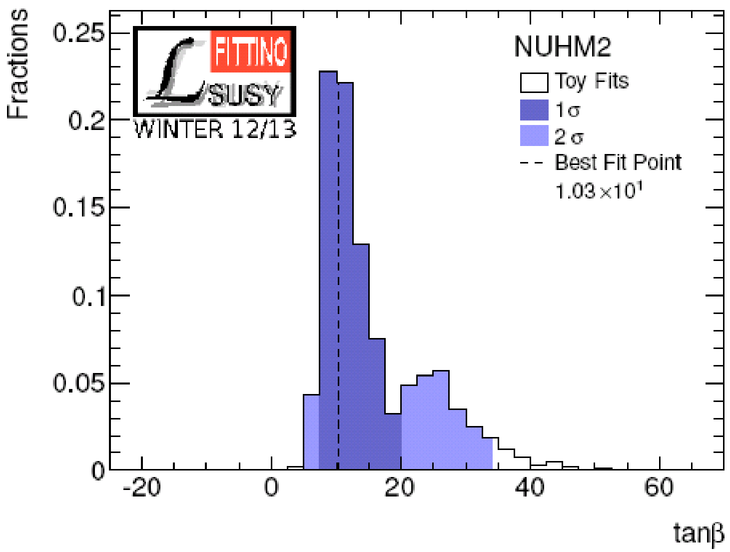




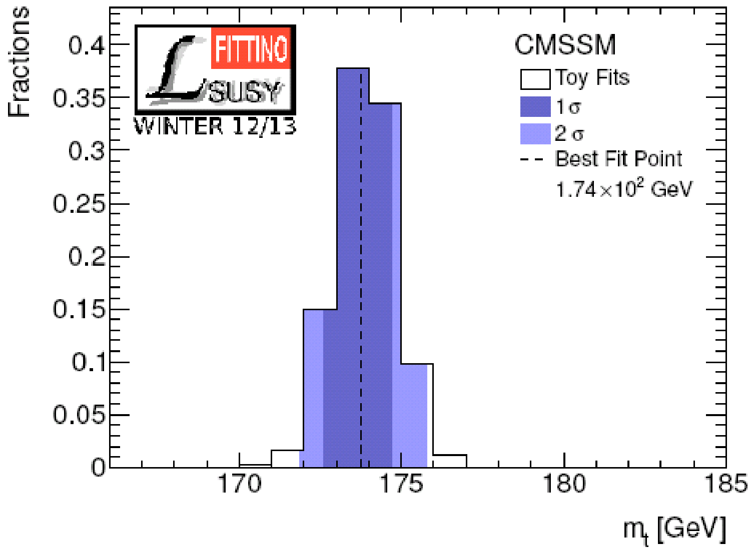
Toy fits (parameters)

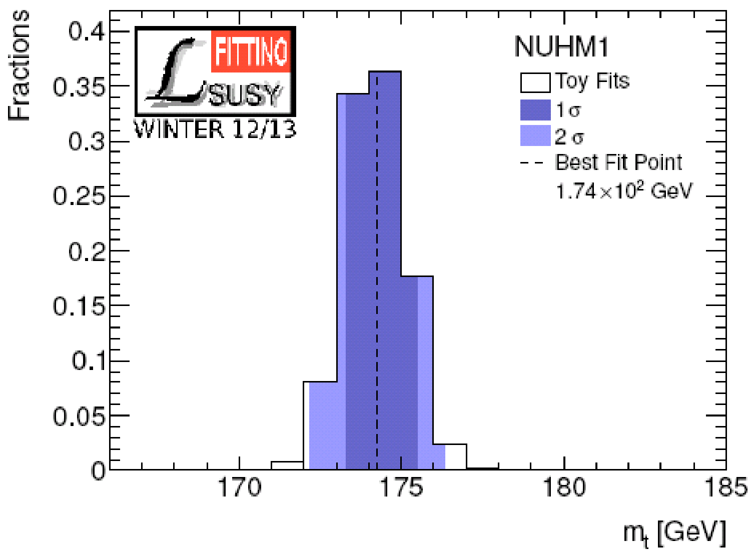


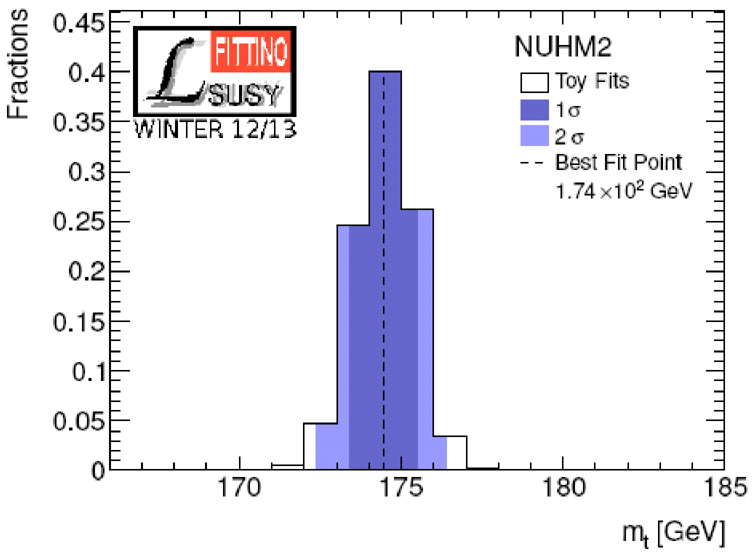




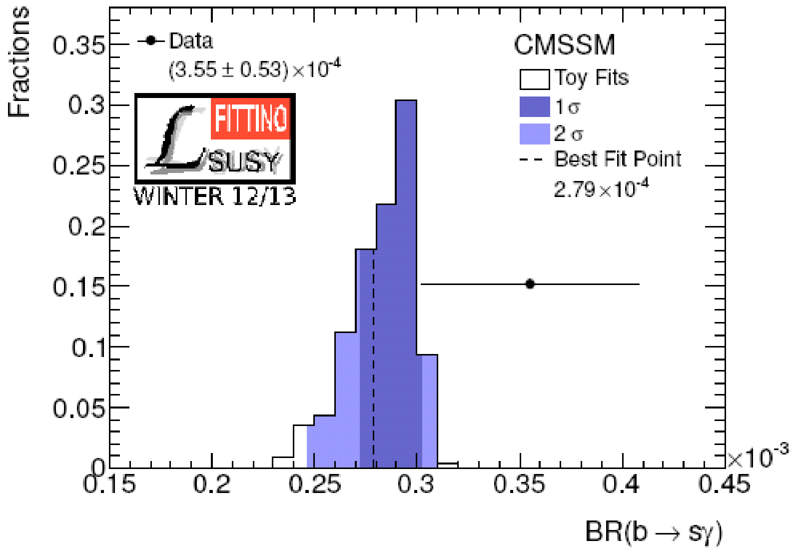
Toy fits (parameters)

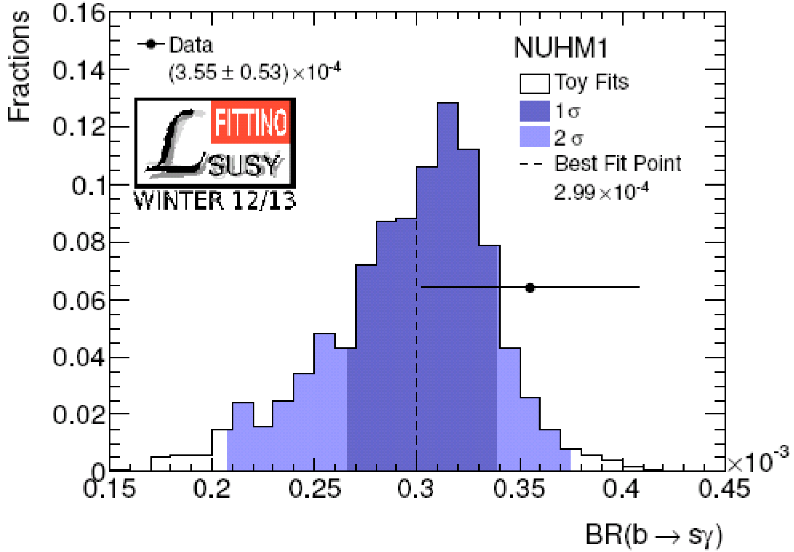


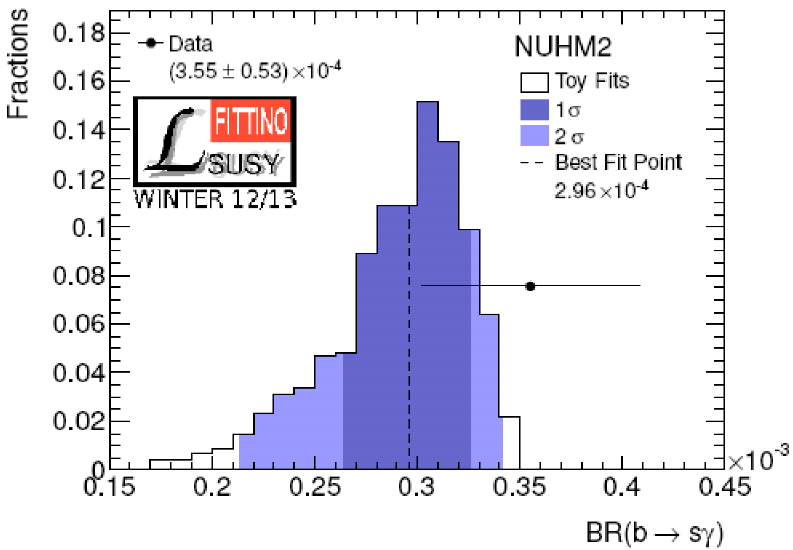




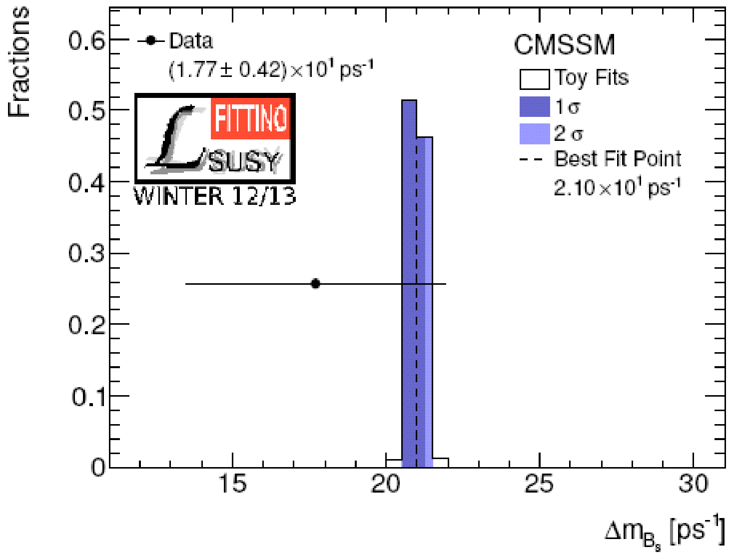
Toy fits (observables)

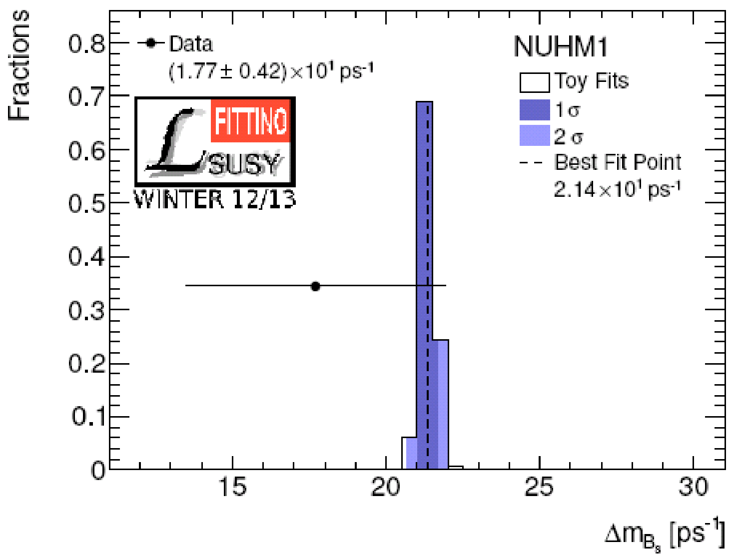


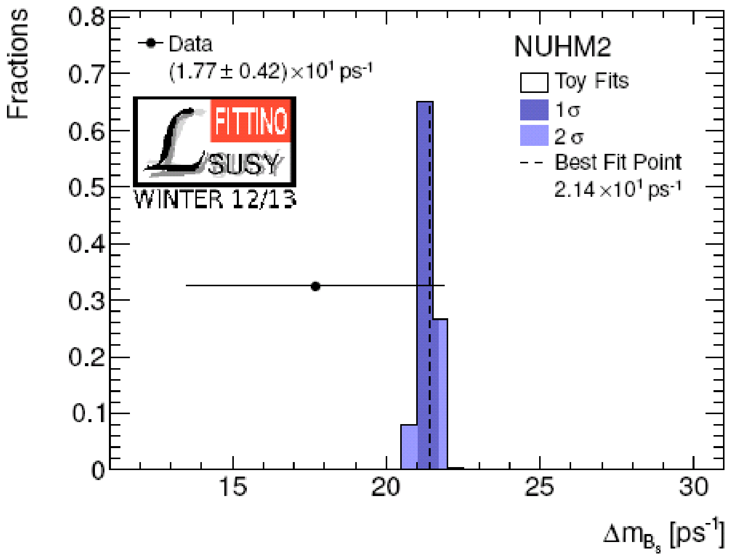




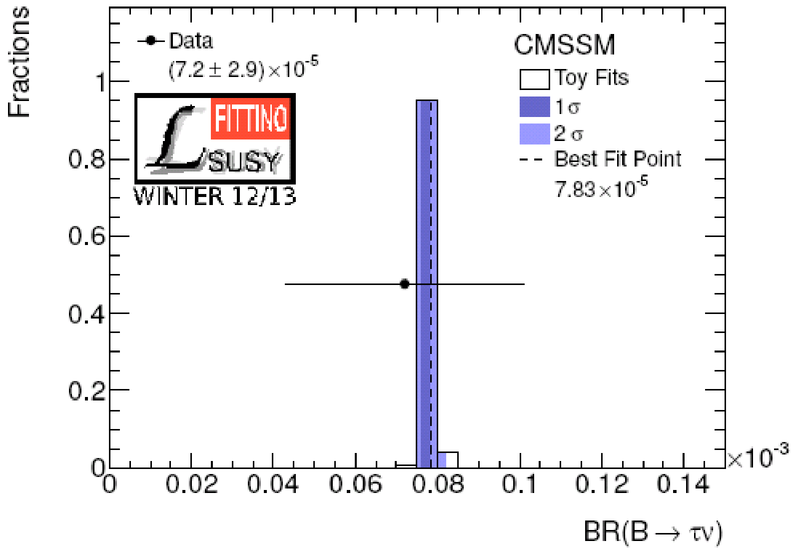
Toy fits (observables)

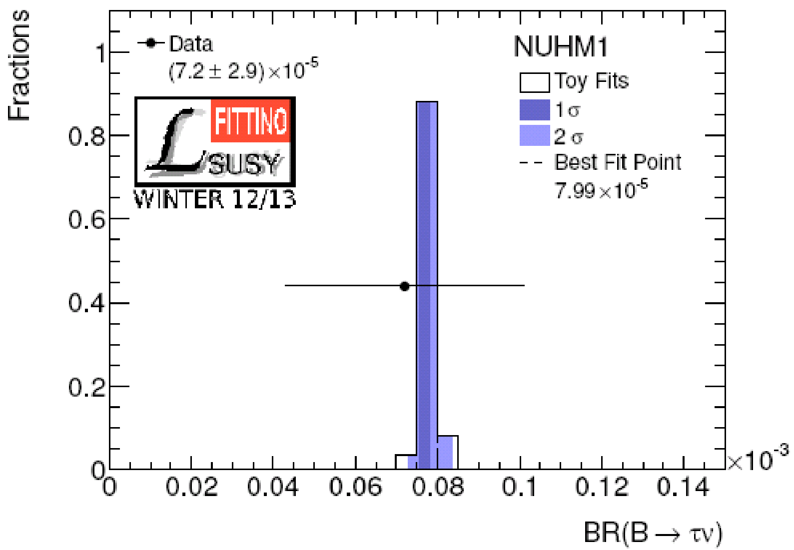


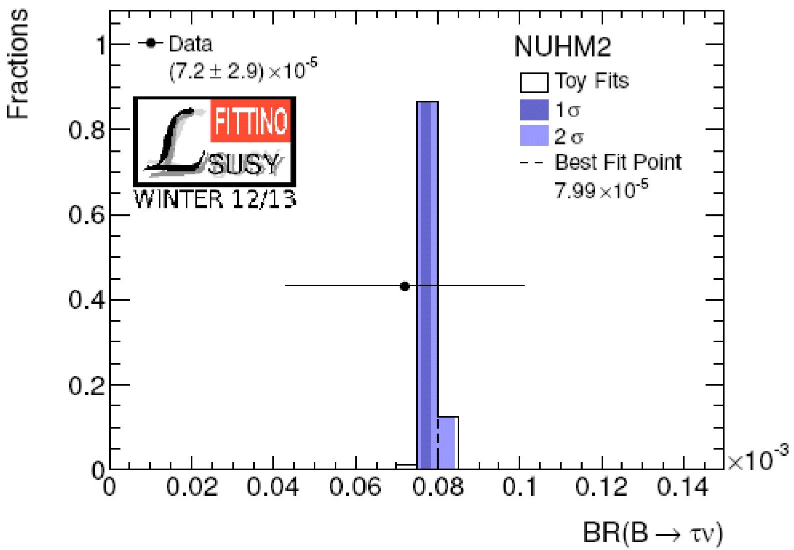




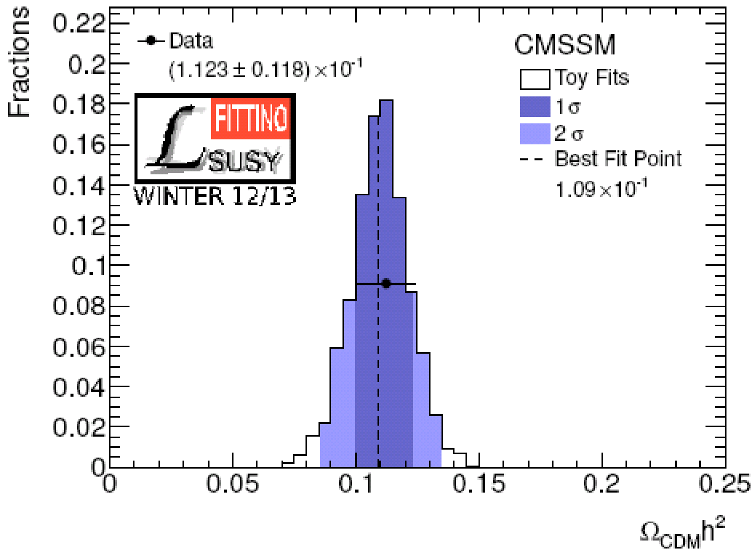
Toy fits (observables)

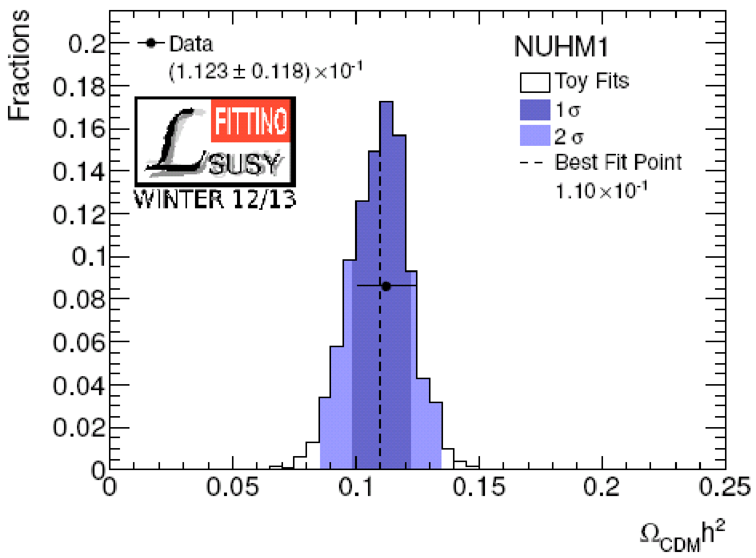


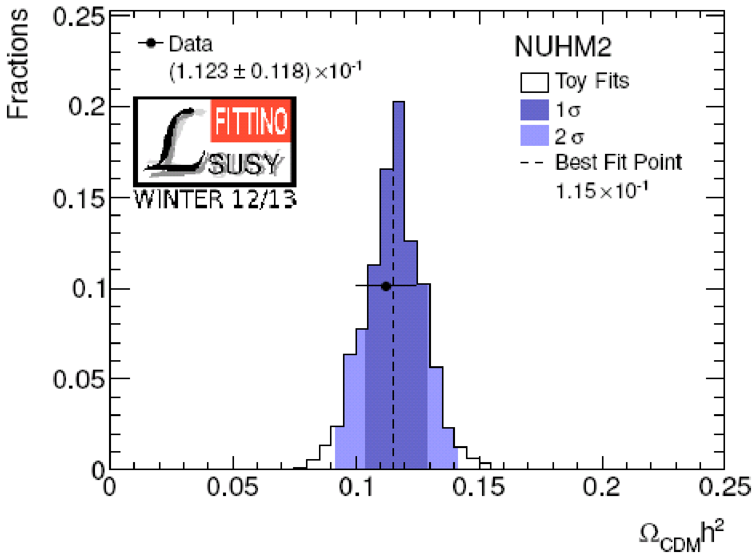




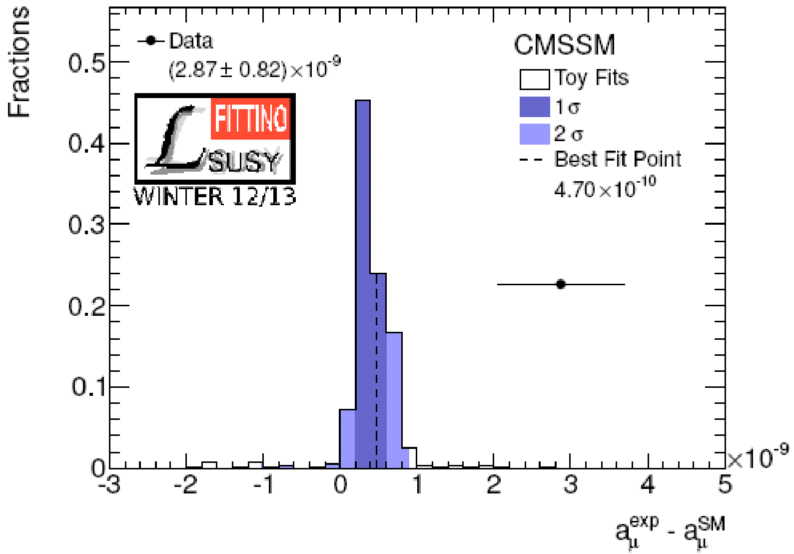
Toy fits (observables)

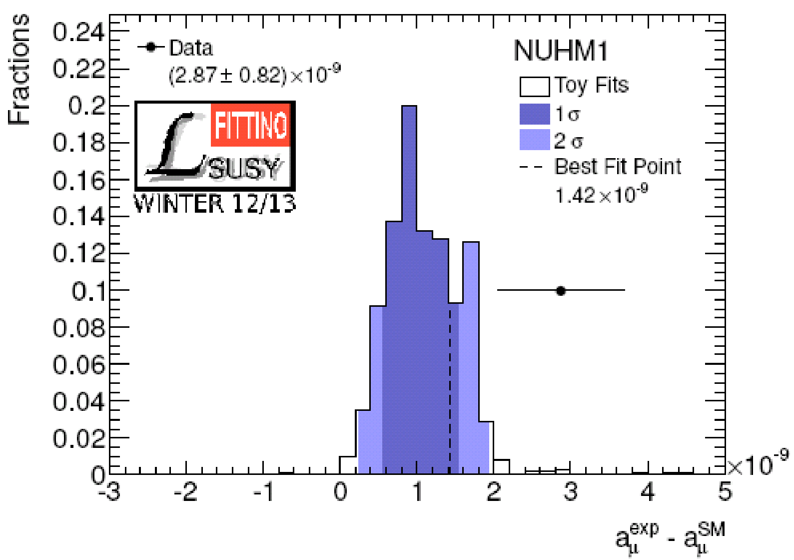


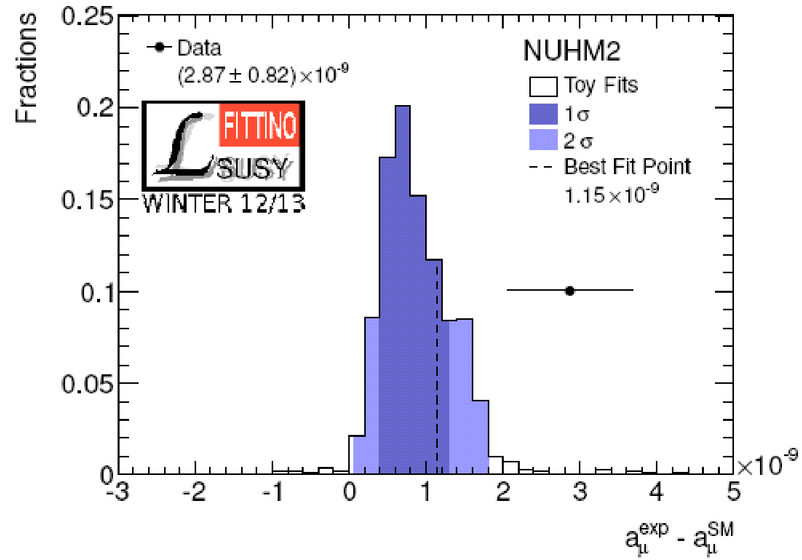


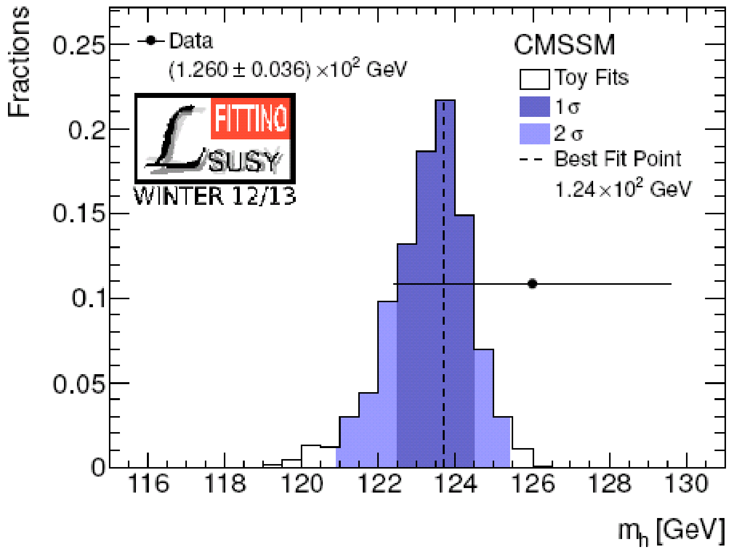


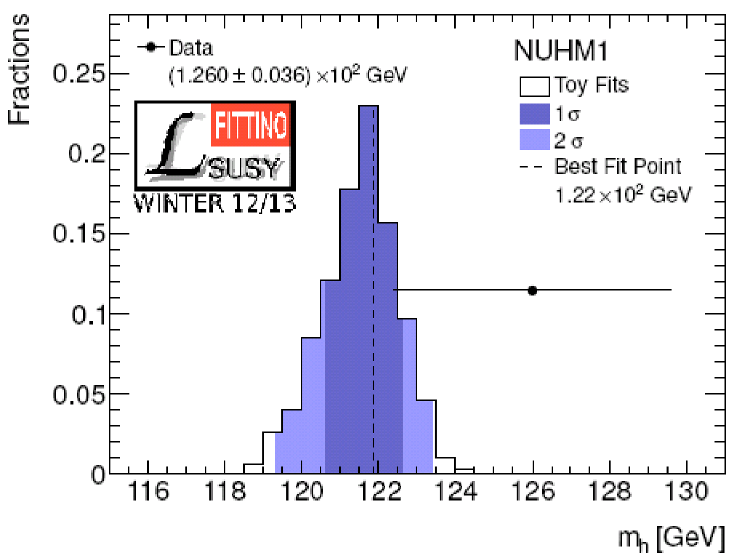
Toy fits (observables)

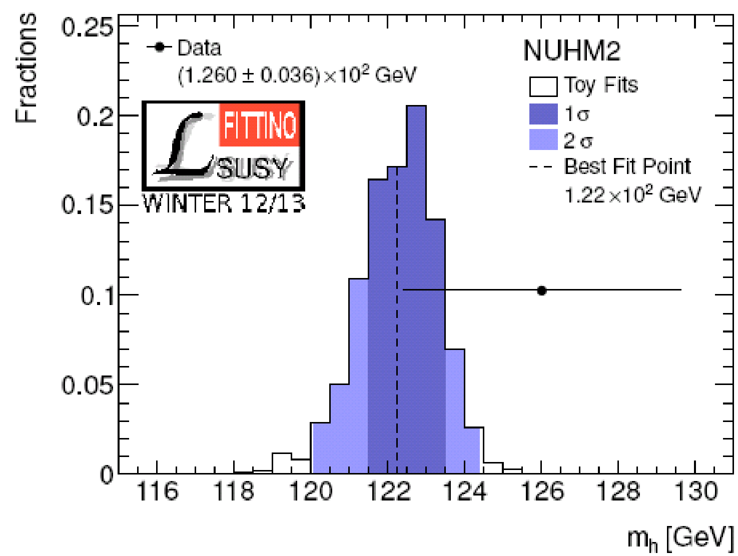




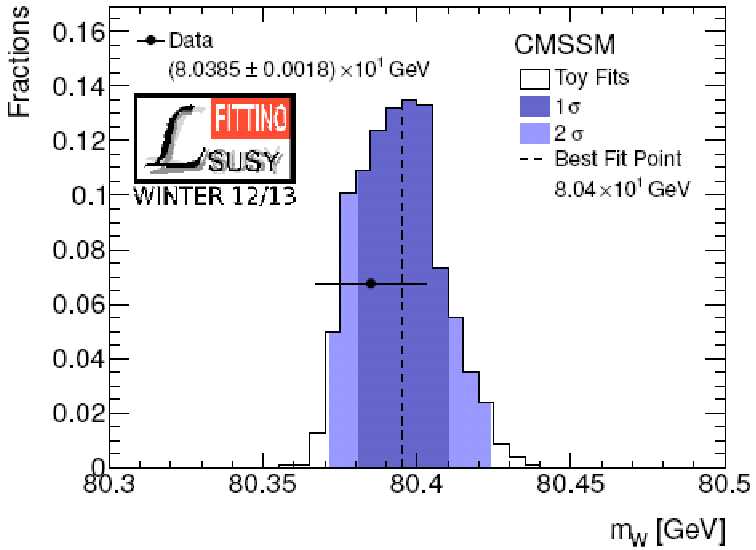


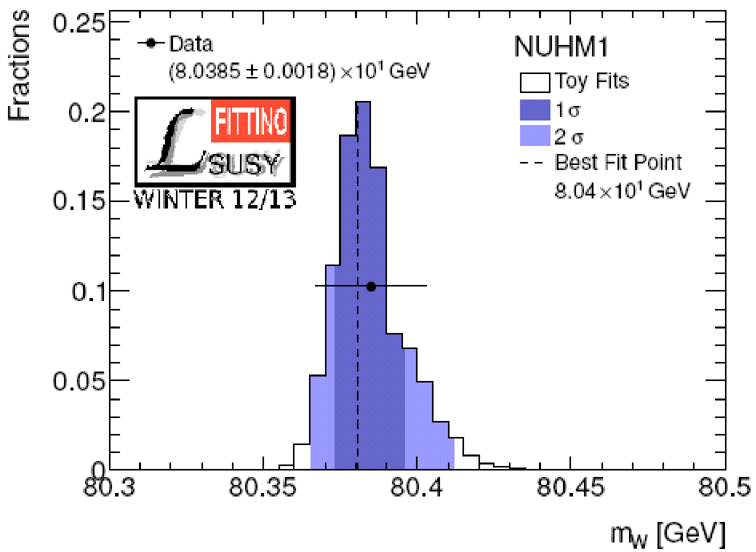

Fractions
Data
(1.260 ± 0.036) ×10² GeV
CMSSM
Toy Fits
1σ
2 σ
Best Fit Point
1.24×10² GeV
FITTINO
SUSY
WINTER 12/13
m_h [GeV]

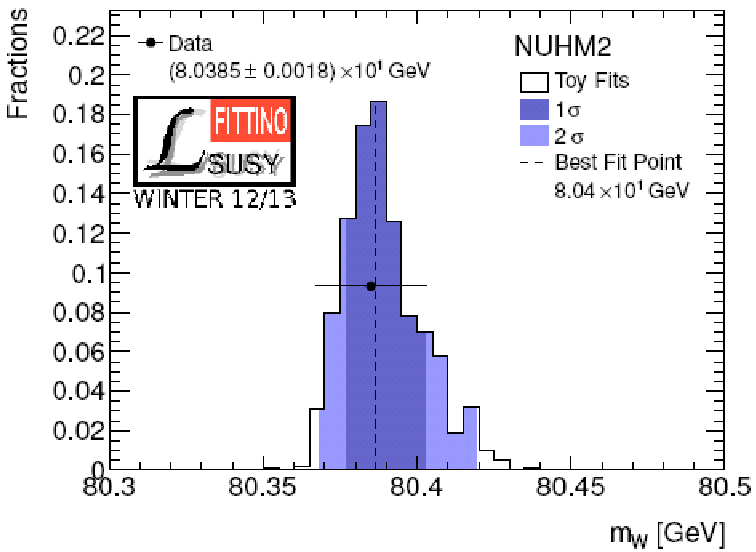

Fractions
Data
(1.260 ± 0.036) ×10² GeV
NUHM1
Toy Fits
1σ
2 σ
Best Fit Point
1.22×10² GeV
FITTINO
SUSY
WINTER 12/13
m_h [GeV]


Fractions
Data
(1.260 ± 0.036) ×10² GeV
NUHM2
Toy Fits
1σ
2 σ
Best Fit Point
1.22×10² GeV
FITTINO
SUSY
WINTER 12/13
m_h [GeV]

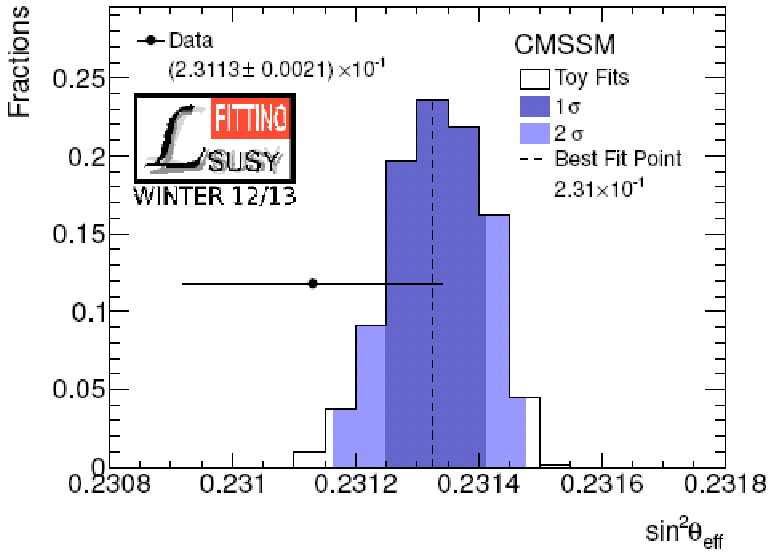
Toy fits (observables)

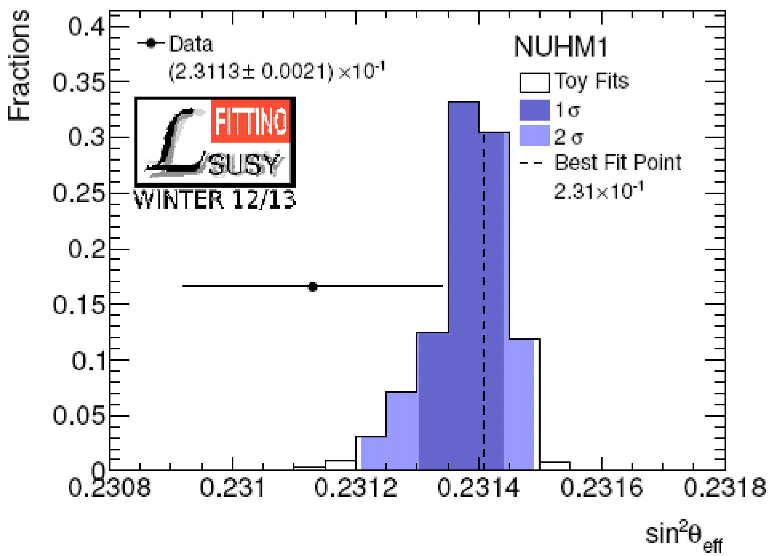


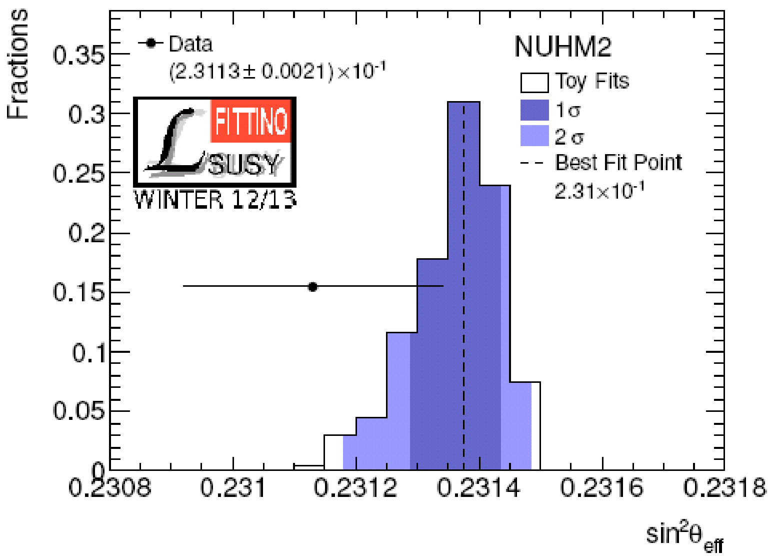




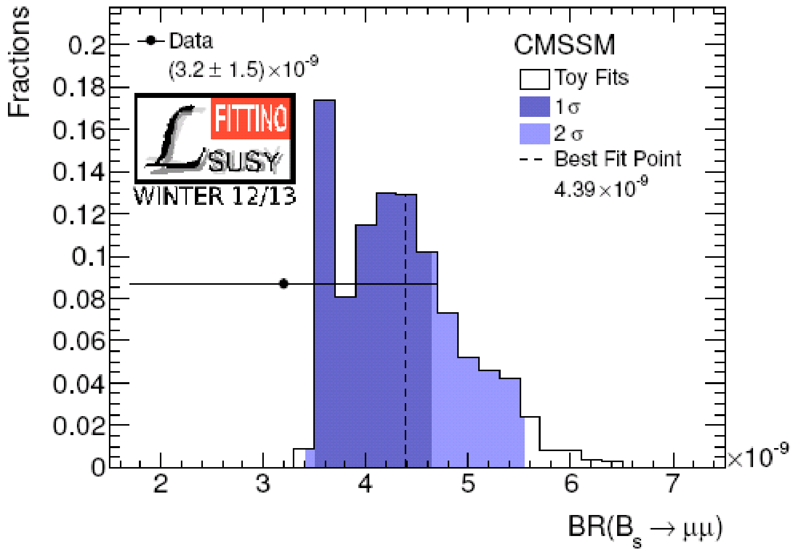
Toy fits (observables)

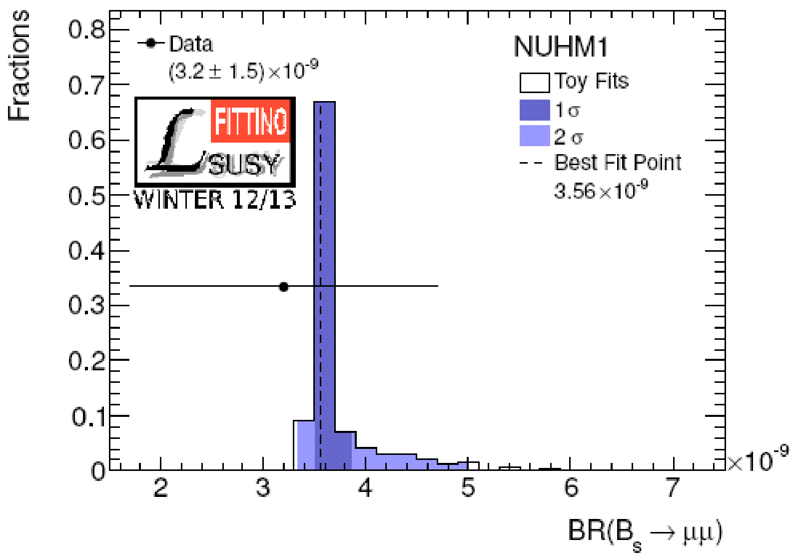


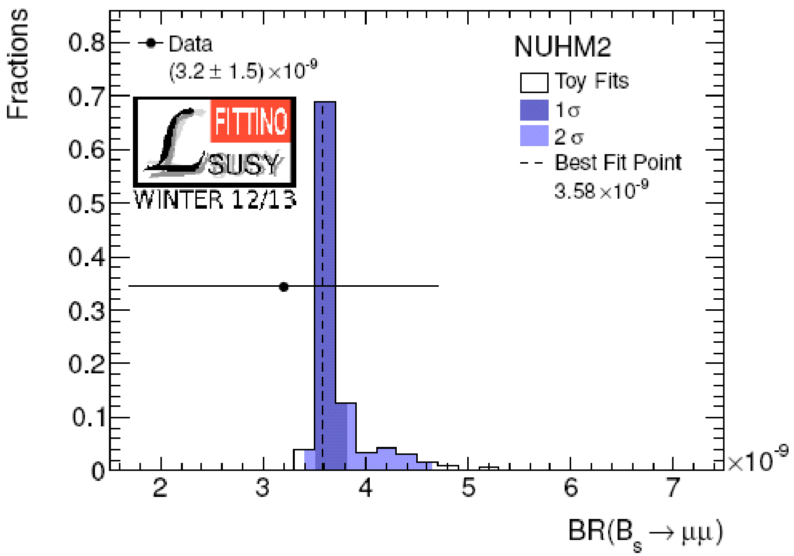




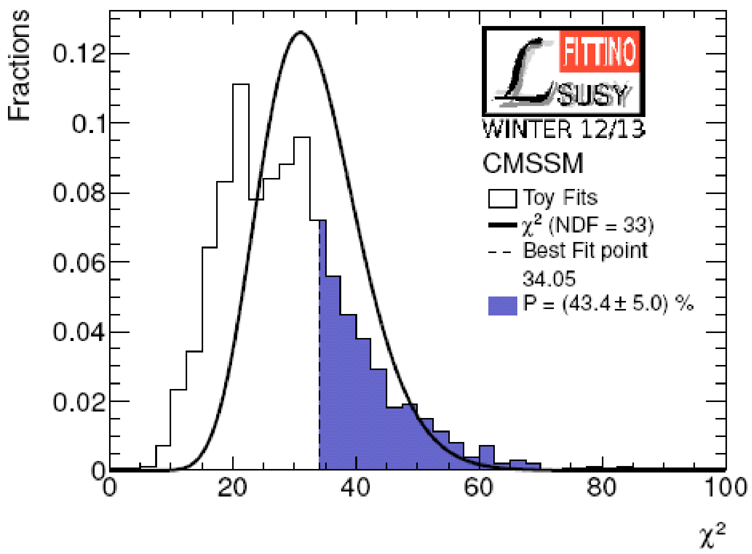
Toy fits (observables)



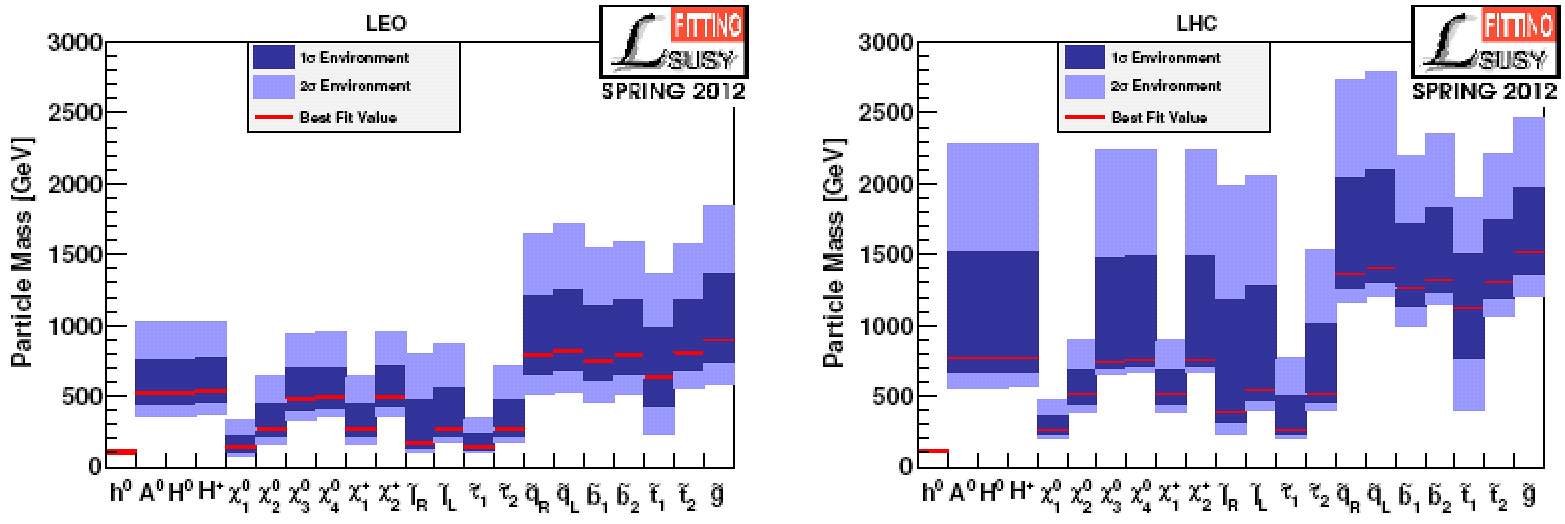




Toy fits (χ^2 distribution)



Mass spectra from the previous Fittino results: JHEP06(2012)098



(a) (b)

Figure 9. Predicted distribution of sparticle and Higgs boson masses from (a) the LEO fit and (b) the LHC fit. The full uncertainty band gives the 1-dimensional 2σ uncertainty of each mass defined by the region $\Delta\chi^2 < 4$ after profiling over all hidden dimensions.

Mass spectra from the previous Fittino results: JHEP06(2012)098

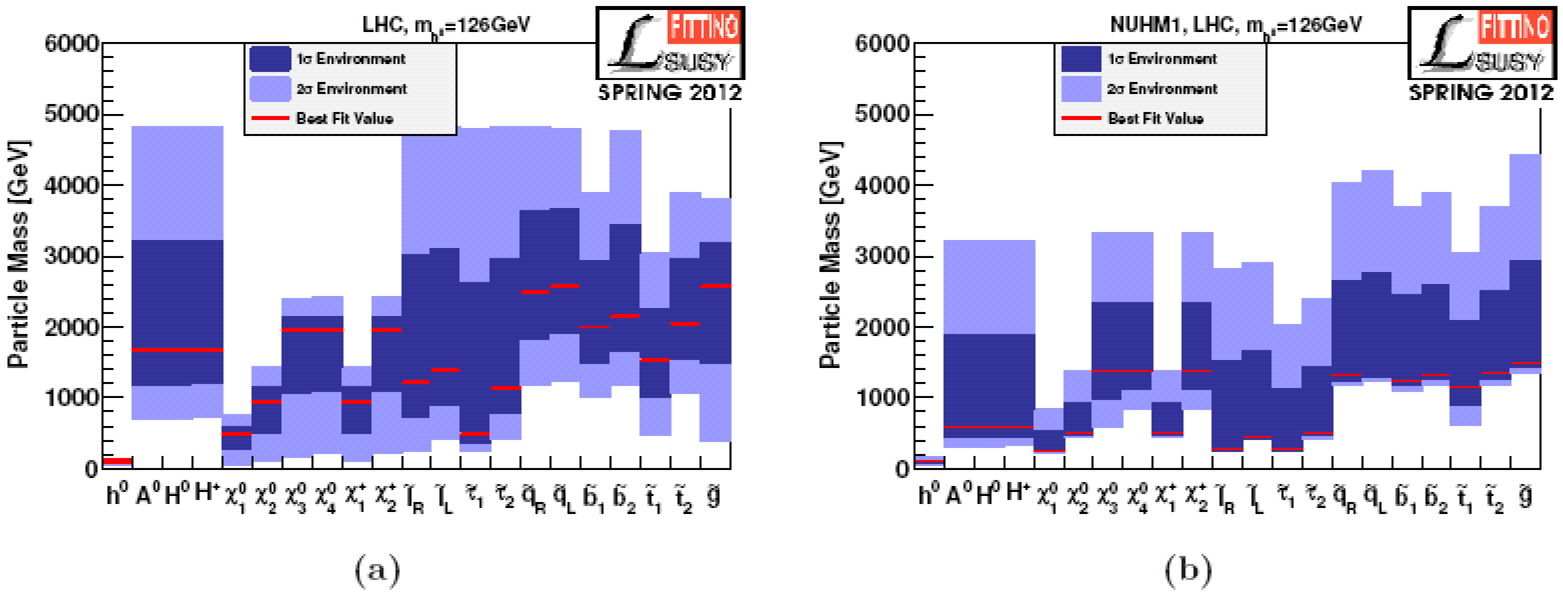
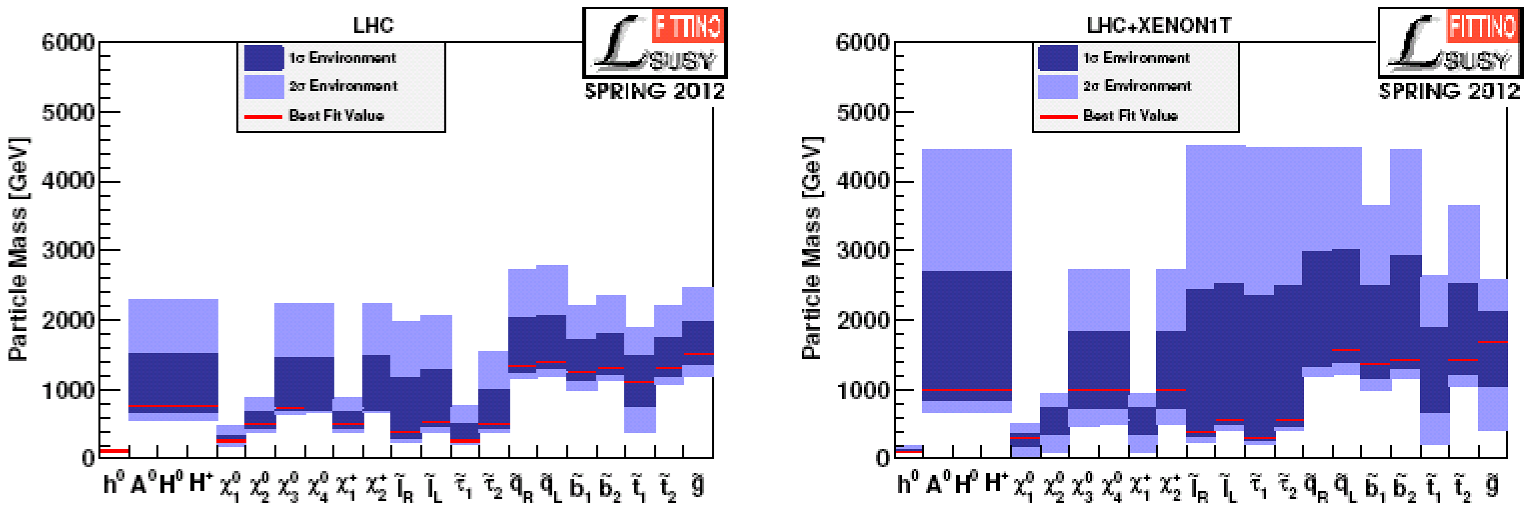


Figure 13. Predicted distribution of sparticle and Higgs boson masses from the CMSSM fit with $m_h = (126 \pm 2 \pm 3)$ GeV in (a) and the NUHM1 fit to the same observable set in (b).

Mass spectra from the previous Fittino results: JHEP06(2012)098



(a) (b)

Figure 16. Predicted distribution of sparticle and Higgs boson masses from (a) the LHC and (b) the LHC+XENON1T fit. The full uncertainty band gives the 1-dimensional 2σ uncertainty of each mass defined by the region $\Delta\chi^2 < 4$ after profiling over all hidden dimensions. Note the different scales on the ordinate (mass) axis compared to figure 9.