Towards gauge coupling unification in minimal SU(5) at 3-loop accuracy

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Outline

- Georgi-Glashow SU(5)
- A minimal extension
- Neutrino masses
- Unification
- Why 3-loops accuracy ?
- Results

Georgi Glashow SU(5)

- Rank-4 Lie Groups with only one coupling strength:

$[SU(2)]^4 \quad [O(5)]^2 \quad [SU(3)]^2 \quad [G_2]^2 \quad O(8) \quad O(9) \quad Sp(8) \quad F_4 \quad SU(5)$

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↓

do not contain $SU(3)_C$

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do not have complex representations

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$[SU(3)]^2$ → cannot describe quarks

$U(1)_Q \subset SU(3) \Rightarrow \sum Q(\text{quarks}) = 0$

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- Gauge sector: SM embedding

$$24_V \longrightarrow G(8,1,0) \oplus W(1,3,0) \oplus B(1,1,0) \oplus X(3,2,-\tfrac{5}{6}) \oplus \overline{X}(\overline{3},2,+\tfrac{5}{6})$$

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- SM fermions live in the same GUT multiplets:

$$\overline{5}_F = \underbrace{(\overline{3},1,+\tfrac{1}{3})_F}_{d^c} \oplus \underbrace{(1,2,-\tfrac{1}{2})_F}_{\ell} \qquad 10_F = \underbrace{(\overline{3},1,-\tfrac{2}{3})_F}_{u^c} \oplus \underbrace{(3,2,+\tfrac{1}{6})_F}_{q} \oplus \underbrace{(1,1,+1)_F}_{e^c}$$

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- Symmetry breaking sector: $24_H \oplus 5_H$

$$SU(5) \xrightarrow[M_X]{\langle 24_H \rangle} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[M_Z]{\langle 5_H \rangle} SU(3)_C \otimes U(1)_Q$$

Georgi Glashow SU(5)

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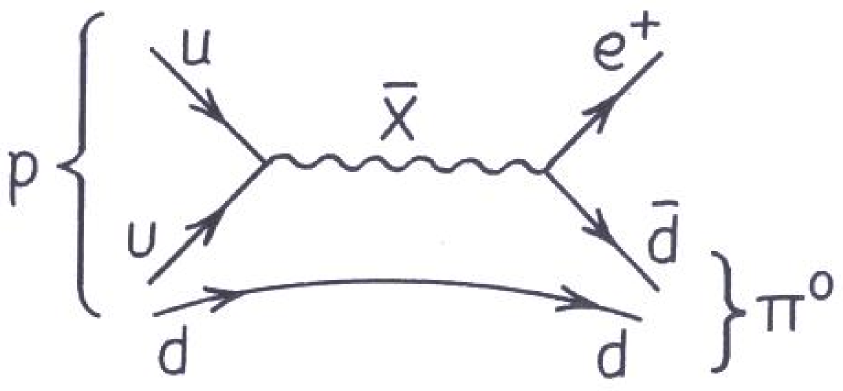
- X connects quarks and leptons: proton is unstable !



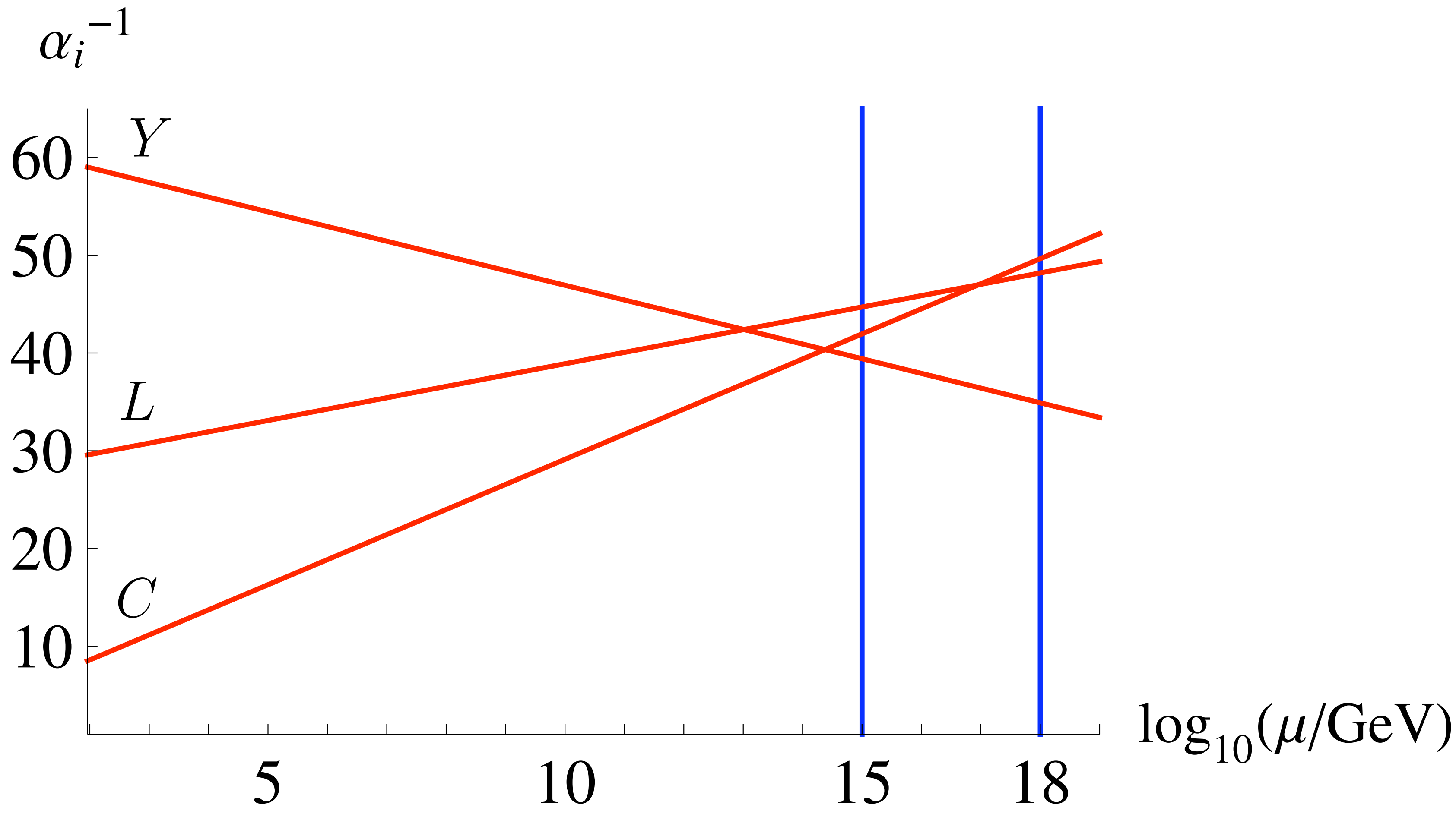
$$\tau_p^{\rm exp} \gtrsim 10^{33}\ {\rm yr}$$

$$\tau_p^{\rm th} \sim \alpha_G^{-2} \frac{M_X^4}{m_p^5}$$

$$\Longrightarrow \quad M_X \gtrsim 10^{15}\ {\rm GeV}$$

Why is the GG SU(5) ruled out ?

- Gauge couplings do not unify (even after including scalar thresholds)



Why is the GG SU(5) ruled out ?

- Gauge couplings do not unify (even after including scalar thresholds)
- Neutrinos are (practically) massless

$$\mathcal{L}_Y = \underbrace{Y_1 \overline{5}_F 10_F 5_H^*}_{M_D = M_E^T} + \underbrace{Y_2 10_F 10_F 5_H}_{M_U} + \frac{1}{\Lambda}\left[Y_3 \overline{5}_F \overline{5}_F 5_H 5_H + \ldots\right]$$

$$m_\nu \sim Y_3 \frac{v^2}{\Lambda} \lesssim 10^{-4} \text{ eV} \qquad \longleftrightarrow \qquad m_{\nu_3} \gtrsim \sqrt{\Delta m^2_{\text{atm}}} \sim 0.05 \text{ eV}$$

for $\Lambda \approx 100 \times M_G \approx 10^{17}$ GeV (b-tau + perturbativity)

Add a fermionic 24_F

- Solves both the problems at once*

[Bajc, Senjanovic (2006)]
[Bajc, Nemevsek, Senjanovic (2007)]

$$24_F = \underbrace{(1,1,0)_F}_{S_F} \oplus \underbrace{(1,3,0)_F}_{T_F} \oplus \underbrace{(8,1,0)_F}_{O_F} \oplus \underbrace{(3,2,-\tfrac{5}{6})_F}_{X_F} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_F}_{\overline{X}_F}$$

- Neutrino masses through seesaw

- RGEs are modified

*Minimal extension of GG SU(5) is not unique: e.g. add a 15_H

[Dorsner, Fileviez Perez (2005)]

[Dorsner, Fileviez Perez, Gonzalez Felipe (2005)]

Neutrino masses

- New Yukawa terms with 24_F

$$\delta\mathcal{L} = L_i \left(y_T^i T_F + y_S^i S_F \right) H + m_T T_F T_F + m_S S_F S_F + \text{h.c.}$$

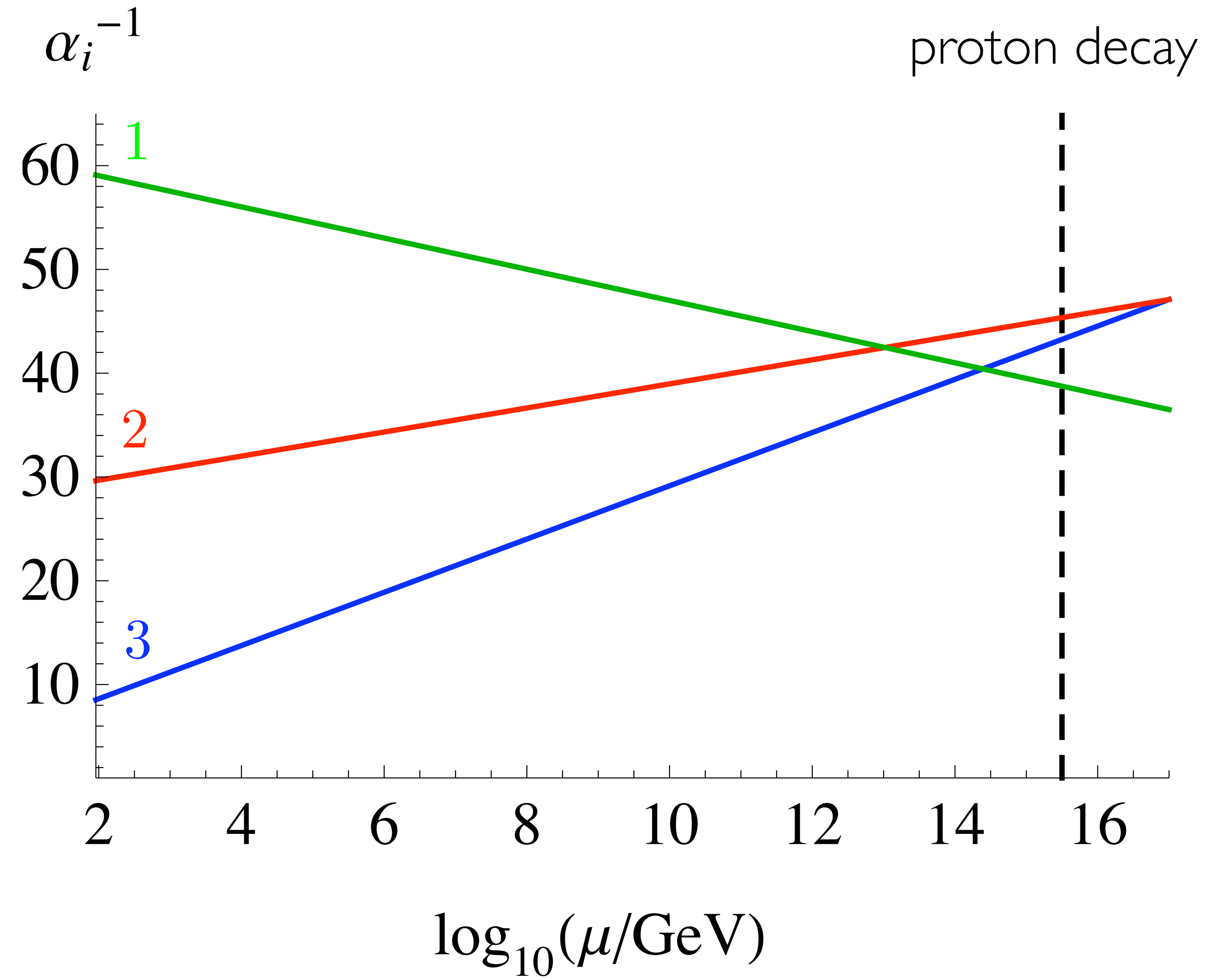
- Mixed type-III + type-I seesaw

$$(m_\nu)^{ij} = v^2 \left(\frac{y_T^i y_T^j}{m_T} + \frac{y_S^i y_S^j}{m_S} \right)$$

Unification patterns

- States which can contribute to the running btw M_Z and M_G :

SU(5) origin	Field content	(b_3, b_2, b_1)
5_H	$(3, 1, -\frac{1}{3})_H$	$(\frac{1}{6}, 0, \frac{1}{15})$
24_H	$(1, 3, 0)_H$	$(0, \frac{1}{3}, 0)$
24_H	$(8, 1, 0)_H$	$(\frac{1}{2}, 0, 0)$
24_F	$(1, 3, 0)_F$	$(0, \frac{4}{3}, 0)$
24_F	$(8, 1, 0)_F$	$(2, 0, 0)$
24_F	$(3, 2, -\frac{5}{6})_F$	$(\frac{4}{3}, 2, \frac{10}{3})$



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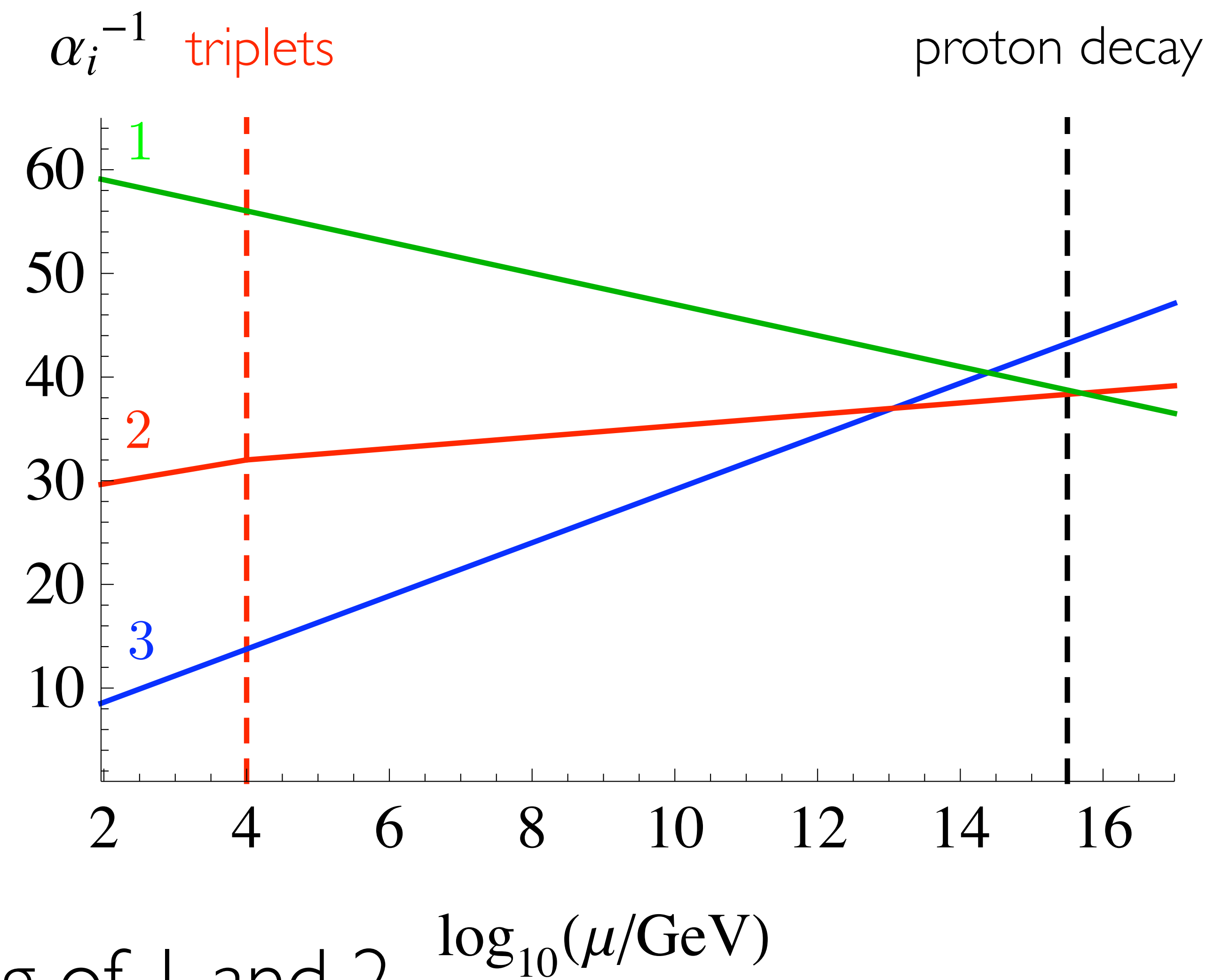

- $b_2 > b_1$ in order to delay the meeting of 1 and 2

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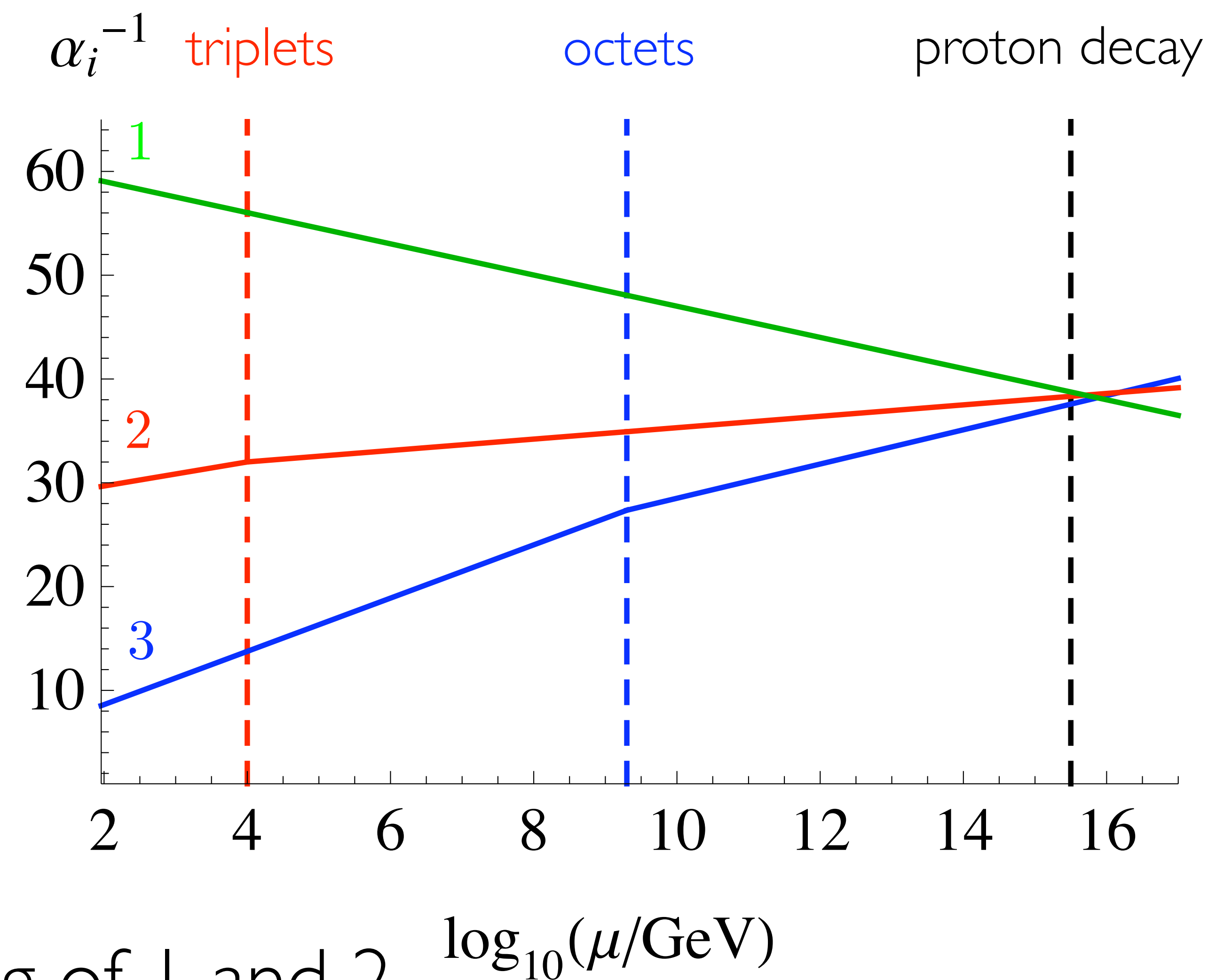
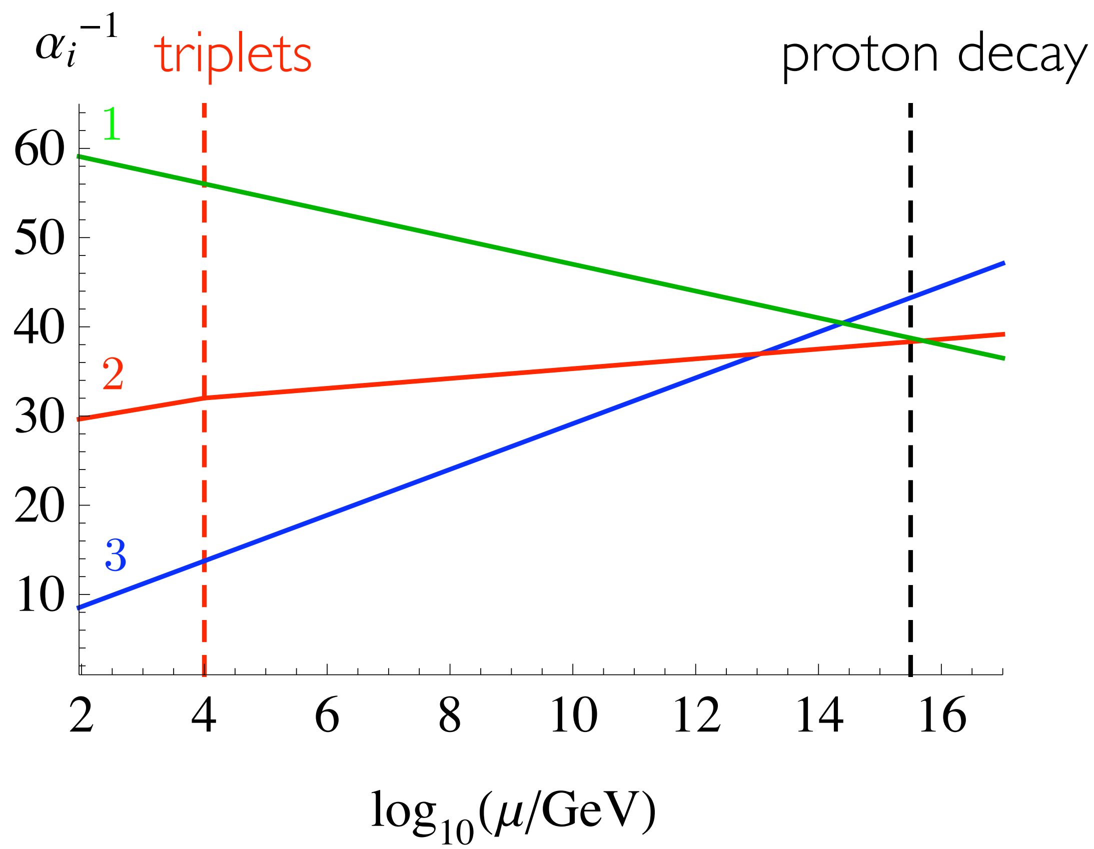


- $b_2 > b_1$ in order to delay the meeting of 1 and 2
- $b_3 > b_1$ and b_2 for the convergence of 3 with 1 and 2
- unification patterns require $m_T \ll m_O \ll M_G$

How heavy the triplets can be ?

- RGEs constrain the quantity $m_3 = \left(m_{T_F}^4 m_{T_H}\right)^{1/5}$
- which is the maximum value allowed for m_3 ?

  - maximize the mass of the extra thresholds with b1 > b2
  - depends on the convergence of α_1 and α_2 (precision observable !)



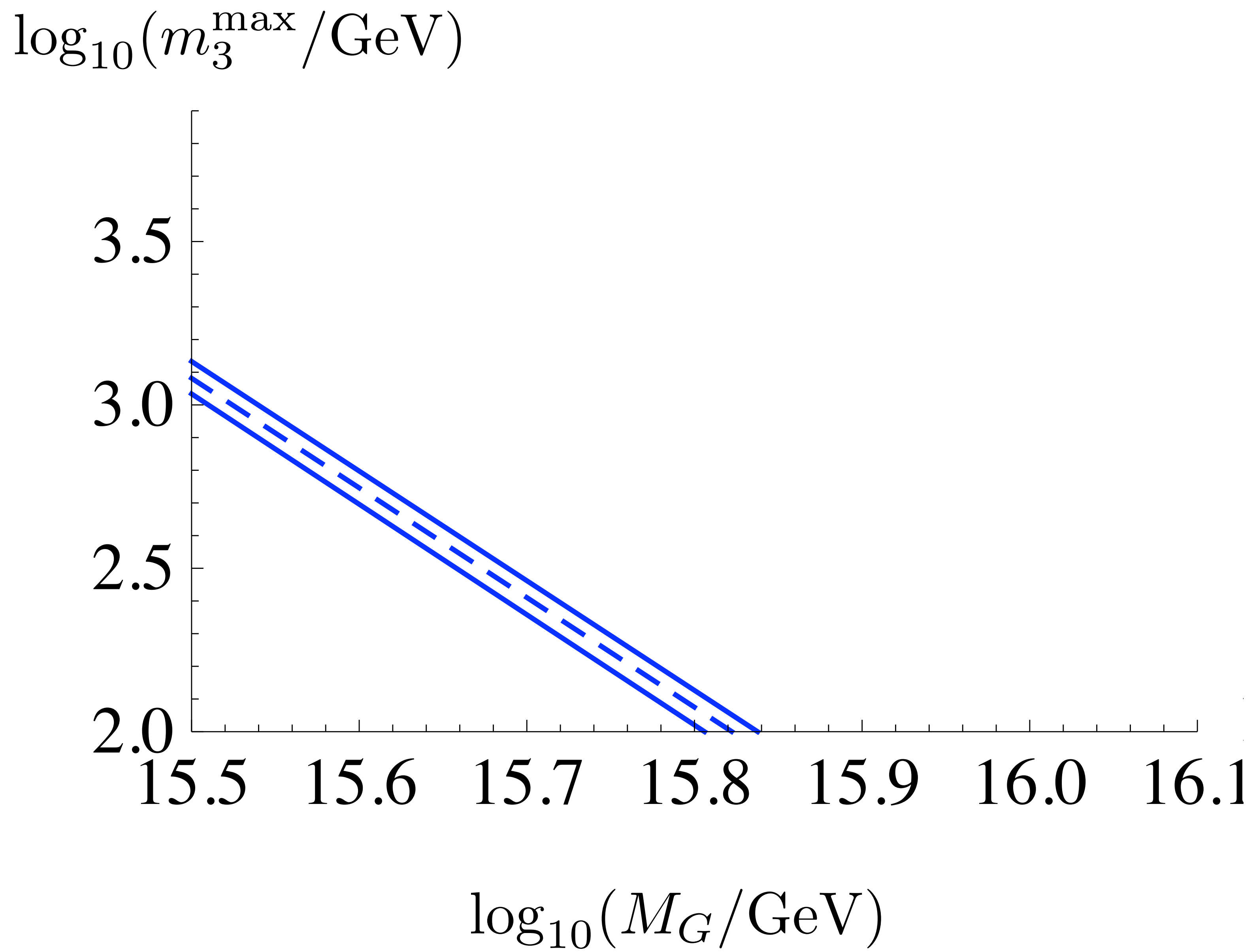
$$\frac{\Delta\alpha_1}{\alpha_1}(M_Z) \approx 0.02\%$$

$$\frac{\Delta\alpha_2}{\alpha_2}(M_Z) \approx 0.06\%$$

$$\frac{\Delta\alpha_3}{\alpha_3}(M_Z) \approx 0.6\%$$

m_3^{max} - M_G correlation

- one-loop

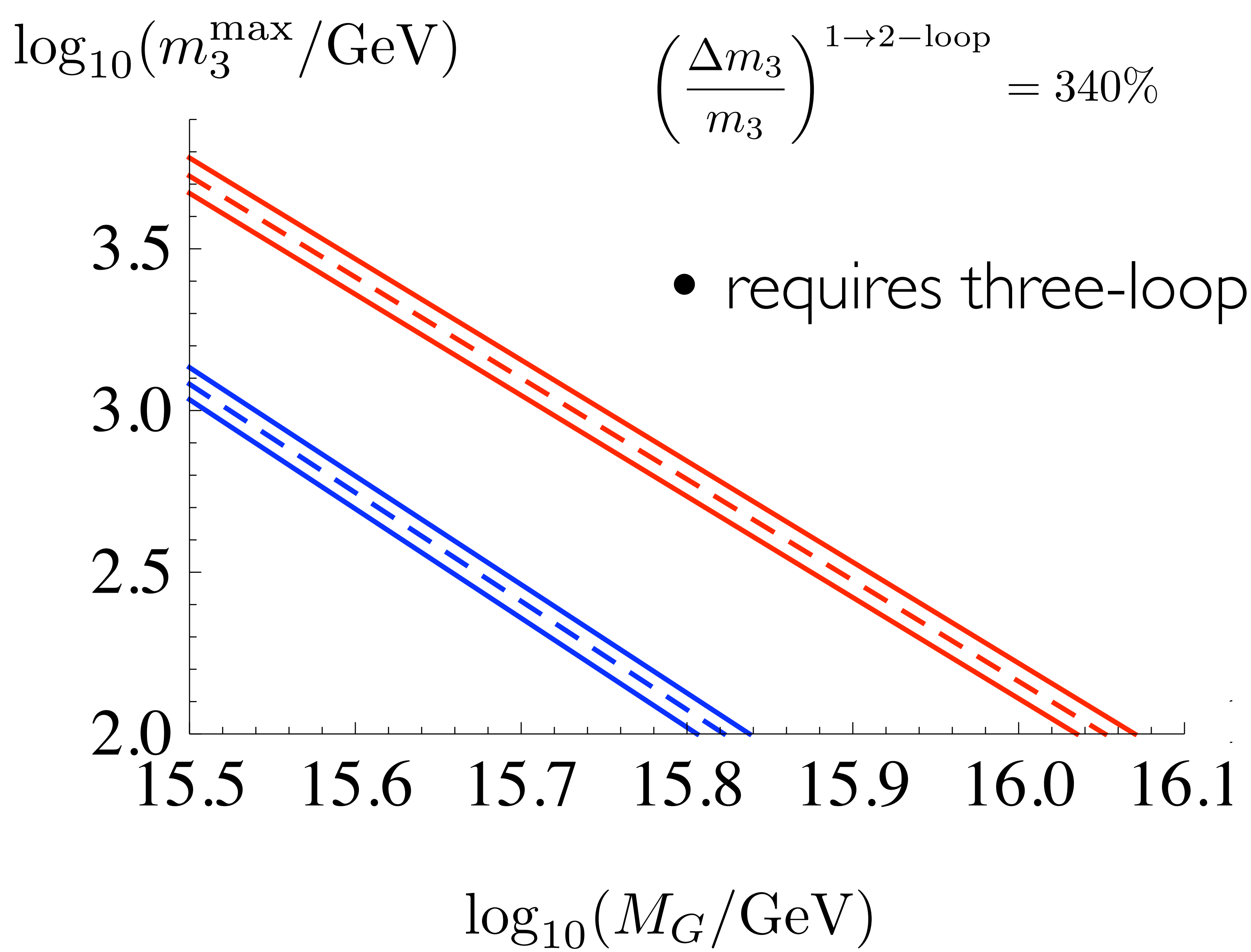


m_3^{max} - M_G correlation

- one-loop ⟶ two-loops

$$\left(\frac{\Delta M_G}{M_G}\right)^{1\to 2-\text{loop}} = 70\% \qquad \left(\frac{\Delta M_G}{M_G}\right)^{\text{exp}} = 9\%$$

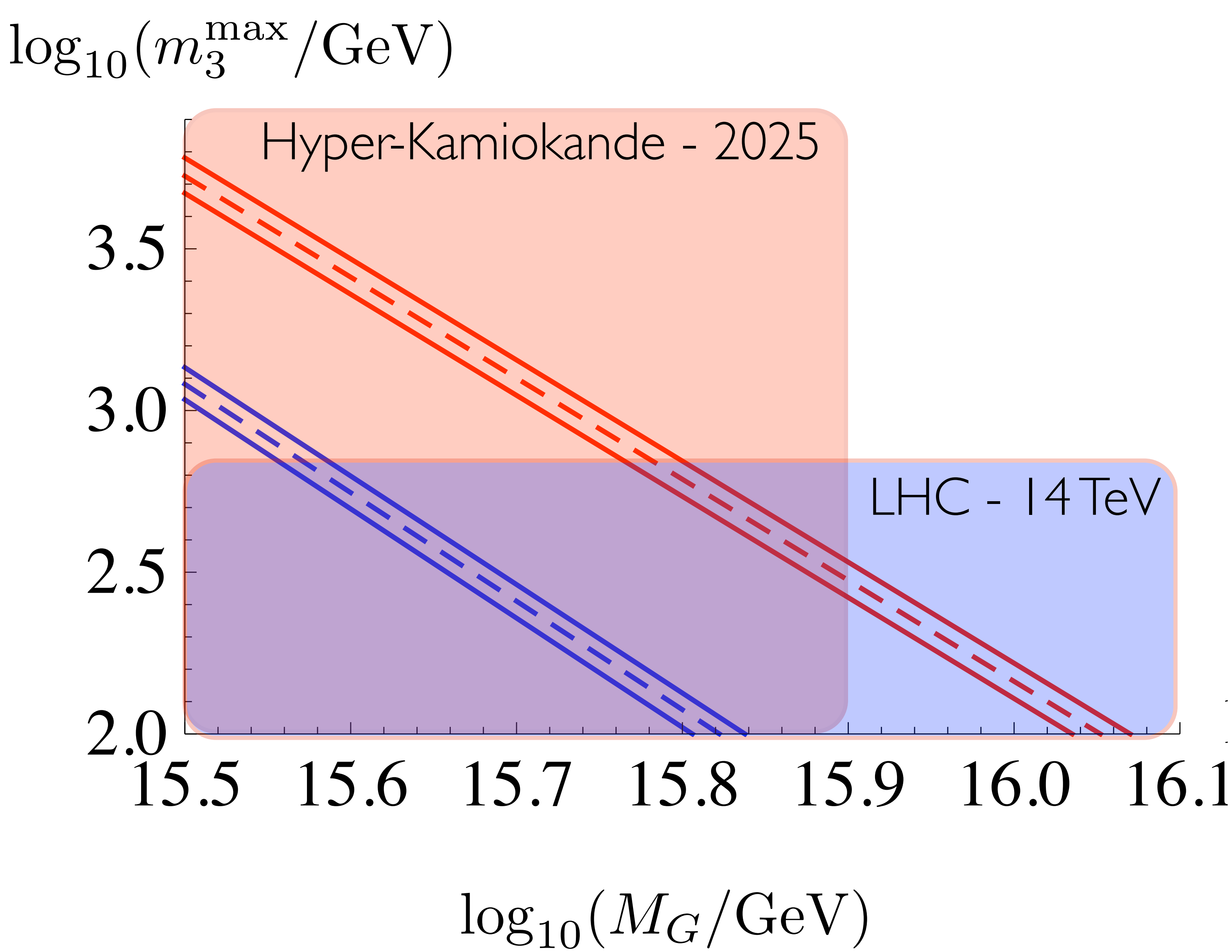
$$\left(\frac{\Delta m_3}{m_3}\right)^{1\to 2-\text{loop}} = 340\% \qquad \left(\frac{\Delta m_3}{m_3}\right)^{\text{exp}} = 25\%$$



- requires three-loop accuracy !

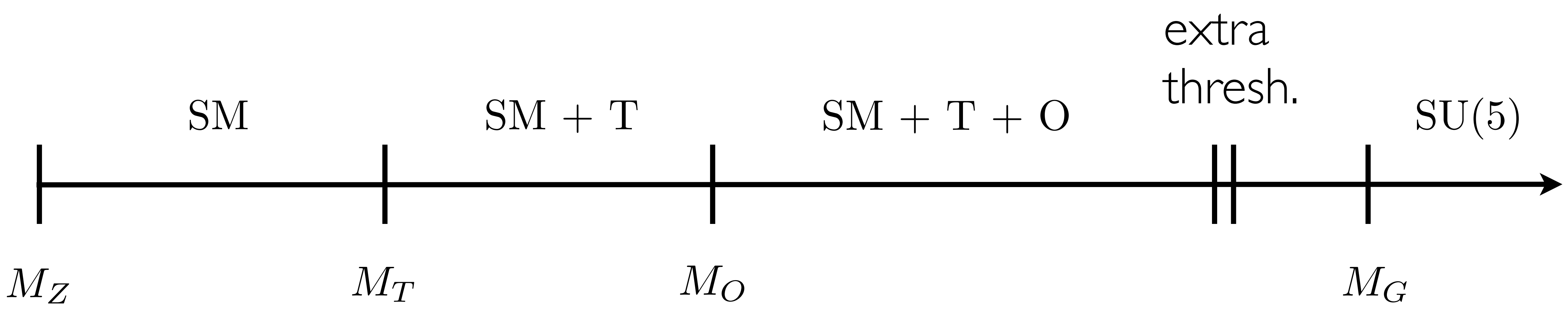
m_3^{max} - M_G correlation

- Interplay btw LHC and HK will cover most of the parameter space



Ingredients for a 3-loop analysis

- Effective field theories: n-loop running + (n-1)-loop matching



- 3-loop beta functions in the SM [Mihaila, Salomon, Steinhauser (2012)]
- 2-loop matching for SM ⟶ SM + T ⟶ SM + T + O (here)
- 3-loop beta functions in SM + T and SM + T + O (here)
- 2-loop matching at M_G (missing)

Analytical results

- 2-loop matching for SM ⟶ SM + T

$$\alpha_i^{\rm SM}(\mu) = \zeta_{\alpha_i}\left(\mu, \alpha_i(\mu), m_{T_{H,F}}(\mu)\right)\, \alpha_i(\mu)$$

$$\begin{aligned}
\zeta_{\alpha_2} = 1 &+ \frac{\alpha_2}{\pi}\left(-\frac{1}{6}C(G_L)\ln\frac{\mu^2}{m_{T_F}^2}N_{T_F} - \frac{1}{24}C(G_L)\ln\frac{\mu^2}{m_{T_H}^2}N_{T_H}\right) \\
&+ \frac{\alpha_2^2}{\pi^2}\left[\left(-\frac{7}{288}C(G_L)^2 - \frac{1}{12}C(G_L)^2\ln\frac{\mu^2}{m_{T_F}^2} + \frac{1}{36}C(G_L)^2\ln^2\frac{\mu^2}{m_{T_F}^2}N_{T_F}\right)N_{T_F}\right. \\
&+ \left(\frac{37}{576}C(G_L)^2 - \frac{11}{96}C(G_L)^2\ln\frac{\mu^2}{m_{T_H}^2} + \frac{1}{576}C(G_L)^2\ln^2\frac{\mu^2}{m_{T_H}^2}N_{T_H}\right)N_{T_H} \\
&\left.+\frac{1}{72}C(G_L)^2\ln\frac{\mu^2}{m_{T_F}^2}\ln\frac{\mu^2}{m_{T_H}^2}N_{T_F}N_{T_H}\right] \\
&+ \frac{\alpha_2}{\pi}\frac{\alpha_{\lambda_T}}{\pi}\left(-\frac{1}{48}C(G_L)N(G_L) - \frac{1}{24}C(G_L) - \frac{1}{48}C(G_L)N(G_L)\ln\frac{\mu^2}{m_{T_H}^2} - \frac{1}{24}C(G_L)\ln\frac{\mu^2}{m_{T_H}^2}\right)N_{T_H}^2
\end{aligned}$$

Analytical results

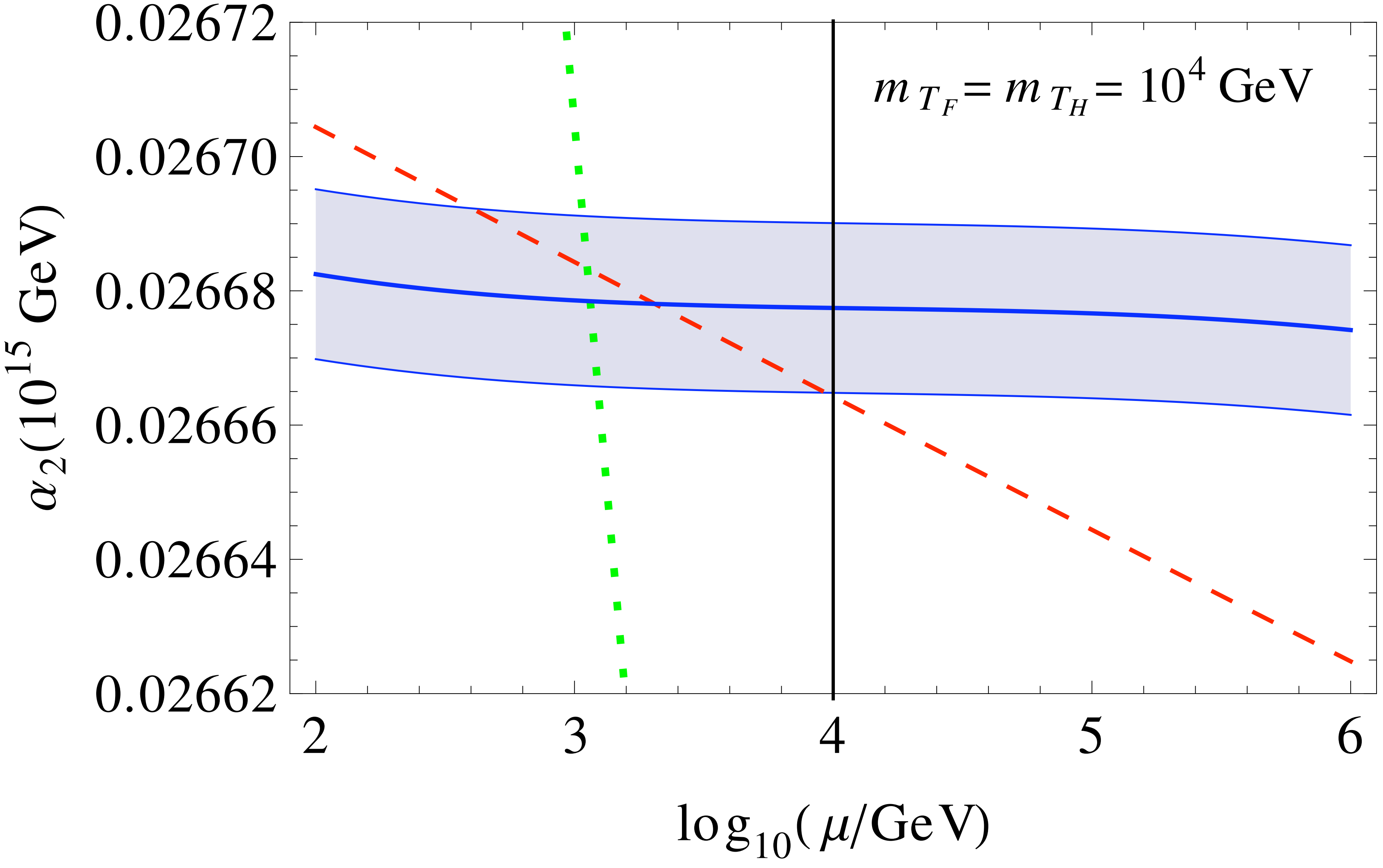
- 3-loop beta-functions in SM + T

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_i}{\pi} = \beta_i(\{\alpha_j\})$$

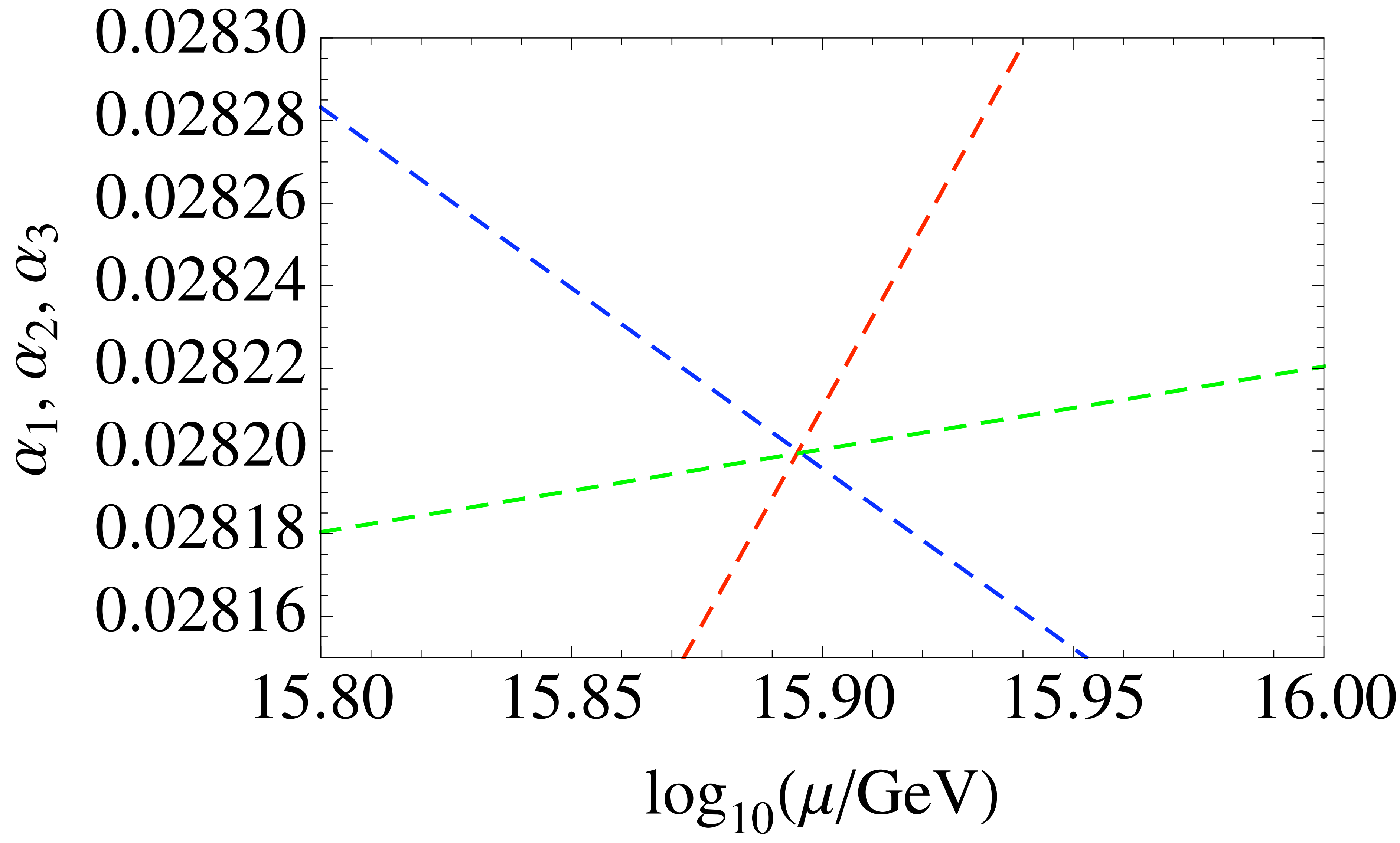
$$\begin{aligned}
\Delta\beta_2 = \frac{\alpha_2^2}{\pi^2} \Bigg\{ & \frac{1}{6} C(G_L)\, N_{T_F} + \frac{1}{24} C(G_L)\, N_{T_H} + \frac{\alpha_2}{\pi} \left(\frac{1}{3} C(G_L)^2 N_{T_F} + \frac{7}{48} C(G_L)^2 N_{T_H} \right) \\
& + \frac{\alpha_2^2}{\pi^2} \Bigg[\Bigg(\frac{247}{432} C(G_L)^3 - \frac{7}{108} C(G_L)^2 T(R_L) (N(R_C) N_q + N_\ell) - \frac{11}{576} C(G_L) C(R_L) T(R_L) (N(R_C) N_q + N_\ell) \\
& - \frac{127}{3456} C(G_L)^2 T(R_L) N_h - \frac{25}{576} C(G_L) C(R_L) T(R_L) N_h - \frac{145}{3456} C(G_L)^3 N_{T_F} - \frac{277}{6912} C(G_L)^3 N_{T_H} \Bigg) N_{T_F} \\
& + \Bigg(\frac{2749}{6912} C(G_L)^3 - \frac{13}{432} C(G_L)^2 T(R_L) (N(R_C) N_q + N_\ell) - \frac{23}{2304} C(G_L) C(R_L) T(R_L) (N(R_C) N_q + N_\ell) \\
& - \frac{143}{6912} C(G_L)^2 T(R_L) N_h - \frac{49}{2304} C(G_L) C(R_L) T(R_L) N_h - \frac{145}{13824} C(G_L)^3 N_{T_H} \Bigg) N_{T_H} \Bigg] \\
& + \frac{\alpha_2}{\pi} \frac{\alpha_{\lambda_T}}{\pi} \frac{5}{64} C(G_L)^2 N_{T_H}^2 + \frac{\alpha_2}{\pi} \frac{\alpha_{\lambda_{HT}}}{\pi} \frac{1}{16} C(G_L) T(R_L) N_h N_{T_H} + \frac{\alpha_{\lambda_T}^2}{\pi^2} \left(-\frac{1}{32} C(G_L) N(G_L) - \frac{1}{16} C(G_L) \right) N_{T_H}^3 \\
& + \frac{\alpha_{\lambda_{HT}}^2}{\pi^2} \left(-\frac{1}{64} C(G_L) N(R_L) N_h - \frac{1}{64} N(G_L) T(R_L) N_h \right) N_{T_H} \Bigg\}
\end{aligned}$$

Scale dependence

$$\text{SM} \longrightarrow \text{SM} + \text{triplets}$$



Sample Unification



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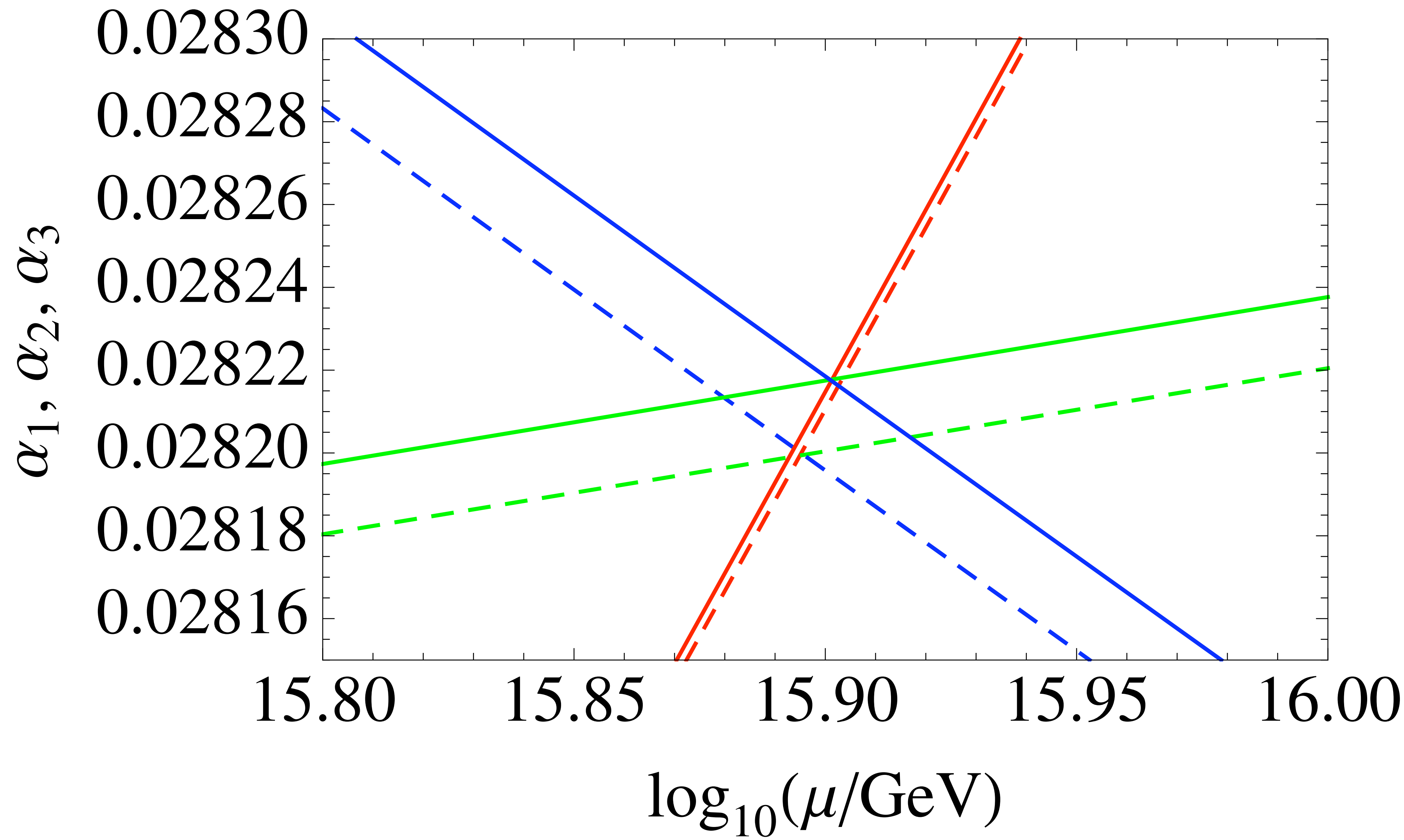
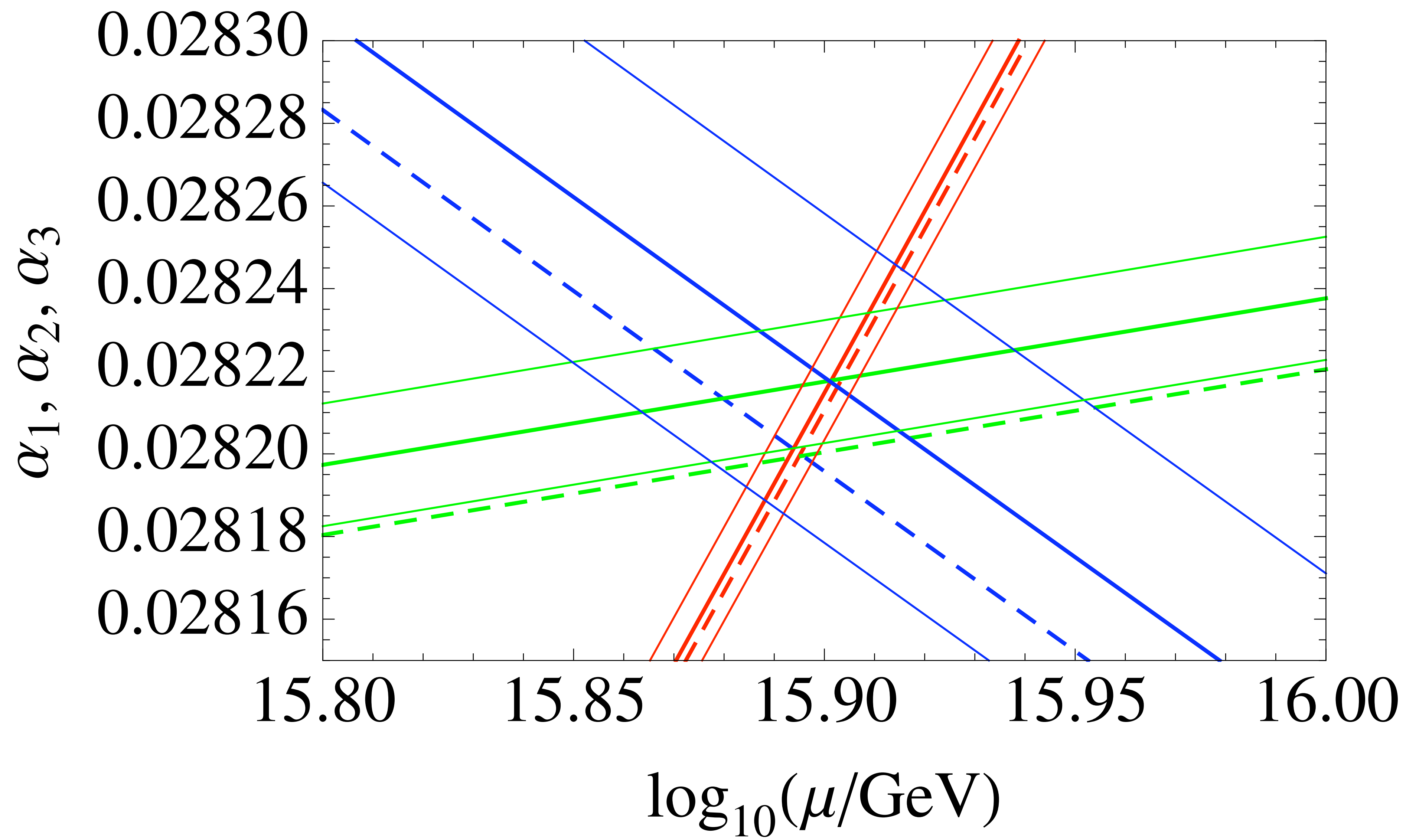


$$\left(\frac{\Delta\alpha_1}{\alpha_1}\right)^{2\to 3-\mathrm{loop}} = 0.015\% \qquad \left(\frac{\Delta\alpha_2}{\alpha_2}\right)^{2\to 3-\mathrm{loop}} = 0.061\% \qquad \left(\frac{\Delta\alpha_3}{\alpha_3}\right)^{2\to 3-\mathrm{loop}} = 0.08\%$$

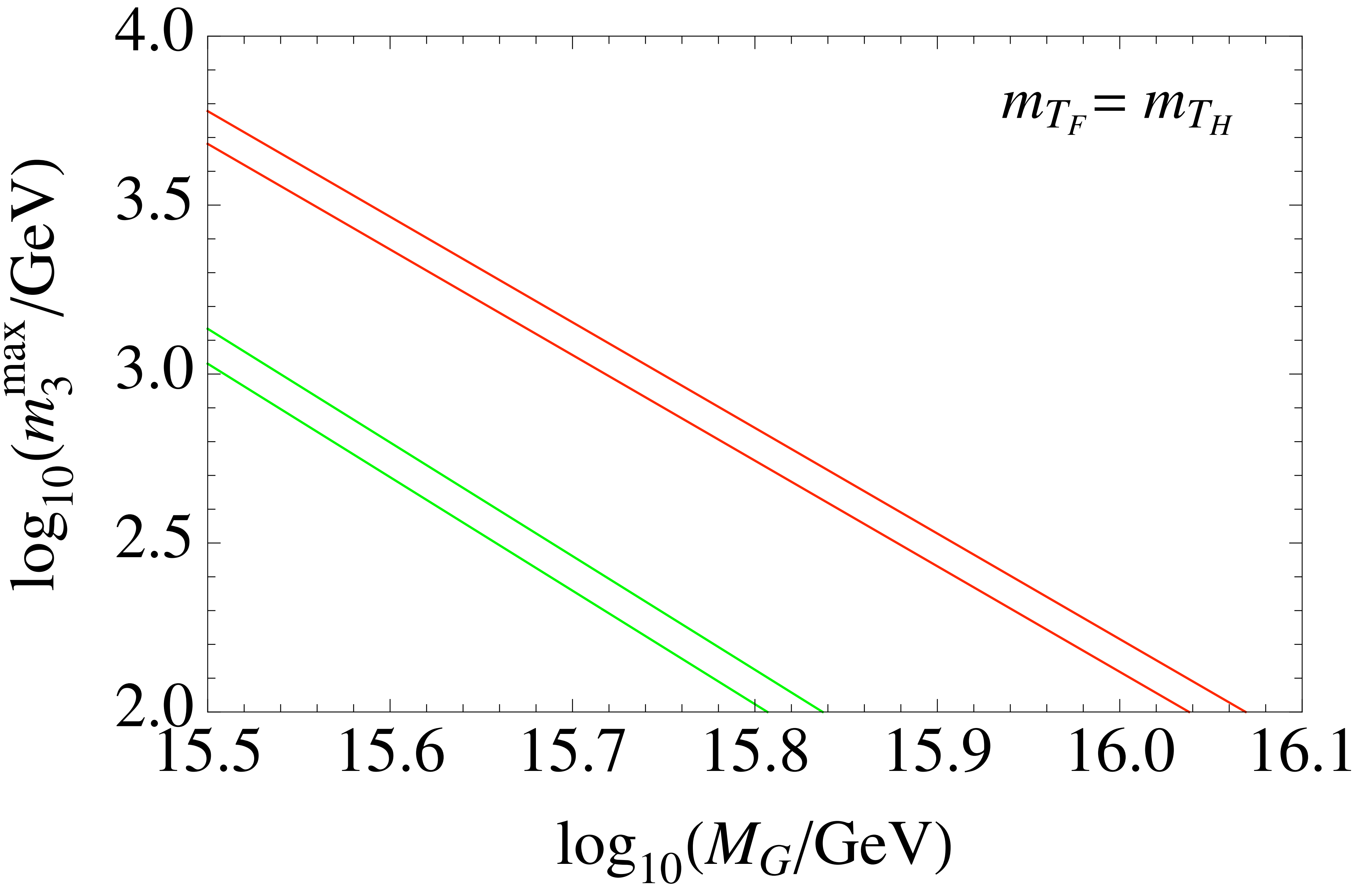
Sample Unification



$$\left(\frac{\Delta\alpha_1}{\alpha_1}\right)^{2\to 3-\mathrm{loop}} = 0.015\% \qquad \left(\frac{\Delta\alpha_2}{\alpha_2}\right)^{2\to 3-\mathrm{loop}} = 0.061\% \qquad \left(\frac{\Delta\alpha_3}{\alpha_3}\right)^{2\to 3-\mathrm{loop}} = 0.08\%$$

$$\left(\frac{\Delta\alpha_1}{\alpha_1}\right)^{\mathrm{exp}} = 0.023\% \qquad \left(\frac{\Delta\alpha_2}{\alpha_2}\right)^{\mathrm{exp}} = 0.059\% \qquad \left(\frac{\Delta\alpha_3}{\alpha_3}\right)^{\mathrm{exp}} = 0.59\%$$

m_3^{max} - M_G correlation @ 3-loops



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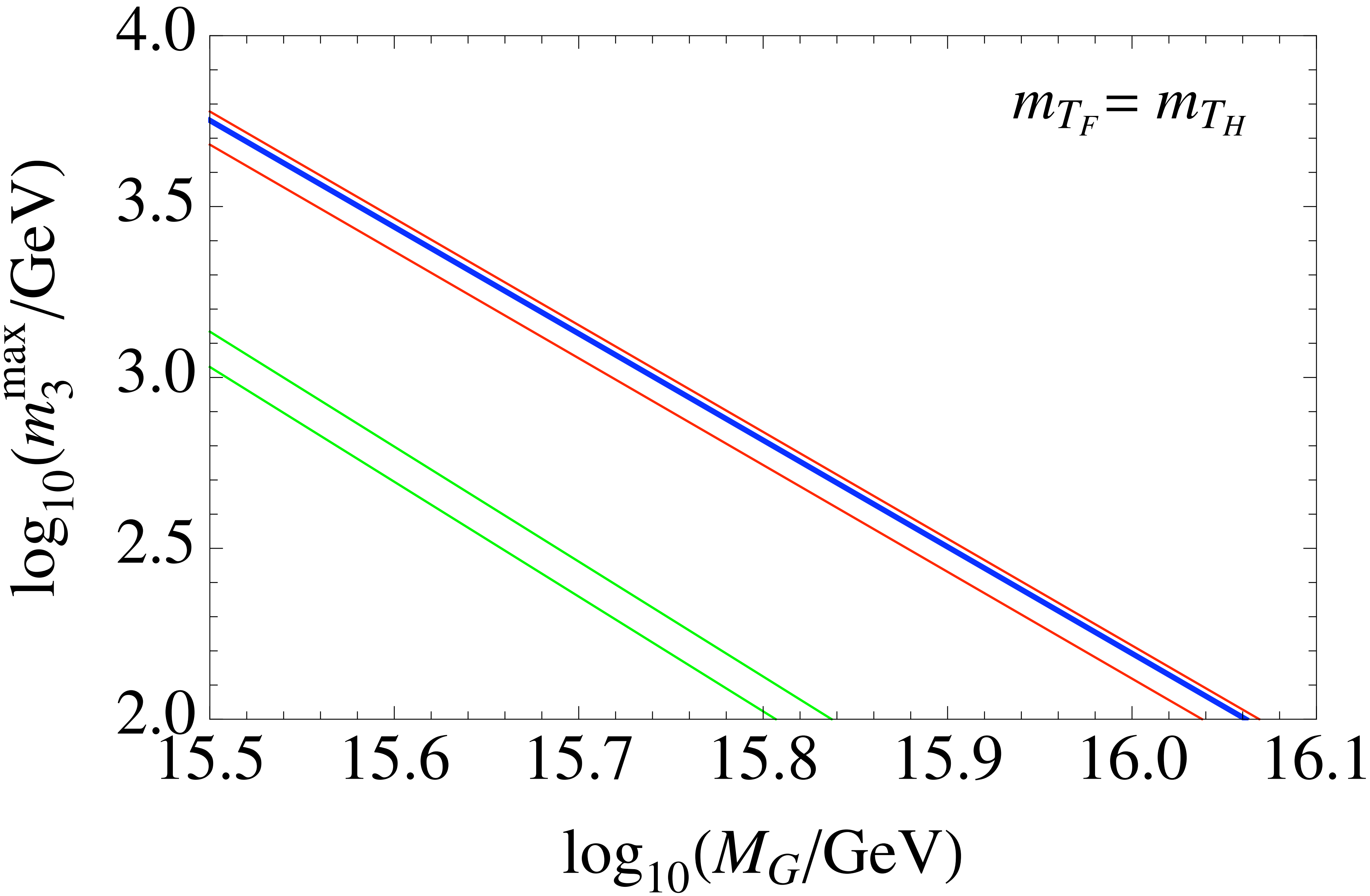


Conclusions

- Minimal extension of GG SU(5) surprisingly predictive (falsifiable)

  - light O(TeV) electroweak triplets

  - unification scale < 10^{16} GeV

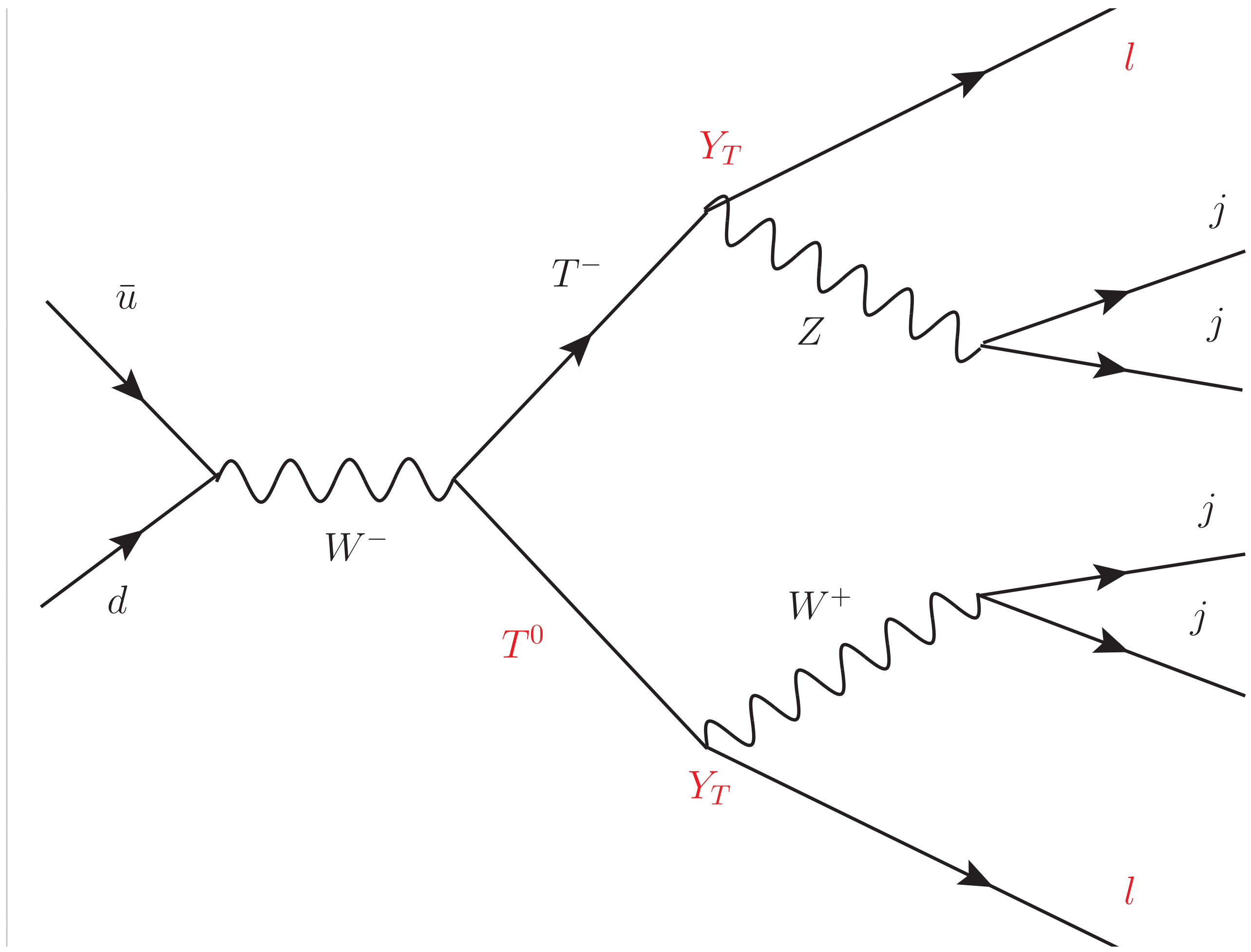
- Joint effort btw experiments (LHC, HK, ...) and theory

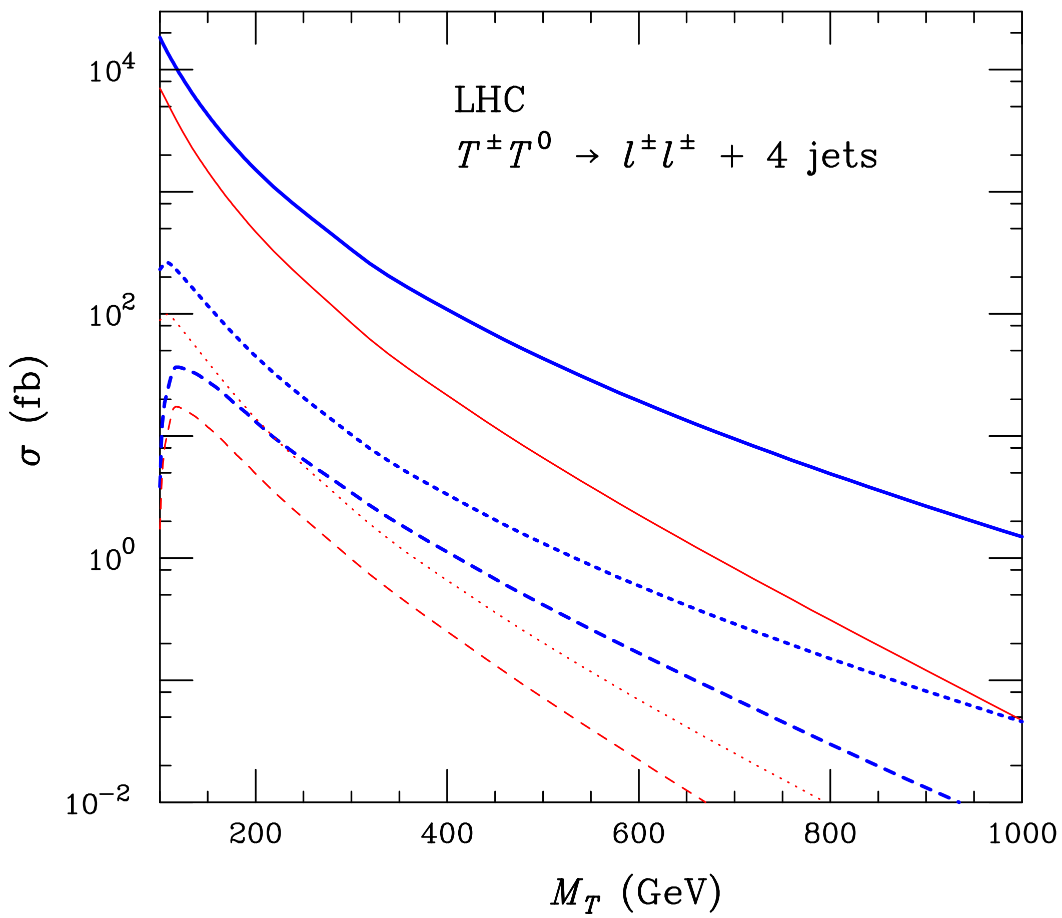
- On the theory side 3-loops needed to match exp precision

Backup slides

Triplet decay

$$pp \longrightarrow T^{\pm}T^{0} \longrightarrow l^{\pm}l^{\pm} + 4 \text{ jets}$$





[Arhrib, Bajc, Ghosh, Han, Huang, Puljak, Senjanovic (2010)]

- $\Delta L = 2$ process (SM background free)

- (fermionic) triplet mass can be probed up to 700 GeV for 14 TeV and 100 fb^{-1}

F / H ratio dependence