Multi-Jet cross section at NLO accuracy in QCD with NJet

Benedikt Biedermann
Humboldt-Universität zu Berlin

In collaboration with
Simon Badger, Peter Uwer, Valery Yundin

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Multi-Jet Production

Signal Process:

- Test of QCD
- Constrain coupling α_s and PDFs

Background Process:

- crucial for physics search beyond the standard model

Multi-jet production at leading order (LO):

- well automated tools like Alpgen, Madgraph, Sherpa+Comix
- multiplicities up to 12 jets possible
- rough estimate, though residual dependence on renormalisation scale

For precision jet physcis, next-to-leading order (NLO) accuracy required

Multi-Jet Production @ NLO

2-jet production:	[Ellis, Kunszt, Soper 1992] [Giele, Glover, Kosower 1993]	
3-jet production:	[Nagy 2002, 2003] [Trocsanyi 1996] [Kilgore, Giele 1997]	(all channels) (gluon channel) (gluon channel)
4-jet production:	[Bern et al. 2012] [Badger, BB, Uwer, Yundin 2013]	(7 TeV) (8 TeV)

arXiv:1209.0098, accepted for Publication PLB

In this talk:

- 3-jet and 4-jet production at the LHC for 8 TeV at NLO
- massless QCD (massless b-quark in the initial state, top quark integrated out)

pp $\longrightarrow$ n jets @ NLO

$$d\sigma_n = \underbrace{d\sigma_n^{\mathrm{LO}}}_{\sim\,\alpha_s^n} + \underbrace{\delta d\sigma_n^{\mathrm{NLO}}}_{\sim\,\alpha_s^{n+1}} + O(\alpha_s^{n+2})$$

NLO contribution with dipole subtraction: [Catani, Seymour 1996]

$$\delta\sigma^{\mathrm{NLO}} = \int_n \left(d\sigma_n^{\mathrm{V}} + \int_1 d\sigma_{n+1}^{\mathrm{S}}\right) + \int_n d\sigma_n^{\mathrm{Fac.}} + \int_{n+1} \left(d\sigma_{n+1}^{\mathrm{R}} - d\sigma_{n+1}^{\mathrm{S}}\right)$$

$$\delta\sigma^{\mathrm{NLO}} = \int_n \left(d\sigma_n^{\mathrm{V}} + \int_1 d\sigma_{n+1}^{\mathrm{S}} \right) + \int_n d\sigma_n^{\mathrm{Fac.}}$$

$$+ \int_{n+1} \left(d\sigma_{n+1}^{\mathrm{R}} - d\sigma_{n+1}^{\mathrm{S}} \right)$$

$$\delta d\sigma_n^{\mathrm{NLO}} = d\bar{\sigma}_n^{\mathrm{V}} + d\bar{\sigma}_n^{\mathrm{I}} + d\sigma_{n+1}^{\mathrm{RS}}$$

Finite part of the virtual corrections	Finite part of the integrated subtraction terms + factorisation	Real corrections with subtraction terms
NJet	**Sherpa with Amegic++**	

Virtual Corrections

Rapid grow in complexity with increasing number of external legs

e.g. 6-gluon one-loop amplitude around 15000 diagrams

⟶ use method of generalised unitarity

Ansätze for automation of virtual corrections:

NJet [Badger, BB, Uwer, Yundin 2012]	***public code***
GoSam, Golem95, Samurai, FeynArts,	*public codes*
Helac-1loop, Cuttools	*public codes*
Blackhat, MadLoop, Rocket, OpenLoops	*private codes*
“numerical loop integration” [Becker et al.]	*private codes*

NJet – what you get

provides full colour summed 1-loop amplitudes for all channels of 2-jet, 3-jet, 4-jet and 5-jet production in massless QCD

Based on NGluon [Badger, BB, Uwer 2011]
generalised Unitarity with tree-level amplitudes as input
to compute ordered 1-loop amplitudes with arbitrarily many legs

Equipped with Binoth Les Houche accord interface [Binoth et al. 2009]
to be linked trivially with standard Monte Carlo Programs

Download at: www.bitbucket.org/njet/njet/downloads
www.physik.hu-berlin.de/pep/tools

arXiv:1209.0100

- detailed analytical checks (IR-, UV-poles, [Nagy,Bern, Dixon, Kosower,Forde…]
- checks against individual phase space points from existing codes (Helac, Gosam, BlackHat)

Colour, loop, primitives & co

Colour summed squared matrix element

$$\sim 2\mathrm{Re}\left[\mathcal{M}_{1-\mathrm{loop}} \cdot (\mathcal{M}_{\mathrm{Born}})^*\right]$$

$$\sim \vec{\mathcal{A}}^{\dagger}_{\mathrm{prim}} \hat{\mathcal{C}}\, \vec{\mathcal{A}}_{\mathrm{tree}}$$

Full-colour amplitude

$$\sim \mathcal{M}(\{p_i, a_i, h_i\}) \quad i \in \{1, ..., n\}$$

Partial amplitudes

$$\sim \mathrm{Tr}[T_{a_1} \cdots T_{a_n}]\, \mathcal{A}_n(\{p_1, h_1\}, \ldots, \{p_n, h_n\})$$

Primitives

$$\mathcal{A}_n(\{p_1, h_1\}, \ldots, \{p_n, h_n\}) = \sum_{P(\sigma)} \mathcal{A}_{\mathrm{prim}}(\sigma(1, ..., n))$$

[Bern, Dixon, Dunbar, Kosower 1994]
[Bern, Dixon, Kosower 1995]

NJet – how it works

- Primitive amplitudes computed with generalised unitarity / integrand reduction techniques [Ossola,Papadopoulos,Pittau] [Ellis, Giele, Kunszt] [Britto, Cachazo, Feng, Mastrolia]
- Berends-Giele recursion for the tree amplitudes [Berends, Giele 1986]
- Construction of the amplitudes in terms of primitives along the lines of [Ellis, Giele, Kunszt, Melnikov, Zanderighi]
- Sophisticated cache system to reuse tree-amplitudes when doing the permutation and helicity sums
- Dynamical switch on the fly between double and quadruple precision [Hida,Li,Bailey]
- QCDLoop and FF for scalar integrals [van Oldenborgh 1990; Ellis, Zanderighi 2008]
- Written in C++ and in Python

Timings for the Virtuals

2-jet	T_{sd}[s]	3-jet	T_{sd}[s]	4-jet	T_{sd}[s]	5-jet	T_{sd}[s]
4g	0.030	5g	0.22	6g	6.19	7g	171.3
2u2g	0.032	2u3g	0.34	2u4g	7.19	2u5g	195.1
2u2d	0.011	2u2d1g	0.11	2u2d2g	2.05	2u2d3g	45.7
4u	0.022	2u2d1g	0.11	4u2g	4.08	4u3g	92.5
				2u2d2s	0.38	2u2d2s1g	7.9
				2u4d	0.74	2u4d1g	15.8
				6u	2.16	6u1g	47.1

everything in full colour + full helicity + scaling test

	$gg \to 2g$	$gg \to 3g$	$gg \to 4g$	$gg \to 5g$
standard sum	0.03	0.22	6.19	171.31
de-symmetrized	0.03	0.07	0.57	3.07

Basic channels for 4-jet production:

$$0 \to gggggg \qquad 0 \to q\bar{q}q'\bar{q}'gg$$

$$0 \to q\bar{q}gggg \qquad 0 \to q\bar{q}q'\bar{q}'q''\bar{q}''$$

Numerical set up

- MSTW2008 PDF set

- anti-kt jet algorithm as implemented in FastJet with jet radius R=0.4
[Cacciari, Salam, Soyez 2012]

- Massless QCD, 5-flavour scheme

- set $\mu_f = \mu_r \equiv \mu$ and use dynamical scale base on sum of the transverse momentum of the final state partons

$$\hat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}} \qquad \mu = \hat{H}_T/2$$

- Scale variation: $\hat{H}_T/4 \leq \mu \leq \hat{H}_T$

- Kinematical cuts: Transverse momentum of the first jet pt > 80 GeV, subsequent jets at least pt > 60 GeV, Rapidity: eta < 2.8
[ATLAS 2011, BlackHat 2011]

Cross Sections

$$\sigma_n^{\mathrm{NLO}} = \sigma_n^{\mathrm{LO}} + \delta\sigma_n^{\mathrm{NLO}}$$

Confirmed results at 7 TeV:

$$\sigma_3^{\mathrm{7TeV\text{-}LO}} = 93.40(0.03)^{+50.37}_{-30.34}\,\mathrm{nb} \qquad \sigma_4^{\mathrm{7TeV\text{-}LO}} = 9.97(0.02)\,\mathrm{nb}$$

$$\sigma_3^{\mathrm{7TeV\text{-}NLO}} = 53.74(0.16)^{+2.06}_{-20.72}\,\mathrm{nb} \qquad \sigma_4^{\mathrm{7TeV\text{-}NLO}} = 5.56(0.17)\,\mathrm{nb}$$

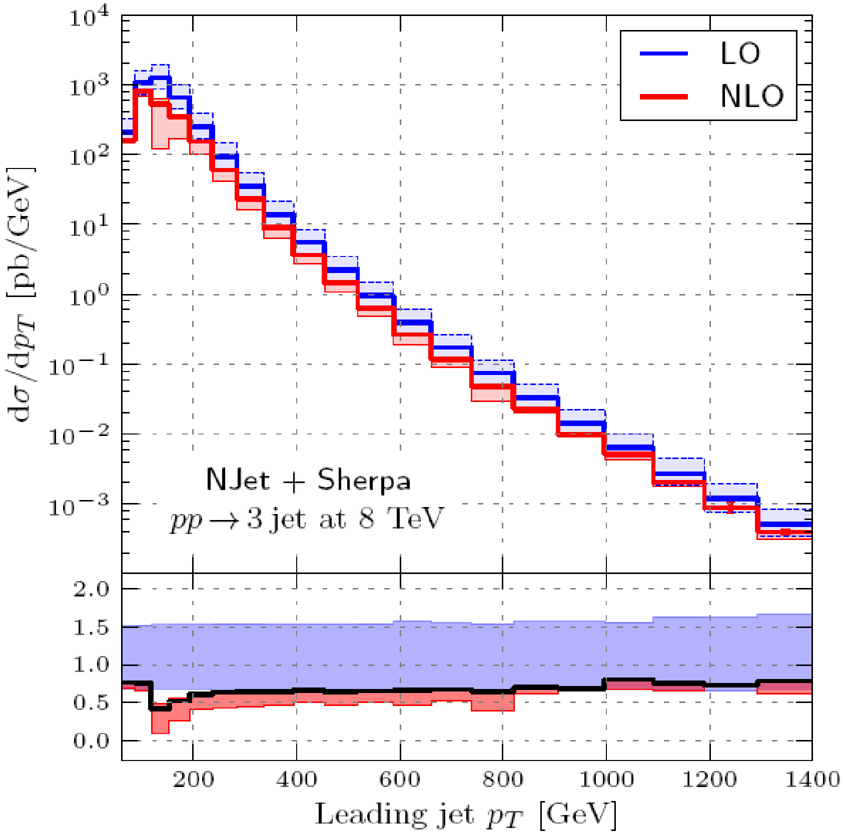
$\longrightarrow$ Excellent agreement with [Bern et. al. 2011]

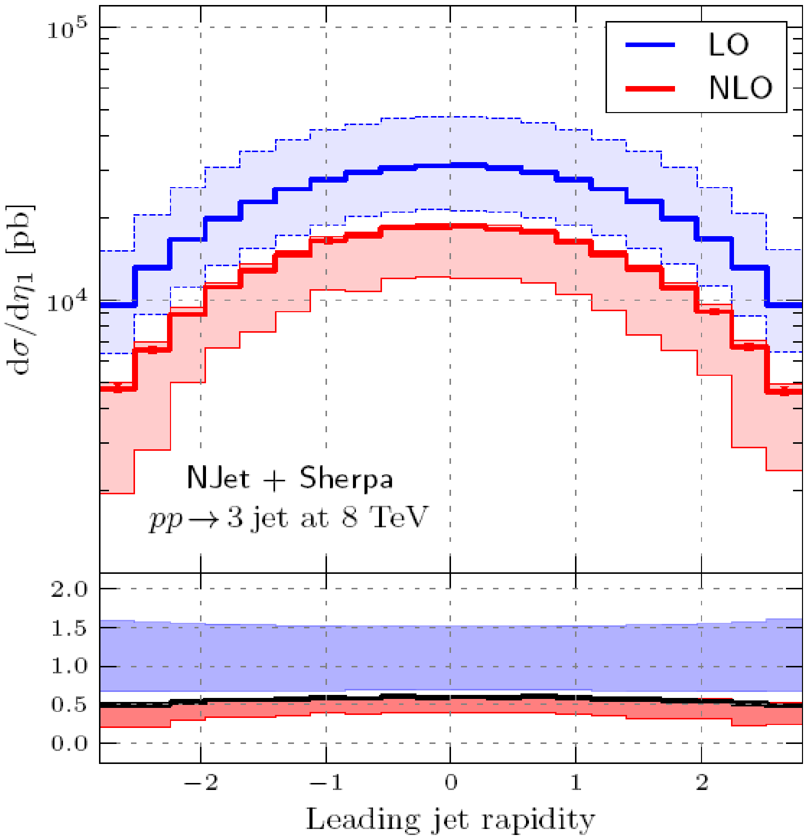
New results at 8 TeV:

$$\sigma_3^{\mathrm{8TeV\text{-}LO}} = 126.65(0.05)^{+66.56}_{-40.40}\,\mathrm{nb}, \quad \sigma_4^{\mathrm{8TeV\text{-}LO}} = 14.36(0.01)^{+10.38}_{-5.6}\,\mathrm{nb},$$

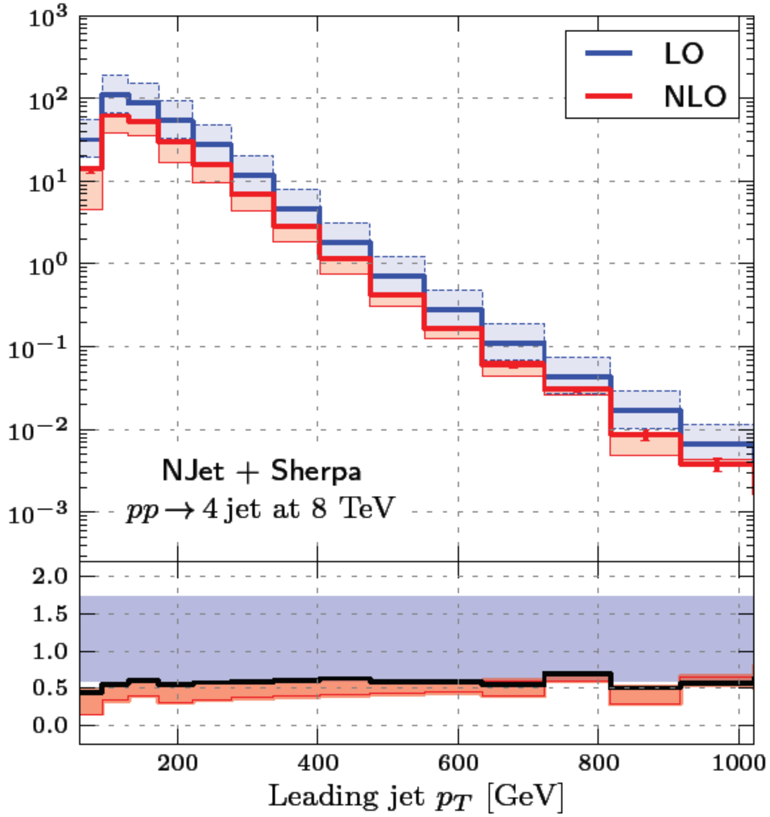
$$\sigma_3^{\mathrm{8TeV\text{-}NLO}} = 72.57(0.16)^{+2.71}_{-28.08}\,\mathrm{nb}, \quad \sigma_4^{\mathrm{8TeV\text{-}NLO}} = 8.15(0.09)^{+0.0}_{-3.24}\,\mathrm{nb}.$$

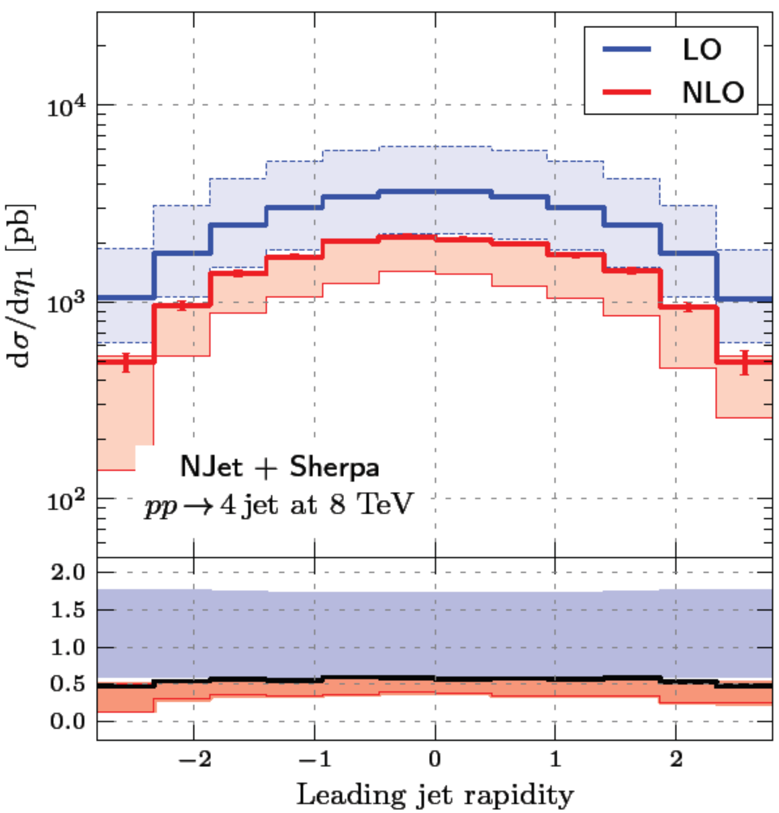
Distributions for 3-jets





Distributions for 4-jets





The total jet cross section

In addition to the 3-jet and 4-jet cross section, we evaluated also the 2-jet cross section:

$$\sigma_2^{8\mathrm{TeV\text{-}LO}} = 1234.9(1.2)\,\mathrm{nb}$$
$$\sigma_2^{8\mathrm{TeV\text{-}NLO}} = 1524.9(2.8)\,\mathrm{nb}$$

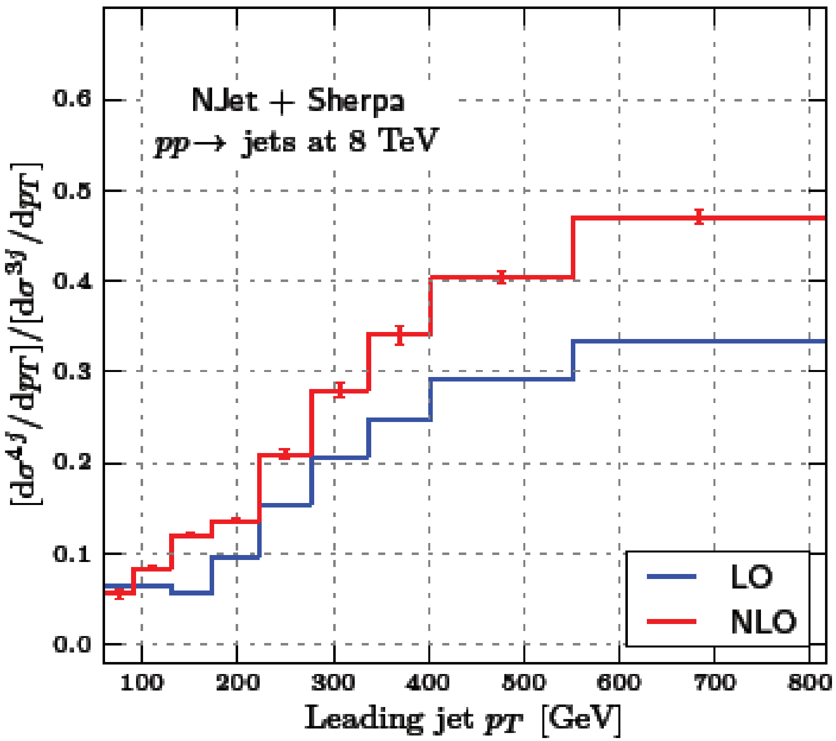
Estimate of the total jet cross section: appr. **1600 nb**

Ratio between 2-jet, 3-jet and 4-jet cross section:

$$1 \;:\; 0.05 \;:\; 0.005$$

Is the four jet rate relevant?
Answer: YES!!! At the level of differential distributions the ratios become less extreme.

Ratio between 3 and 4 jets



Small ratio of total four jet and three jet cross section mainly due to low pt region.
At high pt the ratio increases up to 0.5 (at NLO)

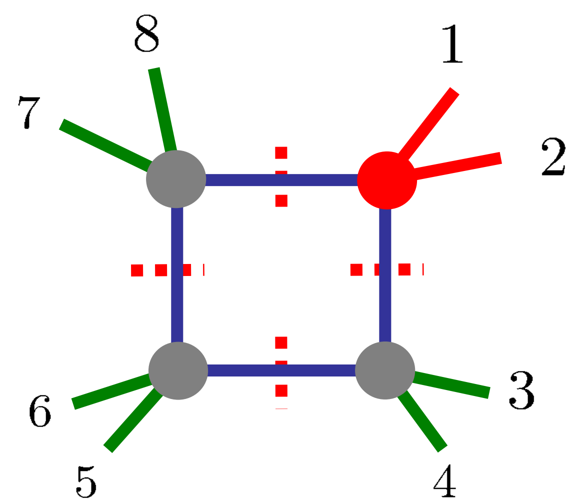
Conclusion

- 3-jet and 4-jet production at 8 TeV for the LHC with NJet and Sherpa have been presented
- NLO cross sections reduce LO result by around 45%
- Results at 8 TeV increased around 35 % - 50 % with respect to 7 TeV due to larger parton flux at higher energies
- Dynamical scale setting gives almost constant K-factor for differential distributions
- Large corrections at low pt may require beyond fixed order calculation All ingredients for matching NLO to a parton shower are publicly available
- Agreement of individual phase space points of NJet with other existing codes and perfect agreement of full cross sections with results from BlackHat
- NJet is publicly available, includes 2-jet, 3-jet, 4-jet and 5-jet production

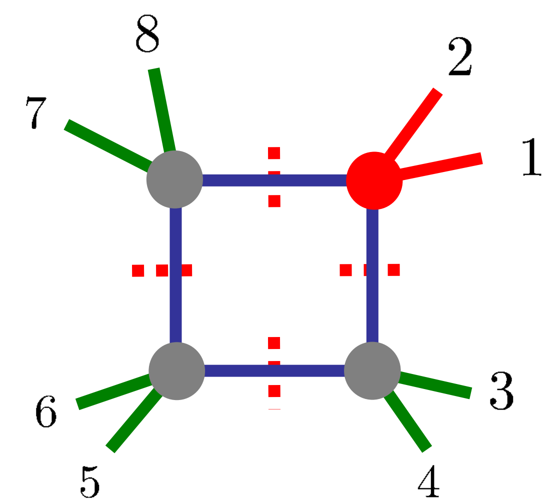
Extra slides

Caching full tree amplitudes

$\mathcal{A}(1,2,3,\ldots,8):$



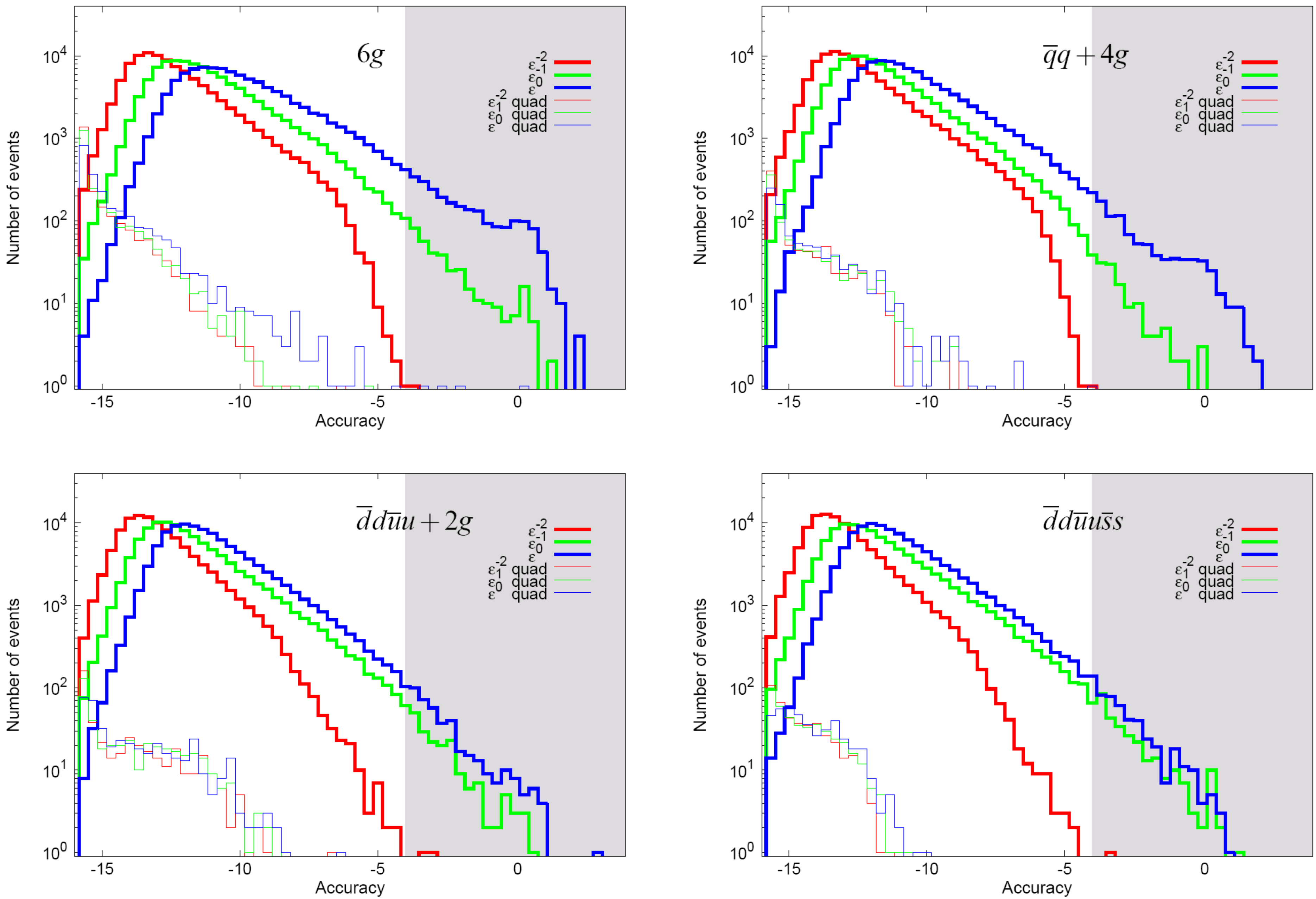
$\mathcal{A}(2,1,3,\ldots,8):$



Only the red partial tree amplitudes are different!

Necessary conditions:

1. Loop Momentum must agree
2. External and loop flavours must agree
3. External helicities must agree



$6g$
$\bar{q}q + 4g$
$\bar{d}d\bar{u}u + 2g$
$\bar{d}d\bar{u}u\bar{s}s$
ε^{-2}
ε^{-1}
ε^{0}
ε^{-2} quad
ε^{-1} quad
ε^{0} quad
Number of events
Accuracy

Scalar Integral Basis

Decomposition of an arbitrary one-loop amplitude: [Passarino,Veltman1979]

$$\mathcal{A}_n^{\text{loop}} = \sum_{\{ijkl\}} d_{ijkl}^{[\mathrm{d}]}\,\mathcal{I}_{ijkl}^{(4)} + \sum_{\{ijk\}} c_{ijk}^{[\mathrm{d}]}\,\mathcal{I}_{ijk}^{(3)} + \sum_{\{ij\}} b_{ij}^{[\mathrm{d}]}\,\mathcal{I}_{ij}^{(2)} + \cancel{\sum_{\{i\}} a_i^{[\mathrm{d}]}\,\mathcal{I}_i^{(1)}}$$

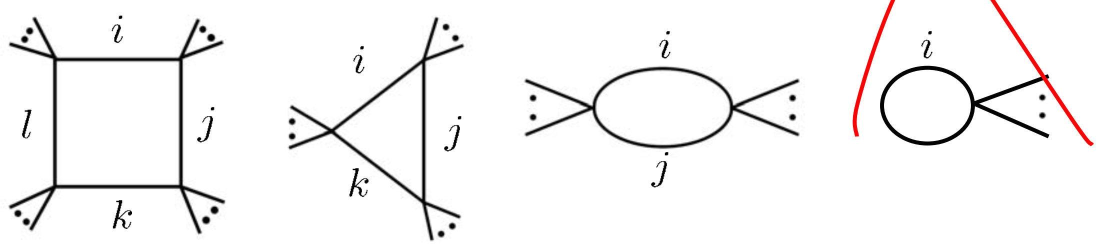
Dimension:

$$[\mathrm{d}] = 4 - 2\varepsilon$$

Topology:

$$\{ijkl\}$$

boxes triangles bubbles ~~tadpoles~~



Scalar integrals:

$$\mathcal{I}_{ijkl}^{(4)} = \int \mathrm{d}^{[\mathrm{d}]} l \frac{1}{D_i D_j D_k D_l}$$

$$D_i = (p_i + l)^2 - m_i^2$$

computation of one-loop amplitudes = determination of integral coefficients

No tadpoles in massless theories

Integrand Properties

Focus on the **integrand** $\mathcal{F}_n(l)$ of the amplitudes

$$\mathcal{A}_n^{\text{loop}} = \int \mathrm{d}^4 l \, \mathcal{F}_n(l) + \mathcal{A}_n^{\text{rat}}$$

[Ossola,Papadopoulos,Pittau]
[Ellis, Giele, Kunszt]
[Britto, Cachazo, Feng, Mastrolia]

$$\mathcal{F}_n(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

Numerators: loop-momentum independent part **+** spurious terms

Spurious terms: loop-momentum tensors which vanish after integration

$\longrightarrow$ Loop-momentum independent part is the desired integral coefficient

Box Example

Integrand $\mathcal{F}_n(l)$ **:**

$$\mathcal{F}_n(l) = \sum_{\{ijkl\}} \frac{\overline{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\overline{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\overline{b}_{ij}(l)}{D_i D_j}$$

Tensor structure of a general box part $\overline{d}(l)$ well known:

$$\begin{aligned} \overline{d}(l) &= d_0 + \tilde{d}(l) \\ &= d_0 + d_1 \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho \, l^\sigma \end{aligned}$$

$$\int \mathrm{d}^4 l \frac{d_0 + \tilde{d}(l)}{D_i D_j D_k D_l} = d_0 \int \mathrm{d}^4 l \frac{1}{D_i D_j D_k D_l} = d_0 \, \mathcal{I}^{(4)}$$

Compute $\overline{d}(l)$ for two different l

$\hookrightarrow$ system of equations $\longrightarrow$ determine d_0

How do we get $\overline{d}(l)$?

Partial fractioning the integrand

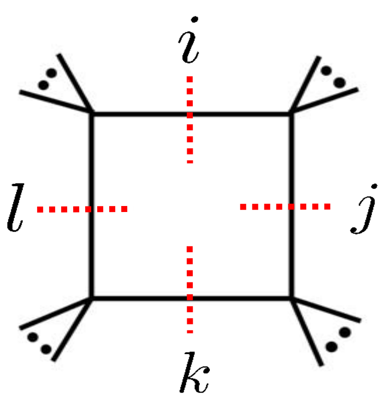
$$\mathcal{F}_n(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

Known!

- multiply with $D_i D_j D_k D_l$
- set loop momentum **on-shell**:

$$l = l_c^{\pm} \longrightarrow D_i = D_j = D_k = D_l = 0$$

$$\begin{aligned} \bar{d}_{ijkl}(l_c^{\pm}) &= \mathcal{F}_n(l_c^{\pm})\, D_i D_j D_k D_l \\ &\equiv \mathcal{A}^{\text{tree}}_{i,j-1}(l_c^{\pm}) \mathcal{A}^{\text{tree}}_{j,k-1}(l_c^{\pm}) \mathcal{A}^{\text{tree}}_{k,l-1}(l_c^{\pm}) \mathcal{A}^{\text{tree}}_{l,i-1}(l_c^{\pm}) \\ &\equiv \text{ Product of four } \textbf{tree amplitudes} \end{aligned}$$



Integral coefficient:

$$\longrightarrow \quad d_0 = \frac{1}{2}\left(\bar{d}_{ijkl}(l_c^{+}) + \bar{d}_{ijkl}(l_c^{-})\right)$$

Rational Part

1. Absorb epsilon dependence in effective (complex) mass:

[Bern, Dixon, Dunbar, Kosower 1997]
[Bern, Morgan 1996]

$$l_{[4-2\varepsilon]} = l_{[4]} + l_{[-2\varepsilon]}$$

$$l^2_{[4-2\varepsilon]} = l^2_{[4]} - l^2_{[-2\varepsilon]} \stackrel{!}{=} 0 \longrightarrow l^2_{[-2\varepsilon]} = -\mu^2$$

Integrand for the rational part is a polynomial in μ^2.

2. Expand integral basis in higher integer dimension and take ε → 0 limit

$$\mathcal{A}_n^{rat} = -\frac{1}{6}\sum_{i,j,k,l} C^{[4]}_{4;i|j|k|l} - \frac{1}{2}\sum_{i,j,k} C^{[2]}_{3;i|j|k} - \sum_{i,j} \frac{s_{i,j-1}}{6} C^{[2]}_{2;i|j}$$

"constant" integral

Integral coefficient

[Giele, Kunszt, Melnikov 2008]
[Badger 2009]

- Additional hidden pentagon contributions
- Use SUSY relations to interpret "rational gluons" as scalar contributions
- Use the same four dimensional techniques