SIMPLICITY IN MULTI-JETS
(JET SCALING)

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OUTLINE

- Motivation for exclusive jet rates
- The concept of scaling
- Experimental applications/speculation
- Conclusions

Work in collaboration with:
Christoph Englert, Ben Gripaios, Tilman Plehn, Peter Schichtel, Steffen Schumann, Bryan Webber

Gerwick, Schumann, Gripaios, Webber; to appear.

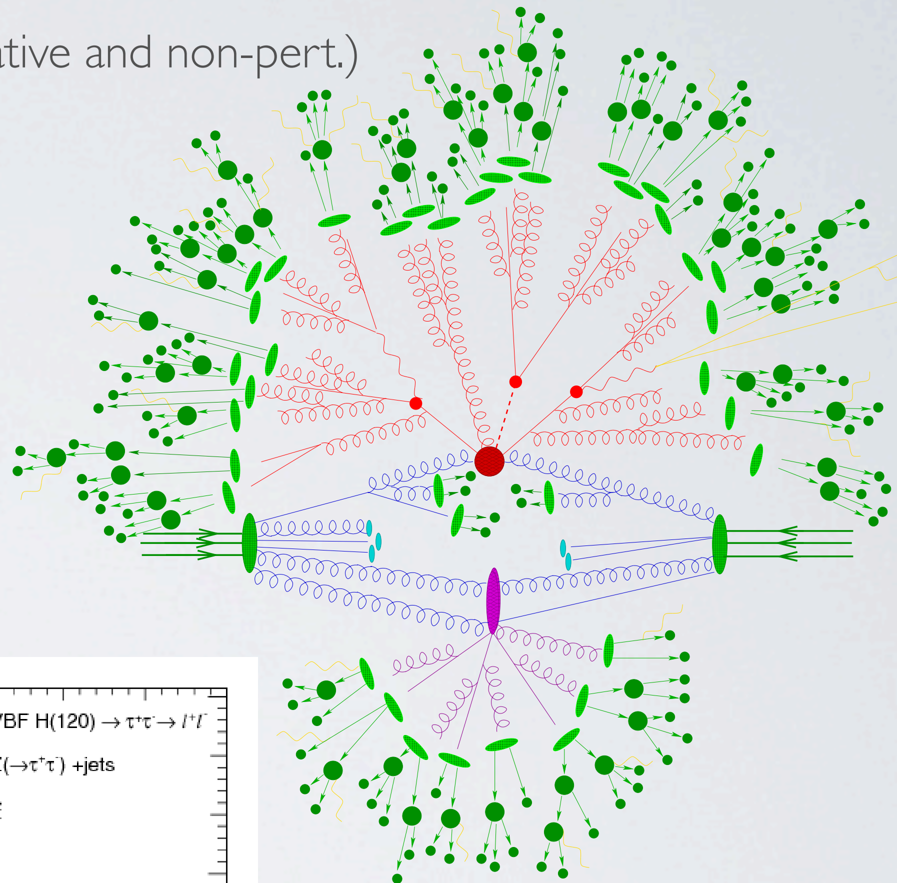
Gerwick, Plehn, Schumann and Schichtel, "Scaling patterns for QCD jets", JHEP, **arXiv:1208.3676**

Gerwick, Plehn and Schumann, Phys. Rev. Lett. 108, 032003 (2012) **arXiv:1108.3335**

QCD @ LHC: DIFFICULTY OF NEW PHYSICS

LHC as a QCD machine

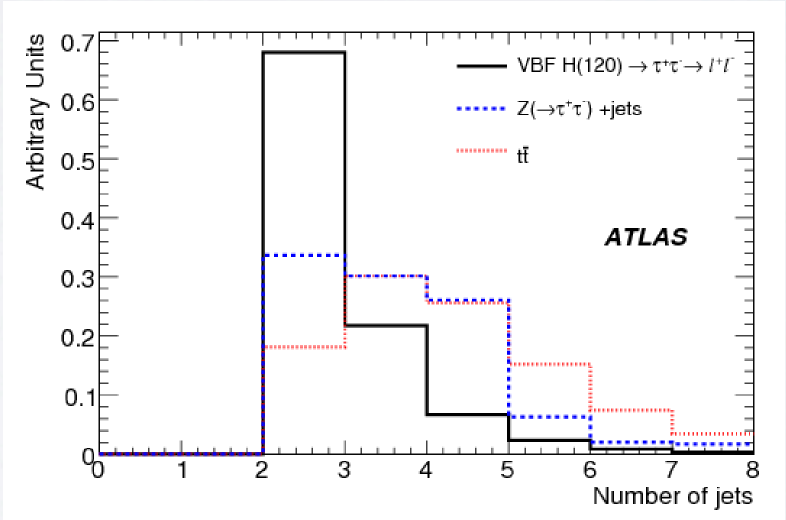
- The goal for the LHC is to discover new physics beyond the SM (also via precision tests of the SM)
- Every analysis depends on understanding jets (both perturbative and non-pert.)



Exclusive n-jet final states

- Many analyses rely on dividing the event sample into jet multiplicity bins and perform (or optimize) analysis bin by bin.

Analysis type	Excl. jet bin
Higgs WW*	0,1 jet
Higgs WBF	2 jet
Di-boson	0,1 jet
Top mass	4 jet
New physics	4,8,12? jet



Theoretical challenge

- There may be many other uses for dividing analyses according to jet bins, but predictions of exclusive jet rates for both signal and background (usually harder) are in some cases already the dominant uncertainty.

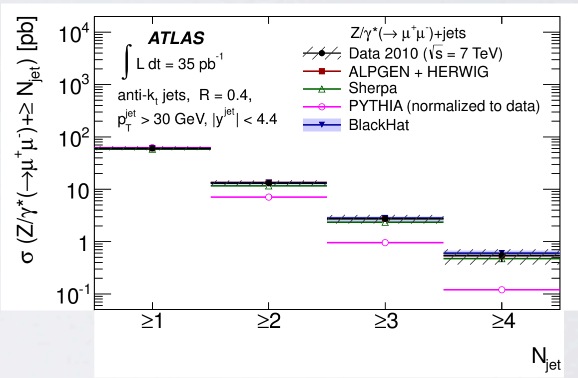
RATIOS OF EXCLUSIVE JET RATES

Exclusive jet rates via jet scaling

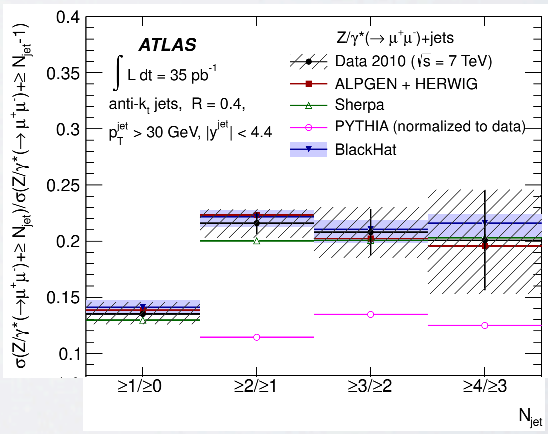
- Our idea: use knowledge of lower multiplicity jet rates (as control region).
- But, we need a well defined prescription for extrapolation.

Reasons for studying ratio instead of rate

$$R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n}$$



- Experimentally and theoretically systematic tend to cancel.
- Visually easier to identify physics



key point: the counting of jets σ_n is throughout this talk the number of jet in addition to the core-process (radiated jets)

OBSERVED SCALING PATTERNS

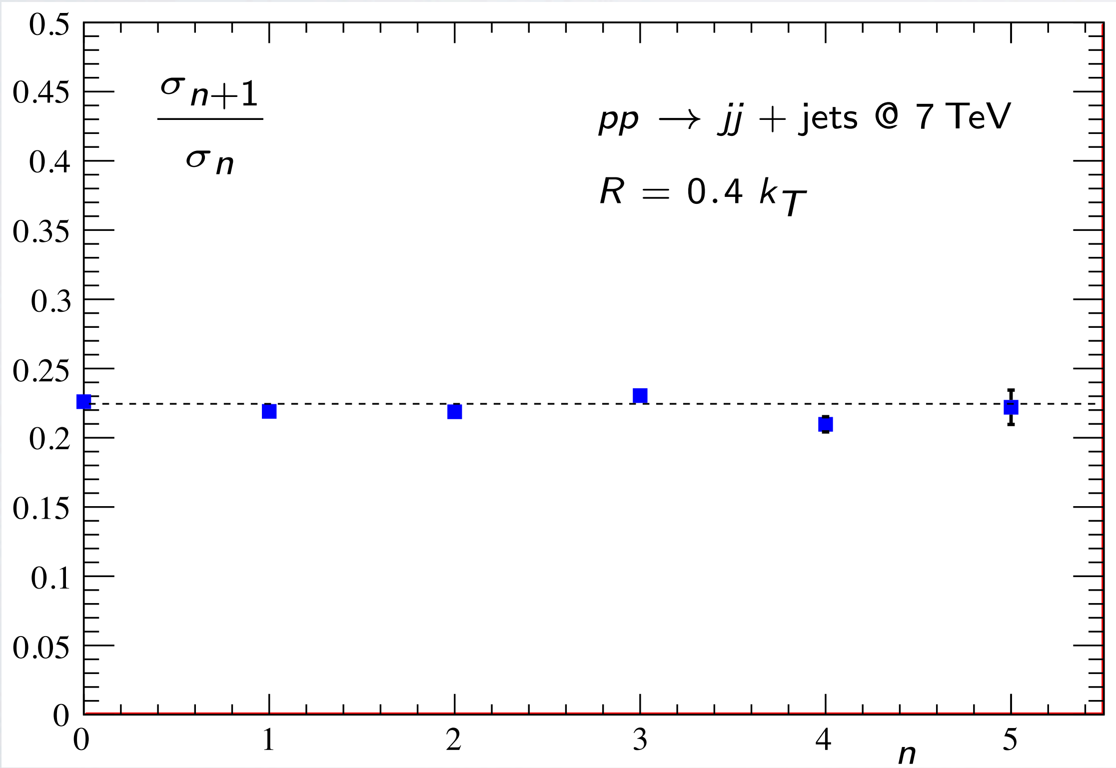
Staircase [Steve Ellis, Kleiss, Stirling (1985), Berends (1989)]

$$\sigma_n^{\rm excl} = R_{1/0}^n \equiv e^{-bn}$$

- Ratios are constant (geometric)

$$\frac{\sigma_{n+1}}{\sigma_n} = e^{-b}$$

- Observed: UA1, Tevatron, LHC



Poisson [Peskin & Schroder; Rainwater, Zeppenfeld (1997)]

$$\sigma_n^{\rm excl} = \frac{e^{-\bar{n}}\bar{n}^n}{n!}$$

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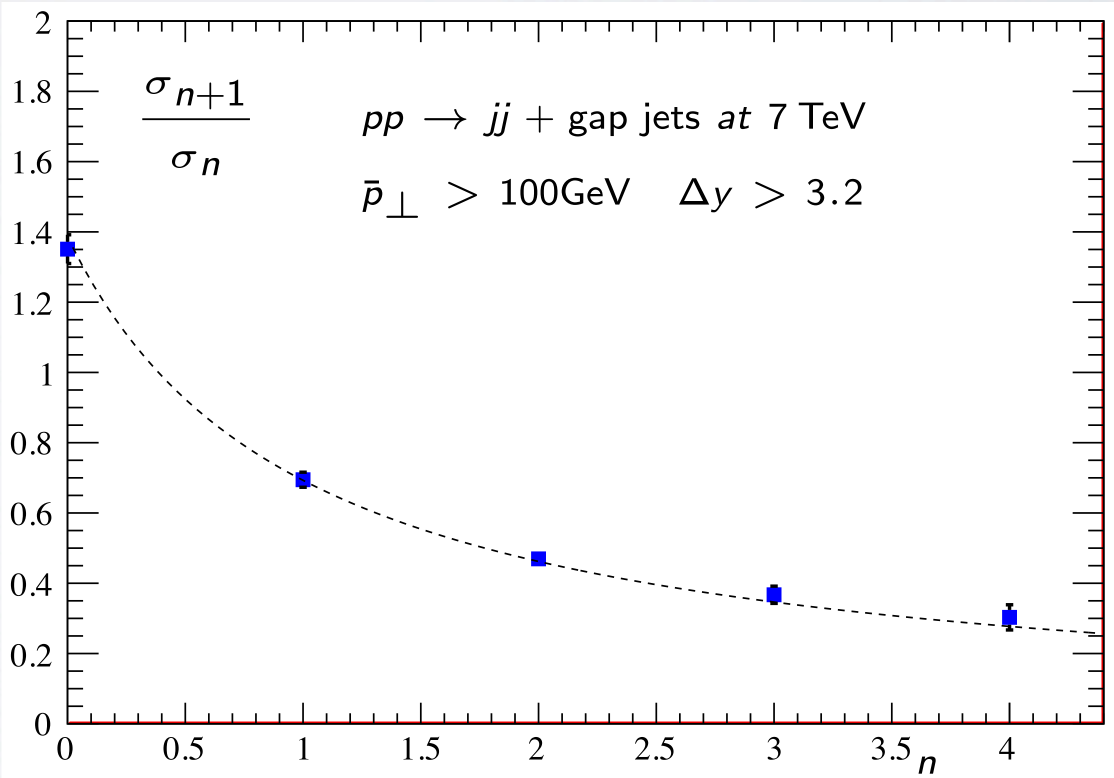

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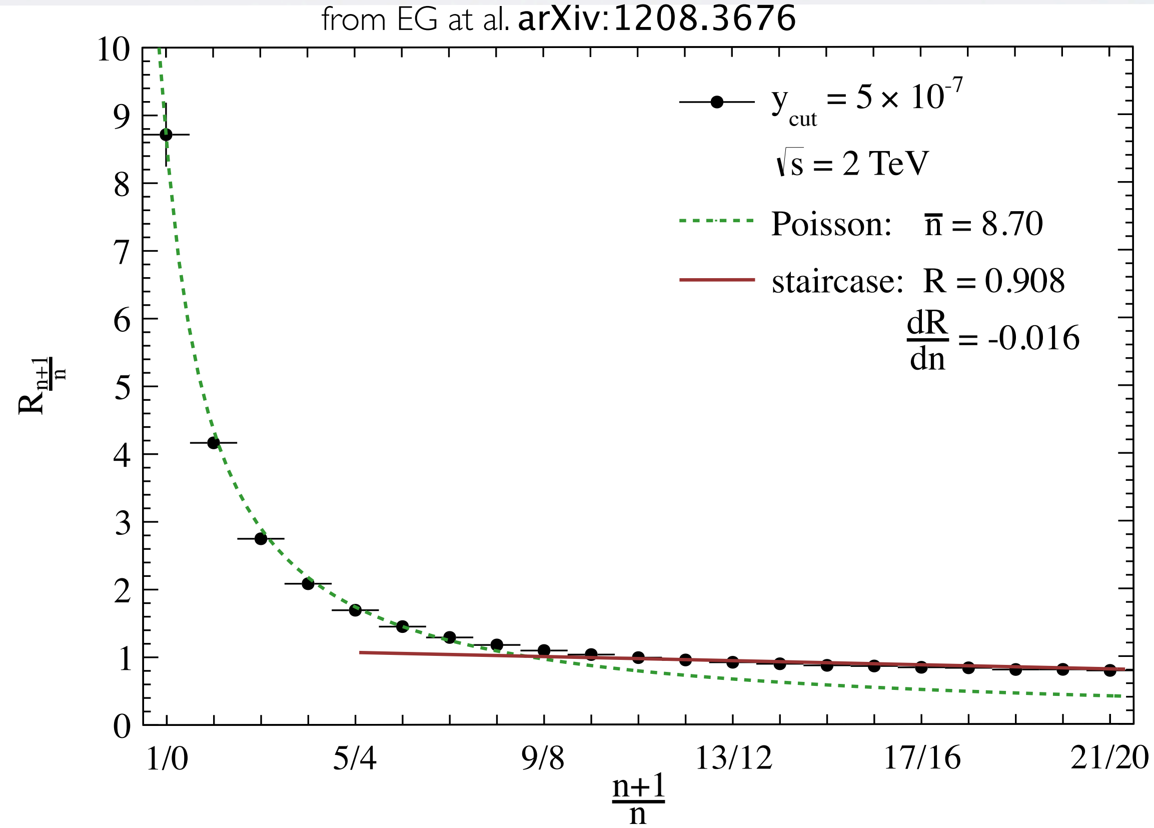
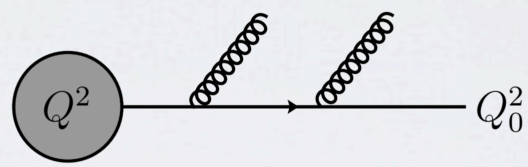
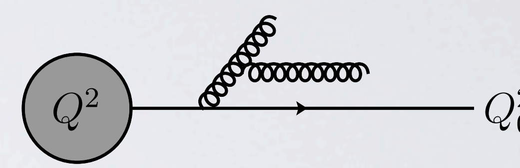
from EG at al. arXiv:1208.3676



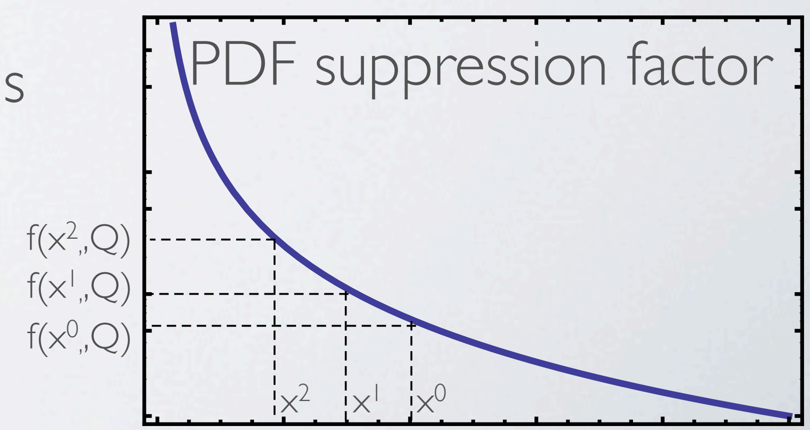
ORIGIN OF SCALING PATTERNS

1. The ratio of the size of the primary emission to subsequent emission amplitude.

	Poisson	Staircase
Diagrammatic representation		
Durham Algorithm [Catani, Dokshitzer, Olsson, Turnock, Webber]	$\sigma^{\text{primary}}(Q^2, Q_0^2) = c^{\text{primary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2, t)\Delta_g(t) \int_{Q_0^2}^{Q^2} dt'\, \Gamma(Q^2, t')\Delta_g(t')$	$\sigma^{\text{secondary}}(Q^2, Q_0^2) = c^{\text{secondary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2, t)\Delta_g(t) \int_{Q_0^2}^{t} dt'\, \Gamma(t, t')\Delta_g(t')$
Generalized k_T Algorithm [EG, Gripaios, Schumann, Webber]	$\sigma_2(\kappa, \lambda) = \Delta_q^2 \left(\int_0^{\kappa} d\bar{\lambda} \int_0^{\lambda} d\bar{\kappa}\, \Gamma_g(\bar{\kappa}, \bar{\lambda})\, \Delta_g(\bar{\kappa}, \bar{\lambda}) \right)^2$	$\sigma_2(\kappa, \lambda) = \Delta_q^2 \int_0^{\kappa} d\bar{\lambda} \int_0^{\lambda} d\bar{\kappa}\, \Gamma_g(\bar{\kappa}, \bar{\lambda})\, \Delta_g(\bar{\kappa}, \bar{\lambda}) \int_0^{\bar{\lambda}} d\lambda' \int_0^{\bar{\kappa}} d\kappa'\, \Gamma_g(\kappa', \lambda')\, \Delta_g(\kappa', \lambda')$

(relative size depends on energy scale difference, jet algorithm/size, color structure...etc)

2. Ratio of relative PDF suppression from producing one versus two additional jets. General effect is to suppress lower multiplicity ratios.



SUMMARY ON ORIGIN OF SCALING PATTERNS

1. We expect Poisson scaling for processes/selections with large scale separation at low multiplicities.

2. Staircase (geometric) scaling takes over for $n > \bar{n}$ ($\bar{n}$ obtained from Poisson fit).

3. Poisson extrapolation breaks down in the case of the generalized k_T for small jets.

4. High multiplicity geometric scaling is a very generic prediction of QCD.

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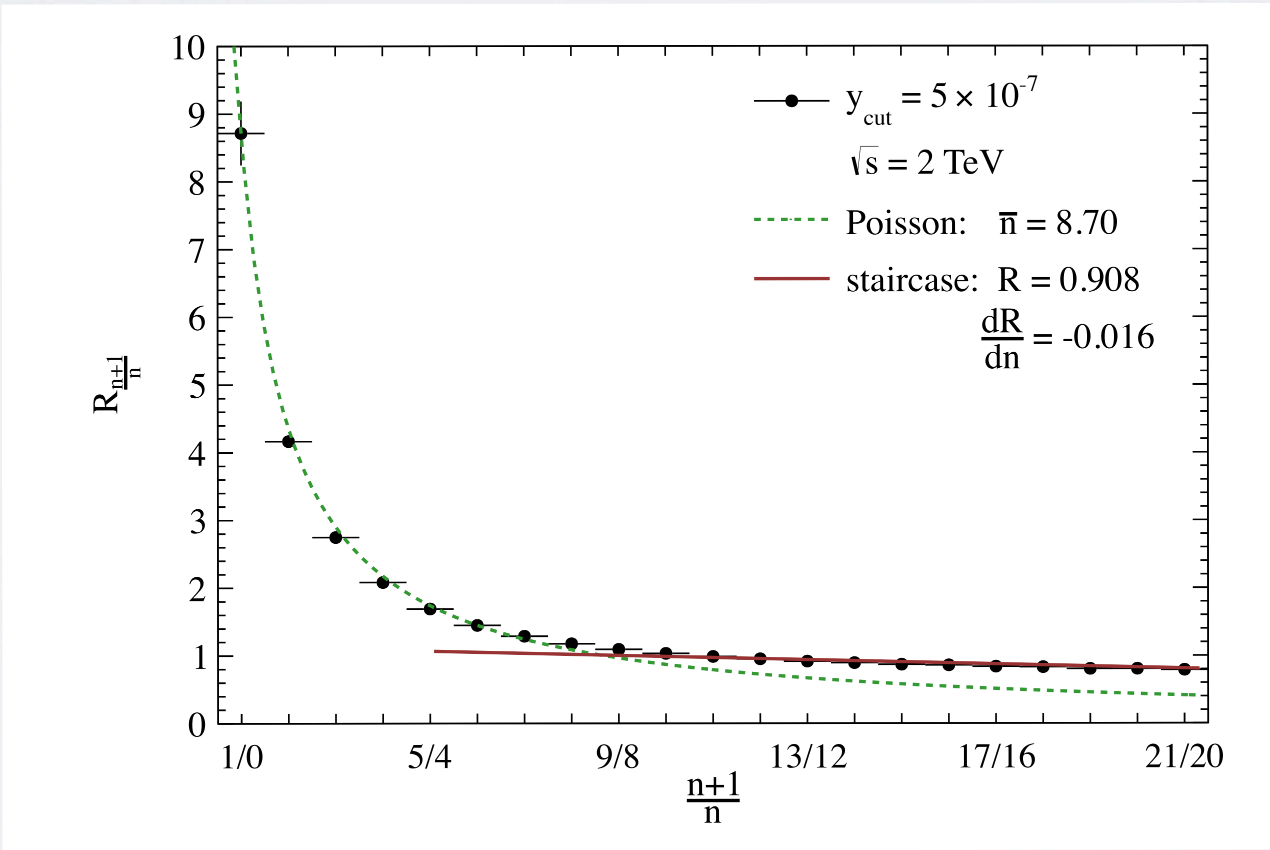
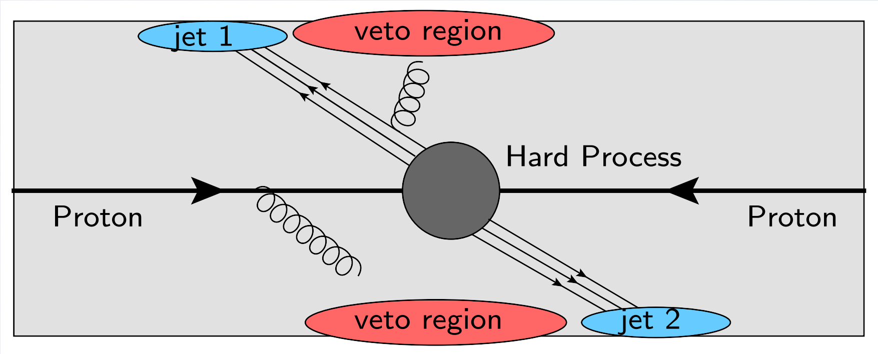


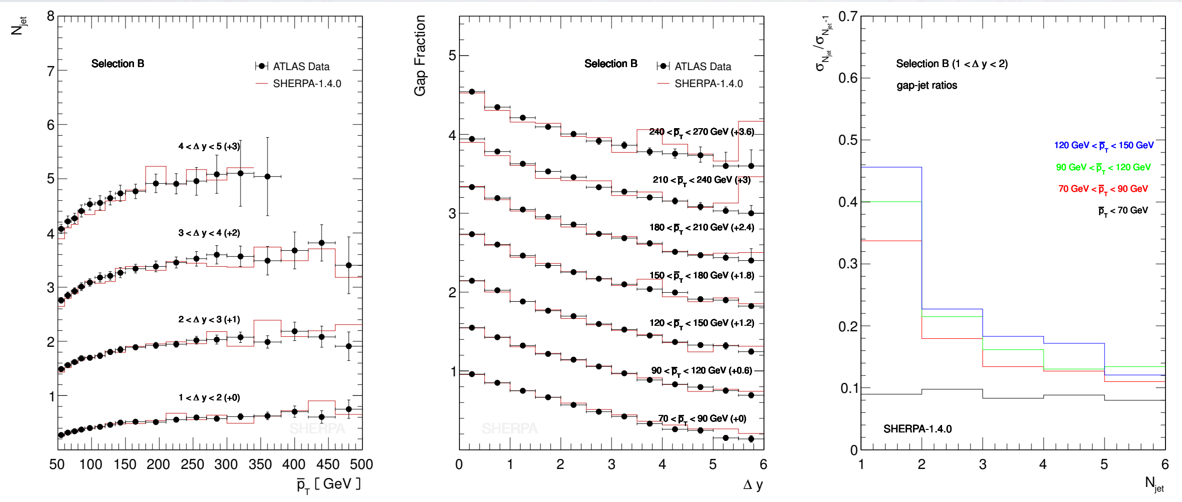
APPLICATION FOR JET SCALING

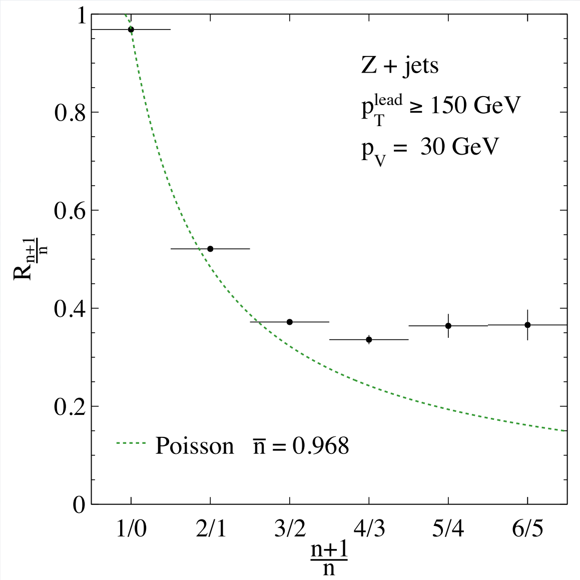
Scaling in the current data



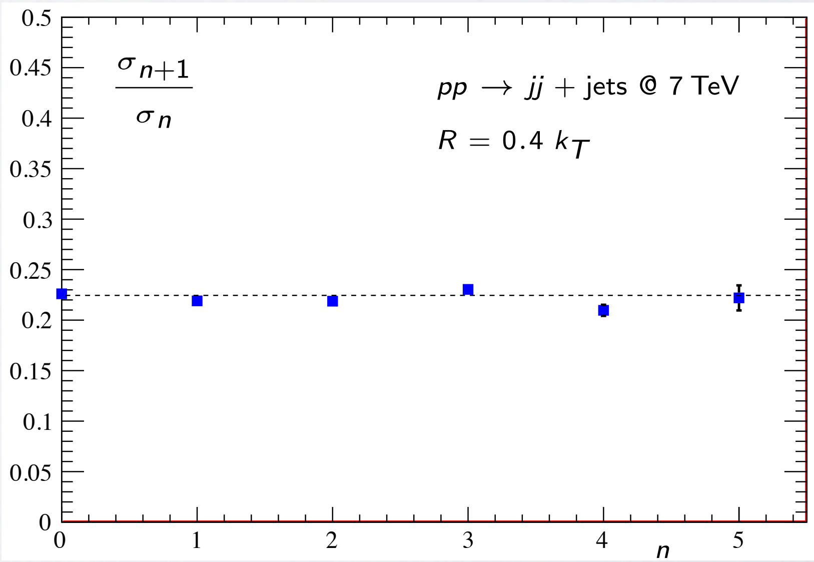
- Atlas public analysis on jet activity in rapidity gaps between "tagging" jets (ATLAS)

- Gap fraction observable sensitive to many different types of QCD effects.





Deviation from scaling as a search for NP



- Scenarios of NP giving an excess in jets starting at some multiplicity (8?).
- Standard background subtraction at high multiplicity many drawback.
- Looking for a deviation from scaling a possible way forward.

CONCLUSIONS

- N_{jets} (ratio) distribution interesting from a theory perspective (and generically important to many analyses).

- The combination (or coincidence) of secondary splittings (at high multiplicity) with PDF effects cause the LHC jet rates to be mostly constant over the whole range.

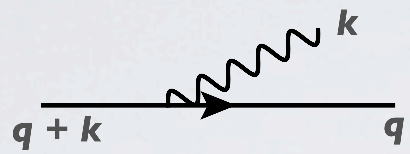
- Strong need for resummed jet rates at Hadron colliders...our generalized kT GF is an attempt at moving in this direction.

- Offers simple search techniques (not depending on MonteCarlo) but we're still thinking about other applications.

BASIS OF THE POISSON DISTRIBUTION

Prototypical example: Soft-photon radiation in QED

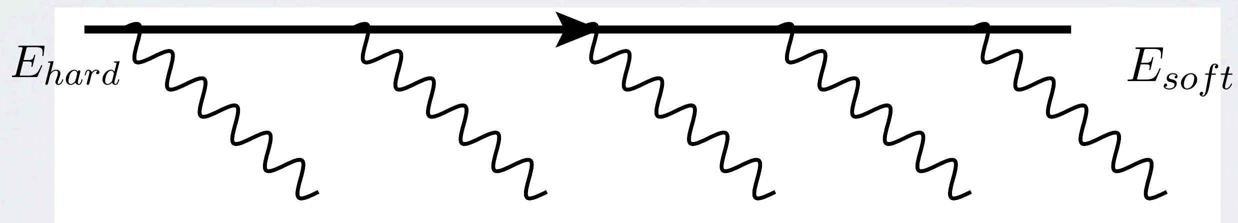
- Fully factorized form of the matrix element (eikonal approximation) [e.g. Peskin and Schroeder]



$$\gamma^\mu \frac{\not{q} + \not{k}}{(q+k)^2} \quad \to \quad \frac{q^\mu}{q \cdot k}$$

- Integrating over phase space, including 1/n! for identical bosons in the final state,

$$\Rightarrow \sigma_n \sim \frac{L^n}{n!} e^{-L} \qquad \text{with} \qquad L \sim \frac{\alpha}{\pi} \log^2 \left(\frac{E_{hard}}{E_{soft}} \right)$$



- Adding together (independent) Poisson processes generates another Poisson process (rate parameters simply add together)

- Matrix element corrections of course important for the rates (unless very log enhanced), but small effect on the scaling.

THE POISSON DISTRIBUTION IN QCD

Fixed order calculation for jet fractions

- Using the Durham algorithm in e+e- [Catani, Dokshitzer, Olsson, Turnock, Webber (1991)]

$$\frac{2\min(E_i^2, E_j^2)}{s}(1-\cos\theta_{ij}) > y_{\rm cut} \qquad\qquad L \equiv \log\frac{1}{y_{\rm cut}} \quad \text{and} \quad a \equiv \frac{\alpha_S}{\pi}$$

- Expand in powers of aL^2, equivalent to not too large single emission probability.

$$f_2^D = 1 - a\frac{C_F}{2}L^2 + a^2\frac{C_F^2}{8}L^4 - a^3\frac{C_F^3}{48}L^6 + a^4\frac{C_F^4}{384}L^8$$

$$f_3^D = a\left(\frac{C_F}{2}\right)L^2 - a^2\left(\frac{C_F^2}{4} + \frac{C_F C_A}{48}\right)L^4 + a^3\left(\frac{C_F^3}{16} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960}\right)L^6 - a^4\left(\frac{C_F^4}{96} + \frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504}\right)L^8$$

$$f_4^D = a^2\left(\frac{C_F^2}{8} + \frac{C_F C_A}{48}\right)L^4 - a^3\left(\frac{C_F^3}{16} + \frac{C_F^2 C_A}{48} + \frac{7 C_F C_A^2}{2880}\right)L^6 + a^4\left(\frac{C_F^4}{64} + \frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120}\right)L^8$$

$$f_5^D = a^3\left(\frac{C_F^3}{48} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720}\right)L^6 - a^4\left(\frac{C_F^4}{96} + \frac{C_F^3 C_A}{128} + \frac{3 C_F^2 C_A^2}{1280} + \frac{41 C_F C_A^3}{161280}\right)L^8$$

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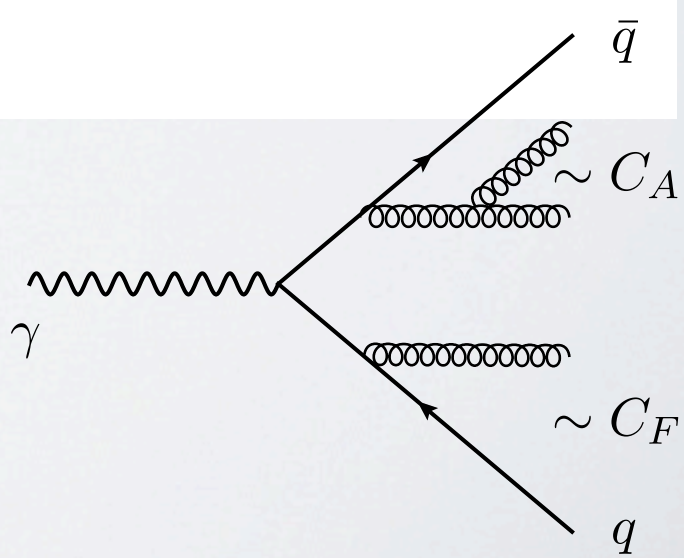
- Deviation from the Poisson in the fixed order expansion purely due to <u>secondary</u> (vs. <u>primary</u>) emissions



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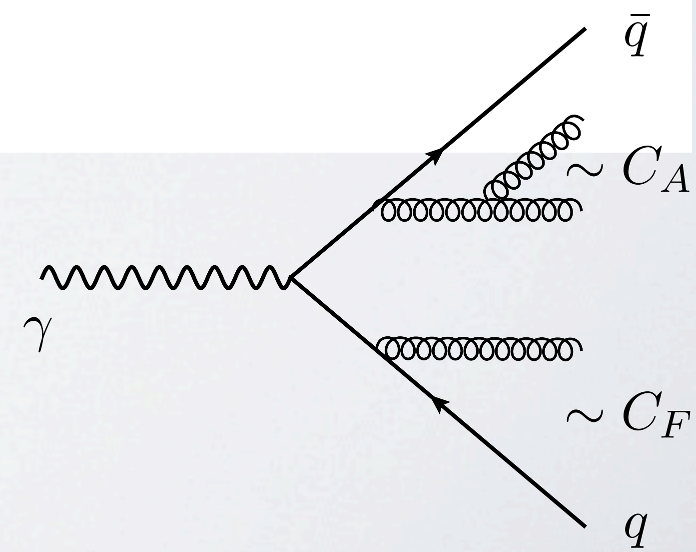

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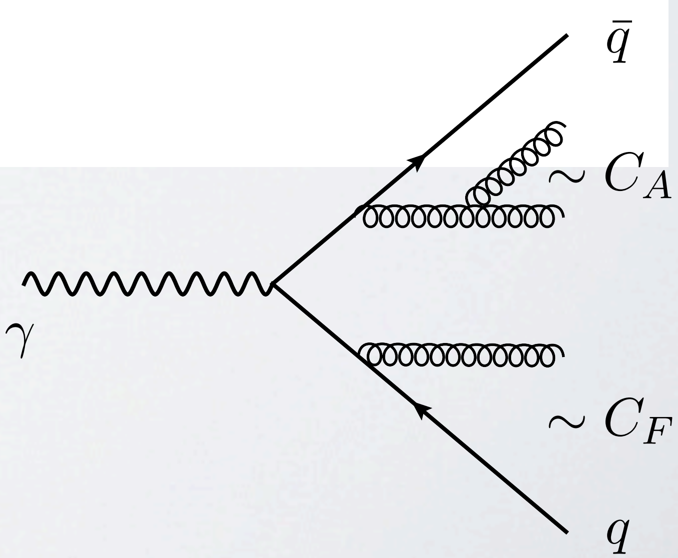
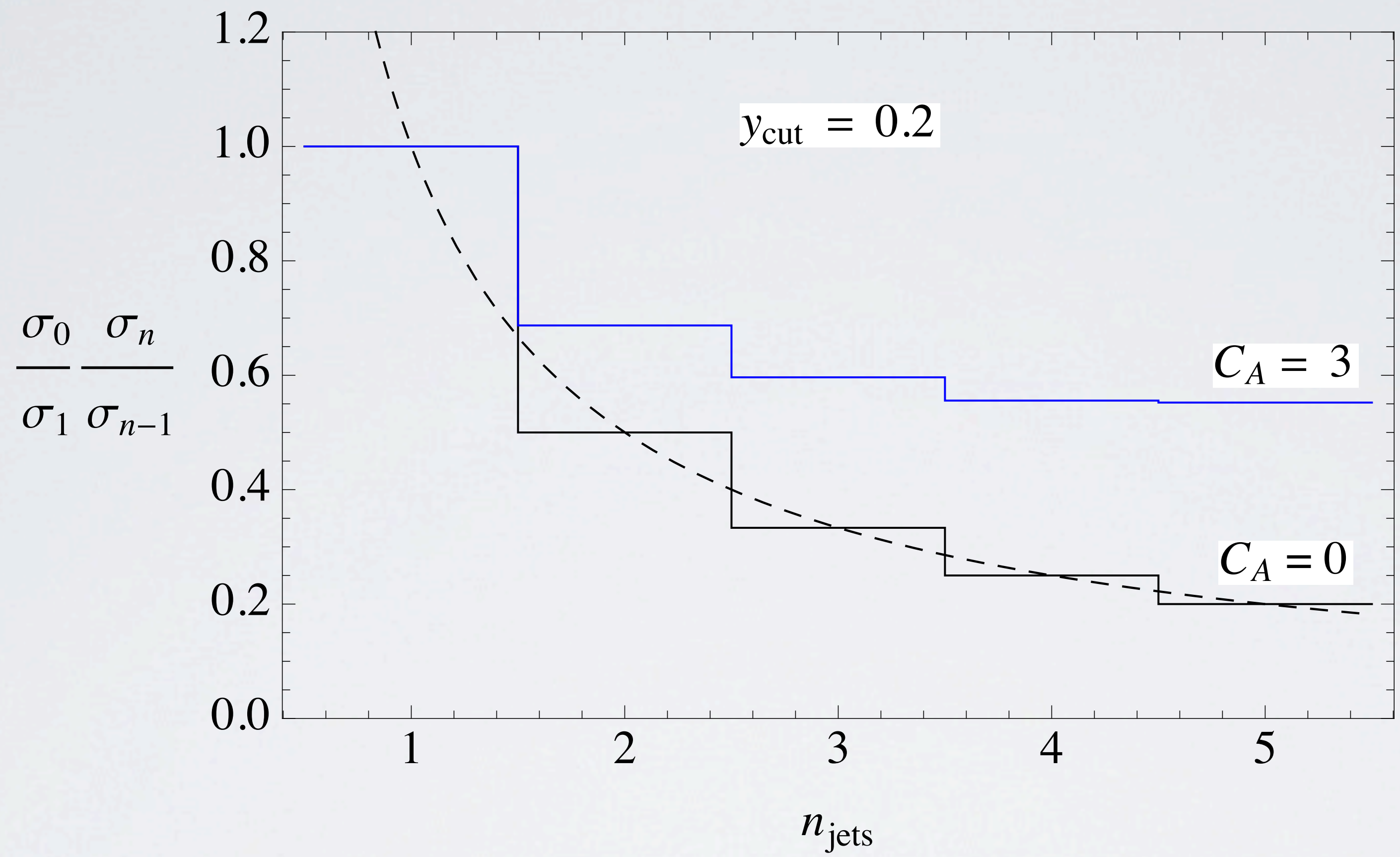


THE POISSON DISTRIBUTION IN QCD

Comparison of fixed order analytic results for jet fractions



Remarks from the fixed order calculations

1. At high multiplicities subsequent splittings take-over and seem to lead to geometric scaling.
2. Lower multiplicities neither Poisson or staircase like behavior.
3. Doesn't really answer our questions in either regime! (need resummed rates)

RESUMMED RATES IN QCD: SOME FORMALISM

Generating functionals for jet rates (resummed to NLL)

- For the Durham algorithm [Catani, Dokshitzer, Olsson, Turnock, Webber (1991)]

$$\Phi_q(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\, \Gamma_q(Q^2,t)\,(\Phi_g(t)-1)\right]$$

$$\Phi_g(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\left(\Gamma_g(Q^2,t)\,(\Phi_g(t)-1)+\Gamma_f(t)\left(\frac{\Phi_q^2(t)}{\Phi_g(t)}-1\right)\right)\right]$$

- Derivatives with respect to "source" u at u=0, produce (resummed) exclusive multiplicities. (first moment corresponds to average jet multiplicity)

$$f_{n-1} = \frac{1}{n!}\frac{d^n}{du^n}\Phi\bigg|_{u=0}$$

- Jet rates include the unresolved components to all-orders (are physically valid even when $\sigma_{n+1} \gg \sigma_n$).

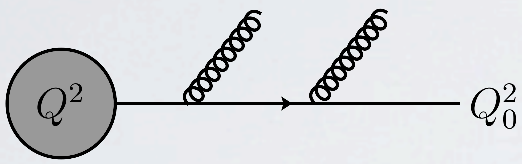
As close as one can get to an analytic description of a Parton shower.

RESUMMED RATES IN QCD: SOME FORMALISM

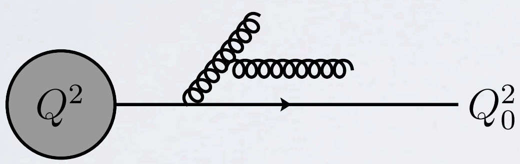
Resummed 2-gluon emission

$$\frac{1}{4!}\frac{d}{du}|\Phi_u|^2 = 2[\Delta_q(Q)]^2\left[\left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\right)^2 + \left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(q')\right)\right]$$

- From a scaling point of view even the primary emission provide a Poisson pattern.



$$\sigma^{\text{primary}}(Q^2,Q_0^2) = c^{\text{primary}}\int_{Q_0^2}^{Q^2} dt\,\Gamma(Q^2,t)\Delta_g(t)\int_{Q_0^2}^{Q^2} dt'\,\Gamma(Q^2,t')\Delta_g(t')$$



$$\sigma^{\text{secondary}}(Q^2,Q_0^2) = c^{\text{secondary}}\int_{Q_0^2}^{Q^2} dt\,\Gamma(Q^2,t)\Delta_g(t)\int_{Q_0^2}^{t} dt'\,\Gamma(t,t')\Delta_g(t')$$

- Key point is that the primary emissions are enhanced wrt secondary emissions as the size of the overall logarithm grows (effect completely missing in the fixed order calculation)

In kinematic limit	Primary emission	Secondary emissions
$\frac{\alpha_s}{\pi}\log^2\frac{Q}{Q_0} \ll 1$	$c^{\text{primary}}\ \log^4\frac{Q}{Q_0}$	$c^{\text{secondary}}\ \log^4\frac{Q}{Q_0}$
$\frac{\alpha_s}{\pi}\log^2\frac{Q}{Q_0} \gg 1$	$\frac{\alpha_s}{C_A}\log^2\frac{Q}{Q_0}$	$\sqrt{\frac{\alpha_s}{C_A^3}}\log\frac{Q}{Q_0}$

DURHAM GENERATING FUNCTIONAL

All multiplicity proof for the scaling patterns

- For Poisson this is more or less simple; the integral for large Q is dominated by region in *t* space close to Q_0.

$$\Phi_j(Q^2) = u\ \exp\left[\int_{Q_0^2}^{Q^2} dt\ \Gamma_j(Q^2,t)\,(u-1)\right] = \frac{u\,\Delta_j(Q^2)}{\Delta_j(Q^2)^u}$$

Gives Poisson ratios: $\quad R_{(n+1)/n} = \frac{|\log \Delta_j(Q^2)|}{n+1}$

- In pure YM keeping leading powers of $(Q - Q_0)/Q$, corresponds to not too large single emission probability (still need log enhancement).

$$\frac{d\Phi_g(Q^2)}{dQ^2} \approx \Phi_g(Q^2)\,\tilde{\Gamma}_g(Q^2,Q_0^2)\,(\Phi_g(Q^2)-1) \longrightarrow \Phi_g(Q^2) = \frac{1}{1+\frac{(1-u)}{u\tilde{\Delta}_g(Q^2)}}$$

Gives Staircase ratios: $\quad R_{(n+1)/n} = 1 - \tilde{\Delta}_g(Q^2)$

- High multiplicity proof of Staircase tale in large emission probability limit an empirical fact, sets in at $n_{\mathrm{trans}} \approx \bar{n}$ (number of Poisson breaking terms grows as a function of n).

SUMMARY: RATES AND GEN. FUNCTIONAL

Fixed-order

- Fixed order calculation tells us that Poisson distribution altered by secondary emissions.
- At high multiplicity start to see the on-set of geometric scaling.

Resummed jet rates

- From the 2-jet rate (2 gluon emission) able to see how the (Poisson-making) primary comments are dynamically enhanced with respect to the secondaries.

Generating Functional

- Able to derive the desired patterns in two opposing limits, in the case of the staircase limit only able to solve the PDE analytically for pure YM.

However, the Durham algorithm is somewhat special, in that there is no resolution scale in physical energy or angle. Therefore, we would like to study an algorithm which mimics LHC relevant jet. i.e Generalized kt algorithm.

GENERALIZED K_T ALGORITHM

Generating functional

- For the Generalized class of algorithms (more analogous to LHC algorithms of choice) [EG et al.]

$$d_{ij} = \min\{E_i^{2p}, E_j^{2p}\}\frac{(1-\cos\theta_{ij})}{(1-\cos R)} \equiv \min\{E_i^{2p}, E_j^{2p}\}\xi_{ij}/\xi_R\ ,$$

- Generating functional solved via iteration for the rates, can be compared with Parton shower.

$$\Phi_q(u,E,\xi) = u\exp\left\{\frac{\alpha_S}{2\pi}\int_{\xi_R}^{\xi}\frac{d\xi'}{\xi'}\int_{E_R/E}^{1} dz\, P_{gq}(z)\,[\Phi_g(u,zE,\xi')-1]\right\}$$

$$\Phi_g(u,E,\xi) = u\Delta_g(E,\xi) + \frac{\alpha_S}{2\pi}\int_{\xi_R}^{\xi}\frac{d\xi'}{\xi'}\frac{\Delta_g(E,\xi)}{\Delta_g(E,\xi')}\int_{E_R/E}^{1} dz\{P_{gg}(z)\Phi_g(u,E,\xi')\Phi_g(u,zE,\xi') + P_{qg}(z)[\Phi_q(u,E,\xi')]^2\}$$

- Splitting apart of soft (energy) and collinear (angle) singularity leads to important differences (already evident looking at the Sudakov form factor).

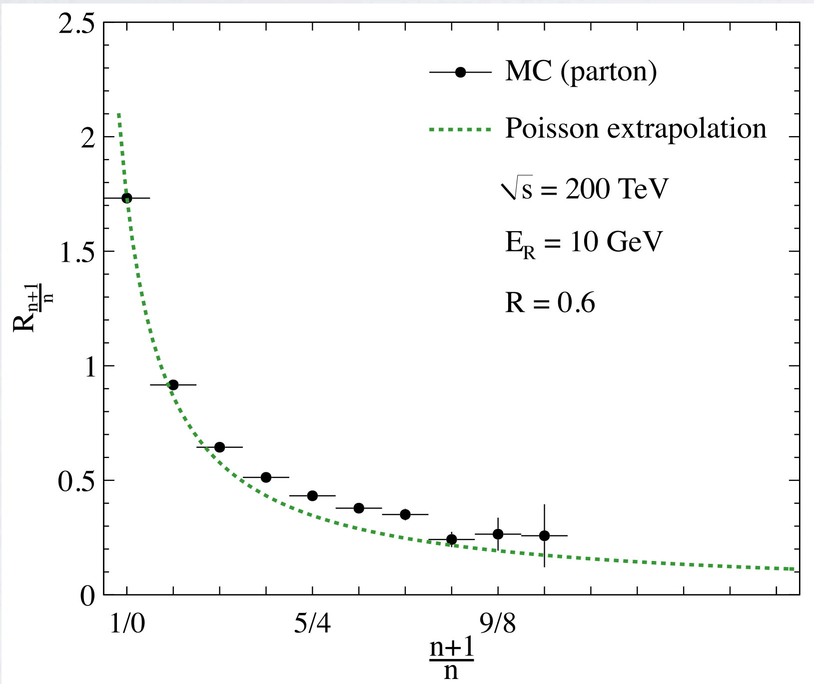
$$\Delta_g(E,\xi) = \exp\left\{-\frac{\alpha_S}{\pi}\ln\left(\frac{\xi}{\xi_R}\right)\left[C_A\ln\left(\frac{E}{E_R}\right) - \frac{11C_A - 2n_f}{12}\right]\right\}$$

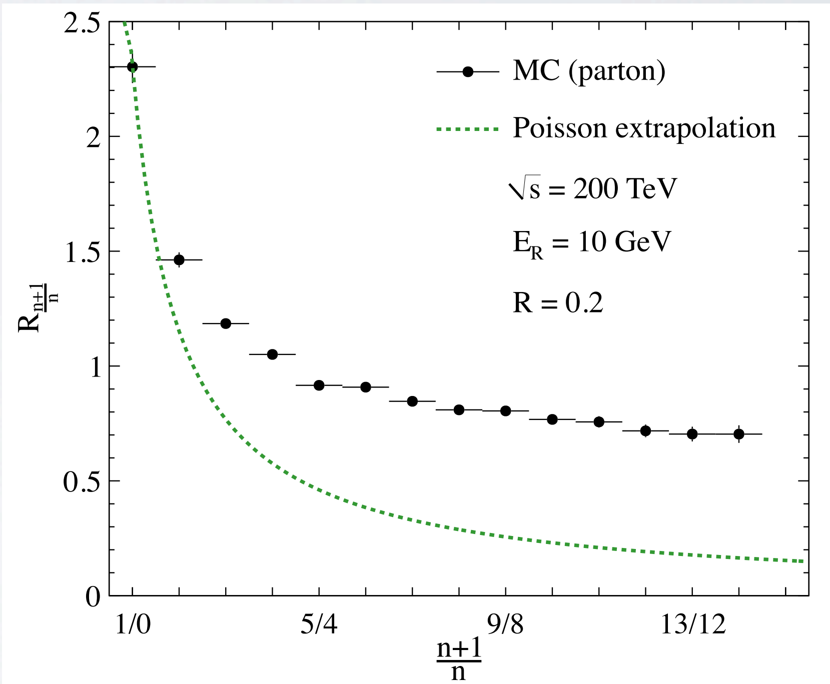
GENERALIZED K_T ALGORITHM

Scaling as a function of the jet area

- With the Durham measure, smaller average jets (i.e smaller y_{cut}) raised the size of the size of the overall logarithms, and increases the goodness of the Poisson fit.

- However, with the generalized algorithm, overall logarithm again increases, although the goodness of the Poisson fit is significantly worse.





Angular dependence of the emission types

- The resummed jet rates (and gen func.) contain no phase space dependence of the two emission types (primary vs secondary), but the parton shower does (through kinematics).

CONCLUSIONS ON e+e- SCALING

We expect the following for the distribution of jet rates in e+e-

Small emission probability (small log):

1. Staircase tail, not a good Poisson at low multiplicity.

Large emission probability (large log):

1. Increasingly good Poisson fit at multiplicities up to <N>.
2. Staircase tail sets in after <N>.
3. Strong deviation from 1. for small jet sizes (in Gen. k_T).

CONCLUSIONS ON e+e- SCALING

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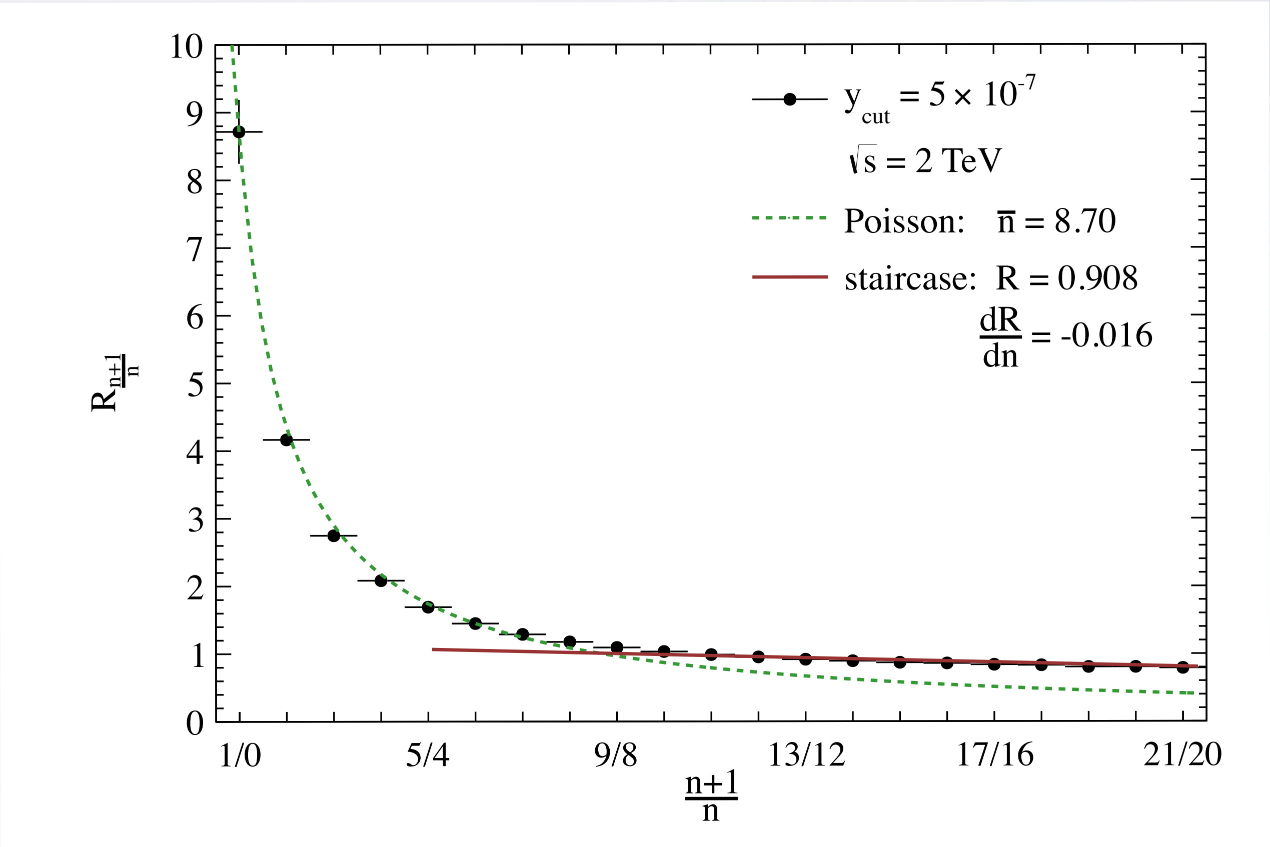


PDF EFFECTS ON SCALING

Factorization at leading logarithmic level

- Generating functional for initial state evolution [Catani, Webber, Dokshitzer (1993)].
- Factorization of the Generating functional at the leading logarithmic level.

$$\Phi_{\text{Drell-Yan}} = \sum_{a,b} f_a(x^{(n)}, p_V^2)\Phi_a(Q^2, p_V^2)\; f_b(x^{(n)}, p_V^2)\Phi_b(Q^2, p_V^2)$$

- Factorization scale $\mu_F \equiv p_V$ avoids possible large logs or double counting.

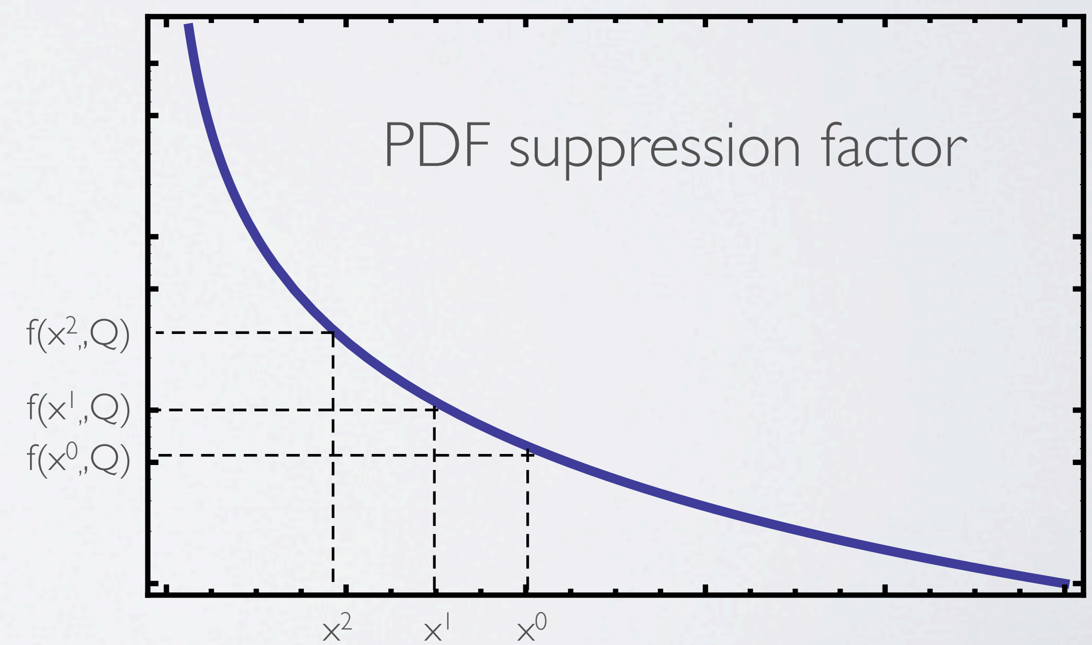
Estimating the PDF suppression

- Assume threshold kinematics on additional jet.

$$R_{(n+1)/n} \sim \left| \frac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)} \right|^2$$

- Effect on scaling essentially discretized second derivative with respect to x.

$$B_n = \left| \frac{\frac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)}}{\frac{f(x^{(n+2)}, p_V)}{f(x^{(n+1)}, p_V)}} \right|^2$$



PDF EFFECTS ON SCALING

Estimating the threshold kinematics

- Most naive estimate not sufficient for good agreement with the data.
- Slightly more sophisticated choice is to include the recoil of the boson from $x^{(0)} \longrightarrow x^{(1)}$

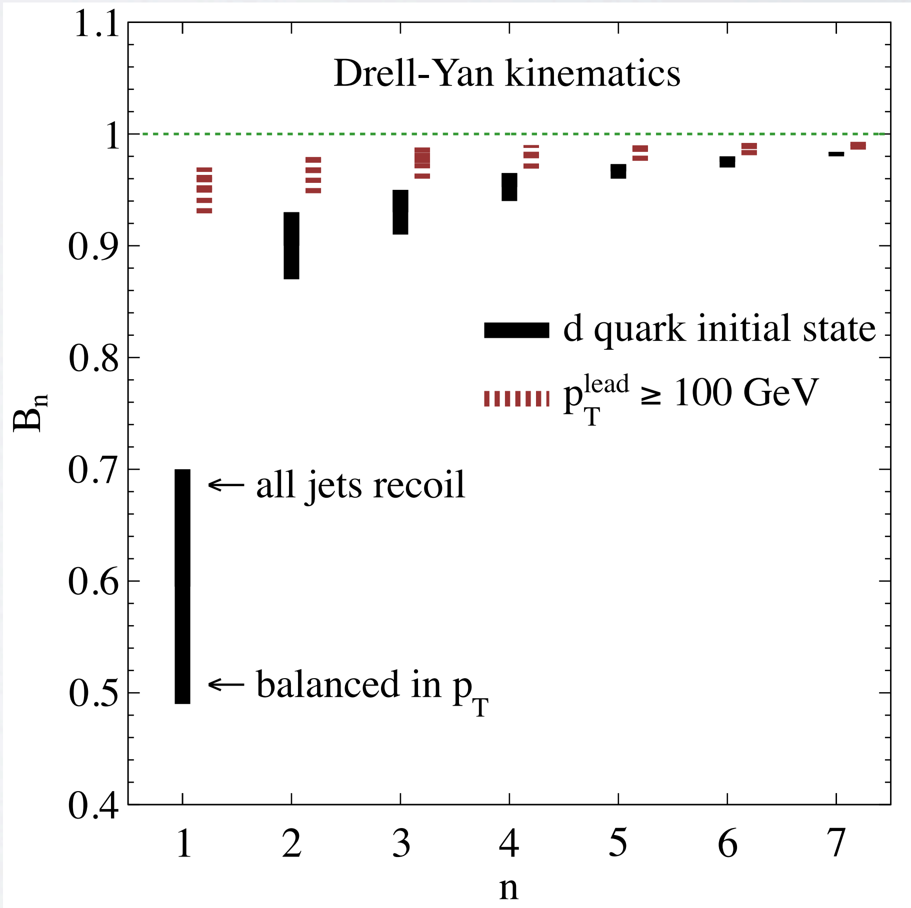
$$x^{(0)} \approx \frac{m_Z}{2\,E_{\text{beam}}} \qquad \longrightarrow \qquad x^{(1)} = \frac{\sqrt{m_Z^2 + 2\left(p_V\sqrt{p_V^2 + m_Z^2} + p_V^2\right)}}{2\,E_{\text{beam}}}$$

Estimating the PDF suppression

- Assume threshold kinematics on additional jet.

$$B_n = \left| \frac{\dfrac{f(x^{(n+1)}, p_V)}{f(x^{(n)}, p_V)}}{\dfrac{f(x^{(n+2)}, p_V)}{f(x^{(n+1)}, p_V)}} \right|^2$$

- Ambiguity starting at the kinematics of the second jet (recoil against the Z-boson or not)
- Almost identical effect for ggH (gluons vs m_{higgs})
- Key result: PDFs push down the lower multiplicity jet ratios ⟶ more staircase-like.



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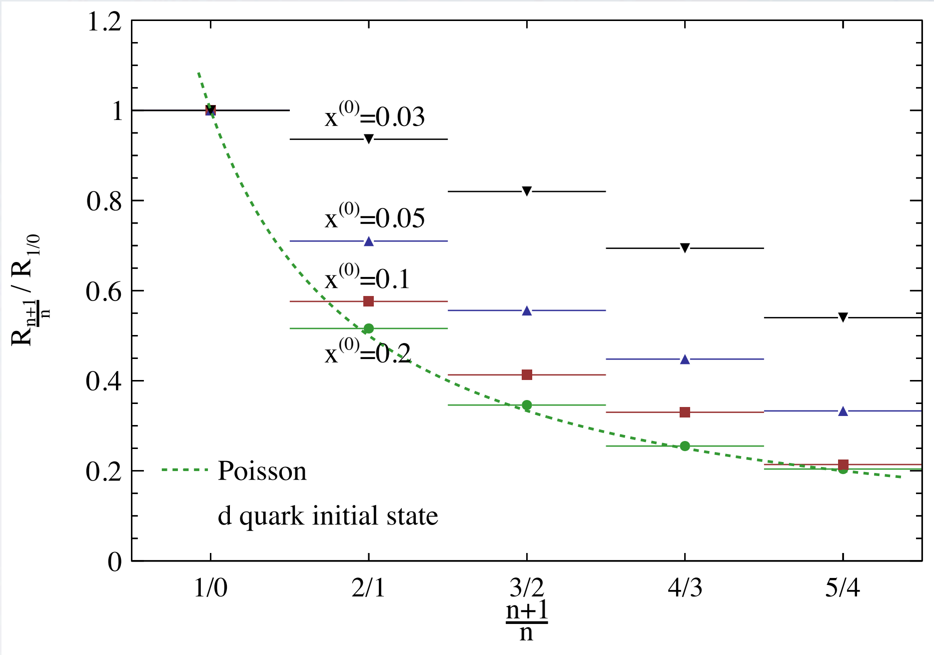
OTHER EFFECTS IN PP

Scaling from the backward evolution form-factor

- Forward evolution Sudakov always gives Poisson along a single line. For backward evolution depends on starting value for x.

$$\Pi(t_1, t_2; x) = \exp\left\{ -\int_{t_1}^{t_2} \frac{dt}{t} \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{q\to qg}(z) \frac{f_q(x/z,t)}{f_q(x,t)} \right\}$$



Scaling in the context of BFKL dynamics

- Generating functional in the small x regime in the Multi-Regge-Kinematics limit [Webber 1998]

$$\Phi(Q^2, p_V^2)_{\rm BFKL} = \exp\left(-\frac{2C_A\alpha_s}{\pi w} \log\frac{Q}{p_V} \right) \left[1 + (1-u)\frac{2C_A\alpha_s}{\pi w} \log\frac{Q}{p_V} \right]^{u/(1-u)}$$

where again we have $f_{n-1} = \frac{1}{n!}\frac{d^n}{du^n}\Phi\Big|_{u=0}$

In kinematic limit	Emission pattern behaves as	Scaling pattern
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \gg 1$	$\sigma_n \approx \frac{1}{n!}\log^n\left(1+\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)$	Poisson
$\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V} \ll 1$	$\sigma_n \approx \left(\frac{2C_A\alpha_s}{\pi w}\log\frac{Q}{p_V}\right)^n$	Geometric

CONCLUSIONS ON SCALING AT PP

- Final state radiation effects from e+e- carry-over and give Poisson in large-log limit with staircase tail as usual
- PDF effect suppresses the lower multiplicity rates, flattening out the overall distribution.

"APPLICATIONS"

DESERTED ISLAND PHYSICS

Task: Calculate the normalized jet ratios for Drell-Yan at the LHC.

Direct Approach

1. Find favorite MonteCarlo
2. Wait a while (days/weeks?)
3. Count rates in each bin
4. Divide to obtain ratios

Scaling Approach

1. Everything starts of as Poisson
2. Add first inhomogenous term [from g->gg splitting]

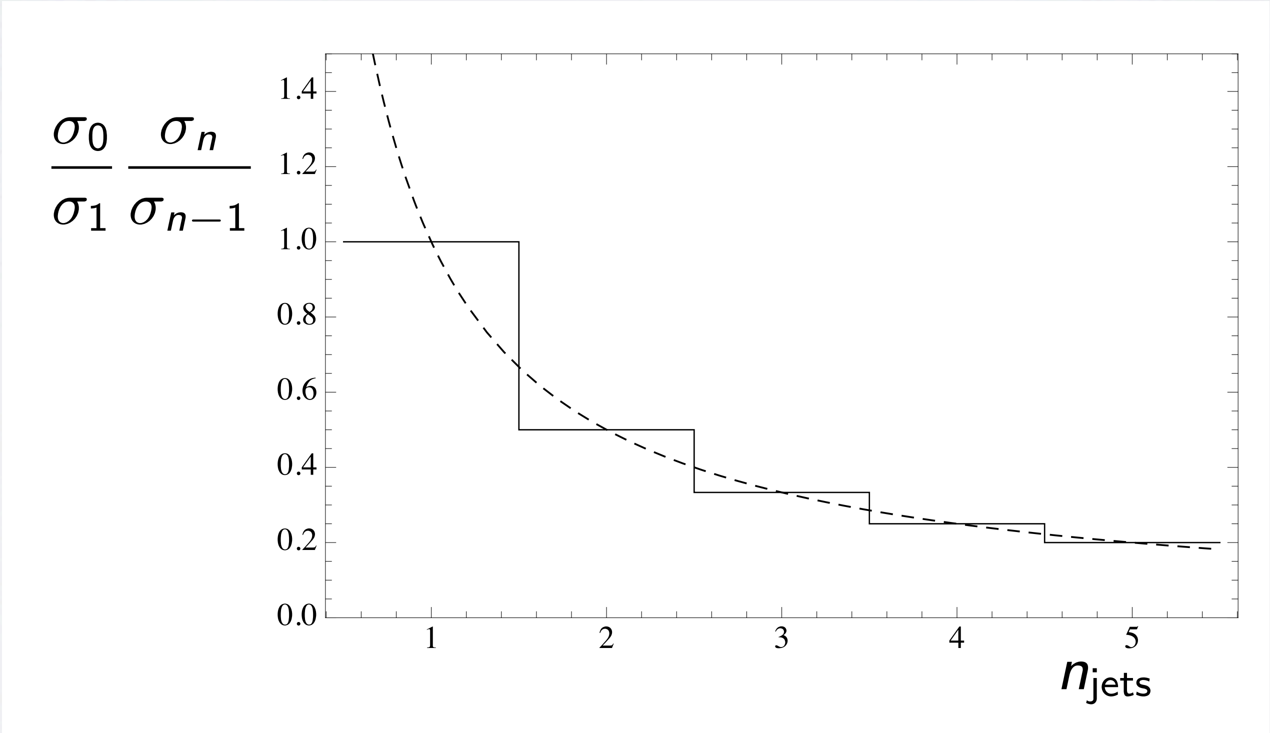
$$\bar{n} \sim 1 \qquad \bar{n}' \sim \frac{C_A}{12 C_F}$$

3. Evaluate PDF function B
4. Fold together!

DESERTED ISLAND PHYSICS

Task: Calculate the normalized jet ratios for Drell-Yan at the LHC.

1. Everything starts of as Poisson
2. Add first inhomogenous term
3. Fold in PDF effect (strong suppression of first bin)



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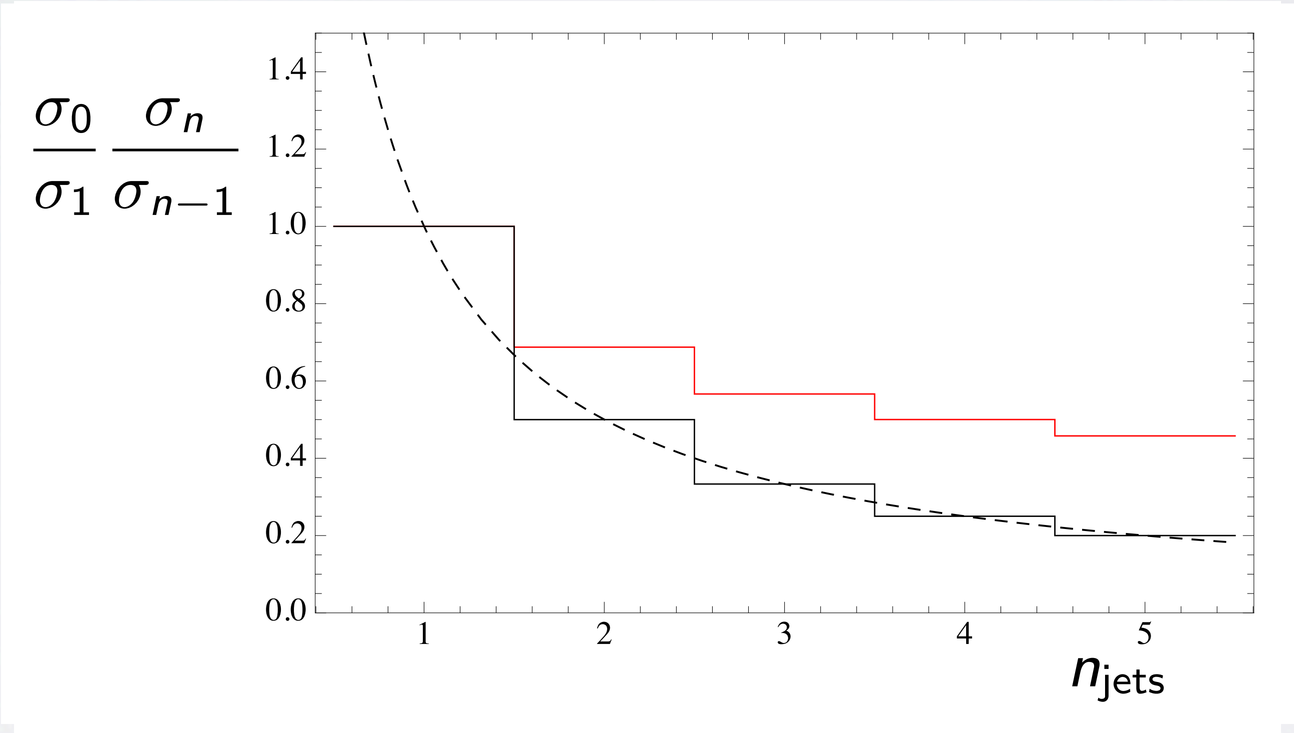

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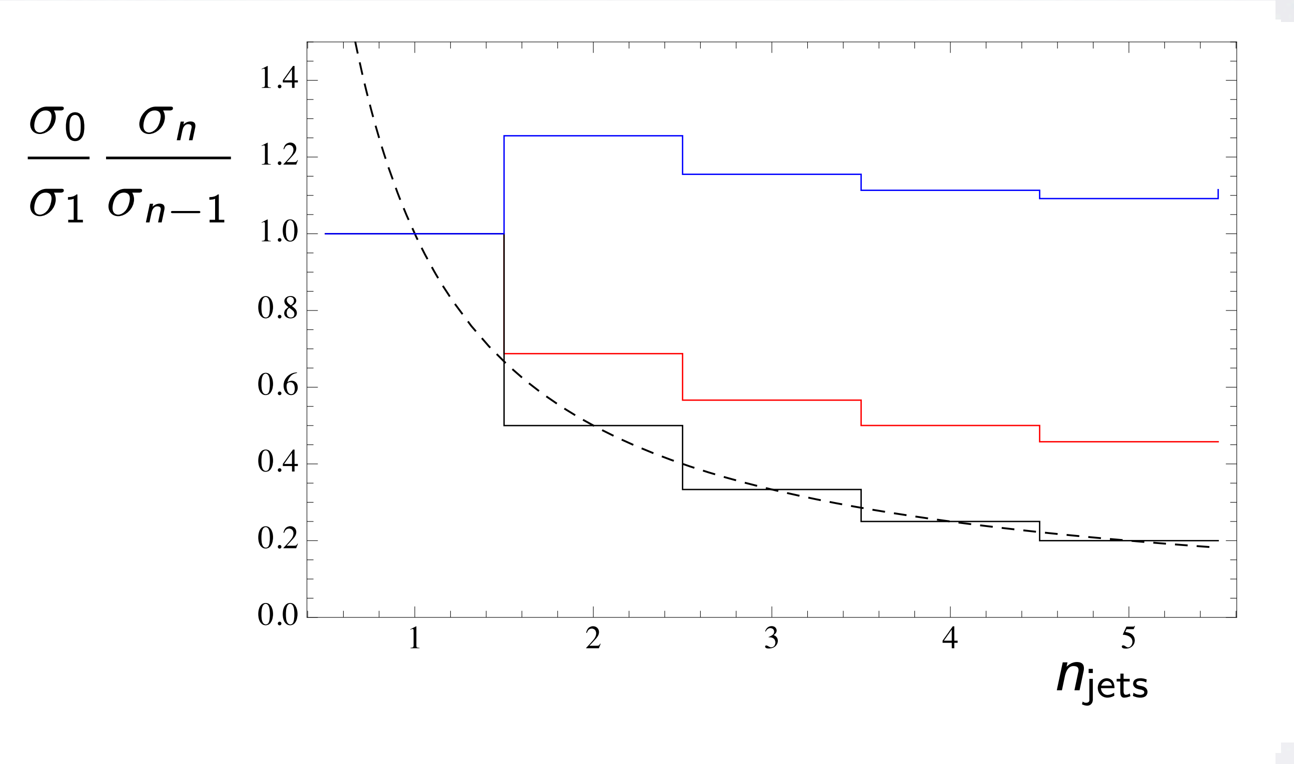

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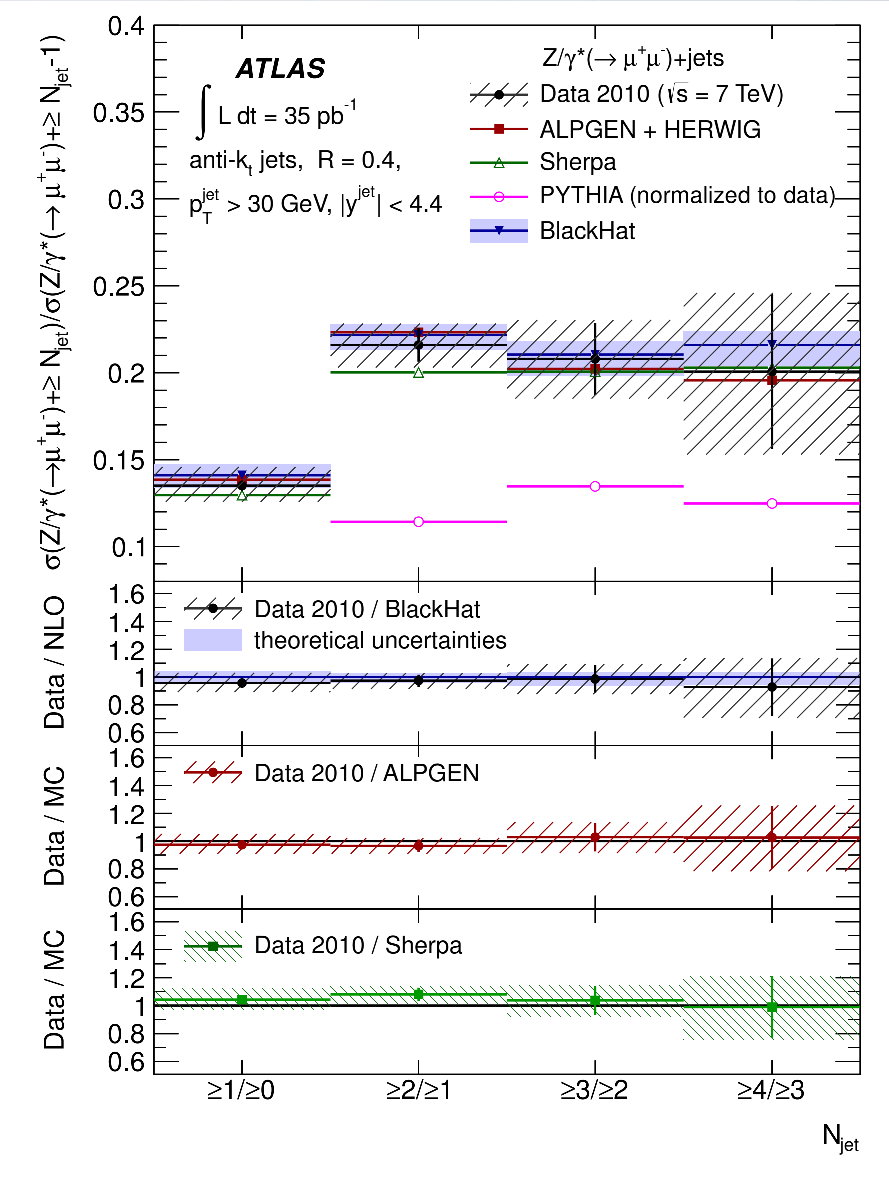

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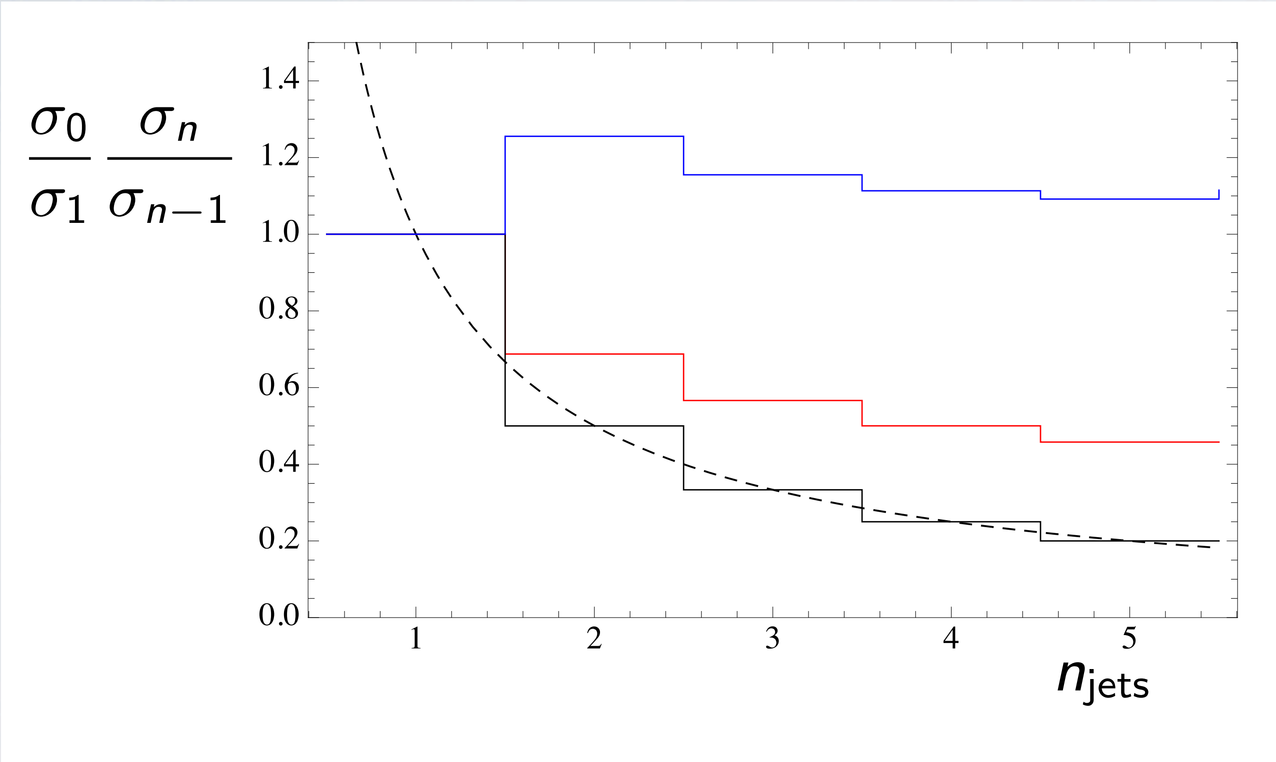
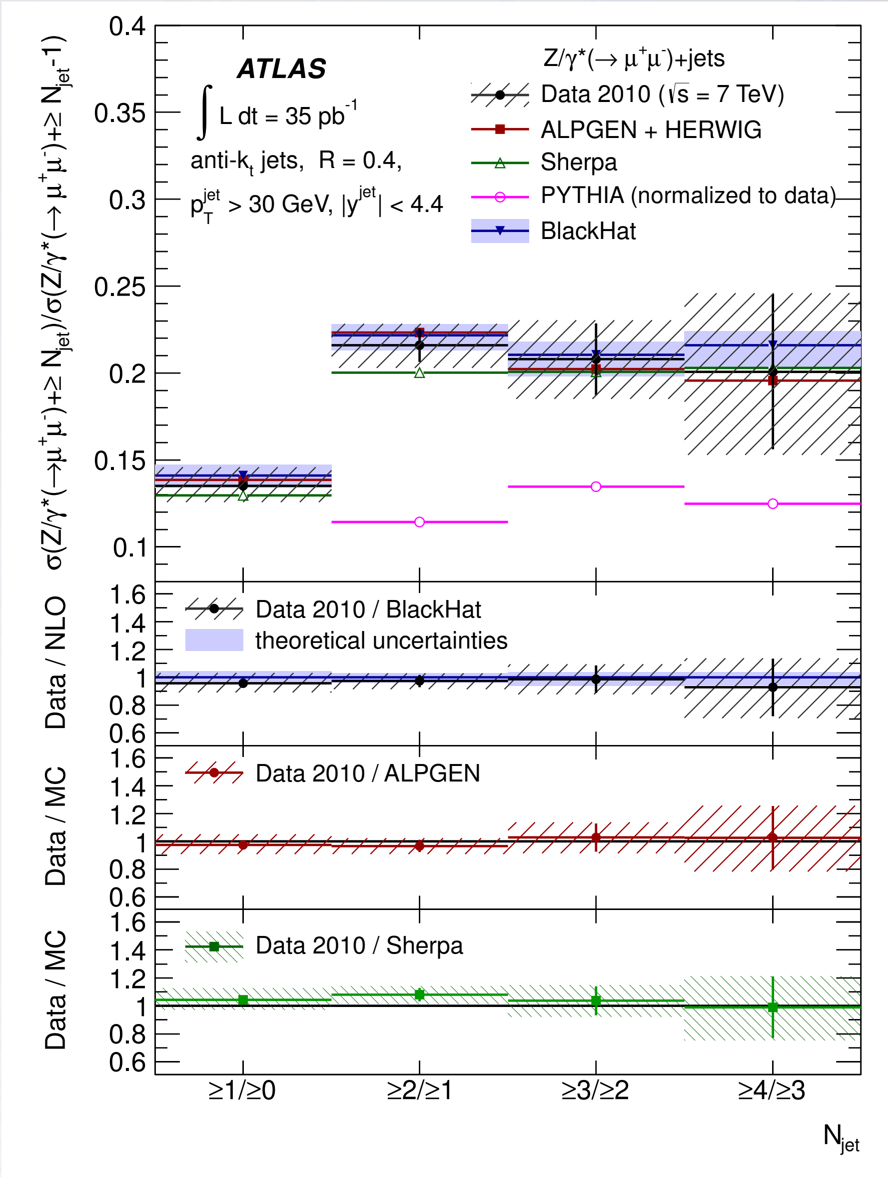




APPLICATIONS

Automation of analyses

- Proposal by CMS to automate NP search via automation ~100,000 different observables and direct comparison to MonteCarlo.
- Limitation is generating enough (high statistic) MonteCarlo, and my personal opinion is that there will be prohibitively many false positive.

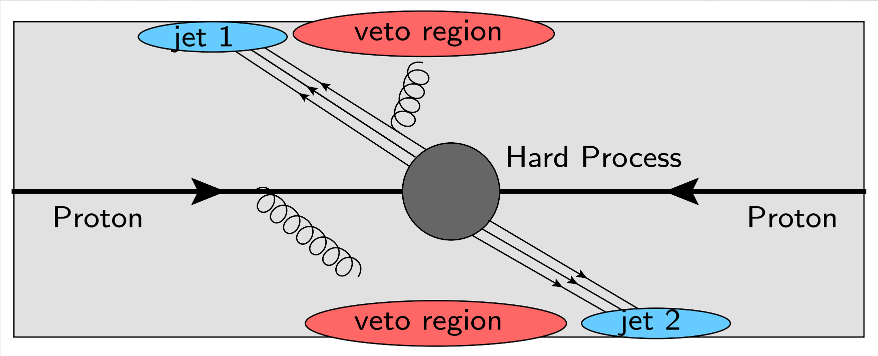
General searches via a scaling hypothesis

- QCD continuum background produces staircase scaling ratios (no structure)
- Many models of new physics produce an excess of jets starting at a certain multiplicity.
- Seeing an excess in e.g. 8 jet bin via automated MonteCarlo a very tedious task. Many models of new physics produce an excess of jets starting at a certain multiplicity.

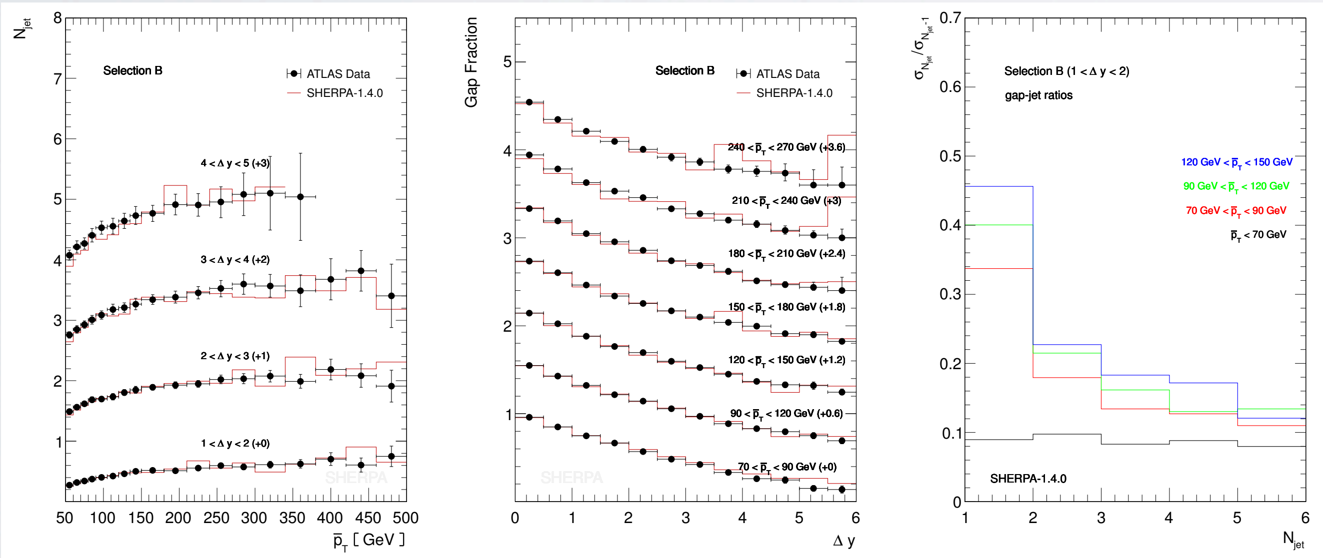
PREDICTIONS IN DI-JET GAPS

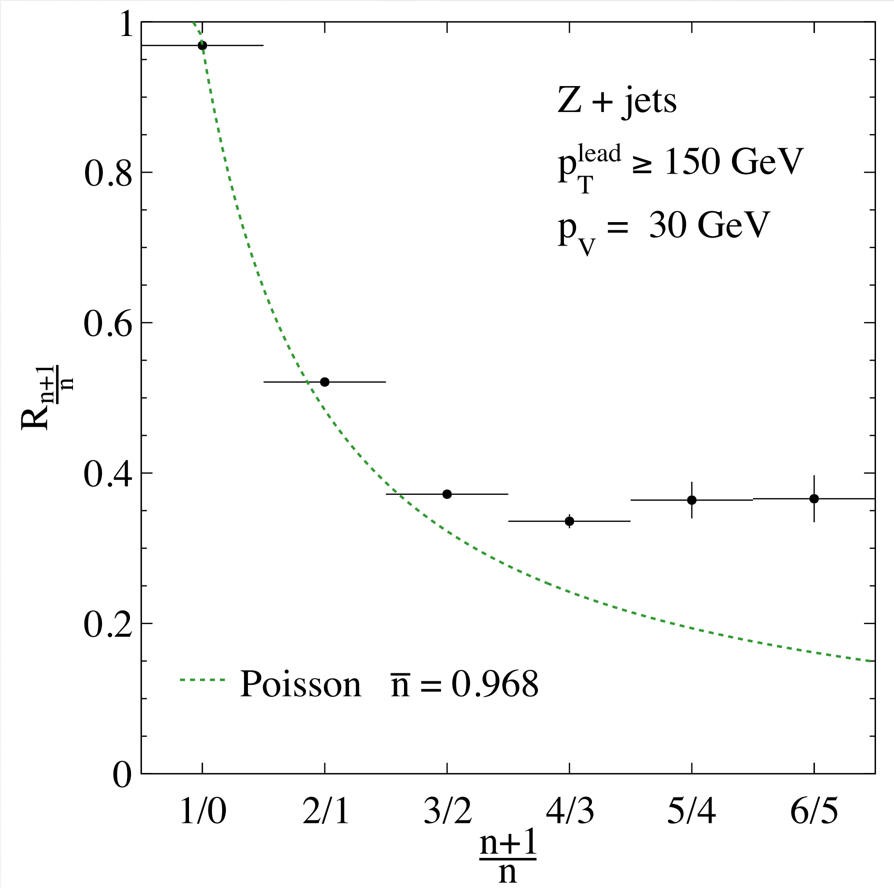
- Atlas public analysis on jet activity in rapidity gaps between "tagging" jets (ATLAS)

- Gap fraction observable sensitive to many different types of QCD effects.



- Ideal testing ground for tools to predict veto efficiencies in Higgs WBF process.





More on Staircase

Theoretical Basis

- Black-hat + Sherpa NLO Z and W + jets (Anti-kt; $R = .4$; $E_T > 30$ GeV) [Berger et al.]
- Staircase improves for NLO versus LO
- No solid theoretical motivation

$R_n = \sigma_n/\sigma_{n-1}$	LO	NLO
R_2	.2805	.235
R_3	.2483	.223
R_4	.2394	.226

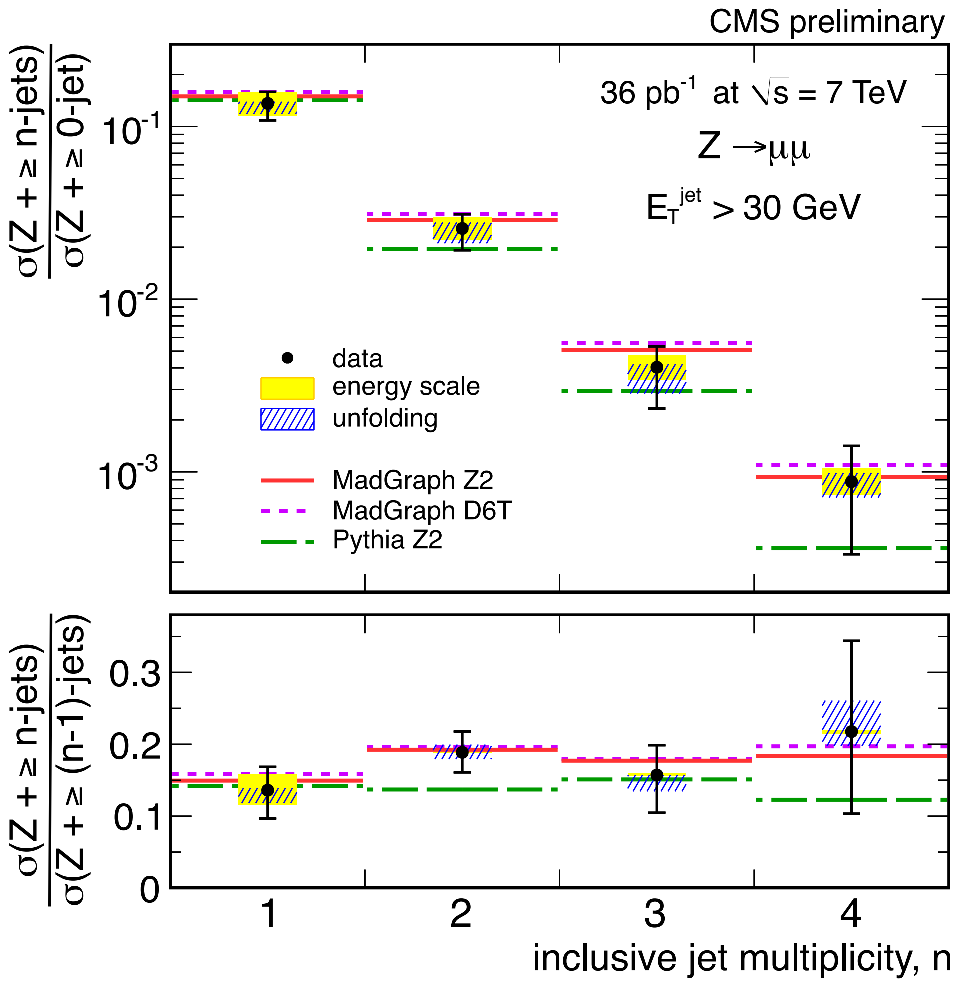
Experimental Observations

- Inclusive = exclusive ratios (for perfect staircase)

$$R_{incl} = \frac{\hat{\sigma}_{n+1}}{\hat{\sigma}_n} = \frac{\sigma_{n+1}\sum R^j_{excl}}{\sigma_n + \sigma_{n+1}\sum R^j_{excl}}$$

$$= \frac{R_{excl}\sigma_n}{(1 - R_{excl})\sigma_n + R_{excl}\sigma_n)}$$

$$= R_{excl}$$



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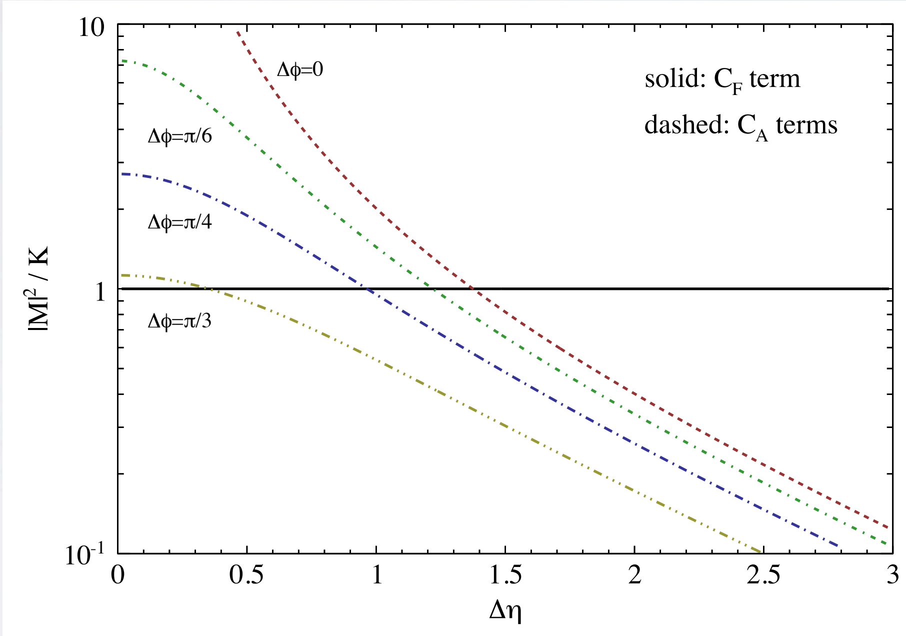
GENERALIZED K_T ALGORITHM

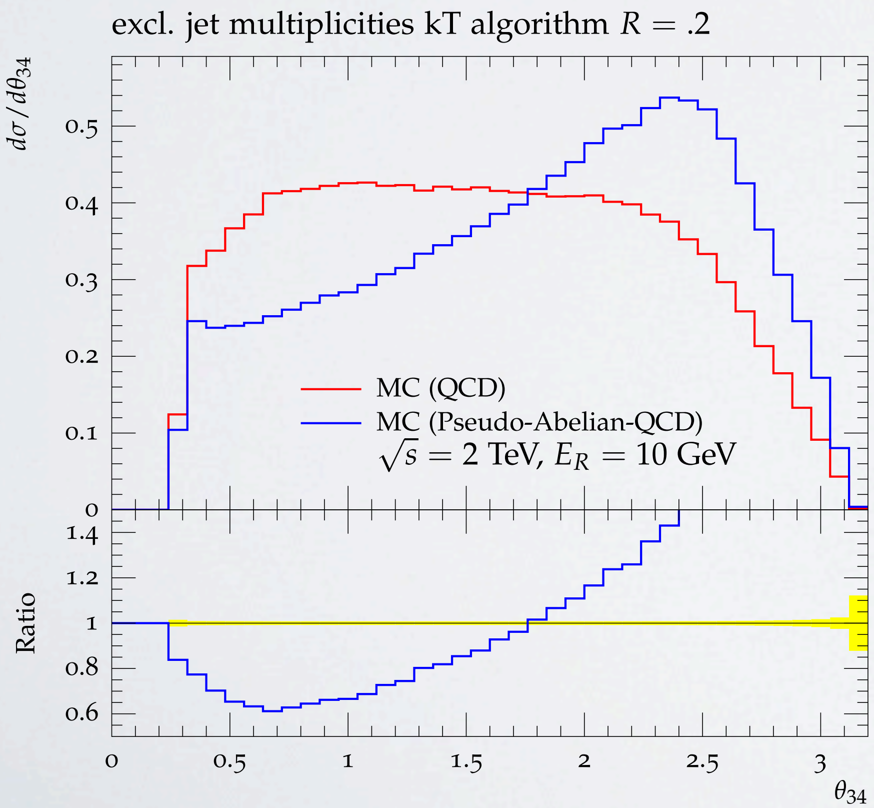
Angular dependence on the emission type

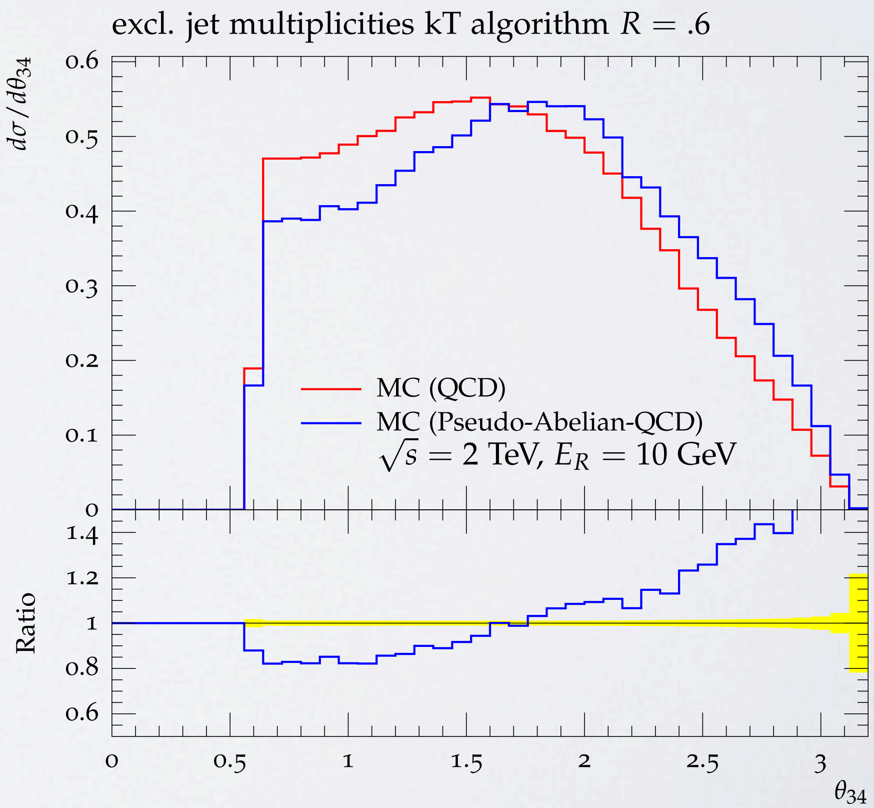
- Squared matrix element (still in the eikonal limit) tells us that secondary emission tend to be closer together in angular space.



$$|\mathcal{M}(p_1,p_2)|^2 = \frac{32C_F}{p_{T,1}p_{T,2}}\left[C_A\left(\frac{\cosh(\eta_1-\eta_2)}{\cosh(\eta_1-\eta_2)-\cos(\phi_1-\phi_2)}-1\right)+2\,C_F\right]$$

- Secondary emission for large (small) jets correspond to intra-jet (jet) evolution.





OTHER EFFECTS ON SCALING FROM e+e-

Phase space suppression

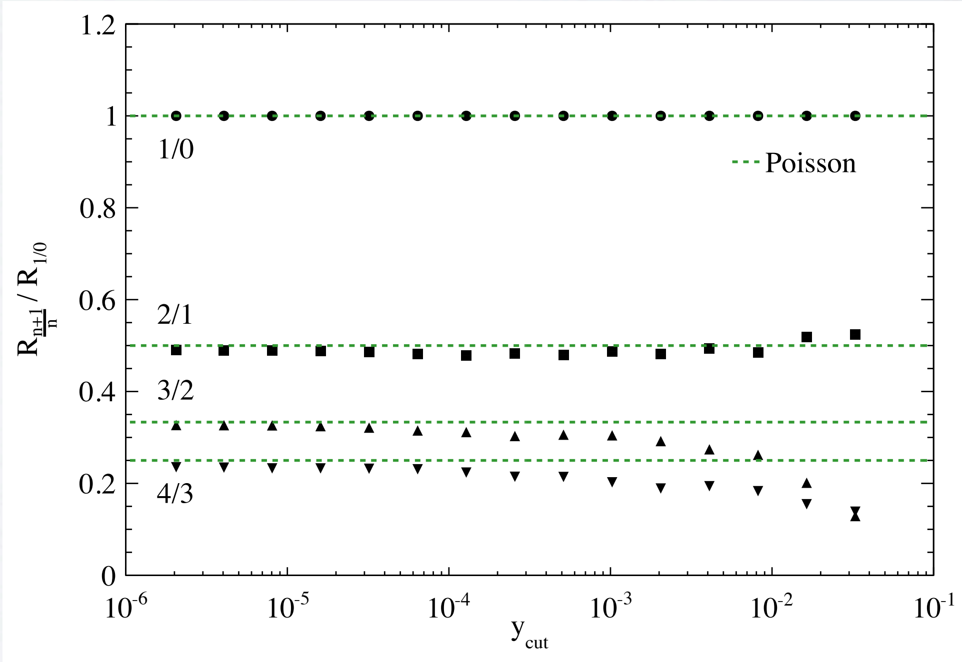
- Clearly there is a maximal number of jets which are energetically or geometrically possible in a certain process and selection.
- Most naive estimate of phase space suppression too small to explain the tilting in the staircase tail. (e.g. for R=.4, 10% over 20 multiplicities)

$$\frac{dR}{dn} \sim \frac{1-(n+1)R/4\pi}{1-nR/4\pi}$$

- But jets are preferentially emitted in the direction of the emitter, real phase space suppression much larger (though hard to estimate)

Matrix element corrections

- The full matrix elements of course do not have a reason to follow an exact scaling pattern. However, while the rates and ratios depend on these, the scaling (shape of the ratios in general does not).



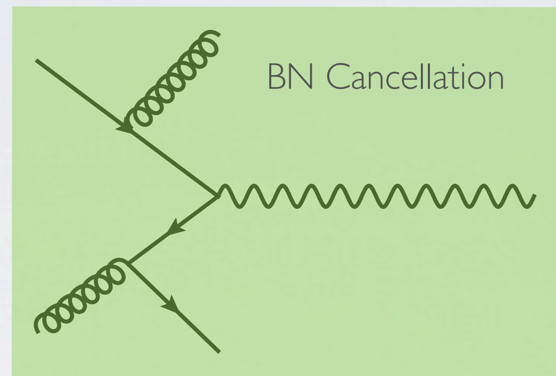
PREDICTIONS FOR EXCLUSIVE RATES

Rate for producing exactly n-jets accompanying a given hard process

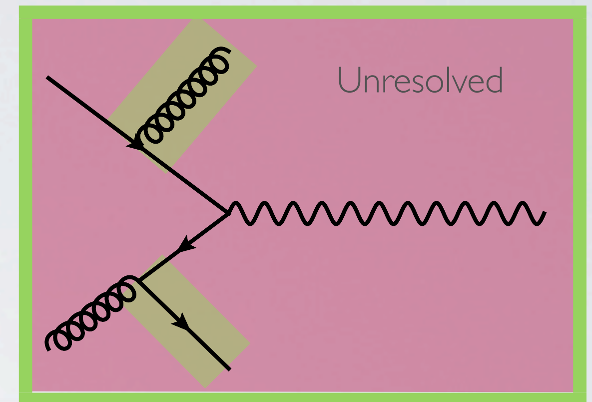
Bloch-Nordsieck theorem

- Rule of thumb: if the jet bin in question has a large unresolved component, then a fixed order calculation will not suffice

2 -jet inclusive



2 -jet exclusive



- High order calculation (NLO, NNLO...) contain unresolved components (in practice additional logarithms), but may still be a problem if too large (all orders approach necessary)

Analytic resummation

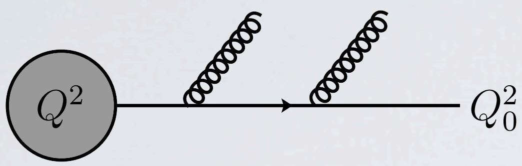
- A great amount of success in predictions for 0-jet exclusive cross sections (e.g. H + 0 jets) [Banfi, Salam, Zanderighi; Becher, Neubert ; Stewart, Tackmann, Walsh]
- Some work on exclusive H + 2 jet rates [Forshaw et al.]
- Generalizing these techniques to higher multiplicities still work in progress.

Parton Shower

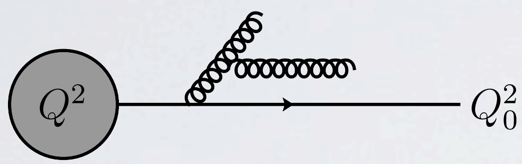
- In principle predictions for jet rates to arbitrarily high multiplicity.
- but...limitations on the formal accuracy (LL and NLL) of any prediction.
- Inherent limitations on PS evolution is the largest uncertainty in some analyses [POWHEG vs. MC@NLO]

RESUMMED RATES IN QCD AND SCALING

Resummed 2-gluon emission



$$\sigma^{\text{primary}}(Q^2, Q_0^2) = c^{\text{primary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2, t)\Delta_g(t) \int_{Q_0^2}^{Q^2} dt'\, \Gamma(Q^2, t')\Delta_g(t')$$



$$\sigma^{\text{secondary}}(Q^2, Q_0^2) = c^{\text{secondary}} \int_{Q_0^2}^{Q^2} dt\, \Gamma(Q^2, t)\Delta_g(t) \int_{Q_0^2}^{t} dt'\, \Gamma(t, t')\Delta_g(t')$$

- Key point is that the primary emissions are enhanced wrt secondary emissions as the size of the overall logarithm grows (effect completely missing in the fixed order calculation)

In kinematic limit	Primary emission	Secondary emissions
$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{Q_0} \ll 1$	$c^{\text{primary}} \log^4 \frac{Q}{Q_0}$	$c^{\text{secondary}} \log^4 \frac{Q}{Q_0}$
$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{Q_0} \gg 1$	$\frac{\alpha_s}{C_A} \log^2 \frac{Q}{Q_0}$	$\sqrt{\frac{\alpha_s}{C_A^3}} \log \frac{Q}{Q_0}$