# Precise predictions for squark and gluino production with soft and Coulomb resummation

Christian Schwinn

— Univ. Freiburg —

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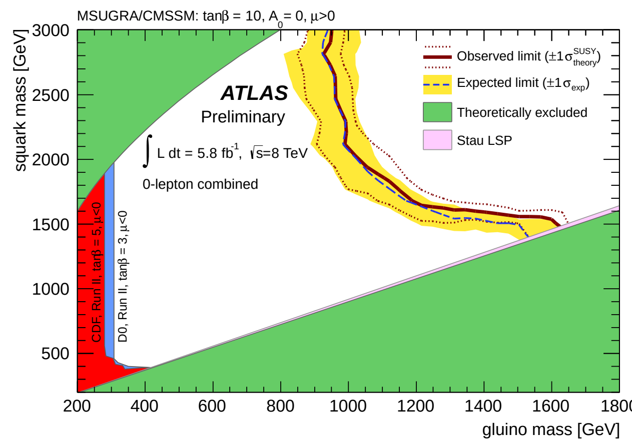
(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], arXiv:1007.5414 [hep-ph]

P.Falgari, CS, C.Wever, arXiv:1202.2260 [hep-ph], arXiv:1211.3408 [hep-ph]. )

**Pre-LHC expectation:** coloured sparticle production
dominant SUSY signal

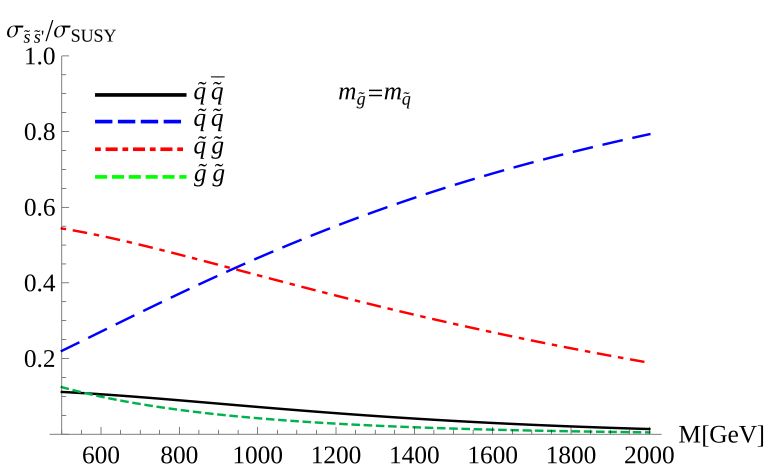
$$pp \to \{\tilde{q}\bar{\tilde{q}}\,,\quad \tilde{q}\tilde{q}\,,\quad \tilde{q}\tilde{g}\,,\quad \tilde{g}\tilde{g}\,,\quad \tilde{t}_i\bar{\tilde{t}}_i\}$$

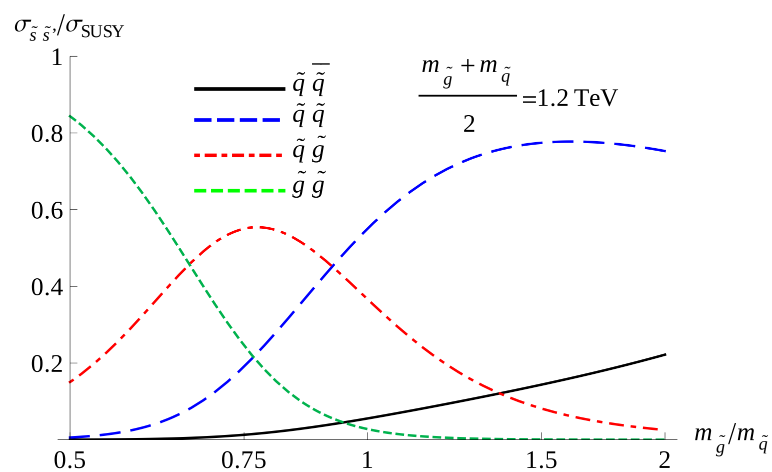



**Precise knowledge of cross sections:**

- can help to distinguish models
  (if new particles observed)
- improve exclusion bounds
  (if no new particles observed)

**Contributions** to total squark/gluino production



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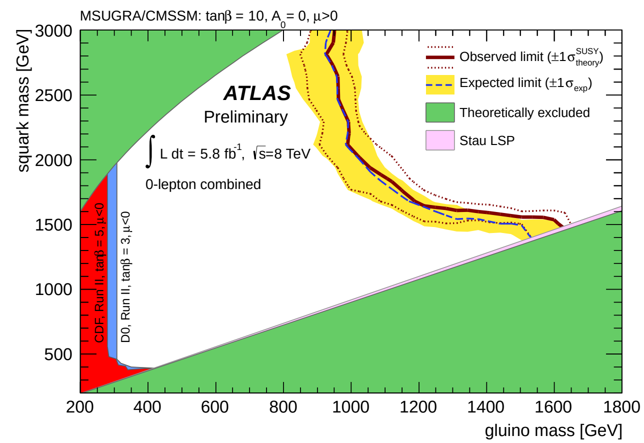


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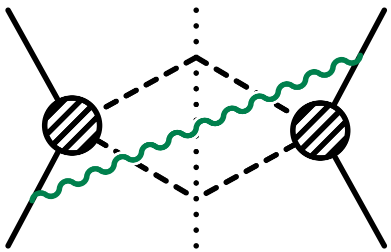
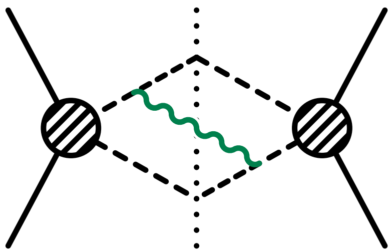
**Theory status:**

- NLO SUSY-QCD (Beenakker et al. 97, PROSPINO; Goncalves-Netto et al. 12, MADGOLEM)
- (N)NLL soft-gluon resummation (Beenakker et al. 09)
  NNLO$_{\text{approx}}$ for $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\tilde{g}$ (Langenfeld et al. 09–12)
- Bound state effects (Hagiwara, Yokoya 09, Kauth et al. 11)
- EW corrections (Bornhauser et al. 07; Germer/Hollik/Mirabella/Trenkel 08-11)

**Soft corrections:** (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)

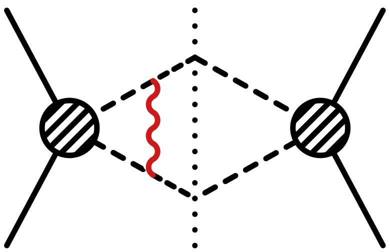


$\Rightarrow \alpha_s \, \log^2(8\beta^2)$



$\Rightarrow \alpha_s \, \log(8\beta^2)$

**Coulomb gluon corrections** (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

**Counting** of threshold corrections:

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\} :$$

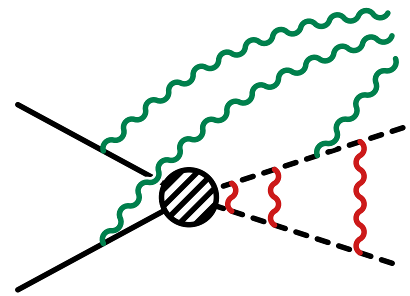
**Combination of Coulomb- and soft effects?**

Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \qquad |\vec{p}| \sim m\beta$$

soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation



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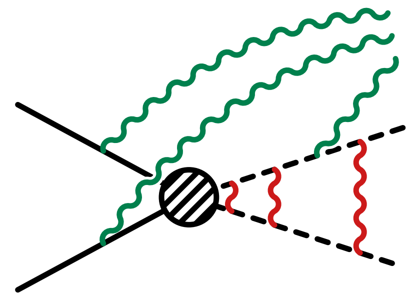

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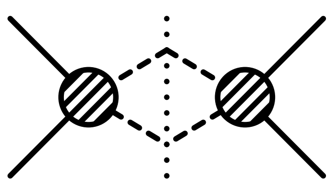
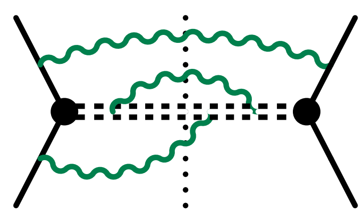
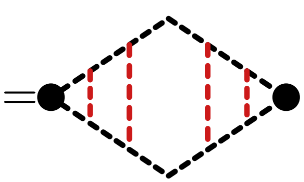
**Factorization** of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

Soft radiation "sees" only total colour charge $R$ of heavy particles

(Singlet, octet,...)

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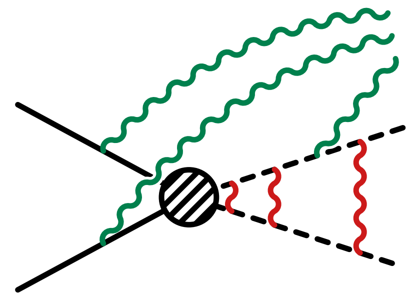

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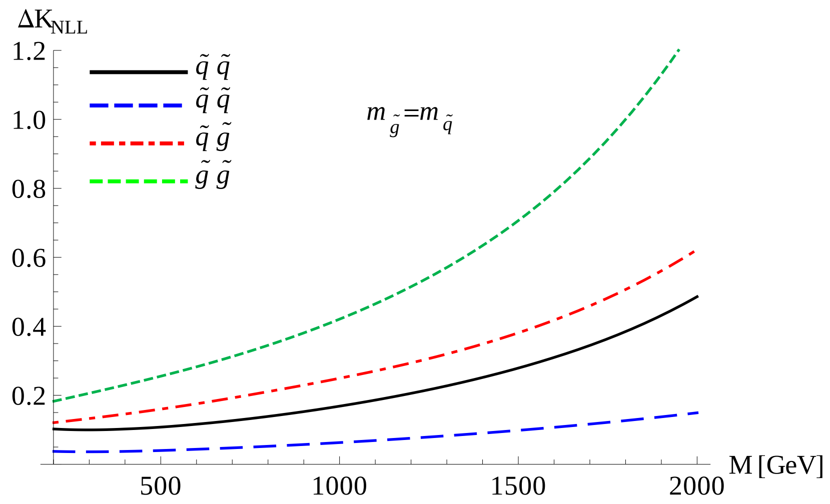
Can perform simultaneous summation:

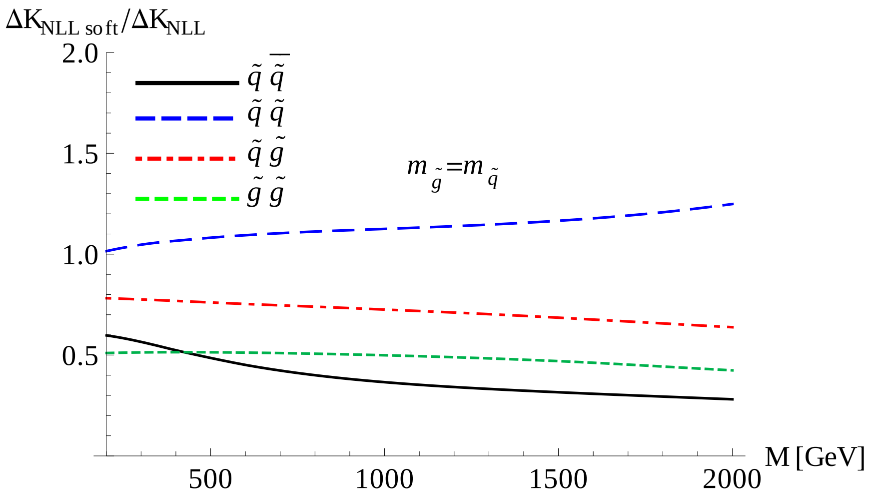
- threshold logs (using RGE running of $H$ and $W$, Becher, Neubert; Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)
- Coulomb corrections (using non-rel. Green Function, NRQCD,Fadin/Khoze 87; Beneke/Signer/Smirnov 99, . . . )

**NLL soft/Coulomb resummation**

for all squark/gluino production processes (Falgari/CS/Wever 12)

- Large corrections depending on process:
  $K_{\text{NLL}} = \sigma_{\text{NLL}}/\sigma_{\text{NLO}} \sim 10$–$120\%$
- Coulomb effects can be large



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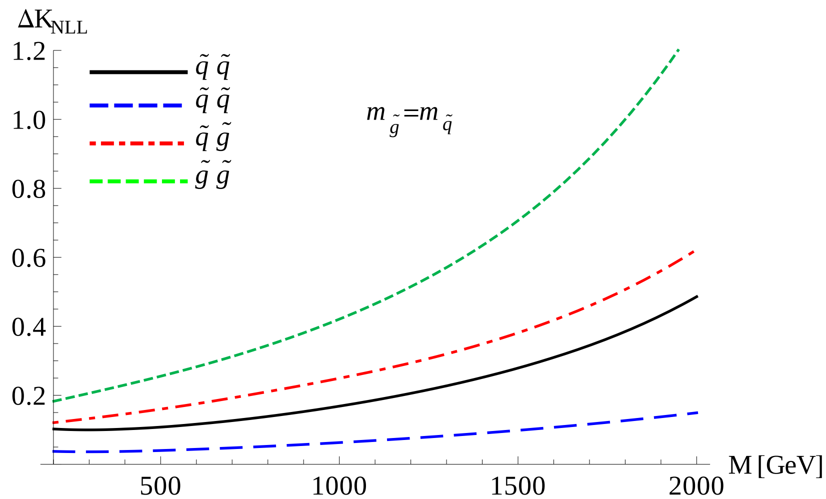
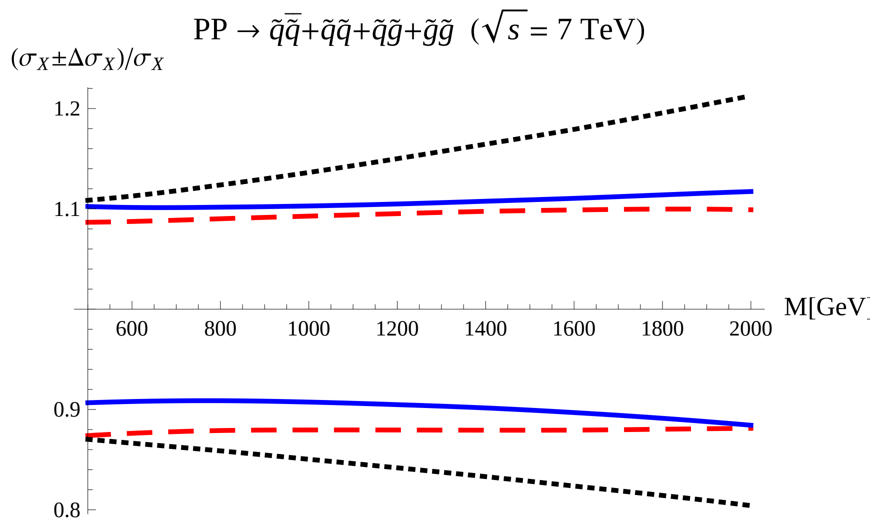
- Reduced scale dependence:

  $\pm 20\text{–}30\%$ (NLO)

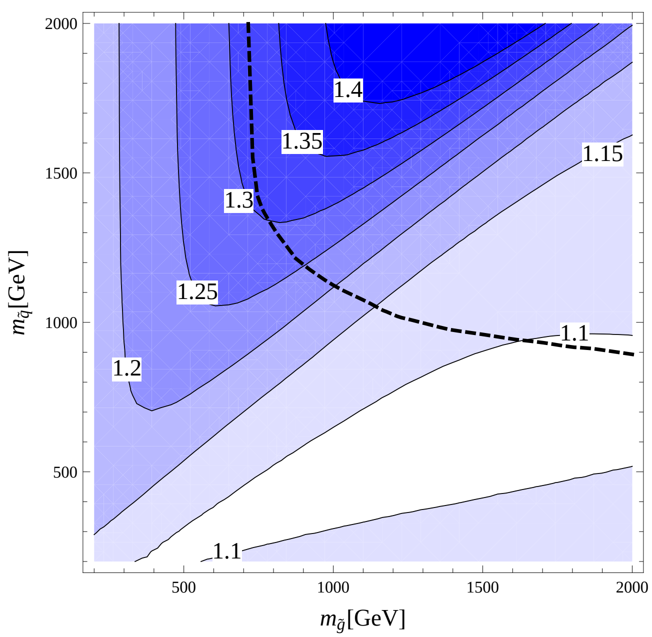
  $\Rightarrow \pm 10\text{–}15\%$ (NLL)

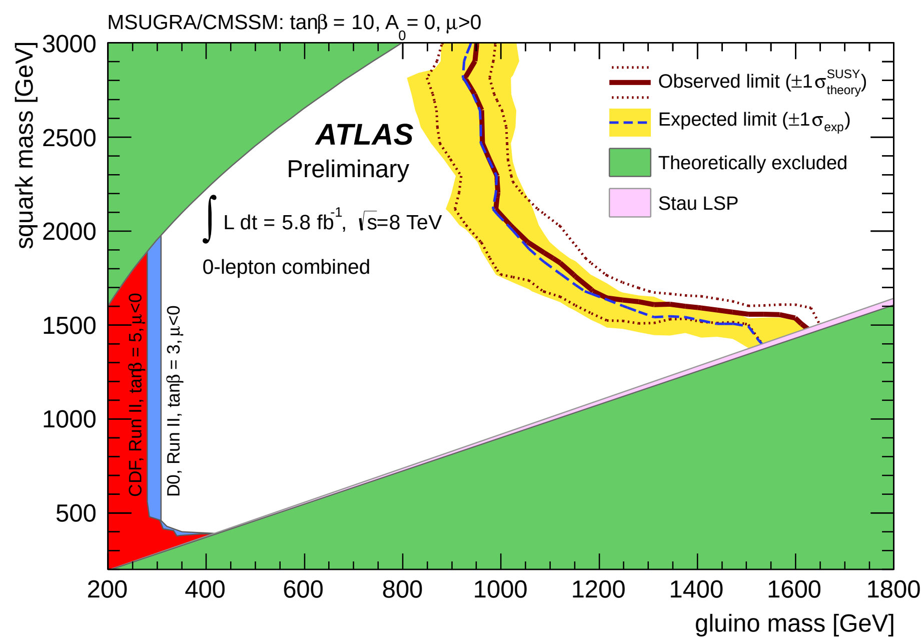


**Total coloured sparticle production**:

$\sigma_{\mathrm{NLL}}/\sigma_{\mathrm{NLO}}$ for LHC7:



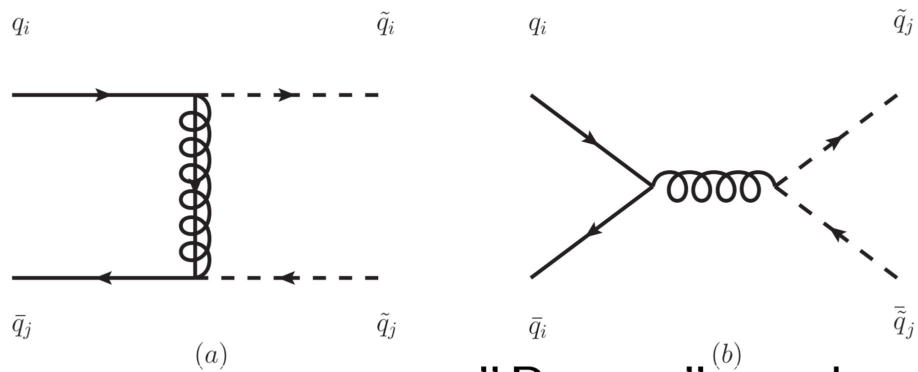




**Grids and interpolations for LHC7/8**

included with `arXiv:1202.2260` and available at

`http://omnibus.uni-freiburg.de/~cs1010/susy.html`

Difference to light-flavour squarks for $q\bar{q}$ initial state:

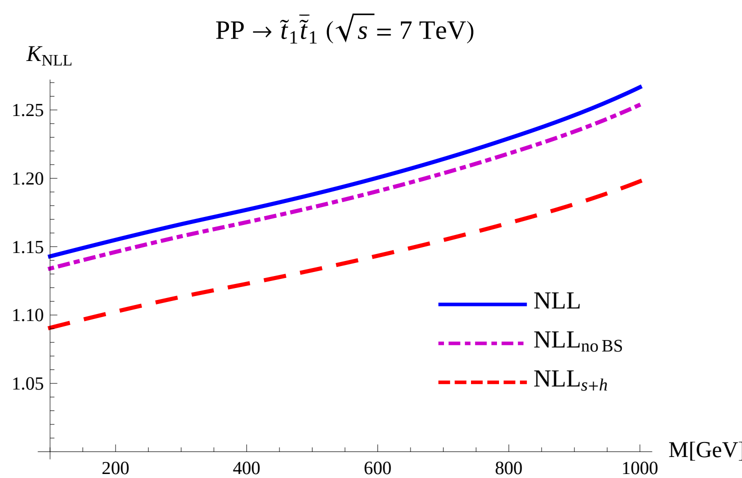



$\Rightarrow$ "P-wave" production, $\sim \beta^3$

absent for stops

Resummation formalism works also for $q\bar{q}$ channel (Falgari, CS, Wever 12)

use Coulomb Green function for $P$-waves (Bigi/Fadin/Khoze 92)



**Preliminary results**

for combined soft/Coulomb resummation for $\tilde{q}\bar{\tilde{q}}$:

- NLO hard function

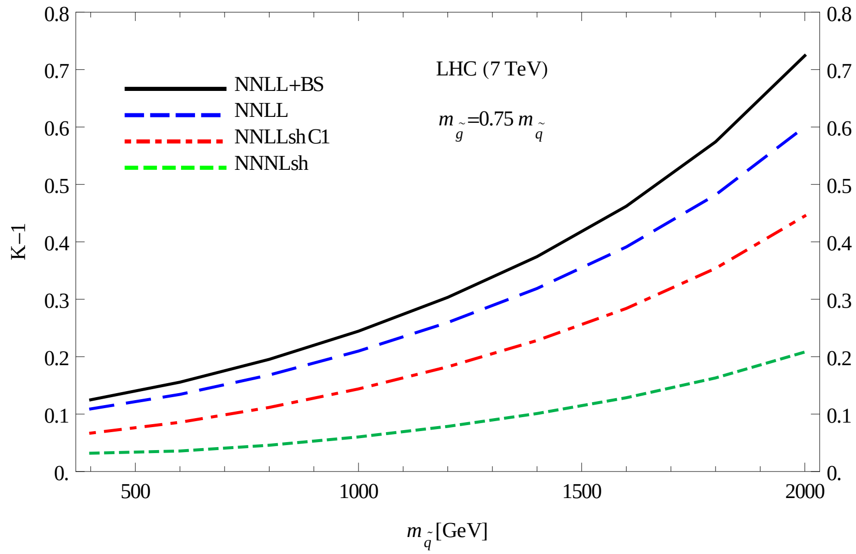
  (Beenakker et al. 11)

  ($\tilde{g}\tilde{g}$: Kauth et al. 11; Pfoh et al. 12;

  other channels ⇒ talk by S.Thewes)

- LO Coulomb function

  (NLO known up to non-Coulomb terms for scalars)




- fixed soft scale

⇒ Results with only first Coulomb: promising agreement with Mellin-space results (Beenakker et al. 11)

⇒ Higher-order Coulomb corrections non-negligible

(but expected to be lowered by NLO Coulomb function)

# Finite width effects

**Finite lifetimes** screen threshold singularities $\log\beta$, $\frac{1}{\beta}$ (Fadin, Khoze 87)

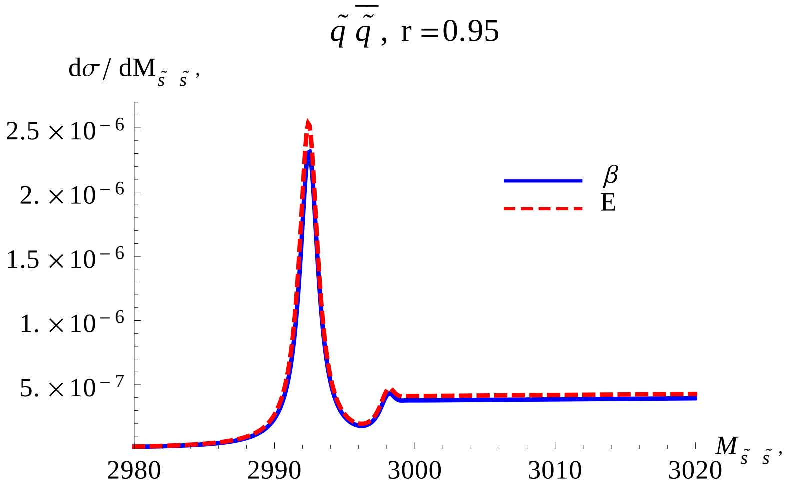
$$E \to E + i\Gamma\,, \qquad E = \sqrt{\hat{s}} - 2m_t \approx m_t\beta^2$$

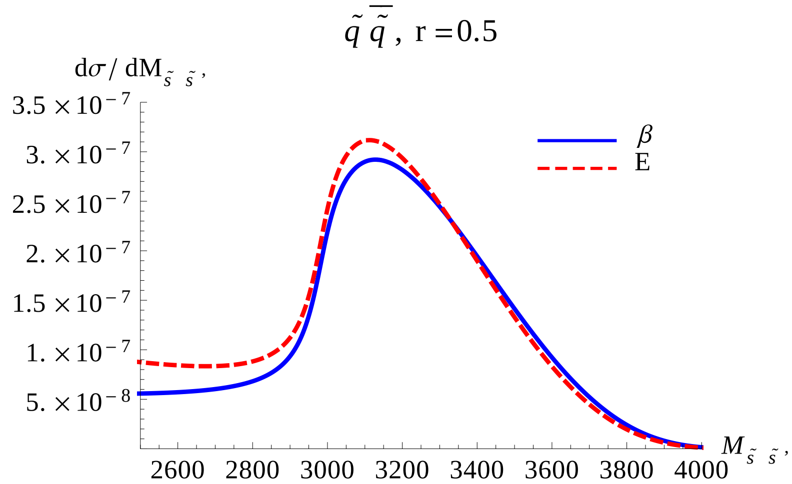
Bound-state poles $E_n = -\frac{\alpha_s^2 D_R^2 m_t}{4n^2}$ smeared-out by finite width

(Hagiwara,Yokoya 09; Kauth et al. 11, Falgari, CS, Wever 12)

E.g. $\tilde{q}\bar{\tilde{q}}$, ($m_{\tilde{q}} = 1.5$ TeV, LHC8)

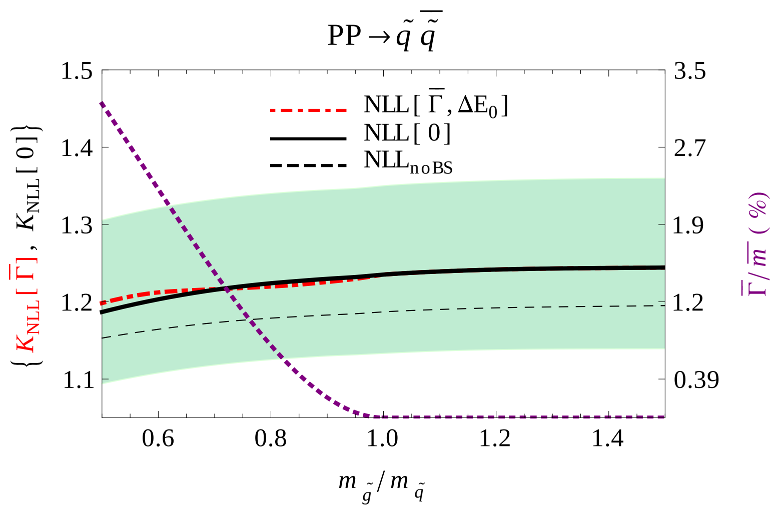
$$\frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 0.05\% \qquad\qquad \frac{\Gamma_{\tilde{q}}}{m_{\tilde{q}}} = 3.5\%$$

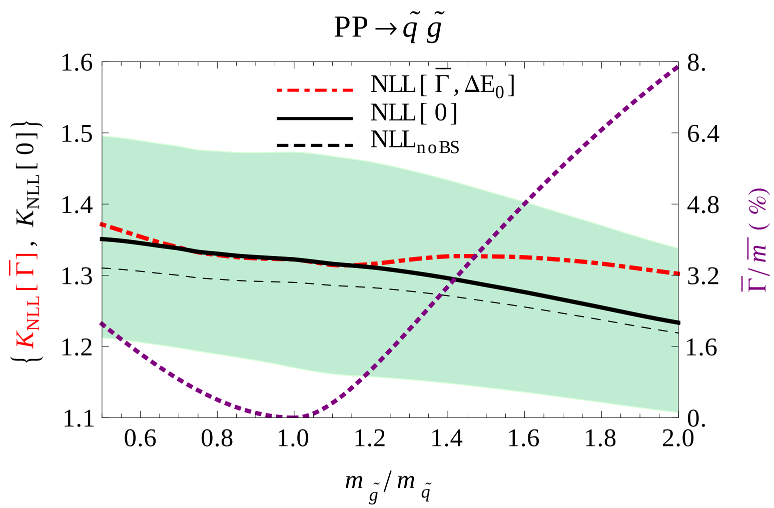


**Total cross sections**, LO-SQCD decays as example:

$$\Gamma_{\tilde{q}\to q\tilde{g}} = \frac{\alpha_s C_F m_{\tilde{q}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^2)^2 \,, \qquad m_{\tilde{q}}>m_{\tilde{g}}, \qquad \Gamma_{\tilde{g}\to q\bar{\tilde{q}}} = \frac{\alpha_s n_f m_{\tilde{g}}}{2}(1-(\frac{m_{\tilde{g}}}{m_{\tilde{q}}})^{-2})^2 \,, \qquad m_{\tilde{q}}<m_{\tilde{g}}.$$



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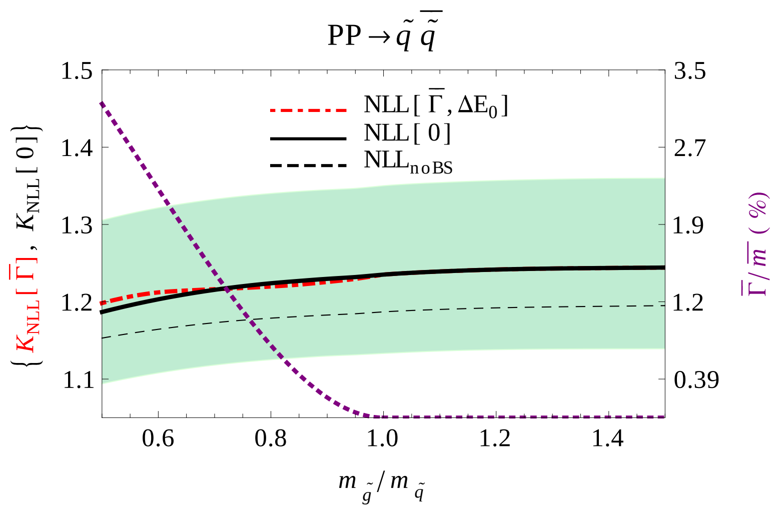
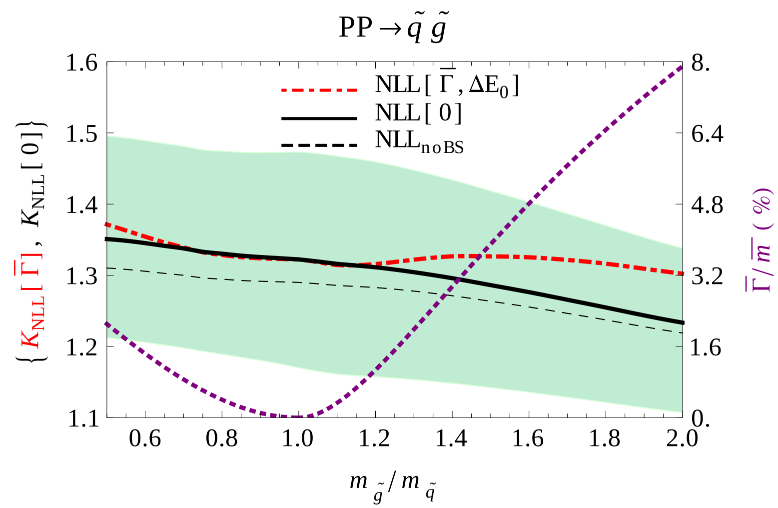
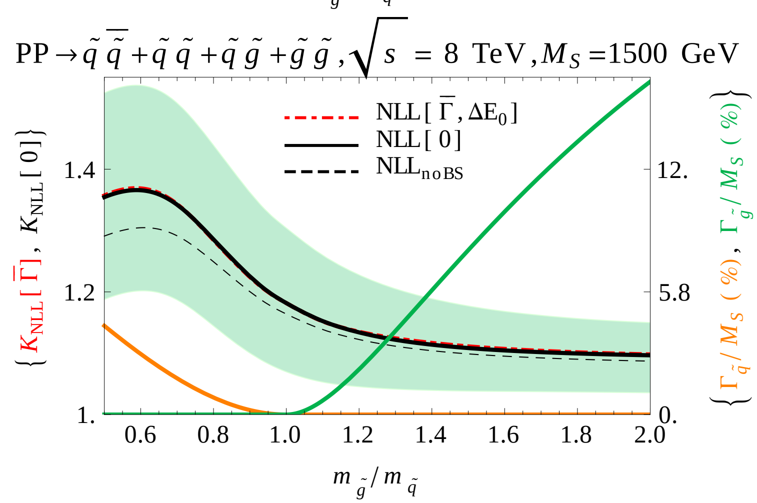



**Negligible effect**

on total SUSY production rate

(but bound-state corrections can be relevant)



**Threshold corrections** $\sim \log^n \beta, \frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible

**NLL resummation** for squark and gluino production

- Corrections from $10 - 15\%$ $(\tilde{q}\tilde{q})$ to $20 - 120\%$ $(\tilde{g}\tilde{g})$
- Coulomb corrections can be sizable
- scale dependence reduced to $10$–$15\%$

**NNLL resummation**

- preliminary results for squark-antisquark production

**Finite width effects**

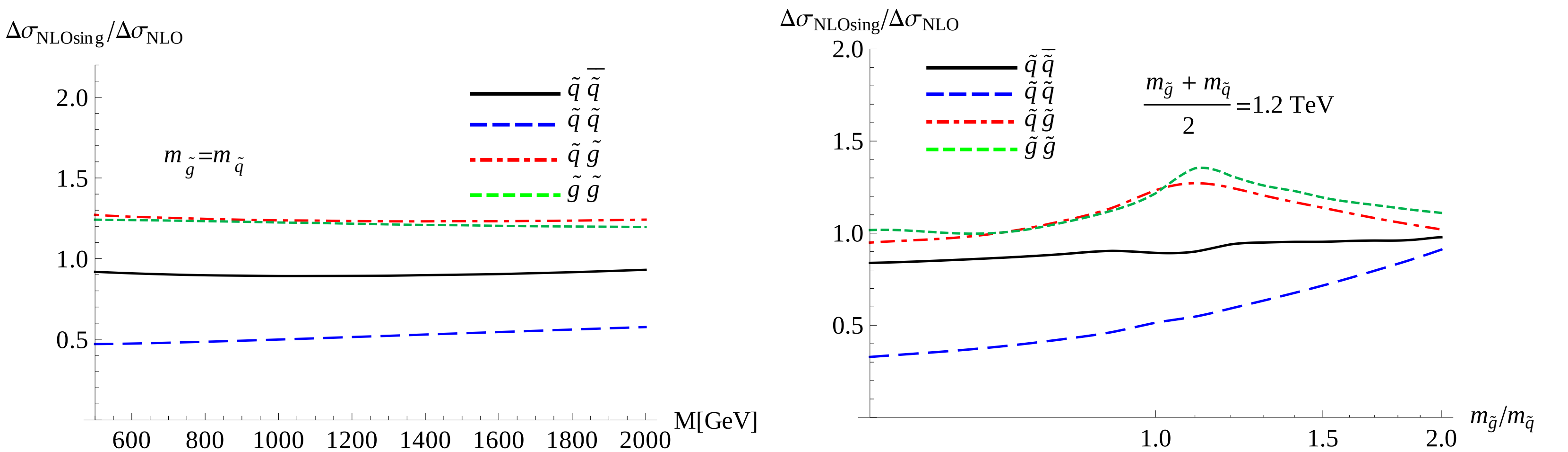
- Small effects on total cross section for $\frac{\Gamma}{m} \lesssim 5\%$

**Universal Threshold behaviour** (Beenakker et al. 97 , Beneke, Falgari, CS 09)

of process $pp' \to \tilde{s}\tilde{s}'$, $p, p' \in \{q, \bar{q}, g\}$ , $\tilde{s}, \tilde{s}' \in \{\tilde{q}, \bar{\tilde{q}}, \tilde{g}\}$

$$\sigma^{(R_\alpha)}_{\text{NLO,app}} = \sigma^{(0)} \frac{\alpha_s}{(4\pi)} \Bigg\{ -\frac{\pi^2 (C_{R_\alpha} - C_R - C_{R'})}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \ln^2\left(\frac{8M\beta^2}{\mu_f}\right) - 4(C_{R_\alpha} + 4(C_r + C_{r'})) \ln\left(\frac{8M\beta^2}{\mu_f}\right) \Bigg\}$$

**Accuracy of threshold approximation:** (NLO:PROSPINO, Plehn et al.)



# Resummation of threshold logarithms

Factorization scale dependence of $H$, $W$ cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher/Neubert; Ferroglia et al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

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**Resummation:**

Mellin-space approach:

(Sterman 87; Catani, Trentadue 89; Korchemsky, Marchesini 92)

Solve RGE using Mellin transform: ($\rho = 4m_t^2/\hat{s}$)

$$\sigma^N = \int_0^1 d\rho \rho^{N-1} \hat{\sigma}(4m_t^2/\rho) \ , \quad \int_0^1 d\rho \rho^N \beta \log^n \beta \propto \ln^n N + \ldots$$

Numerical inverse transformation

Factorization scale dependence of $H$, $W$ cancels against PDFs:

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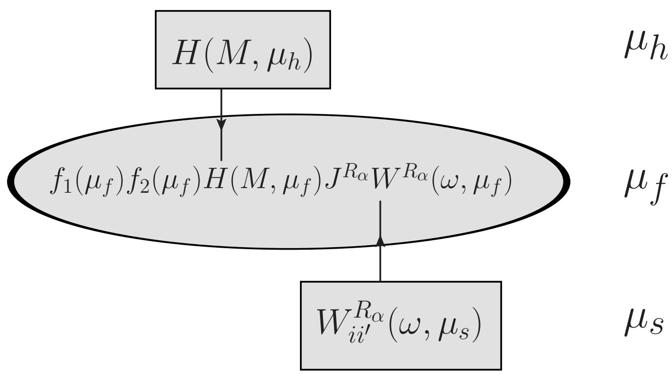
**Resummation:**

- Momentum-space solution to RGE ( Becher, Neubert, Pecjak 07)
- evolve hard function from $\mu_h \sim 2m_t$ to $\mu_f$
- evolve soft function from $\mu_s \sim m_t\beta^2$ to $\mu_f$



**Soft scale** choice in momentum-space resummation

Structure of resummation formula (NLL, no Coulomb summation)

$$\hat{\sigma}^{\text{NLL}} = \sum_{i,R_\alpha} \hat{\sigma}^{i,(0)} U_i^{R_\alpha}(\mu_s, \mu_f, \mu_h, m_t) \beta^{4\eta} \frac{\sqrt{\pi} e^{-2\eta\gamma_E}}{2\Gamma(2\eta + \frac{3}{2})}$$

$$U_i^{R_\alpha} = e^{-\frac{\alpha_s \Gamma_{\text{cusp}}}{2\pi} \log^2(\frac{\mu_s}{\mu_f}) + \ldots}, \qquad \eta = \frac{\alpha_s \Gamma_{\text{cusp}}}{2\pi} \log(\frac{\mu_s}{\mu_f}) + \ldots$$

**Expansion** in $\alpha_s$ generates all logs in $\hat{\sigma}$ for $\mu_s \sim m_t \beta^2$

**Resummation:** Cannot convolute $\exp\left[+c \log(1 - \frac{4m_t^2}{\hat{s}})\right]$ with PDFs

- multiply with PDFs in Mellin space (Catani et.al. 96)
- Introduce cutoffs (Berger/Contopagnaos 96; Bonvini/Forte/Ridolfi 10)

**RGE approach:** fixed $\mu_s$ that minimizes soft corrections to hadronic $\sigma$ (Becher, Neubert, Xu 07)

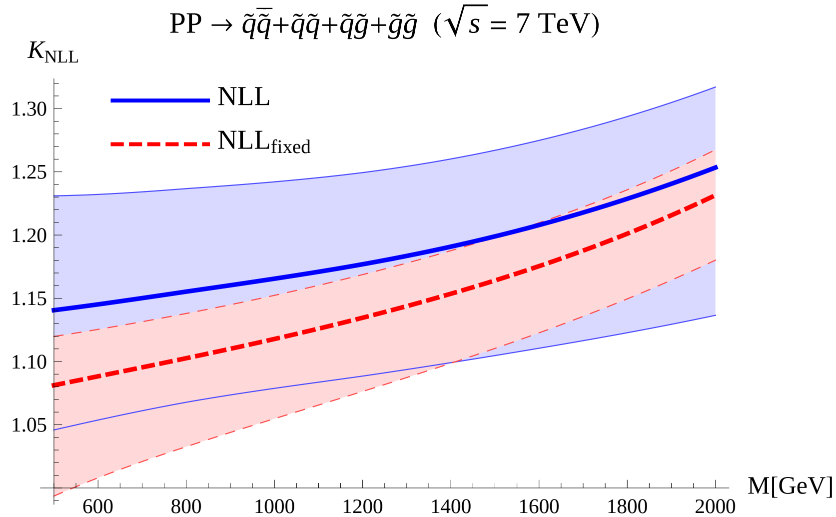
**Running scale** frozen at $\beta_{\text{cut}}$ (Beneke/Falgari/Klein/CS 11)

$$\mu_s = 2m_t \max\{\beta^2, \beta_{\text{cut}}^2\}$$

## Ambiguities in resummation

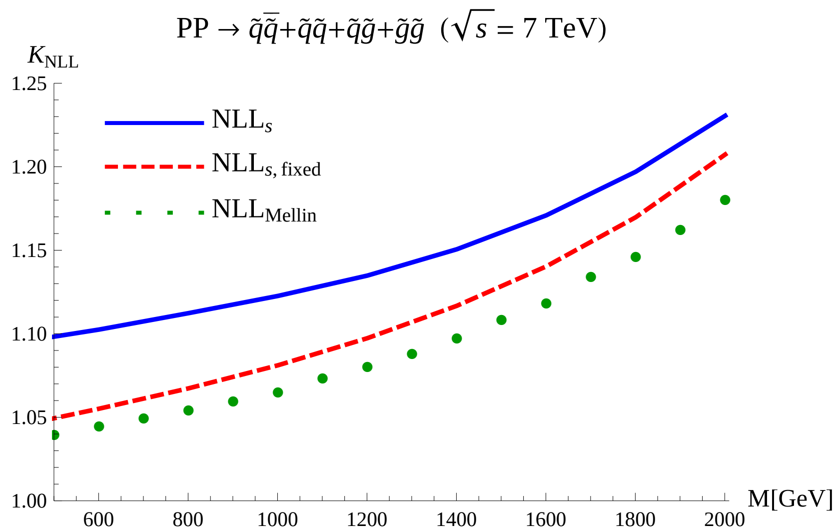
Running-scale and fixed-scale implementations agree within resummation uncertainties

($\beta_{\text{cut}}$, $\mu_s$ variation...)




For soft resummation reasonable consistency with Mellin-space resummation

(Beenakker et al. 09)




(LHC7, $m_{\tilde{q}} = m_{\tilde{g}}$)

# Coulomb effects

**Potential function** related to

Coulomb Green function: (Fadin, Khoze 87; Peskin, Strassler 90,...)

$$J_R(E) = 2\text{Im}\, G_C^R(0,0;E) = \begin{cases} \frac{m_t^2 \pi D_R \alpha_s}{2\pi}\left(e^{\pi D_R \alpha_s \sqrt{\frac{m_t}{E}}} - 1\right)^{-1} & E > 0 \\ \sum_{n=1}^{\infty} \delta(E - E_n) 2R_n & E < 0 \end{cases}$$

$$E = \sqrt{\hat{s} - 2m_t} \approx m_t \beta^2.$$

Bound-state poles at

$$E_n = -\frac{\alpha_s^2 D_R^2 m_t}{4n^2}$$

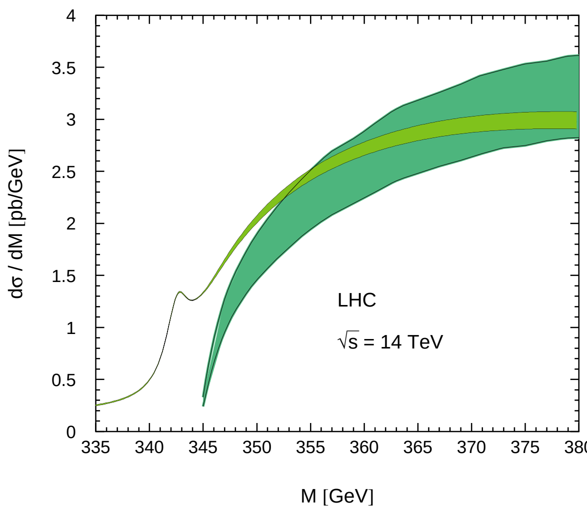
Smeared out by finite decay with

$$E \to E + \text{i}\Gamma$$

For total cross section:

use bound states for $\Gamma = 0$

(Top quark: $\Gamma_t/m_t < 1\%$)




(Hagiwara et.al. 08, Kiyo et.al. 08)