A Generalised Narrow-Width Approximation

for interference and higher-order effects in the MSSM.

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DESY

in collaboration with
Silja C. Thewes and Georg Weiglein

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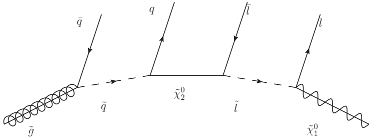
"Physics at the Terascale"

HELMHOLTZ GEMEINSCHAFT

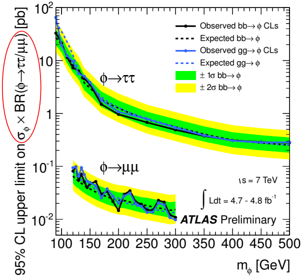
DESY

Motivation: Useful approximation

- SUSY: extended spectrum → typical cascade decays
- many-particle final state not always technically feasible
- ⇝ simplified by factorisation into **production**×**decay**
- application in MC generators, experimental limits



$$\sigma_{production} \times BR_1 \times BR_2 \times BR_3 \times BR_4$$



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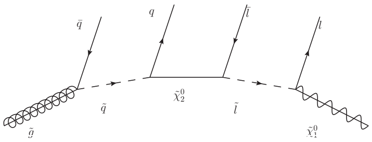

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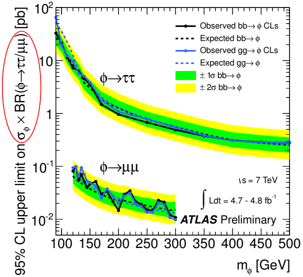

Extension necessary!

include interfering diagrams for intermediate particles with similar masses

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> **Generalised Narrow-Width Approximation (NWA)**
- Standard NWA
- Generalised NWA including interference term

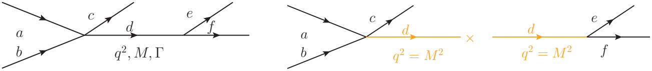
> **Tree-level results for example process** $\Gamma(\chi_4^0 \to \chi_1^0 \tau^+ \tau^-)$

> **Towards a generalised NWA at 1-loop**

> **Conclusion**

Standard NWA: Conditions and limitations

generic example:
$ab \xrightarrow{d} cef$



Factorisation: production × decay

- narrow width $\Gamma \ll M$
- kinematically open
- no interference with other processes

- on-shell production of particle with mass M, and subsequent decay:

$$\sigma_{ab\to cef} \approx \sigma_{ab\to cd}(q^2 = M^2) \cdot BR_{d\to ef}$$

- error of order $\mathcal{O}\left(\frac{\Gamma}{M}\right)$ [Uhlemann, Kauer '09]

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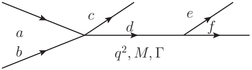

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Breakdown for mass degeneracy: interference effect

- NWA not applicable for $|M_i - M_j| \leq \Gamma_i, \Gamma_j$ (Breit-Wigner overlap)
- MSSM: for some parameters, h^0, H^0, A^0 have similar masses
- also relevant for other models
- extension of NWA required for interference term [Fowler, PhD Thesis '10]

Generalised NWA with interference term

$$\sigma(ab \to cef) = \frac{1}{F}\int d\Phi \left(\frac{|\mathcal{M}(ab \to ch)|^2 |\mathcal{M}(h \to ef)|^2}{(q^2 - M_h^2)^2 + M_h^2\Gamma_h^2} + \frac{|\mathcal{M}(ab \to cH)|^2 |\mathcal{M}(H \to ef)|^2}{(q^2 - M_H^2)^2 + M_H^2\Gamma_H^2} \right.$$

$$\left. + \, 2Re\left\{ \frac{\mathcal{M}(ab \to ch)\mathcal{M}^*(ab \to cH)\mathcal{M}(h \to ef)\mathcal{M}^*(H \to ef)}{(q^2 - M_h^2 + iM_h\Gamma_h)(q^2 - M_H^2 - iM_H\Gamma_H)} \right\} \right)$$

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3 steps for approximation of interference term

- matrix elements on-shell $\mathcal{M}(q^2 = M^2)$, but phase space $\Phi(q^2)$
- phase space on-shell $\Phi(q^2 = M^2)$
- approximation $M_h \approx M_H$: interference term as R-factors

$$\sigma \approx \sigma_{P_1} BR_1 \cdot (1 + R_1) + \sigma_{P_2} BR_2 \cdot (1 + R_2)$$
$$R_i = R_i(M, \Gamma, \sigma_P, BR, couplings, I)$$
$$I = \int \frac{dq^2}{2\pi} \Delta_1^{BW}(q^2) \cdot \Delta_2^{*BW}(q^2)$$

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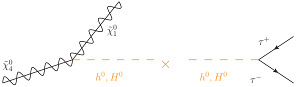
Example process: $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$



- $\mathbb{R}$MSSM ($\mathcal{CP}$): $h^0 - H^0$
- scenario with $m_h \lesssim m_H$

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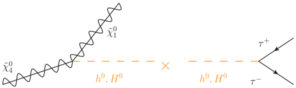

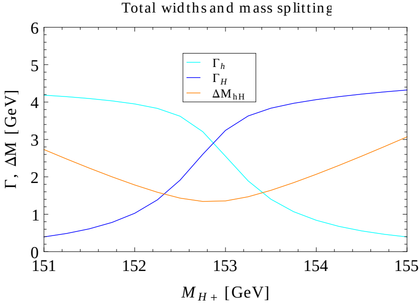

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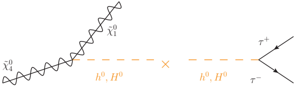

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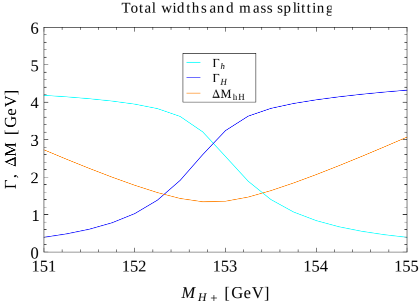


expect sizable interference around $\Delta M_{hH} \lesssim \Gamma_{h/H}$

Calculation

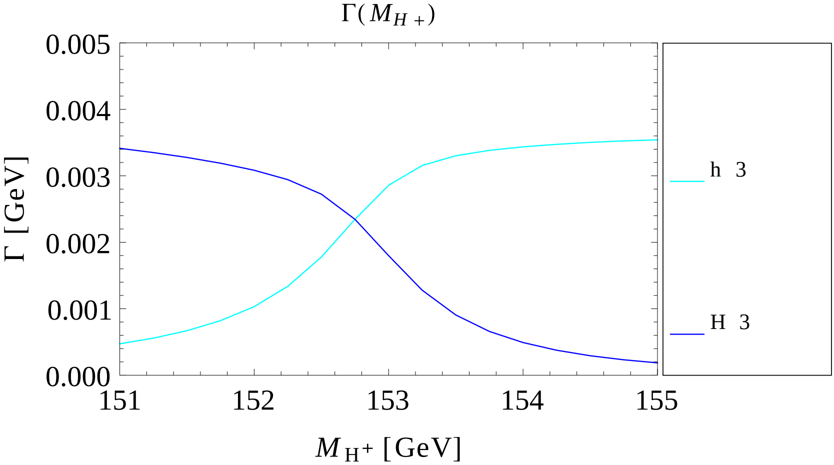
> 2-/3-body decays with FeynArts/ FormCalc

> interference term implemented in different approximations

> tree-level amplitudes with Breit-Wigner propagators

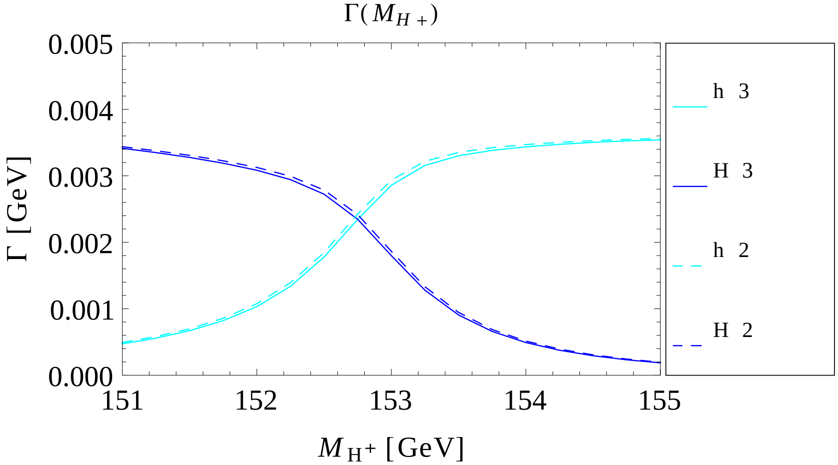
> 2-loop Higgs masses and widths from FeynHiggs

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



3-body decays with h^0 and H^0

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



dashed: 2-body decays×BR with h^0 and H^0

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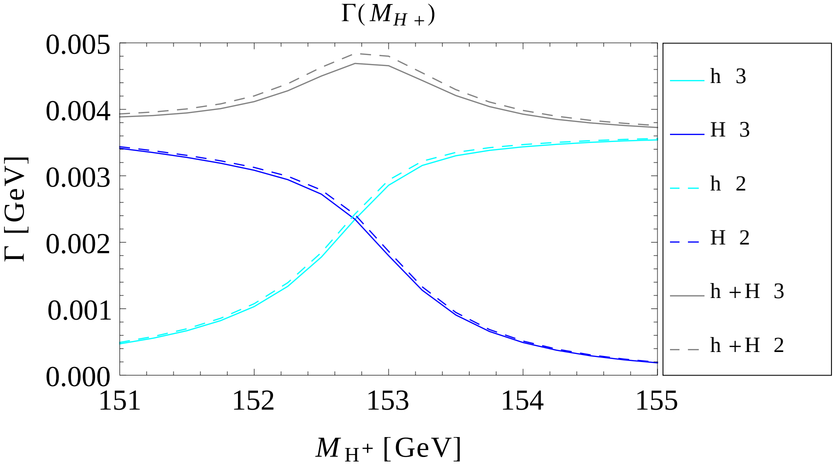
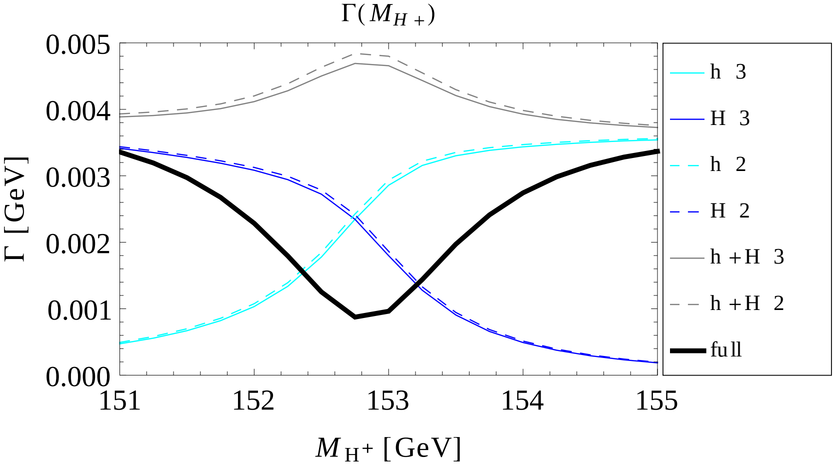


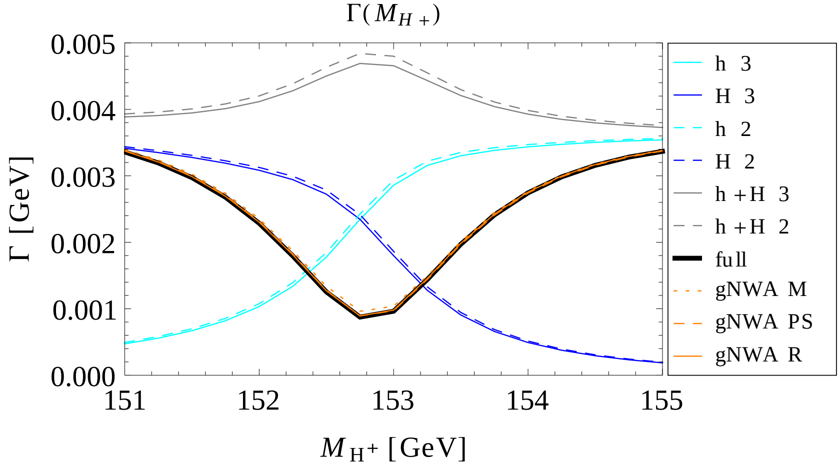
incoherent sum $|h^0|^2 + |H^0|^2$ of 3-/2-body decays

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



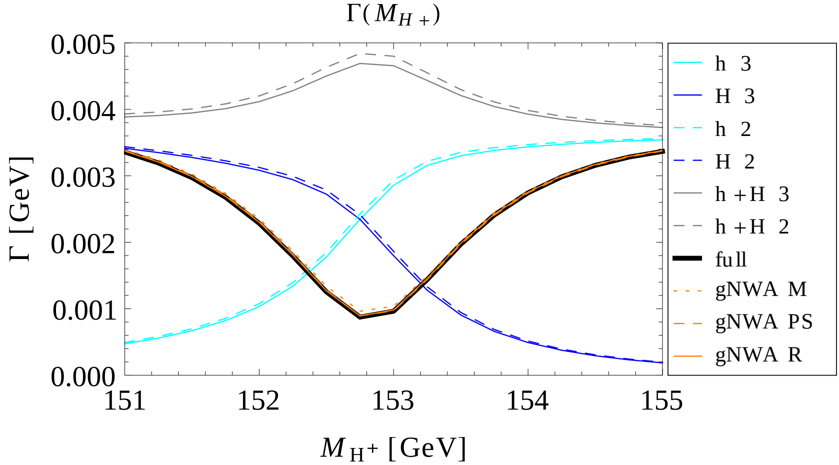
full 3-body decay $|h^0 + H^0|^2$ including interference term

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



generalised NWA including interference term

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



large interference effect neglected in **sNWA**, but well approximated by **gNWA**

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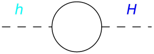
Renormalisation scheme

Motivation

radiative corrections to sub-processes possibly relevant

Higgs sector: $\overline{DR}$ [Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '07]

> correct on-shell properties of external Higgs bosons with mixing: $\hat{Z}_{ij}$



$$\Gamma_{h_i}^{(Z)} = \hat{Z}_{h_i h}\Gamma_h + \hat{Z}_{h_i H}\Gamma_H + ...$$

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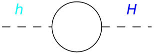


$$\Gamma_{h_i}^{(Z)} = \hat{Z}_{h_i h}\Gamma_h + \hat{Z}_{h_i H}\Gamma_H + \ldots$$

Neutralino sector: on-shell [Fowler, Weiglein '09] [Bharucha, Fowler, Moortgat-Pick, Weiglein '12]

[Chatterjee, Drees, Kulkarni, Xu '11] [Bharucha, Heinemeyer, Pahlen, Schappacher '12]

> 3 out of 6 $\tilde{\chi}^0, \tilde{\chi}^\pm$ masses on-shell

> choose most bino-, wino- and higgsino-like states as input
> $\rightarrow$ 3 parameters $|M_1|, |M_2|, |\mu|$ properly fixed

> otherwise: huge counterterms and unphysically large mass corrections

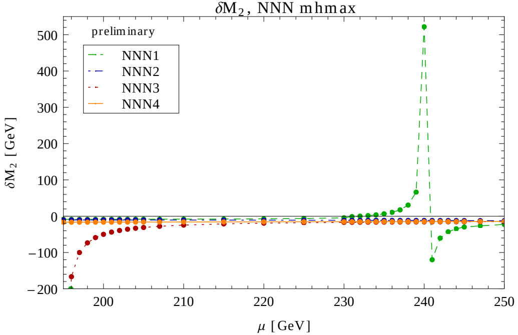
Neutralino renormalisation

Neutralino and chargino matrices

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \quad X = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}$$

Renormalisation constants and ΔM with 3 neutralinos on-shell

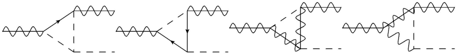
- **NNNi scheme:** $m_{\tilde{\chi}_i^0}$ and $m_{\tilde{\chi}_{1,2}^\pm}$ receive loop correction
- scenario with $\mu = M_2 = 200\,\text{GeV}$
- stable schemes: *here* NNN2, NNN4 with $\tilde{\chi}_2^0/\tilde{\chi}_4^0$ shifted



stability of scheme (proper parameter fixing): parameter dependent

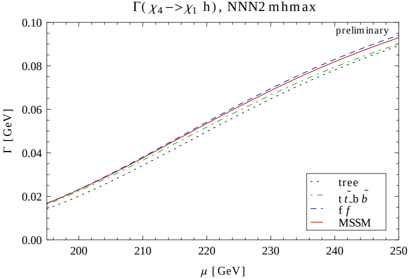
Vertex corrections to $\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 h$

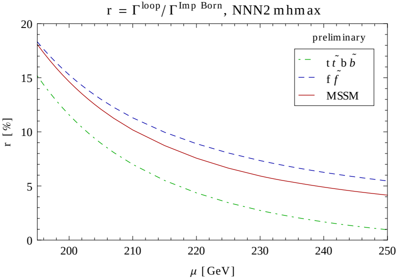
Triangle diagrams: (s)fermions and bosons/gauginos, e.g.



$\rightarrow$ LoopTools

Loop contibution from various sectors





significant loop corrections to Higgs production in $\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 h$ in a stable scheme

Higher-order corrections in the generalised NWA

- > separate calculation of loop corrections to production and decay
- > approximation of interference term based on NLO matrix elements
- > calculate Γ, M and couplings at high precision (e.g. FeynHiggs)

$\Longrightarrow$

combination of higher-order corrections to sub-processes in **generalised NWA**

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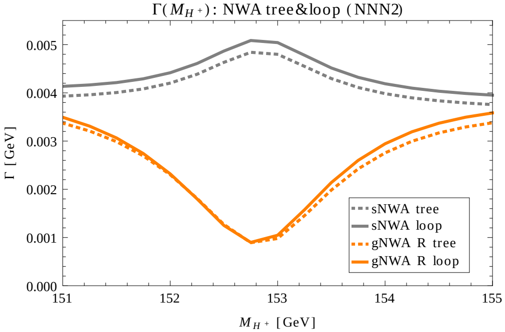

large interference effect also in the approximated 1-loop result

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Summary: significant improvement of accuracy by generalised NWA

- example: decay $\tilde{\chi}_4^0 \overset{h^0,H^0}{\rightarrow} \tilde{\chi}_1^0 \tau^+ \tau^-$ with interference
  - standard NWA overestimates Γ for $\Delta M_{hH} \leq \Gamma_{h,H}$
  - generalised NWA approximates full width to a few percent accuracy
- approximation extended to include loop-corrections
  - vertex corrections to $\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^0 h/H$: renormalisation of $\tilde{\chi}^0, \tilde{\chi}^{\pm}$-sector

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Outlook: $\mathcal{CP}$-violating mixing, more complicated processes

- $\mathbb{C}$MSSM, $\mathcal{CP}$-violation $\Rightarrow H^0 - A^0$ interference
- relevant for $\sigma_{H^0} + \sigma_{A^0} \xrightarrow{\not{\mathcal{CP}}} \sigma_{H^0+A^0} \approx (2\sigma_{H^0})_{\mathcal{CP}} + \sigma_{int,\not{\mathcal{CP}}}$
- combination of most advanced results for production and branching ratios with appropiate prediction for the interference term

Appendix

Thank you for your attention!

... questions?

NWA basics

Factorisation of the n-particle phase space $d\Phi_n$

> $d\Phi_n \equiv dlips\,(P; p_1, ..., p_f) = (2\pi)^4 \delta^{(4)}(P - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f}$

> here: kinematics of 3-body decay $\rightarrow$ 2-body
$d\Phi = dlips(\sqrt{s}; p_c, p_e, p_f) = dlips(\sqrt{s}; p_c, q) \frac{dq^2}{2\pi} dlips(q; p_e, p_f)$

Production$\times$decay

> instead of Breit-Wigner propagator $\frac{1}{q^2 - M^2 + iM\Gamma}$

> on-shell production of particle with mass M, and subsequent decay:

$$\sigma_{ab\rightarrow cef} \approx \sigma_{ab\rightarrow cd}(q^2 = M^2) \cdot BR_{d\rightarrow ef}$$

> error of order $\mathcal{O}\left(\frac{\Gamma}{M}\right)$ [Uhlemann, Kauer '09]

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$$\left. + 2Re \left\{ \frac{\mathcal{M}(ab \to ch)\mathcal{M}^*(ab \to cH)\mathcal{M}(h \to ef)\mathcal{M}^*(H \to ef)}{(q^2 - M_h^2 + iM_h\Gamma_h)(q^2 - M_H^2 - iM_H\Gamma_H)} \right\} \right)$$

$$\overset{\mathcal{M}\ on-shell}{\approx} \sigma_{ab\to ch} BR_{h\to ef} + \sigma_{ab\to cH} BR_{H\to ef}$$

$$+ \frac{2}{F} \mathrm{Re} \left\{ \int \frac{dq^2}{2\pi} \left(\Delta_1^{BW}(q^2) \Delta_2^{*BW}(q^2) \left[\int d\Phi_P(q^2) \mathcal{M}_{P_1}(M_1^2) \mathcal{M}^*_{P_2}(M_2^2) \right] \right. \right.$$

$$\left. \left. \left[\int d\Phi_D(q^2) \mathcal{M}_{D_1}(M_1^2) \mathcal{M}^*_{D_2}(M_2^2) \right] \right) \right\}$$

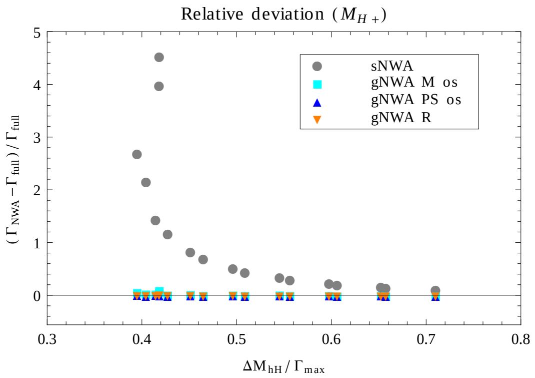
$$\overset{M_h \simeq M_H}{\approx} \sigma_{P_1}\, BR_1 \cdot (1 + R_1) + \sigma_{P_2}\, BR_2 \cdot (1 + R_2)$$

$$R_i := 2M_i \Gamma_i w_i \cdot 2\mathrm{Re}\{x_i I\}$$

$$w_i := \frac{\sigma_{P_i}\, BR_i}{\sigma_{P_1}\, BR_1 + \sigma_{P_2}\, BR_2}$$

$$x_i := \frac{g_{P_i} g^*_{P_j} g_{D_i} g^*_{D_j}}{|g_{P_i}|^2 |g_{D_i}|^2} \qquad (g_{P/D} : \text{couplings in production/ decay})$$

Evaluation of the interference term



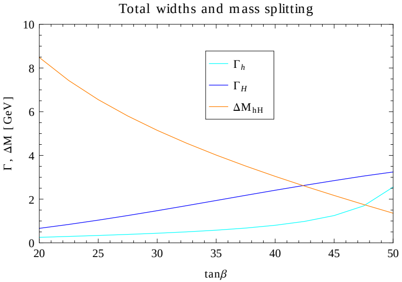
$$\frac{\Delta M_{hH}}{\Gamma_{max}} \equiv \frac{M_H - M_h}{max\{\Gamma_h, \Gamma_H\}}$$

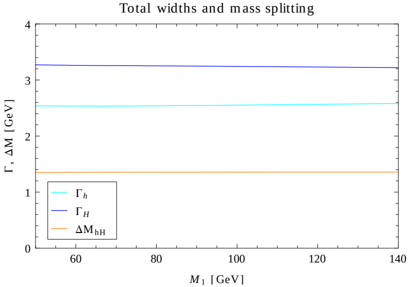
Improvement of accuracy

- standard NWA: overestimation of full width by up to factor 5
- generalised NWA: approximation of interference term to $2 - 3\%$

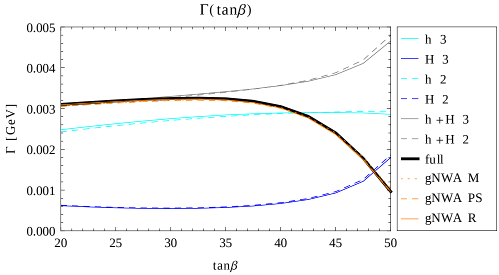
Masses and widths

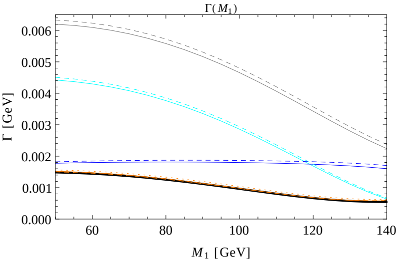
Higgs masses and widths depending on $\tan\beta$ and M_1





Dependence on $\tan\beta$ and M_1





Parameter dependence of the interference term

- more sensitive to parameters from Higgs sector than from neutralino sector
  - interference effect large only at high $\tan\beta$ (smaller mass difference)
  - for fixed $M_{H^+}, \tan\beta$: large effect throughout M_1 interval
- good performance of the generalised NWA in the analysed parameter space