

# Towards a theory for multiple hard scattering in QCD

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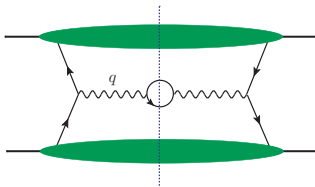
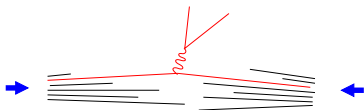
## Hadron-hadron collisions

- ▶ standard description based on factorization formulae

cross sect = parton distributions  $\times$  parton-level cross sect

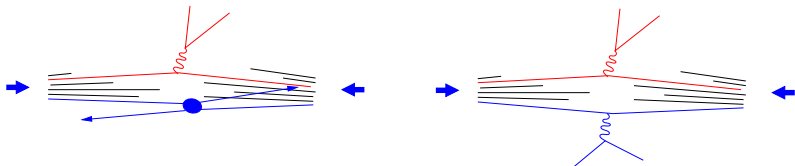
example:  $Z$  production

$$pp \rightarrow Z + X \rightarrow \ell^+ \ell^- + X$$



- ▶ factorization formulae are for **inclusive** cross sections  $pp \rightarrow Y + X$  where  $Y =$  produced in parton-level scattering, specified in detail  
 $X =$  summed over, no details
- ▶ have also interactions between “spectator” partons
  - typically affect unspecified system  $X \rightsquigarrow$  **underlying event**  
these effects cancel in inclusive cross sections thanks to unitarity
  - **but** can also produce particles with **large**  $p_T$  or **large** mass  
 $\rightsquigarrow$  **multiparton interactions**

## Multiparton interactions (MPI)



- ▶ characteristic final-state signature:  
individual particles or groups of particles have  
 $p_T \ll \text{large scale of process}$
- ▶ can affect both signal and background processes  
e.g.  $pp \rightarrow H + Z \rightarrow b\bar{b} + Z$  and  $pp \rightarrow b\bar{b} + Z$  Del Fabbro, Treleani 1999
- ▶ expected to be important for many processes at LHC  
see e.g. Procs. of MPI 2011 (DESY-PROC-2012-03)  
significant effect found in  $pp \rightarrow W + 2 \text{ jets} + X$ , ATLAS-CONF-2011-160
- ▶ in the following restrict ourselves to **double parton interactions**

## Double parton interactions (DPI)

- phenomenology based on simple, physically intuitive formula
  - cross sect = double parton distributions  $\times$  parton-level cross sect's
  - and simple ansatz for double parton distributions

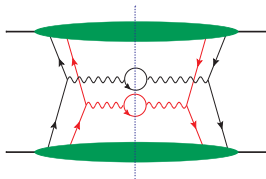
- typically assume

$$F_{ab}(x_1, x_2, \mathbf{y}) = f_a(x_1) f_b(x_2) F(\mathbf{y})$$

where  $\mathbf{y}$  = transverse distance  
between partons 1 and 2

- then get pocket formula  $\sigma_{\text{double}} = \sigma_1 \sigma_2 / \sigma_{\text{eff}}$   
with  $\sigma_{\text{eff}}$  depending on  $F(\mathbf{y})$

experimentally find:  $\sigma_{\text{eff}} \sim 11 \dots 15 \text{ mb}$



Tevatron, LHC

## Double parton interactions (DPI)

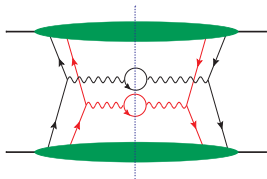
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with  $\sigma_{\text{eff}}$  depending on  $F(\mathbf{y})$
  - but independence of long. mom. and transv. positions is approximate  
for single partons have evidence for correlations from
    - diffractive vector meson production at HERA ( $x \sim 10^{-3}$ )
    - lattice calculations (dominated by  $x \sim 0.2 \dots 0.4$ )
- correlations have observable effects in DPI



Corke, Sjöstrand 2011

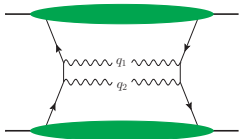
## Single versus double hard scattering

- ▶ consider gauge boson pair prod'n (analogous results for other processes)

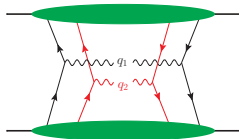
$$s \frac{d\sigma}{dx_1 d\bar{x}_1 d^2\mathbf{q}_1 dx_2 d\bar{x}_2 d^2\mathbf{q}_2} \sim \frac{1}{Q^2 \Lambda^2}$$

$$Q^2 \sim q_i^2 \gg \Lambda^2 \sim q_i^2$$

for both



and



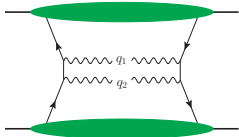
⇒ double scattering **not** power suppressed

## Single versus double hard scattering

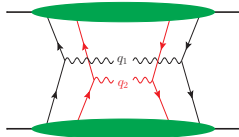
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- but if integrate over  $\mathbf{q}_1$  and  $\mathbf{q}_2$  then

$$\text{single: } s \frac{d\sigma}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} \sim 1 \quad \text{since } \int d^2(\mathbf{q}_1 + \mathbf{q}_2) \sim \Lambda^2$$

$$\text{and } \int d^2(\mathbf{q}_1 - \mathbf{q}_2) \sim Q^2$$

$$\text{double: } s \frac{d\sigma}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2} \quad \text{since } \int d^2\mathbf{q}_1 \int d^2\mathbf{q}_2 \sim \Lambda^4$$

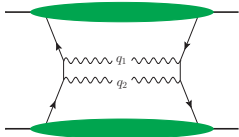
i.e. single hard scattering has **larger phase space** for transv. momenta

## Single versus double hard scattering

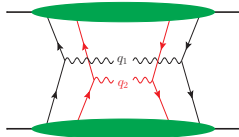
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- ▶ at small  $x_1 \sim x_2 \sim x$  expect

- single scattering  $\propto x^{-\lambda}$
- double scattering  $\propto x^{-2\lambda}$

with  $xf(x) \sim x^{-\lambda}$

⇒ double scattering **enhanced** at small  $x$



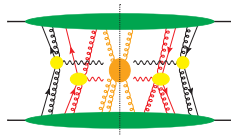
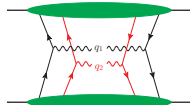
## Toward a factorization formula

- ▶ master formula for measured transverse momenta

$$\frac{d\sigma}{dx_1 d\bar{x}_1 d^2\mathbf{q}_1 dx_2 d\bar{x}_2 d^2\mathbf{q}_2} = \hat{\sigma}_{ac}(q_1^2) \hat{\sigma}_{bd}(q_2^2) \times \left[ \prod_{i=1,2} \int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i) \right] \int d^2\mathbf{y} F_{ab}(x_i, \mathbf{k}_i, \mathbf{y}) F_{cd}(\bar{x}_i, \bar{\mathbf{k}}_i, \mathbf{y})$$

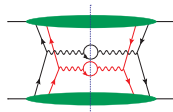
can be derived at tree level from Feynman graphs and kinematic approx's  
no semi-classical arguments required MD, Ostermeier, Schäfer 2011

- ▶  $F(x_i, \mathbf{k}_i, \mathbf{y}) = k_T$  dependent double parton distribution  
recover standard formalism by  $\int$  over transv. momenta
- ▶ for factorization proof must show how to treat additional gluons
  - partly under theoretical control  
derivation of Sudakov factors from soft/collinear gluons
  - unsolved problem: gluons from Glauber region could spoil factorization



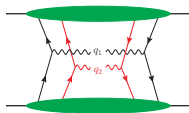
## Spin structure

$$\frac{d\sigma}{dx_1 d\bar{x}_1 d^2\mathbf{q}_1 dx_2 d\bar{x}_2 d^2\mathbf{q}_2} = \hat{\sigma}_{ac}(q_1^2) \hat{\sigma}_{bd}(q_2^2) \times \left[ \prod_{i=1,2} \int d^2\mathbf{k}_i d^2\bar{\mathbf{k}}_i \delta(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i) \right] \int d^2\mathbf{y} F_{ab}(x_i, \mathbf{k}_i, \mathbf{y}) F_{cd}(\bar{x}_i, \bar{\mathbf{k}}_i, \mathbf{y})$$



- ▶ even in unpolarized proton can have spin correlations between partons
  - unpolarized, longitudinal pol., transverse pol. quarks
  - unpol., longitudinal pol., linear pol. gluons
- ▶ in general **not** suppressed in hard scattering consequences for rate and distributions
- ▶ detailed calc'n for gauge boson pair production followed by leptonic decay  
Kasemets, MD 2012; see also Manohar, Waalewijn 2011
- ▶ transverse quark spin correlation  
 $\rightsquigarrow$  **cos 2φ modulation** between decay planes of the two bosons  
 in general: **correlated** scattering planes
- ▶ how large are spin correlations?  
 find **large** effects in constituent quark region  
 MD, Ostermeier, Schäfer 2011; Chang, Manohar, Waalewijn 2012

## Color structure



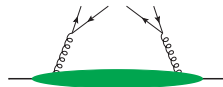
- ▶ quark lines with given  $x_i$  in amplitude and conjugate amp. can couple to color singlet or color octet  
 for two-gluon dist's more color structures:  $1, 8_S, 8_A, 10, \overline{10}, 27$ 
  - ▶ color singlet distributions: analog to single-parton case
  - ▶ color octet distributions essentially unknown  
 no probability interpretation as a guide  
 not implemented in phenomenology
- ▶ for  $k_T$  integrated cross sections/distributions: color octet suppressed by Sudakov logarithms Mekhfi 1988; Manohar, Waalewijn 2011  
 but not necessarily negligible for moderately hard scales
- ▶ in  $k_T$  unintegrated case:  
 Sudakov factors for both color singlet and color octet  
 slightly stronger suppression for octet MD, Ostermeier, Schäfer 2011

## Problems at short distance

- ▶ for short interparton distance  $\mathbf{y}$  can compute double parton distribution from splitting graphs

$$F(x_i, \mathbf{k}_i, \mathbf{y}) \sim \log \mathbf{y}^2$$

$$F(x_i, \mathbf{y}) \sim 1/\mathbf{y}^2$$



- ▶ changes scale evolution

Kirschner 1979; Shelest, Snigirev, Zinovev 1982, Gaunt, Stirling 2009

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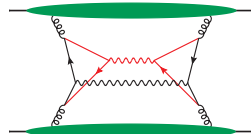
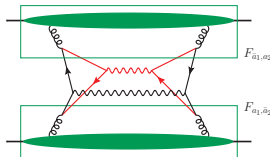
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- contribution from splitting graphs in cross section gives **divergent** integrals  $\int d^2\mathbf{y} F(x_i, \mathbf{k}_i, \mathbf{y}) F(\bar{x}_i, \bar{\mathbf{k}}_i, \mathbf{y})$



- double counting** problem between double scattering with splitting and single scattering at loop level

Cacciari, Salam, Sapeta 2009

MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012

Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012

## Summary

- ▶ multiple hard interactions are **not** suppressed in specific parts of phase space
- ▶ multi-parton dist's depend on **transverse distance  $y$**  between partons correlations with  $x$  have consequences for MPI
- ▶ nontrivial **spin** and **color** structure  
size of these effects presently unknown  
     $\rightsquigarrow$  extra uncertainty in quantitative description  
        **in addition to uncertainty on "usual" multi-parton dist's**
- ▶ should remove small  $y$  contribution in order to avoid divergences and double counting
- ▶ **opportunity at LHC:**  
investigate in detail final states susceptible to MPI  
     $\rightsquigarrow$  help to sort out which effects are important and which are not