An alternative subtraction scheme for NLO QCD calculations

Tania Robens

.

based on

.

C.H.Chung, TR (arXiv:1001.2704, arXiv:1209.1569)
C.H.Chung, M. Krämer, TR (arXiv:1012.4948, arXiv:1105.5327)
M. Bach, TR (work in progress)

IKTP, TU Dresden

Helmholtz Alliance Meeting 2012
DESY Hamburg

4.12.2012

NLO corrections: general structure

Masterformula

for m particles in the final state

$$\sigma_{\text{NLO,tot}} = \sigma_{\text{LO}} + \sigma_{\text{NLO}},$$

$$\sigma_{\text{LO}} = \int d\Gamma_m \, |\mathcal{M}^{(m)}_{\text{Born}}|^2(s) \quad \text{leading order contribution}$$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virt}},$$

$$\sigma_{\text{real}} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2 \quad \text{real emission}$$

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*) \quad \text{virtual contribution}$$

with $d\Gamma$: phase space integral, $\mathcal{M}$ matrix elements
(here: flux factors etc implicit)

Infrared divergencies and NLO subtraction schemes: ingredients

$$\sigma_{\text{NLO,tot}} = \underbrace{\int d\Gamma_m \, |\mathcal{M}^{(m)}_{\text{Born}}|^2}_{\sigma_{\text{LO}}} + \underbrace{2 \int d\Gamma_m Re(\mathcal{M}^{(m)}_{\text{Born}} (\mathcal{M}^{(m)}_{\text{virt}})^*)}_{\sigma_{\text{virt}}(\varepsilon)} + \underbrace{\int d\Gamma_{m+1} |\mathcal{M}^{(m+1)}|^2}_{\sigma_{\text{real}}(\varepsilon)}$$

- **infrared poles** $1/\varepsilon$, $1/\varepsilon^2$ cancel in $\sum \sigma_{\text{real}} + \sigma_{\text{virt}}$
- **matrix elements factorize in singular limits, unique behaviour** (depending on nature of splitting)

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j) \, |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j}$$

- D_{ij}: **dipoles**, contain complete singularity structure

$$\Longrightarrow \quad \int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \textbf{finite}$$

- general idea of dipole subtraction:
  shift singular parts from $m+1$ to m particle phase space

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C\right),$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, (\mathbf{K}(x) + \mathbf{P}(x;\mu_F))\, |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \right\}$$

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} \left(d\sigma^R - d\sigma^A\right) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C\right),$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\sigma_m^{\text{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\text{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\text{Born}}(s)|^2 + \int_0^1 dx \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\text{Born}}(x,s)|^2 \right\}$$

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C \right),$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \left\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, (\mathbf{K}(x) + \mathbf{P}(x;\mu_F))\, |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \right\}$$

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C$$

$$= \int_{m+1} \left(d\sigma^R - d\sigma^A\right) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C\right),$$

⇒ effectively added "0"; both integrals finite

$$\sigma_m^{\mathrm{NLO}}(s) = \int_m \Big\{ |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 + \int_0^1 dx\, \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \Big\}$$

Ingredient for subtraction schemes: momentum mapping

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \widetilde{F}_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2$$

- addition and subtraction takes place in different phase spaces

$$p^{(m)}_{\tilde{a}} = F\left(p^{(m+1)}_a, p^{(m+1)}_b, \ldots\right)$$

This function is highly scheme dependent !!!

requirements: keep total energy/ momentum conserved, all particles onshell

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a,\; p_i^2 = \tilde{p}_i^2 = m_i^2$$

(sum over outgoing particles only)

Nagy Soper subtraction scheme

- many different subtraction schemes are around (best known: Catani, Seymour, 1996)
- all schemes: poles have to be the same; finite parts can differ

Main motivation for new scheme

- proposal of improved parton shower: Nagy, Soper (arXiv:0706.0017, 0801.1917, 0805.0216)
- basic idea: can use the splitting functions of the new shower as dipole subtraction terms

⇒ (cf Catani Seymour Showers in Sherpa (Schumann ea '07), Herwig++ (Plätzer ea '11), ... (Winter ea, Dinsdale ea '07, ...))

- introduce new mapping between m and $m+1$ phase spaces

⇒ leads to a much smaller number of subtraction terms especially important for large number of external particles (same dipoles in shower and subtraction scheme: facilitates matching with NLO calculations)

Interim: Status of NLO tools

(I already apologize for non-completeness of the list)

- two main ingredients, living in different phase spaces:

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*); \; \sigma_{\text{real}} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2$$

- **virtual contribution:** many many [(new) (non) public] tools on the market, within MC or as standalone "virtual" generators (eg Openloops (Pozzorini ea, '12), Golem/Gosam (Cullen ea,'11), Blackhat(Bern ea, '08), aMC@NLO (Frixione ea, '11), and many more)

$\Rightarrow$ this part handled by above tools ("NLO revolution")

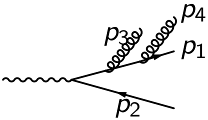
- **real emission:**
  (typically) handled by the interfacing Monte Carlo

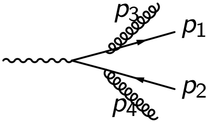
$\Rightarrow$ **this is where new scheme becomes important** $\Leftarrow$

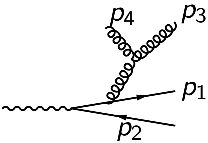
Shifting momenta: Example

$\gamma^* \longrightarrow q(p_1)\bar{q}(p_2)\,g(p_3)$ **(@ NLO)**

$q\,\bar{q}\,g\,g$ **real emission contributions:**







CS: 1 momentum shift/ spectator

$p_2,\ p_3$: 2 transformations

NS: 1 total transformation

$\Rightarrow$ **from simple counting:** $(+(p_1 \leftrightarrow p_2))$

10 transformations using CS vs 5 using NS dipoles !!

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	**CS**	**NS**
$\sum \sim$	$N^3/2$	$N^2/2$

$\Rightarrow$ **each mapping requires reevaluation of $\mathcal{M}_{\text{Born}}$** $\Leftarrow$

large (computational) effects as N increases

Final state mapping: Catani Seymour vs Nagy Soper

- CS mapping (per spectator $\tilde{p}_k$)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \quad \tilde{p}_k^\mu = \frac{1}{1-y} p_k$$

- NS mapping

$$\tilde{p}_i = \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a} Q, \quad \tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu \quad \textbf{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu (K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu \widetilde{K}^\nu}{K^2}, \; K = Q - p_i - p_j, \; \widetilde{K} = Q - \tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\mathrm{CS}} = \frac{(2\,\tilde{p}_i \tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}} \, dz\, dy\, (1-y)^{1-2\,\varepsilon} y^{-\varepsilon} \, [z\,(1-z)]^{-\varepsilon},$$

$$[dp_j]_{\mathrm{NS}} = \frac{(2\,\tilde{p}_i\, Q)^{1-\varepsilon}}{16\,\pi^2} \frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}} \, dz\, dy\, \lambda^{1-2\,\varepsilon} y^{-\varepsilon} \, [z\,(1-z)]^{-\varepsilon}, \; [\lambda\,(p_{\tilde{i}}, Q)$$

Scheme validation (C.H.Chung, M. Krämer, TR, JHEP 1106 (2011) 144; C.H.Chung, TR, arXiv:1209.1569)

- **main reason** for improved scaling: **different mapping**
- leads to **more complicated integrated subtraction terms**

✓ **all done and veryfied:** (final result independent of subtraction scheme)

Test-processes

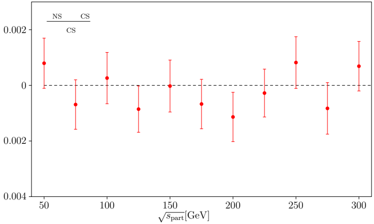
- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \longrightarrow H$ and $H \longrightarrow g\,g$
- DIS
- $e\,e \longrightarrow$ 3 jets

DIS: Catani Seymour vs Nagy Soper - numerical result

considered process:

$$e(p_{in})\, q(p_1) \longrightarrow e(p_{out})\, q(p_4)\ [g(p_3)]$$

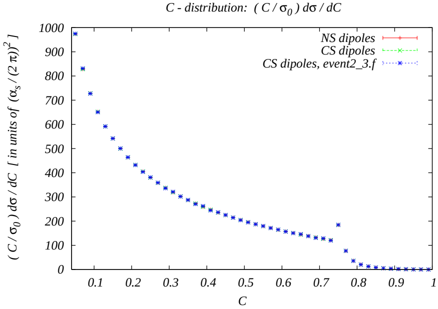
apply both schemes: get the same result



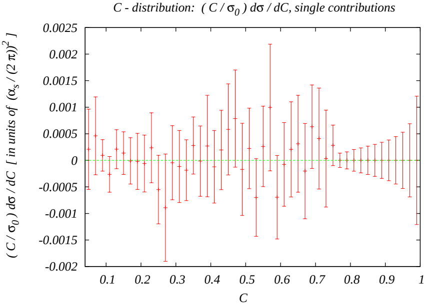
relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$

agree on the sub-permill level ✓

$e^+ e^- \longrightarrow 3$ jets ($N_C^2 C_F$ component) - numerical result



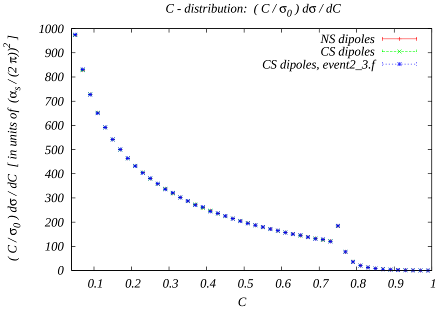
Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



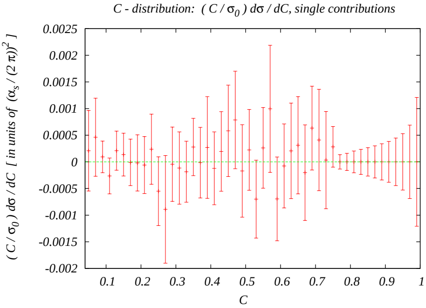
Relative difference between NS and CS implementation

- results agree on the permill level, compatibel with 0 ✓
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

$e^+ e^- \longrightarrow 3$ jets ($N_C^2\, C_F$ component) - numerical result



Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



Relative difference between NS and CS implementation

- results agree on the permill level, compatibel with 0 ✓
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

Nagy Soper - subtraction in the virtual contribution (here:DIS)

$\Rightarrow$ **major difference in integrated subtraction terms**

$$\int_0^1 \mathbf{dx}\, |\mathcal{M}|_2^2 \;=\; \int_0^1 dx \left\{ \frac{\alpha_s}{2\,\pi}\, C_F\, \delta(1-x) \left[-9 + \frac{1}{3}\pi^2 - \frac{1}{2}\mathrm{Li}_2[(1-\tilde{z}_0)^2] + 2\ln 2 \ln \tilde{z}_0 + 3\ln \tilde{z}_0 + 3\,\mathrm{Li}_2(1-\tilde{z}_0) + \mathbf{I}_{\text{fin}}^{\text{tot},0}(\tilde{\mathbf{z}}_0) + \mathbf{I}_{\text{fin}}^{1}(\tilde{\mathbf{a}}) \right] + K_{\text{fin}}^{\text{tot}}(x;\tilde{z}) + P_{\text{fin}}^{\text{tot}}(x;\mu_F^2) \right\} |\mathcal{M}|^2_{\text{Born}}(x\,p_1),$$

$$\mathbf{K}_{\text{fin}}^{\text{tot}}(\mathbf{x};\tilde{\mathbf{z}}) \;=\; \frac{\alpha_s}{2\,\pi}\, C_F \left\{ \frac{1}{x}\left[2\,(1-x)\ln(1-x) - \left(\frac{1+x^2}{1-x}\right)_+ \ln x + 4\,x \left(\frac{\ln(1-x)}{1-x}\right)_+ \right] + \mathbf{I}_{\text{fin}}^{1}(\tilde{\mathbf{z}},\mathbf{x}) \right\},$$

$\Rightarrow$ **contains integrals which need to be evaluated numerically** $\Leftarrow$

Nagy Soper - integrals to be evaluated numerically

$\Rightarrow$ **Integrals contain nontrivial functions depending on m and $m+1$ four-momenta** $\Leftarrow$

$$\mathbf{I}_{\text{fin}}^{\text{tot},0}(\tilde{\mathbf{z}}_0) = 2\int_0^1 \frac{dy}{y}\left\{\frac{\tilde{z}_0}{\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}} \times \ln\left[\frac{2\,z\,\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\,(1-y)}{\left(2\,y+\tilde{z}_0-2\,y\,\tilde{z}_0+\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\right)^2}\right]+\ln 2\right\}.$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{a}}) = 2\int_0^1 \frac{du}{u}\int_0^1\frac{dx}{x} \times\left[\frac{\mathbf{x}\,(\mathbf{1}-\mathbf{x}+\mathbf{u}\,\mathbf{x}\,[(\mathbf{1}-\mathbf{u}\,\mathbf{x})\,\tilde{\mathbf{a}}+\mathbf{2}])}{\mathbf{k}(\mathbf{u},\mathbf{x},\tilde{\mathbf{a}})}-\frac{1}{\sqrt{1+4\,\tilde{a}_0\,u^2\,(1+\tilde{a}_0)}}\right].$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{z}},\mathbf{x}) = \frac{2}{(1-x)_+}\,\frac{1}{\pi}\int_0^1\frac{dy'}{y'}\left[\int_0^1\frac{dv}{\sqrt{v\,(1-v)}}\,\frac{\tilde{\mathbf{z}}}{\mathbf{N}(\mathbf{x},\mathbf{y}',\tilde{\mathbf{z}},\mathbf{v})}-1\right],$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m+1$ variables have to be reconstructed

$\Rightarrow$ difference wrt standard scheme(s) $\Leftarrow$

in initial state subtraction terms

$$\mathbf{N} = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{\mathbf{p}}_3 = \underbrace{\frac{(1-x)(1-y')}{x}}_{\alpha} \mathbf{p}_1 + \underbrace{(1-x)\,y'}_{\beta} \mathbf{p}_i - \mathbf{k}_\perp, \hat{\mathbf{p}}_4^\mu = \Lambda^\mu{}_\nu(\hat{K}, K)\hat{\mathbf{p}}_4^\nu$$

$$k_\perp^2 = -2\,\alpha\,\beta\,p_1 \cdot p_i,\ k_\perp = -|k_\perp| \begin{pmatrix} 0 \\ 1-2v \\ 2\sqrt{v(1-v)} \\ 0 \end{pmatrix},$$

in final state subtraction terms

$$\mathbf{k}^2(\mathbf{x}, \mathbf{u}, \tilde{\mathbf{a}}) = \left[(1+ux-x)(z-z') + ux\left((1-ux)\,\tilde{a}+1\right)\right]^2 + 4\,u\,x\,z'(1-z)(1+u\,x-x)\left((1-ux)\,\tilde{a}+1\right)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-y)p_o)}$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m+1$ variables have to be reconstructed

⇒ difference wrt standard scheme(s) ⇐

in initial state subtraction terms

$$\mathbf{N} = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{\mathbf{p}}_3 = \underbrace{\frac{(\mathbf{1-x})\,(\mathbf{1-y'})}{\mathbf{x}}}_{\alpha} \mathbf{p_1} + \underbrace{(\mathbf{1-x})\,\mathbf{y'}}_{\beta} \mathbf{p_i} - \mathbf{k_\perp}, \hat{\mathbf{p}}_4^\mu = \mathbf{\Lambda}^\mu{}_\nu(\hat{\mathbf{K}}, \mathbf{K}) \hat{\mathbf{p}}_4^\nu$$

$$k_\perp^2 = -2\,\alpha\,\beta\,p_1 \cdot p_i,\; k_\perp = -|k_\perp| \begin{pmatrix} 0 \\ 1-2\,v \\ 2\sqrt{v\,(1-v)} \\ 0 \end{pmatrix},$$

in final state subtraction terms

$$\mathbf{k^2(x,u,\tilde{a})} = \left[(1+ux-x)(z-z') + ux\,((1-ux)\,\tilde{a}+1)\right]^2 + 4\,u\,x\,z'\,(1-z)\,(1+u\,x-x)\,((1-ux)\,\tilde{a}+1)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-y)p_o)}$$

Numerical integrals - approximations through grids/ polynomials (1) (work done by M. Bach)

status as on arXiv:

scheme has in total **7** integrals which need to be evaluated numerically

- **5** depend on **1** external parameter
  (leftover finite terms from singular regions, à la $\varepsilon \times \frac{1}{\varepsilon}$)

$$I_3(a),\ I_{\text{fin}}(a),\ I_{\text{fin}}^{(b)}(a),\ I_{\text{fin}}^{(e)}(a),\ I_{\text{fin}}(\tilde{z}_0)$$

- **2** depend on **2** external parameters
  (finite terms for interference integrals in non-singular regions)

$$I_{\text{fin}}^{(d)}(\tilde{a}, a), I_{\text{fin}}(\tilde{z}_0, x)$$

- (not really a problem though...)

Numerical integrals - approximations through grids/ polynomials (2) (work done by M. Bach)

- **work in progress:** approximate these using polynomials and/ or grids
- all 'one-parameter' integrals: **approximated by polynomials, implemented and checked** (apart from $I_{\text{fin}}(\tilde{z}_0)$)
- agreement for approximation: typically $\mathcal{O}(10^{-6})$ **or better**
- set up for **interpolating grids for others:** on the way

$\Rightarrow$ **no additional integrations needed** $\Leftarrow$

$\Rightarrow$ **implementation 'like' Catani Seymour scheme** $\Leftarrow$

(however: the hard part are always the real emissions...)

What about constant scaling ?? (for the experts...)

(this is typically a question from an FKS user/ author...)

- aMC@NLO (Frixione ea): constant scaling for certain processes
- makes use of symmetries in matrix elements and phase space

$\Rightarrow$ **also possible here** $\Leftarrow$

- need to **partition**, but not to **parametrize** à la FKS
  ($\hat{=}$ each partition contains at most one soft/ soft and collinear divergence)
- then dipoles in our scheme which reflect this singularity structure obey **single** mapping $\Rightarrow$ **constant scaling**

!! very preliminary, no implementation yet...

- details on linear scaling in aMC@NLO: Frederix ea, JHEP 0910 (2009) 003

Summary and Outlook

Summary

- scheme works, no phase space reparametrization needed, scaling $\leq N^2$

Outlook

- make generically available for application in new higher order calculations
- finish interpolation of finite integrals
- **implementation in current NLO tool(s)**
- (further down the road: extend to massive scheme, combine w parton shower)

! Thanks for listening !

Appendix

Difference 2: Combining showers and NLO (1)

Very very short...

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- want:hard: matrix element, soft: shower (always talk about 1 jet)
- can be achieved by adding and subtracting a counterterm

$$-\int_{m+1} d\sigma^{\mathrm{PS}}|_{m+1} + \int_{m+1} d\sigma^{\mathrm{PS}}|_{m}$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

Difference 2: Combining showers and NLO (2)

- important: have new terms in $m+1$ phase space

$$\int_{m+1}\left(d\sigma^R \underbrace{-d\sigma^A + d\sigma^{PS}|_m}_{=0} - d\sigma^{PS}|_{m+1}\right)$$

- same splitting functions: second and third term cancel !!
  left with

$$\int_{m+1}\left(d\sigma^R - d\sigma^{PS}|_{m+1}\right)$$

$\Rightarrow$ improves numerical efficiency

- more details on this, also for MC@NLO vs Powheg:
  S. Hoeche ea, "A critical appraisal of NLO+PS matching methods", arXiv:1111.1220

Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs) $f_l(x,\mu_F)$; $l = q, \bar{q}, g$ flavour, x momentum fraction, (μ_F factorization scale)

masterformula

$$\sigma_{\text{hadr}}(p\,\bar{p} \to X) = \sum_{l_1,l_2} \int dx_1 \int dx_2\, f_{l_1}(x_1) f_{l_2}(x_2)\, \sigma_{\text{part}}(x_1, x_2;\, l_1 l_2 \to X)$$

- perturbative, nonperturbative part

Infrared divergences in NLO corrections

- source of infrared divergence: integration over phase space of emitted massless particles in real and virtual contribution
- KNL theorem: infrared poles cancel in $\sigma_{\text{real}} + \sigma_{\text{virt}}$
- appear in matrix elements as terms

$$\frac{1}{p_i p_j} = \frac{1}{E_i E_j \left(1 - \cos\theta_{ij}\right)}$$

$E_j \to 0$: soft divergence, $\cos\theta_{ij} \to 1$: collinear divergence

- matrix element level: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j)\, |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \qquad (1)$$

- D_{ij}: **dipoles**, contain complete singularity structure

$\Rightarrow$ **need to have a good (analytical) parametrization of the singularity structure**

Dipole subtraction: general idea

- here: go from $D = 4$ to $D = 4 - 2\,\varepsilon$ dimensions in integration over phase space
- poles then appear as

$$\sigma_{\text{real}} \sim \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \ldots.$$

- A, B depend on the splitting process, but are independent of the rest of reaction = **general**

$\Rightarrow$ underlying idea of dipole subtraction schemes $\Leftarrow$

- add and subtract a function in the m (=Born, virtual corrections) and $m+1$ (real emission) phase space which mimicks singular behaviour in singular regions
- also implies "smoothing" of the integrand

Dipole subtraction: Real master formula

Real Masterformula $(s = (p_a + p_b)^2)$

$$\begin{aligned}\sigma(s) =& \int_m d\Phi^{(m)}(s)\frac{1}{n_c(a)n_c(b)}\,|\mathcal{M}^{(m)}|^2(s)F_J^{(m)}\\&+\int d\Phi^{(m+1)}(s)\left\{\frac{1}{n_c(a)n_c(b)}\,|\mathcal{M}^{(m+1)}|^2(s))F_J^{(m+1)} - \sum_{\text{dipoles}}(\mathcal{D}\cdot F_J^{(m)})\right\}\\&+\int d\Phi^{(m)}(s)\left\{\frac{1}{n_c(a)n_c(b)}|\mathcal{M}^{(m)}|^2_{1\text{ loop}}(p_a,p_b) + \mathbf{I}(\varepsilon)|\mathcal{M}^{(m)}|^2(s)\right\}_{\varepsilon=0} F_J^{(m)}\\&+\left\{\int dx_a\,dx_b\delta(x-x_a)\,\delta(x_b-1)\int d\Phi^{(m)}(x_ap_a,x_bp_b)|\mathcal{M}^{(m)}|^2(x_ap_a,x_bp_b)\right.\\&\left.\times\left(\mathbf{K}^{a,a'}(x)+\mathbf{P}^{a,a'}(x_ap_a,x_bp_b,x;\mu_F^2)\right)\right\} + (a\leftrightarrow b)\end{aligned}$$

where all colour/ phase space factors have been accounted for

Integrated Dipoles in more details: I, K, P (1)

$m+1$ **phase space: in principle easy**

$$\int d\Gamma_{m+1}\,\left(|\mathcal{M}_{\text{real}}|^2 - \sum D\right),\quad \text{finite}$$

m **particle phase space: more complicated**

need integration variables (emission from p_1):

$$x \;=\; 1 - \frac{p_4(p_1+p_2)}{p_1 p_2}\ \text{softness},\ \ \tilde{v} \;=\; \frac{p_1 p_4}{p_1 p_2}\ \text{collinearity}$$

Integrated Dipoles in more details: I, K, P (2)

- in principle, obtain $\int d\Gamma_1 \, D = \int_0^1 dx \left(\mathbf{I}(\varepsilon) + \tilde{\mathbf{K}}(x,\varepsilon) \right)$
- $\mathbf{I}(\varepsilon) \propto \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x,\varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\epsilon} \left(\frac{4\pi\mu^2}{\mu_F^2} \right)^{\epsilon} P^{qq}(x)$$

  depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)
- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for $q\,g \rightarrow W\,q$ like processes, only singularity which appears

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij},\ \widetilde{F}_{sing} = \int d\Gamma_1\, D_{ij},\ d\Gamma_1 \propto d^4 p_j\, \delta(p_j^2)$$

$$\Longrightarrow \widetilde{F}_{sing} \propto \int d^4 p_j\, \delta(p_j^2)\, D_{ij}$$

- 3 free variables (in D dimensions: $D-1$)

  !! need to be written in terms of m particle variables !!
- now all ingredients:
  total energy momentum conservation, onshellness of external particles, need for integration variables

Catani Seymour vs Nagy Soper: Shifting momenta

- matching between m and $m+1$ particle spaces requires reshuffling of momenta
- for

$$p^{(m)}_{\text{mother}} = p^{(m+1)}_{\text{daughter, 1}} + p^{(m+1)}_{\text{daughter,2}},$$

  not all particles can be onshell simultaneously

$\Rightarrow$ need additional spectators to take over additional momenta

- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

$\Rightarrow$ quite easy integrations; however, for increasing number of particles, huge number of transformations necessary

- Nagy Soper:
  shift momenta to **all** non-emitting external particles
- number of transformations = number of emitters
- leads to more complicated integrals during framework setup
- in general: # of transformations: CS $\sim N^3_{\text{jets}}/2$, NS$\sim N^2_{\text{jets}}/2$

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	**CS**, (ij)	**CS**, k	**NS**, (ij)
fin,fin	$\binom{N'}{2}$	$(N'-2)$	$\binom{N'}{2}$
fin,ini	$\binom{N'}{2}$	2	–
ini,fin	$2\,N'$	$(N'-1)$	$2\,N'$
ini,ini	$2N'$	1	–
total $(\sum_{\text{comb's}}(ij)\times(k))$	$N'^2(N'+3)/2 = (N+1)^2(N+4)/2$		$N'(N'+3)/2 = (N+1)(N+4)/2$
$\sim$	$N^3/2$		$N^2/2$

(N' number of real emission, N number of Born type final state particles)

NS integration measures (1)

Initial state

$$d\xi_p = dx \int_0^1 dy' \int_0^1 dv \frac{(2p_a \cdot p_b)^{1-\varepsilon} x^{\varepsilon-1}}{(4\pi)^2} \frac{\pi^{\varepsilon-\frac{1}{2}}}{\Gamma\left(\frac{1-2\varepsilon}{2}\right)} \times \left[y'(1-y')\right]^{-\varepsilon} \left[v\,(1-v)\right]^{-\frac{1+2\,\varepsilon}{2}} \Theta\left[(1-x)x\right]$$

Final state, $2 \longrightarrow 2$ processes

$$d\xi_p = \frac{(2p_\ell \cdot Q)^{1-\varepsilon}}{16} \frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma\left(\frac{1}{2}-\varepsilon\right)} \times \int_0^1 du u^{-\varepsilon}(1-u)^{-\varepsilon} \int_0^1 dx x^{1-2\varepsilon}(1-x)^{-\varepsilon} \int_0^1 dv\left[v\,(1-v)\right]^{-\frac{1+2\varepsilon}{2}}$$

NS integration measures (2)

Final state, $2 \to n$ processes

$$d\xi_p = \frac{(2\,p_{\tilde{i}}Q)^{1-\varepsilon}}{16}\,\frac{\pi^{-\frac{5}{2}+\varepsilon}}{\Gamma\left(\frac{1}{2}-\varepsilon\right)}$$

$$\int_0^1 du u^{-\varepsilon}\int_0^1 dx \delta^{1-\varepsilon}\gamma^{1-2\varepsilon}\left[(1-x)(x-x_0)\right]^{-\varepsilon}\int_0^1 dv\left[v(1-v)\right]^{-\frac{1+2\,\varepsilon}{2}}.$$

Variables

$$\lambda = \sqrt{(1+y)^2-4\,a\,y},\; a = \frac{Q^2}{2\,p_{\tilde{i}}Q},\; y = \frac{\hat{p}_i\hat{p}_j}{p_{\tilde{i}}Q}$$

$$\gamma = \frac{1}{2}\left(1+y+\lambda\right),\; x_0(y,a) = \frac{1-\lambda+y}{1+\lambda+y},$$

$q \longrightarrow q\, g$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,(s+t+u)}{t\,(t+u)} + (1-\varepsilon)\,\frac{t+u}{t}\right)$$

- matching ($\tilde{p}_2 = p_2$)

$$\tilde{p}_1 = x\,p_1, \quad x = 1 - \frac{p_4\,(p_1+p_2)}{(p_1 p_2)}$$

$$\tilde{p}_k^{\mu} = \Lambda^{\mu}{}_{\nu}\,p_k^{\nu}, \quad (k\text{: final state particles})$$

$$\Lambda^{\mu\nu} = -g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^{\mu}(K+\widetilde{K})^{\nu}}{(K+\widetilde{K})^2} + \frac{2\,K^{\mu}\widetilde{K}^{\nu}}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \widetilde{K} = \tilde{p}_1 + p_2$$

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v = \frac{p_1 p_4}{p_1 p_2},\; x = 1 - \frac{p_4\,(p_1 + p_2)}{(p_1 p_2)}$$

- in p_1, p_2 cm system: $E_4 \longrightarrow 0 \Rightarrow x \longrightarrow 1$ (softness)
  $\cos\theta_{14} \longrightarrow 1 \Rightarrow v \longrightarrow 0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\,\pi\,\alpha_s\,C_F}{v\,x\,s}\left(\frac{1+x^2}{1-x} - \varepsilon(1-x)\right)$$

- integration measure

$$[dp_j] = \frac{(2\,p_1 p_2)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dv\,dx\,(1-x)^{-2\,\varepsilon}\left[\frac{v}{1-x}\left(1-\frac{v}{1-x}\right)\right]^{-\varepsilon}$$

where $v \leq 1 - x$ and all integrals between 0 and 1

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon} \int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi}\, \frac{1}{\Gamma(1-\varepsilon)}\, C_F \left(\frac{2\mu^2\pi}{p_1 p_2}\right)^{\varepsilon} \times \int_0^1 dx \left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{I}(\epsilon) = \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ \quad \text{regularized splitting function}$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,u\,(s+t+u)}{t\,(t^2+u^2)} + (1-\varepsilon)\,\frac{u}{t}\right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s\,C_F}{x\,s} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2\,x}{y\,(1-x)} - \frac{2\,x\,[2\,y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]}\right)$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon} \int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\, C_F \left(\frac{2\,\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times \int_0^1 dx \left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{K}(x) =$$

$$(1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+ -(1-x)$$

- equivalence of dipoles schemes checked analytically

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \longrightarrow q(p_i) + \bar{q}(p_j)$,
  spectator: any other final state parton, p_k
- Dipole (in terms of integration variables):

$$D^{ij,k}_{\text{NS, CS}} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[1 - \frac{z\,(1-z)}{1-\varepsilon}\right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i\,p_j}{(p_i+p_j)Q - p_i\,p_j}, \quad z_{\text{NS}} = \frac{p_j\,\tilde{n}}{p_i\,\tilde{n} + p_j\,\tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda}\,Q - \frac{a}{\lambda}\left(p_i + p_j\right),\ \lambda = \sqrt{(1+y)^2 - 4\,a\,y},\ a = \frac{Q^2}{(p_i+p_j)Q - p_i\,p_j}$$

- CS definitions:

$$y_{CS} = \frac{p_i\,p_j}{p_i\,p_j + p_i\,p_k + p_j\,p_k}, \quad z_{CS} = \frac{p_i\,p_k}{p_i\,p_k + p_j\,p_k}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (2)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k,\ \tilde{p}_k^\mu = \frac{1}{1-y} p_k$$

- NS matching

$$\tilde{p}_i = \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a}\,Q,\ \tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu \quad \textbf{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu (K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu \widetilde{K}^\nu}{K^2},\ K=Q-p_i-p_j,\ \widetilde{K}=Q-\tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\text{CS}} = \frac{(2\,\tilde{p}_i\tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,(1-y)^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon},$$

$$[dp_j]_{\text{NS}} = \frac{(2\,\tilde{p}_i\,Q)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,\lambda^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (3)

- result CS

$$\mu^{2\varepsilon} \int [dp_j]\, D^{ij,k} \;=\; \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,T_R\left(\frac{2\mu^2\pi}{\tilde{p}_i\tilde{p}_k}\right)^{\varepsilon}\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}\right]$$

- result NS

$$\mu^{2\varepsilon} \int [dp_j]\, D^{ij} \;=\; T_R\,\frac{\alpha_s}{2\,\pi}\,\frac{\alpha_s}{\Gamma(1-\varepsilon)}\left(\frac{2\,\pi\,\mu^2}{p_{\tilde{i}}Q}\right)^{\varepsilon}$$

$$\times\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}+\frac{2}{3}\left[(a-1)\ln(a-1)-a\ln a\right]\right],$$

- for $a=1$, reduces completely to Catani Seymour result
- (reason: $a = 1$ implies only 2 particles in the final state, $\tilde{n} \longrightarrow p_k$, $\Rightarrow$ complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- **1 particle phase space (virtual contribution)**

$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}(-8+\frac{2}{3}\pi^2)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned}\mathbf{K}^a(xp_a) = \; & \tfrac{\alpha_s}{2\pi}C_F\tfrac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) \\ & +4x\left(\tfrac{\ln(1-x)}{1-x}\right)_+ - \tfrac{2x\ln x}{(1-x)_+} - \left(\tfrac{1+x^2}{1-x}\right)_+\ln\left(\tfrac{4\pi\mu^2}{2xp_a\cdot p_b}\right) \\ & +(1-x)\Big]\end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

$$\mathcal{D}^{14,2}+\mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- **1 particle phase space (virtual contribution)**

$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned} K^a(xp_a) = & \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x+2(1-x)\ln(1-x) \\ & +4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+\ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) \\ & +(1-x)\Big]\end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

NS, CS-NS, CS= NS+CS-NS

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9} g^2 \alpha_s \pi}_{\text{finite}}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}_v}|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned}\mathbf{K}^a(xp_a) = {} & \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) \\ & +4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+ \ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) \\ & +(1-x)\Big]\end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Single W production: Nagy Soper vs Catani Seymour subtraction (easy)

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- **1 particle phase space (virtual contribution)**

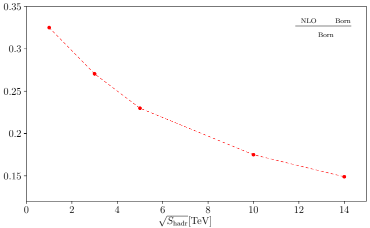
$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\mathbf{K}^a(xp_a) = \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) + 4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+\ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) + (1-x)\Big]$$

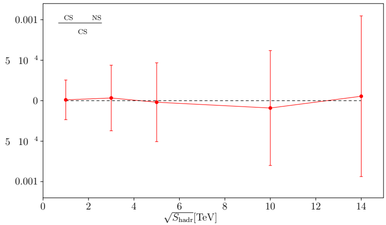
compare to Nagy Soper :
pole structure the same, finite terms differ ✓

Numerical results for single W (slide by C. Chung)

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$

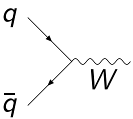


$\frac{\sigma_{NLO}-\sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S_{\mathrm{hadr}}}$
corrections up to 30%

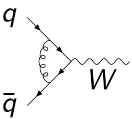


relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$
agree on the sub-permill level ✓

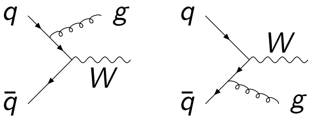
Single W production (slide by C.H. Chung)



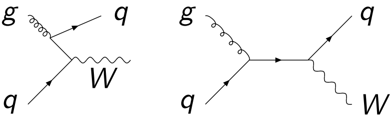
Tree level: $q\bar{q} \to W$



Virtual corrections: $q\bar{q} \to W$



Real corrections: $q\bar{q} \to Wg$



$gq \to Wq$ (+ 2 more diagrams)

$$\frac{1}{4}\frac{1}{9}|\mathcal{M}_B|^2 = \frac{g^2}{12}|V_{qq'}|^2 M_W^2, \quad \frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_R|^2 = \frac{8g^2\pi\alpha_s}{9}|V_{qq'}|^2 \frac{\hat{t}^2+\hat{u}^2+2M_W^2\hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}_V|^2 = |\mathcal{M}_B|^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right\}$$

Deep inelastic scattering (subprocess of...) (hard)

$$e(p_{in})\, q(p_1) \longrightarrow e(p_{out})\, q(p_4)\, [g(p_3)]$$

- CS: spectator for final state gluon emission:

  initial state quark

- NS: spectator for final state gluon emission:

  final state lepton

- ("spectator" = spectator in momentum mapping)

⇒ first nontrivial check of NS scheme ⇐

DIS: Catani Seymour

Real emission subtraction terms

$$\mathbf{D}_{43,1} = \frac{4\pi\alpha_s}{p_3 p_4}\frac{1}{x_{43,1}} C_F \left[\frac{2}{1-\tilde{z}_4 + (1-x_{43,1})} - (1+\tilde{z}_4)\right] |\mathcal{M}|^2_{\text{Born}}(\tilde{p}_1, \tilde{p}_4)$$

$$\mathbf{D}_{13,4} = \frac{4\pi\alpha_s}{p_1 p_3}\frac{1}{x_{34,1}} C_F \left[\frac{2}{1-x_{34,1}+u_3} - (1+x_{34,1})\right] |\mathcal{M}|^2_{\text{Born}}(\tilde{p}_1, \tilde{p}_4)$$

$$\tilde{z}_4 = \frac{p_1 p_4}{(p_3+p_4)p_1}, \quad x_{43,1} = x_{34,1} = \frac{p_i p_o}{p_1 p_4 + p_1 p_3}, \quad u_3 = \frac{p_1 p_3}{(p_3+p_4)p_1}$$

Mapping

$$\tilde{\mathbf{p}}_1 = x_{43,1} p_1, \quad \tilde{\mathbf{p}}_4 = p_3 + p_4 - (1-x_{43,1}) p_1$$

Integrated subtraction terms

$$\int_0^1 \mathbf{dx} |\mathcal{M}|^2_{2,\text{tot}} = \int_0^1 \frac{dx}{x}\left\{-\frac{9}{2}\frac{\alpha_s}{2\pi} C_F \delta(1-x) + K^{\text{eff}}_{\text{fin}}(x) + P^{\text{eff}}_{\text{fin}}(x;\mu_F^2)\right\} |\mathcal{M}|^2_{\text{Born}}(x p_1)$$

$$\mathbf{K}^{\text{eff}}(\mathbf{x}) = \frac{\alpha_s}{2\pi} C_F \left\{\left(\frac{1+x^2}{1-x}\ln\frac{1-x}{x}\right)_+ + \frac{1}{2}\delta(1-x) + (1-x) - \frac{3}{2}\frac{1}{(1-x)_+}\right\}$$

DIS: Nagy Soper - real emission terms

Initial state real emission subtraction

$$\mathbf{D}^{1,3} = \frac{4\,\pi\,\alpha_s}{x\,y\,\hat{p}_1\cdot\hat{p}_i}\;C_F\left(1-x-y+\frac{2\,\tilde{z}\,x}{v\,(1-x)+y}\right)|\mathcal{M}_{\text{Born}}(p)|^2$$

$$x = \frac{\hat{p}_o\cdot\hat{p}_4}{\hat{p}_i\cdot\hat{p}_1},\; y = \frac{\hat{p}_1\cdot\hat{p}_3}{\hat{p}_1\cdot\hat{p}_i},\; \tilde{z} = \frac{\hat{p}_1\cdot\hat{p}_4}{\hat{p}_4\cdot\hat{Q}},\; v = \frac{(\hat{p}_1\cdot\hat{p}_i)\,(\hat{p}_3\cdot\hat{p}_4)}{(\hat{p}_4\cdot\hat{Q})\,(\hat{p}_3\cdot\hat{Q})}.$$

Initial state: mapping

$$\mathbf{p_1} = x\,\hat{p}_1,\; \mathbf{p_i} = \hat{p}_i,\; \mathbf{p}^{\mu}_{\mathbf{o,4}} = \mathbf{\Lambda}^{\mu}{}_{\nu}(\mathbf{K},\hat{\mathbf{K}})\hat{p}^{\nu}_{o,4},\, K = x\,\hat{p}_1+\hat{p}_i,\; \hat{K} = \hat{p}_1+\hat{p}_i-\hat{p}_3\,.$$

Final state real emission subtraction

$$\mathbf{D}^{4,3} = \frac{4\pi\alpha_s\,C_F}{y(\hat{p}_i\cdot\hat{p}_1)}\left[\frac{y}{1-y}F_{\text{eik}}+z+2\frac{(1-v)(1-z(1-y))}{v\,[1-z(1-y)]+y\,[(1-y)\tilde{a}+1]}\right]|\mathcal{M}_{\text{Born}}(p)|^2$$

$$y = \frac{\hat{p}_3\cdot\hat{p}_4}{\hat{p}_1\cdot\hat{p}_i},\; z = \frac{\hat{p}_3\cdot\hat{p}_o}{\hat{p}_3\cdot\hat{p}_o+\hat{p}_4\cdot\hat{p}_o},\; v = \frac{\hat{p}_1\cdot\hat{p}_3}{\hat{p}_1\cdot\hat{p}_3+\hat{p}_1\cdot\hat{p}_4},\; F_{\text{eik}} = 2\,\frac{(\hat{p}_3\cdot\hat{p}_o)\,(\hat{p}_4\cdot\hat{p}_o)}{(\hat{p}_3\cdot\hat{Q})^2}.$$

Final state: mapping

$$\mathbf{p_i} = \hat{p}_i,\; \mathbf{p_1} = \hat{p}_1,\; \mathbf{p_4} = \frac{1}{1-y}\left[\hat{p}_3+\hat{p}_4-y\,(\hat{p}_1+\hat{p}_i)\right],\; \mathbf{p_o} = \frac{\hat{p}_o}{1-y}.$$

$e^+ e^- \longrightarrow$ 3 jets (1)

- consider process

$$e^+ e^- \longrightarrow q\,\bar{q}\,g$$

  at NLO
- real emission contributions:

$$e^+ e^- \longrightarrow q\,\bar{q}\,q\,\bar{q},\; q\bar{q}\,g\,g$$

- number of necessary mappings (in total):
  $(8+10)_{CS}$ vs $(4+5)_{NS}$
- 3 different color structures: $C_A\,C_F^2,\; C_A\,C_F\,n_f\,T_R,\; C_A^2\,C_F$
- singular parts: $C_A\,C_F\,n_f\,T_R$: $q\bar{q}\,q\,\bar{q}$ only,
  $C_A^2\,C_F,\; C_A\,C_F^2$: $q\bar{q}\,gg$ only
- result known for a long time: Ellis ea 1980
  (also Kuijf 1991, Giele ea 1992)

$e^+ e^- \longrightarrow$ 3 jets (2)

- singularity structure in integrated dipoles for all color configurations: done ✓
- $q\bar{q}q\bar{q}$ real emission terms and all finite contributions: done ✓
- $q\bar{q}gg$ real emission terms and all finite contributions: done(✓)

- current work: improve numerics, especially for $g \rightarrow g\,g$ splittings

- infrared safe observable: C-distribution (Ellis ea 1980),

$$C^{(n)} = 3\left\{1 - \sum_{i,j=1,\,i<j}^{n} \frac{s_{ij}^2}{(2\,p_i \cdot Q)\,(2\,p_j \cdot Q)}\right\}$$

$e^+ e^- \longrightarrow$ 3 jets (2)

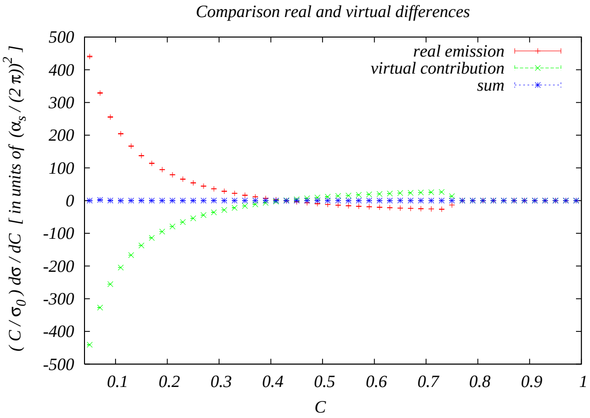
- singularity structure in integrated dipoles for all color configurations: done ✓
- $q\bar{q}q\bar{q}$ real emission terms and all finite contributions: done ✓
- $q\bar{q}gg$ real emission terms and all finite contributions: done(✓)

- current work: improve numerics, especially for $g \rightarrow g\,g$ splittings

- infrared safe observable: C-distribution (Ellis ea 1980),

$$C^{(n)} = 3\left\{1 - \sum_{i,j=1,\,i<j}^{n} \frac{s_{ij}^2}{(2\,p_i\cdot\,Q)\,(2\,p_j\cdot\,Q)}\right\}$$

$e\,e \longrightarrow$ 3jets: single components



differences between real and virtual contributions from CS and NS dipoles respectively