Hybrid conformal field theories

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forthcoming work with M. Bertolini and M.R. Plesser

Focus and Results

we study

a class of d $= 2$ (2,2) SCFTs with $c_L = c_R = 9$ and $\mathbf{q}_L, \mathbf{q}_R \in \mathbb{Z}$

we obtain

- intrinsic definition of these (2,2) hybrid SCFTs
- massless spacetime spectrum in hybrid limit

Motivation

many (2,2) (!) questions

- isolated (2,2) SCFTs? SCFTs without large radius limit?
- $\mathcal{M}_{(2,2)} \subseteq \mathcal{M}_{(0,2)}$?

stringy results for compactifications

- g_s perturbative II/het compactification exact in α'
- a rock&hard place — moduli stabilization vs existence of vacuum
- new computable corners of landscape
- (2,2) — a stepping stone for (0,2)

mathematical physics

moduli spaces, quantum geometry, mirror symmetry, . . .

Outline

- focus & motivation ✓
- (2,2) constructions
- extrinsic and intrinsic hybrids
- massless spectra
- prospects

Constructions of (2,2) SCFTs

- solvable (R)CFTs : e.g. T^6/Γ & Gepner models

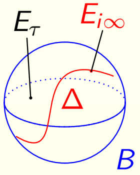
- CY NLSMs

  $\beta_g = \mathrm{Ric}(g) + \alpha'$ corrections

  e.g. elliptic (or K3)–fibered 3-fold M

  $\mathcal{M}_{(2,2)} = \mathcal{M}_{\text{cK}} \times \mathcal{M}_{\text{c-x}}$

  $h^1(T^*_M) + h^1(T_M)$ moduli



$\bar{\partial}\tau = 0$

- RG flow from (2,2) SUSY UV theory
  - Gauged linear sigma models

    $\cup$
  - Landau-Ginzburg orbifolds ($\supset$ Gepner models)

Landau-Ginzburg orbifolds (LGO)

[Zamolodchikov'86; Martinec'89; Vafa&Warner'89; Vafa'89; Kreuzer&Skarke'92, Klemm&Schimmrigk'92. . .]

- (2,2) chiral superfield $\Phi = \phi + \theta_L \chi + \theta_R \eta + \theta_L \theta_R F + \ldots$
- $L = \int d^4\theta \sum_{i=1}^{n} \Phi_i \overline{\Phi}_i + \int d\theta_L d\theta_R W(\Phi) + \text{c.c.}$

  quasi-homogeneous $W(t^{m_i}\Phi_i) = t^N W(\Phi)$

  $\implies \mathrm{U(1)_L} \times \mathrm{U(1)_R}$ chiral symmetry $\mathbf{q}_{L,R}(\Phi_i) = \frac{m_i}{N} \equiv q_i$

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- RG flow: (2,2) SCFT
  - $c_L = c_R = 3\sum_i (1 - 2q_i)$
  - (c,c) ring : $R_W \simeq \mathbb{C}[\phi_1, \ldots, \phi_n]/\langle \partial_1 W, \ldots, \partial_n W \rangle$ "c-x structure"
  - (a,c) ring : $\mathbb{1}$ "(no) cKähler"

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- orbifold by $\Gamma \equiv \mathbb{Z}_N$ $\quad \gamma = e^{2\pi i J_{L,0}} \Longrightarrow \mathbf{q}_{L,R} \in \mathbb{Z}$
  - (a,c) ring $\subset$ twisted sectors
  - $\mathbb{Z}_N$ quantum symmetry
  - II/heterotic GSO $\to$ Minkowski SUSY string vacua.

LGO Example

$$W = \Phi_1^3 + \Phi_2^3 + \Phi_3^3 - 3\xi\Phi_1\Phi_2\Phi_3 \qquad q_i = \frac{1}{3} \qquad \Gamma = \mathbb{Z}_3$$

- $L \supset V(\phi, \overline{\phi}) = |dW|^2$

  $dW^{-1}(0) = \{\phi_i = 0\} \iff \dim R_W < \infty \iff \Delta \equiv \xi^3 - 1 \neq 0$

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  $\implies T^2$ NLSM at special value of $B + iJ \equiv Z_0$

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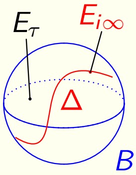
- A special case of CY / LGO correspondence.

Inevitability of hybrids

Elliptic (K3) fibered CY M



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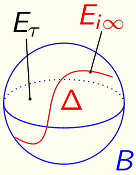

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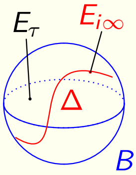

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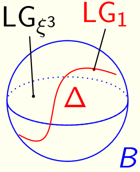
LGO fibered over B



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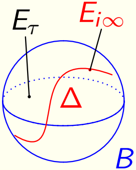

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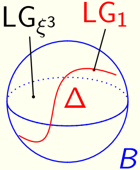

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Hybrid limit : locus in $\mathcal{M}_{\mathrm{cK}}(M)$ with small fiber, large base

with UV model as LGO fibered over B

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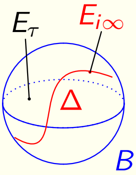

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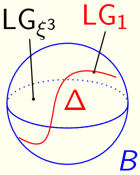

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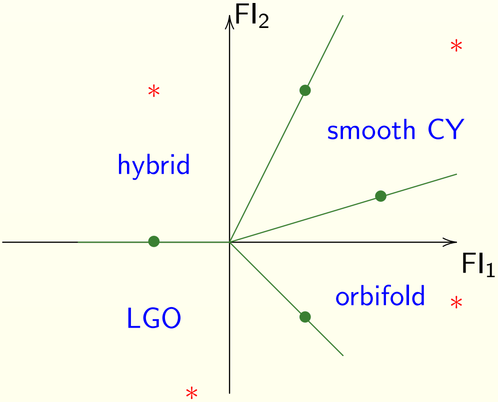
Hybrid SCFT $\equiv$ SCFT with UV model as LGO fibered over B

Extrinsic hybrids via GLSM [Aspinwall et al'93;Witten'93]

GLSM : d=2 gauge theory $G \supseteq \mathrm{U}(1)^r$; $\{i\mathrm{FI} + \theta\} \subseteq \mathcal{M}_{\mathrm{cK}}(M)$

phases $\equiv$ EFT in IR at $\{*\}$

$\{*\}$ smoothly connected



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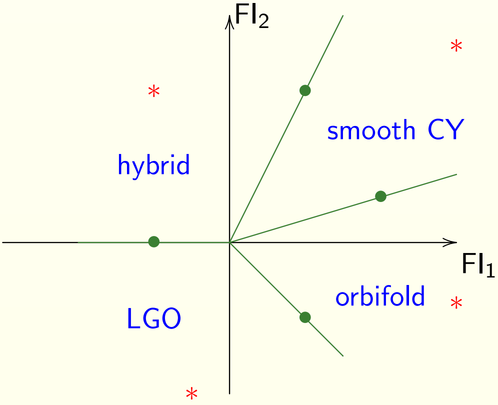

- compute RG-invariants at $*$ in weakly coupled theory
- hybrid phases ubiquitous; now as computable as LGO or CY

Intrinsic hybrids

- (2,2) NLSM with Kähler target $\mathbf{Y}_0$ and $W(y) : \mathbf{Y}_0 \to \mathbb{C}$

  potential condition : $dW^{-1}(0) = B$, B smooth, compact $\dim_{\mathbb{C}} B = d$

- hybrid geometry: model space in IR

$\mathbf{Y} \equiv \mathrm{tot}(X \to B)$,	X is holo rank n bundle
$y = (\phi, u)$	local coordinates
$W = \sum_{\vec{a}} f_{\vec{a}}(u) \prod_i \phi_i^{a_i}$	holo section of $\mathcal{O}_{\mathbf{Y}}$

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- 2 familiar examples: $B =$ compact CY; $B = \text{pt} \implies$ LGO

Intrinsic hybrids : symmetries

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  - for a good hybrid V is vertical vector field $\sum_i q_i \phi_i \partial_i$.

    $\Longrightarrow$ fibered LGO over B a good description
  - pseudo-hybrids : R-symmetry acts on base bosons

    mysterious

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- SUSY $\mathbf{Q}_{L,R}$ $\overline{\mathbf{Q}}_{L,R}$ $\overline{\mathbf{Q}}_R \overset{\text{eom}}{=} \overline{\mathbf{Q}}_0 + \overline{\mathbf{Q}}_W$ $\{\overline{\mathbf{Q}}_0, \overline{\mathbf{Q}}_W\} = 0$

  $\overline{\mathbf{Q}}_0 = \overline{\mathbf{Q}}_R$ in $W = 0$ theory; $\overline{\mathbf{Q}}_W$ is "correction"

  2 nilpotent anti-commuting operators $\rightarrow$ spectral sequence

Intrinsic hybrids: UV $\rightarrow$ IR connection

- $N = 2$ algebra in $\overline{\mathbf{Q}}_R$ cohomology [Fre et al'92;Witten'93;Silverstein&Witten'95]
  - $\overline{\mathbf{Q}}_R$-closed left-moving operators $T, G^{\pm}, J$
  - curved $bc - \beta\gamma$ system realization for good hybrid
  - satisfy $N = 2$ SCA with $c = 3d + 3\sum_i(1 - 2q_i)$
- hybrid limit: large radius limit for B
  - $\implies$ world-sheet instantons suppressed
  - use $bc - \beta\gamma$ to construct $H_{\overline{\mathbf{Q}}}$; grade by L_0 and J_0
  - exact results up to world-sheet instantons in the base!
- curved $bc - \beta\gamma$ perfect tool for (0,2) LGO&NLSM [Kawai&Mohri'94;Nekrasov'05;Witten'05;IVM'09]

Massless heterotic hybrid spectra

- orbifold by $\Gamma = \mathbb{Z}_N$
  - II/het GSO $\Longrightarrow$ SUSY string vacuum

    $\Longrightarrow$ massless fermions : $H_{\overline{\mathbf{Q}}_R} \subset$ (R,R) & (NS,R) sectors
  - hybrid orbi-bundles e.g. $X = \mathcal{O}(-5/2) \oplus \mathcal{O}(-3/2) \to \mathbb{CP}^3$
- massless heterotic $\mathrm{E}_6 \times \mathrm{E}_8$ singlets
  - CY : $h^1(T_M^*) + h^1(T_M) + h^1(\mathrm{End}\ T_M)$ ws instantons in M
  - LGO : twisted (R,R) + (NS,R) sectors [Kachru&Witten'93] exact
  - hybrid : twisted (R,R) + (NS,R) sectors ws instantons in B

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- unprotected quantity determined exactly at limiting points in $\mathcal{M}_{\mathrm{cK}}$
  a window into stringy corrections
  general lesson: jumping mild, unexplained by known non-renormalization results [Silverstein&Witten'95;Basu&Sethi'02;Beasley&Witten'02]

Prospects

- Good (2,2) hybrids — a large playground!
  - natural unifying framework for CY + LGO
  - amenable to quantitative analysis
  - a world-sheet instanton laboratory instantons in $B = \mathbb{P}^1$ vs in quintic
  - (1/2) TFT for hybrids — beyond spectra
- How do hybrids fit into GLSM world?
  [Caldadaru et al'07;Addington,Segal&Sharpe'12;Halverson,Kumar&Morrison'13]
  - may be possible to classify à la Kreuzer&Skarke
  - does every good hybrid have a GLSM embedding?
  - are pseudo-hybrids singular?
- (0,2) hybrids : a much larger world
  - existence of SCFT more delicate
  - interplay between LG and base anomalies
  - potentially new classes of SCFTs
  - insights into (0,2) moduli space
    [Distler&Kachru'93-'95;Kawai&Mohri'94;...;Anderson et al'11;Blumenhagen&Rahn'11; Aspinwall&Plesser'11;Blaszczyk et al'11; IVM&Sharpe'11]