Aspects of Non-geometric String Compactifications

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(Bhg, Deser, Plauschinn, Rennecke, Schmid , arXiv:1304.2784)

(Bhg, Gao, Herschmann, Shukla, arXiv:1306.2761)

Introduction

Introduction

String theory is described by 2D non-linear sigma model

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2 z \left(G_{ab} + B_{ab} \right) \partial X^a \, \overline{\partial} X^b + \ldots \ ,$$

where conformal invariance provides the string equations of motion, which are captured by the effective action

$$S = \frac{1}{2\kappa^2} \int d^n x \sqrt{-|G|} e^{-2\phi} \Big(R - \tfrac{1}{12} H_{abc} H^{abc} + 4 \partial_a \phi \, \partial^a \phi \Big) \ .$$

There exist conformal field theories which cannot be identified with such simple large radius geometries (asymmetric orbifolds). (talk by D. Lüst)

Introduction

Introduction

Applying T-duality using the Buscher rules leads to the chain of fluxes (Shelton, Taylor, Wecht, hep-th/0508133)

$$H_{abc} \leftrightarrow F_{ab}{}^{c} \leftrightarrow Q_{a}{}^{bc} \stackrel{?}{\leftrightarrow} R^{abc} \; ,$$

with geometric flux

$$[e_a, e_b] = F_{ab}{}^{c}\, e_c$$

for n-bein $e_a = e_a{}^{i}\partial_i$. Q and R are non-geometric fluxes.

- What is the formal description of these fluxes?
- Mutual consistency conditions: independence?, Bianchi identities?
- Effects for compactifications with these fluxes turned on?

Superpotential and moduli potential

Superpotential and moduli potential

Via T-duality, it was argued that all these fluxes generate a 4D (type IIA) superpotential (Grimm, Louis, Shelton, Taylor, Wecht, Aldazabal, Camara, Font, Ibanez, Villadoro, Zwirner,Dibitetto, Guarino, Roest, ...)

$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G} \, .$$

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$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G} \, .$$

with

$$\overline{\mathfrak{H}}^C_{ijk} = \overline{H}_{ijk} + 3\, \overline{F}^m{}_{[\underline{ij}} \, (-iJ_c)_{\underline{k}]m} + 3\, \overline{Q}_{[\underline{i}}{}^{mn} (-iJ_c)_{\underline{j}m} \, (-iJ_c)_{\underline{k}]n}$$
$$+ \overline{R}^{mnp} (-iJ_c)_{im} (-iJ_c)_{jn} (-iJ_c)_{kp}$$

- Ω^C depends on complex structure moduli
- $J_c = J + iB$ depends on Kähler moduli

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$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G}\,.$$

There are claims that orientifolds with non-geometric fluxes generically

- stabilize moduli and in particular can lead to de-Sitter vacua (Carlos, Guarino, Moreno, 0911.2876), (Danielsson, Dibitetto, 1212.4984), (Damian, Loaiza-Brito, 1304.0792)
- Contribute “negatively” to R-R tadpoles. (Aldazabal, Camara, Font, Ibanez, hep-th/0602089)

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$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G}\,.$$

- What is the 10D origin of this 4D scalar potential?
- Perform the dimensional reduction 10d$\rightarrow$ 4d a la Taylor/Vafa (hep-th/9912152)
- String phenomenology/cosmology with generic fluxes? So far, we might only have considered a small part of the string landscape.

Generalized geometry

Generalized geometry

Formal developments have led to proposals for effective field theories describing this non-geometric regime of string theory

- Generalized Geometry: manifold M with generalized bundle $E = TM \oplus T^*M$ (Hitchin, Gualtieri, Grana, Minasian, Petrini, Waldram, Coimbra, Strickland-Constable, ...)
- Double field theory: doubled coordinates $X^M = (\tilde{x}_i, x^i)$ tree-level $O(D,D)$ covariant action (Siegel, Hull, Zwiebach, Hohm, Kwak, Aldazabal, Baron, Marques, Nunez, Berman, Blair, Malek, Perry, Musaev, Thompson, Jeon, Lee, Park, ...)
- Generalization to M-theory: U-duality covariant actions

Doubled differential geometry is non-standard.

Generalized geometry

Generalized geometry

- Generalized tangent bundle $E = TM \oplus T^*M$ over a D-dimensional manifold M, $(X+\xi) \in \Gamma(E)$
- The natural bilinear form on the bundle E is

$$\langle X+\xi, Y+\zeta \rangle = \xi(Y) + \zeta(X)\,,$$

- The transformations $\mathcal{M}^t\, \eta\, \mathcal{M} = \eta$ form the group $O(D,D)$.

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\,,$$

Briefly review formal framework for string actions in $O(D,D)$ redefined fields (Andriot, Hohm, Larfors, Lüst, arXiv:1202.3060+1204.1979), (Bhg, Deser, Plauschinn, Rennecke , arXiv:1210.1591+1211.0030+1304.2784)

(more details in talks by E. Plauschinn & F. Rennecke and by D. Andriot & A. Betz)

Generalized metric

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Combine the metric G_{ab} and Kalb-Ramond field B_{ab} into the generalized metric

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$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

Mass formula of toroidal compactification

$$M^2 = P^M \, \mathcal{H}_{MN} \, P^N + \text{oscil.}$$

with $P^M = (w_i, p^i)$.

Generalized metric

Combine the metric G_{ab} and Kalb-Ramond field B_{ab} into the generalized metric

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}.$$

$O(D,D)$ act on $\mathcal{H}$ as

$$\widehat{\mathcal{H}} = \mathcal{M}^t \, \mathcal{H} \, \mathcal{M}\,.$$

Main examples

$$\begin{pmatrix} \mathsf{A} & 0 \\ 0 & (\mathsf{A}^t)^{-1} \end{pmatrix}, \qquad \begin{pmatrix} \mathbb{1} & 0 \\ -\mathsf{d}\xi & \mathbb{1} \end{pmatrix}, \qquad \begin{pmatrix} \mathbb{1} & \beta \\ 0 & \mathbb{1} \end{pmatrix}$$

$G_{\rm geom} = ($diffeomorphisms, B-field gauge trafos$)$ β-transforms

Field redefinition

Field redefinition

Action invariant under G_{geom}:

$$S(G,B) = \frac{1}{2\kappa^2} \int d^n x \sqrt{|G|}\, e^{-2\phi} \Big(R - \tfrac{1}{12} H^2 + 4\partial_a \phi\, \partial^a \phi \Big)$$

String action for $O(D,D)$ transformed fields (G,B)?

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String action for $O(D,D)$ transformed fields (G,B)?

$$\mathcal{M}^t \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix} \mathcal{M} = \mathcal{H}'(G,B)$$

Field redefinition

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String action for $O(D,D)$ transformed fields (G,B)?

$$\mathcal{H}'(G,B) = \mathcal{M}^t\mathcal{H}(G,B)\mathcal{M} \overset{!}{=} \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix}$$

Field redefinition

Action invariant under $G_{\rm geom}$:

$$S(G,B) = \frac{1}{2\kappa^2}\int d^n x\,\sqrt{|G|}\,e^{-2\phi}\Big(R - \tfrac{1}{12}\,H^2 + 4\partial_a\phi\,\partial^a\phi\Big)$$

String action for $O(D,D)$ transformed fields (G,B)?

$$\mathcal{H}'(G,B) = \mathcal{M}^t\mathcal{H}(G,B)\mathcal{M} = \mathcal{H}(g,b)$$

One gets

$$g = \gamma^{-1} G\,(\gamma^{-1})^t \quad \text{with } \gamma = d + (G-B)\,b\,.$$

Field redefinition

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$$S(G,B) = \frac{1}{2\kappa^2}\int d^n x\,\sqrt{|G|}\,e^{-2\phi}\Big(R - \tfrac{1}{12}\,H^2 + 4\partial_a\phi\,\partial^a\phi\Big)$$

String action for $O(D,D)$ transformed fields (G,B)?

$$\mathcal{H}'(G,B) = \mathcal{M}^t\mathcal{H}(G,B)\mathcal{M} = \mathcal{H}(g,b)$$

One gets

$$g = \gamma^{-1}G\,(\gamma^{-1})^t \quad \text{with } \gamma = d + (G-B)\,b\,.$$

and similarly

$$b = \gamma^{-1}\left[\gamma\,(c + (G-B)\,a)^t - G\right](\gamma^{-1})^t\,.$$

Quest for non-geometric actions

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$$S(G,B) = \frac{1}{2\kappa^2}\int d^n x\,\sqrt{|G|}\,e^{-2\phi}\Big(R - \tfrac{1}{12}\,H^2 + 4\partial_a\phi\,\partial^a\phi\Big)$$

$$\Big\downarrow\ \begin{matrix} g(G,B) \\ b(G,B) \end{matrix}$$

$$S(g,b)?$$

- Due to $\gamma = d + (G-B)\,b$, the computation is very cumbersome and the geometric symmetries mix in a complicated manner
- Need an order principle!
- Observation: The differential geometry of Lie-algebroids provides precisely that (more details in talk by E. Plauschinn)

Non-geometric string action

Non-geometric string action

Thus we found:

$$S(G,B) = \int d^n x \sqrt{|G|}\, e^{-2\phi}\Big(R - \tfrac{1}{12} H^2 + 4\partial_a\phi\,\partial^a\phi\Big)$$

$$\Big\downarrow \quad \begin{matrix} g(G,B) \\ b(G,B) \end{matrix}$$

$$S(g,b) = \int d^n x \sqrt{|g|}\,|\det\rho^*|\, e^{-2\phi}\Big(\widehat{R} - \tfrac{1}{12}\,\Theta_{\alpha\beta\gamma}\Theta^{\alpha\beta\gamma} + 4 D_\alpha\phi\, D^\alpha\phi\Big)\,.$$

By construction, this action

- for β-transf. makes visible non-geometric fields
- is invariant under diffeos, 2-form gauge transf.
- but lacks the symmetries needed for a global description of non-geometric backgrounds(more details in talk by F. Rennecke)

Double field theory

Double field theory

(Bhg, Gao, Herschmann, Shukla, arXiv:1306.2761)

Doubled coordinates $X^M = (\tilde{x}_i, x^i) \to$ DFT action

$$S_{\rm DFT} = -\frac{1}{2\kappa^2}\int d^D x\, d^D\tilde{x}\; e^{-2d}\Big(\tfrac{1}{8}\mathcal{H}^{MN}(\partial_M\mathcal{H}^{KL})(\partial_N\mathcal{H}_{KL}) - \tfrac{1}{2}\mathcal{H}^{MN}(\partial_N\mathcal{H}^{KL})(\partial_L\mathcal{H}_{MK}) - 2(\partial_M d)(\partial_N\mathcal{H}^{MN}) + 4\mathcal{H}^{MN}(\partial_M d)(\partial_N d)\Big).$$

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String world-sheet: doubled zero modes

$$X_L^a(z) = \frac{x^a + \tilde{x}^a}{2} + (p^a - \frac{1}{2}w^a)(\tau - \sigma) + \text{osc.}$$

$$X_R^a(z) = \frac{x^a - \tilde{x}^a}{2} + (p^a + \frac{1}{2}w^a)(\tau + \sigma) + \text{osc.}$$

Double field theory

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$$S_{\rm DFT} = -\frac{1}{2\kappa^2}\int d^Dx\, d^D\tilde{x}\; e^{-2d}\Big(\tfrac{1}{8}\mathcal{H}^{MN}(\partial_M\mathcal{H}^{KL})(\partial_N\mathcal{H}_{KL}) - \tfrac{1}{2}\mathcal{H}^{MN}(\partial_N\mathcal{H}^{KL})(\partial_L\mathcal{H}_{MK}) - 2(\partial_M d)(\partial_N\mathcal{H}^{MN}) + 4\mathcal{H}^{MN}(\partial_M d)(\partial_N d)\Big).$$

Strong constraint

$$\partial_i A\,\tilde{\partial}^i B + \tilde{\partial}^i A\,\partial_i B = 0\ .$$

Double field theory

Double field theory

Symmetries:

- Doubled diffeomorphisms, i.e.

$$(\tilde{x}_i, x^i) \to (\tilde{x}_i + \tilde{\xi}_i(X), x^i + \xi^i(X))$$

  x^i dependence: standard diffeomorphisms and B-field gauge transformations
  $\tilde{x}_i$ dependence: β-field gauge transformations.

- Global $O(D,D)$ symmetry:

$$\mathcal{H}' = h^t \mathcal{H} h \,, \qquad d' = d \,,$$
$$X' = hX \,, \qquad \partial' = (h^t)^{-1} \partial \,,$$

DFT Fluxes

DFT Fluxes

Introduce non-holonomic basis

$$\partial_a = e_a{}^i \partial_i \qquad \tilde{\partial}^a = e_i{}^a \tilde{\partial}^i$$

The $e_a{}^i$ depend on $(\tilde{x}_i, x^i)$.

DFT Fluxes

Introduce non-holonomic basis

$$\partial_a = e_a{}^i \partial_i \qquad \tilde{\partial}^a = e_i{}^a \tilde{\partial}^i$$

The $e_a{}^i$ depend on $(\tilde{x}_i, x^i)$.

For the commutator of two partial derivatives one gets

$$[\partial_a, \partial_b] = f^c{}_{ab}\, \partial_c$$

with

$$f^c{}_{ab} = e_i{}^c \Big(\partial_a e_b{}^i - \partial_b e_a{}^i \Big) \ .$$

DFT Fluxes

Introduce non-holonomic basis

$$\partial_a = e_a{}^i \partial_i \qquad \tilde{\partial}^a = e_i{}^a \tilde{\partial}^i$$

The $e_a{}^i$ depend on $(\tilde{x}_i, x^i)$.

Analogously, for the partial winding derivatives one finds

$$[\tilde{\partial}^a, \tilde{\partial}^b] = \tilde{f}_c{}^{ab}\, \tilde{\partial}^c$$

with

$$\tilde{f}_a{}^{bc} = e_a{}^i \left(\tilde{\partial}^b e_i{}^c - \tilde{\partial}^c e_i{}^b \right) .$$

DFT Fluxes

Introduce non-holonomic basis

$$\partial_a = e_a{}^i \partial_i \qquad \tilde{\partial}^a = e_i{}^a \tilde{\partial}^i$$

The $e_a{}^i$ depend on $(\tilde{x}_i, x^i)$.

Consider the two DFT vector fields

$$\mathcal{D}_a = \partial_a + B_{am} \tilde{\partial}^m \,, \qquad \tilde{\mathcal{D}}^a = \tilde{\partial}^a + \beta^{am} \mathcal{D}_m$$

and compute

$$[\mathcal{D}_a, \mathcal{D}_b] = F^c{}_{ab} \, \mathcal{D}_c + H_{abc} \, \tilde{\mathcal{D}}^c$$

$$[\mathcal{D}_a, \tilde{\mathcal{D}}^b] = Q_a{}^{bc} \, \mathcal{D}_c - F^b{}_{ac} \, \tilde{\mathcal{D}}^c$$

$$[\tilde{\mathcal{D}}^a, \tilde{\mathcal{D}}^b] = R^{abc} \, \mathcal{D}_c + Q_c{}^{ab} \, \tilde{\mathcal{D}}^c$$

see also (Geissbuhler, Marquez, Nunez, Penaz, arXiv:1304.1472)

DFT Fluxes

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with the DFT flux

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with the DFT flux

$$H_{abc} = 3\Big(\partial_{[\underline{a}} B_{\underline{bc}]} + f^{m}{}_{[\underline{ab}} B_{\underline{c}]m} + B_{[\underline{a}m} \tilde{\partial}^{m} B_{\underline{bc}]} + B_{[\underline{a}m} B_{\underline{b}n} \tilde{f}_{\underline{c}]}{}^{mn}\Big)\,,$$

DFT Fluxes

with the DFT flux

$$F^c{}_{ab} = f^c{}_{ab} + \tilde{\partial}^c B_{ab} + \tilde{f}_a{}^{cm} B_{mb} + \tilde{f}_b{}^{cm} B_{am} + \beta^{cm} H_{mab}\,,$$

DFT Fluxes

with the DFT flux

$$Q_c{}^{ab} = \tilde{f}_c{}^{ab} + \partial_c \beta^{ab} + f^a{}_{cm} \beta^{mb} + f^b{}_{cm} \beta^{am}$$
$$+ B_{cm} \tilde{\partial}^m \beta^{ab} + 2\beta^{m[\underline{a}} \tilde{\partial}^{\underline{b}]} B_{mc}$$
$$+ 2B_{cm} \tilde{f}_n{}^{m[\underline{a}} \beta^{\underline{b}]n} + 2\beta^{m[\underline{a}} \tilde{f}_c{}^{\underline{b}]n} B_{mn} + \beta^{am} \beta^{bn} H_{mnc}$$

DFT Fluxes

with the DFT flux

$$R^{abc} = 3\Big(\tilde{\partial}^{[\underline{a}}\beta^{\underline{bc}]} + \tilde{f}_m{}^{[\underline{ab}}\,\beta^{\underline{c}]m}\Big) + 3\Big(\beta^{[\underline{a}m}\partial_m\beta^{\underline{bc}]} + \beta^{[\underline{a}m}\beta^{\underline{b}n}f^{\underline{c}]}{}_{mn}\Big)$$
$$+ 3\Big(B_{mn}\beta^{[\underline{a}m}\tilde{\partial}^n\beta^{\underline{bc}]} + \beta^{[\underline{a}m}\beta^{\underline{b}n}\tilde{\partial}^{\underline{c}]}B_{mn} +$$
$$2\beta^{[\underline{a}m}\beta^{\underline{b}n}\tilde{f}_{[\underline{m}}{}^{\underline{c}]k}B_{k\underline{n}]}\Big) + \beta^{am}\beta^{bn}\beta^{cp}H_{mnp}\,.$$

DFT Fluxes

with the DFT flux

$$R^{abc} = 3\Big(\tilde{\partial}^{[\underline{a}}\beta^{\underline{bc}]} + \tilde{f}_m{}^{[\underline{ab}}\beta^{\underline{c}]m}\Big) + 3\Big(\beta^{[\underline{a}m}\partial_m\beta^{\underline{bc}]} + \beta^{[\underline{a}m}\beta^{\underline{b}n}f^{\underline{c}]}{}_{mn}\Big)$$
$$+ 3\Big(B_{mn}\beta^{[\underline{a}m}\tilde{\partial}^n\beta^{\underline{bc}]} + \beta^{[\underline{a}m}\beta^{\underline{b}n}\tilde{\partial}^{\underline{c}]}B_{mn} +$$
$$2\beta^{[\underline{a}m}\beta^{\underline{b}n}\tilde{f}_{[\underline{m}}{}^{\underline{c}]k}B_{k\underline{n}]}\Big) + \beta^{am}\beta^{bn}\beta^{cp}H_{mnp}\,.$$

In a given patch with geometric frame, one can turn on components of H, F and also Q.

Bianchi identies

Bianchi identies

The Jacobi identities for the brackets generate the 5 Bianchi identities for the fluxes.

$$
\begin{aligned}
\mathcal{D}_{[\underline{a}}H_{\underline{bcd}]} - \tfrac{3}{2}H_{m[\underline{ab}}F^{m}{}_{\underline{cd}]} &= 0 \\
-\tfrac{1}{3}\tilde{\mathcal{D}}^{d}H_{abc} + \mathcal{D}_{[\underline{a}}F^{d}{}_{\underline{bc}]} + F^{m}{}_{[\underline{bc}}F^{d}{}_{\underline{a}]m} + H_{m[\underline{ab}}Q_{\underline{c}]}{}^{md} &= 0 \\
2\tilde{\mathcal{D}}^{[\underline{c}}F^{\underline{d}]}{}_{[\underline{ab}]} + 2\mathcal{D}_{[\underline{a}}Q_{\underline{b}]}{}^{[\underline{cd}]} \qquad & \\
-F^{m}{}_{[\underline{ab}]}Q_{m}{}^{[\underline{cd}]} + 4\,F^{[\underline{c}}{}_{m[\underline{a}}Q_{\underline{b}]}{}^{\underline{d}]m} - H_{abm}R^{mcd} &= 0 \\
-\tfrac{1}{3}\mathcal{D}_{d}R_{abc} + \tilde{\mathcal{D}}^{[\underline{a}}Q_{d}{}^{\underline{bc}]} + Q_{m}{}^{[\underline{bc}}Q_{d}{}^{\underline{a}]m} + R^{m[\underline{ab}}F^{\underline{c}]}{}_{md} &= 0 \\
\tilde{\mathcal{D}}^{[\underline{a}}R^{\underline{bcd}]} - \tfrac{3}{2}R^{m[\underline{ab}}Q_{m}{}^{\underline{cd}]} &= 0.
\end{aligned}
$$

Orientifold projection

Orientifold projection

Under world-sheet parity $\Omega : (\sigma, \tau) \to (-\sigma, \tau)$

$$\Omega : \begin{cases} \partial_a \to \partial_a \,, \quad \tilde{\partial}^a \to -\tilde{\partial}^a \\ B_{ab} \to -B_{ab} \,, \quad \beta^{ab} \to -\beta^{ab} \\ f^a{}_{bc} \to f^a{}_{bc} \,, \quad \tilde{f}_a{}^{bc} \to -\tilde{f}_a{}^{bc} \end{cases}$$

so that the fluxes transform as

$$\Omega : \begin{cases} H_{abc} \to -H_{abc} \\ F^a{}_{bc} \to F^a{}_{bc} \\ Q_a{}^{bc} \to -Q_a{}^{bc} \\ R^{abc} \to R^{abc} \end{cases}$$

Half of the components are Ω-invariant.

Superpotential

Superpotential

For toroidal type IIA background, oxidize the scalar potential induced by the superpotential to 10D

$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G} \,.$$

with R-R flux $G = e^{-B}\overline{G}$.

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with R-R flux $G = e^{-B}\overline{G}$.

- Compute the induced scalar potential
- Inspection of appearing terms $\rightarrow$ ansatz for terms quadratic in fluxes with indices contracted via $B, G(T, U)$
- Invoke Bianchi identities

Superpotential

For toroidal type IIA background, oxidize the scalar potential induced by the superpotential to 10D

$$W = -i \int_X \overline{\mathfrak{H}}^C \wedge \Omega^C + \int_X e^{iJ_c} \wedge \overline{G} \, .$$

with R-R flux $G = e^{-B}\overline{G}$.
The following combinations of fluxes appear

$$\mathfrak{H}_{ijk} = \overline{H}_{ijk} + 3\, \overline{F}^m{}_{[\underline{ij}}\, B_{m\underline{k}]} + 3\, \overline{Q}_{[\underline{i}}{}^{mn} B_{m\underline{j}}\, B_{n\underline{k}]} + \overline{R}^{mnp} B_{m[\underline{i}} B_{n\underline{j}} B_{p\underline{k}]}$$

$$\mathfrak{F}^i{}_{jk} = \overline{F}^i{}_{jk} + 2\, \overline{Q}_{[\underline{j}}{}^{mi} B_{m\underline{k}]} + \overline{R}^{mni} B_{m[\underline{j}} B_{n\underline{k}]}$$

$$\mathfrak{Q}_k{}^{ij} = \overline{Q}_k{}^{ij} + \overline{R}^{mij} B_{mk}$$

$$\mathfrak{R}^{ijk} = \overline{R}^{ijk} \, .$$

Oxidized 10D action: NS-NS

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The oxidized action can be written as

$$S = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\Big(\mathcal{L}_1^{\rm NS} + \mathcal{L}_2^{\rm NS} + \mathcal{L}^{\rm R}\Big)\,.$$

with

$$\mathcal{L}_1^{\rm NS} = -\frac{e^{-2\phi}}{12}\Big(\mathfrak{H}_{ijk}\,\mathfrak{H}_{i'j'k'}\,g^{ii'}\,g^{jj'}\,g^{kk'} + 3\,\mathfrak{F}^{i}{}_{jk}\,\mathfrak{F}^{i'}{}_{j'k'}\,g_{ii'}g^{jj'}\,g^{kk'}$$
$$+\,3\,\mathfrak{Q}_k{}^{ij}\,\mathfrak{Q}_{k'}{}^{i'j'}\,g_{ii'}g_{jj'}g^{kk'} + \mathfrak{R}^{ijk}\,\mathfrak{R}^{i'j'k'}\,g_{ii'}g_{jj'}g_{kk'}\Big)$$

and a term containing a single metric factor

$$\mathcal{L}_2^{\rm NS} = -\frac{e^{-2\phi}}{2}\Big(\mathfrak{F}^{m}{}_{ni}\,\mathfrak{F}^{n}{}_{mi'}\,g^{ii'} + \mathfrak{Q}_m{}^{ni}\,\mathfrak{Q}_n{}^{mi'}\,g_{ii'}$$
$$-\,\mathfrak{H}_{mni}\,\mathfrak{Q}_{i'}{}^{mn}\,g^{ii'} - \mathfrak{F}^{i}{}_{mn}\,\mathfrak{R}^{mni'}\,g_{ii'}\Big)\,.$$

Oxidized 10D action: NS-NS

Oxidized 10D action: NS-NS

The contribution from the RR-sector is

$$\mathcal{L}^{\mathrm{R}} = -\frac{1}{2} \sum_{p=0,2,4,6} |G^{(p)}|^2 \,,$$

with the components

$$G^{(0)} = \overline{G}^{(0)} + \frac{1}{6}\,\mathfrak{R}^{mnp} C^{(3)}_{mnp}$$

$$G^{(2)}_{ij} = \overline{G}^{(2)}_{ij} - B_{ij}\overline{G}^{(0)} + \mathfrak{Q}_{[\underline{i}}{}^{mn} C^{(3)}_{mn\underline{j}]}$$

$$G^{(4)}_{ijkl} = \overline{G}^{(4)}_{ijkl} - 6\, B_{[\underline{ij}}\overline{G}^{(2)}_{\underline{kl}]} + 3\, B_{[\underline{ij}}B_{\underline{kl}]}\,\overline{G}^{(0)} - 6\,\mathfrak{F}^{m}{}_{[\underline{ij}}C^{(3)}_{m\underline{kl}]}$$

$$G^{(6)}_{ijklmn} = \overline{G}^{(6)}_{ijklmn} - 15\, B_{[\underline{ij}}\overline{G}^{(4)}_{\underline{klmn}]} + 45\, B_{[\underline{ij}}B_{\underline{kl}}\,\overline{G}^{(2)}_{\underline{mn}]} - 15\, B_{[\underline{ij}}B_{\underline{kl}}B_{\underline{mn}]}\,\overline{G}^{(0)} - 20\,\mathfrak{H}_{[\underline{ijk}}C^{(3)}_{\underline{lmn}]} \,.$$

D-terms

D-terms

Defining a three-form

$$\tau_{ijk} = \overline{H}_{ijk}\,\overline{G}^{(0)} + 3\,\overline{F}^{m}{}_{[\underline{ij}}\overline{G}^{(2)}_{m\underline{k}]} - \frac{3}{2}\overline{Q}_{[\underline{i}}{}^{mn}\overline{G}^{(4)}_{mn\underline{jk}]} - \frac{1}{6}\overline{R}^{mnp}\overline{G}^{(6)}_{mnpijk}\,.$$

the dimensional reduction of 10D action also gives a D-term

$$V_D = -\frac{1}{2}e^{K}t_1t_2t_3\Big[s\,\tau_{135} - u_1\,\tau_{146} - u_2\,\tau_{236} - u_3\,\tau_{245}\Big]$$

where s, u_i complex stucture moduli.
Invoking R-R tadpole cancellation, this term cancels against tensions of the $D6$-branes and $O6$-planes.

DFT NS-NS action

DFT NS-NS action

The NS-NS part can be related to terms appearing the flux formulation of the DFT action (Aldazabel, Marques, Nunez, Rosabal, 1101.5954), (Geissbuhler, 1109.4280), (Grana, Marques, 1201.2924)

$$
\begin{aligned}
S_{\mathrm{DFT}} = \frac{1}{2}\int d^{20}X\, e^{-2d}\Big[&\mathcal{F}_{ABC}\mathcal{F}_{A'B'C'}\Big(\frac{1}{4}S^{AA'}\eta^{BB'}\eta^{CC'} \\
&-\frac{1}{12}S^{AA'}S^{BB'}S^{CC'} - \frac{1}{6}\eta^{AA'}\eta^{BB'}\eta^{CC'}\Big) \\
&+\mathcal{F}_A\mathcal{F}_{A'}\Big(\eta^{AA'} - S^{AA'}\Big)\Big].
\end{aligned}
$$

with $\mathcal{F}_{abc} = H_{abc}\,,\ \mathcal{F}^a{}_{bc} = F^a{}_{bc}\,,\ \mathcal{F}^{ab}{}_c = Q_c{}^{ab}\,,\ \mathcal{F}^{abc} = R^{abc}$.
Using e.g.

$$
\mathcal{F}_{abc} = e_a{}^i\, e_b{}^j\, e_c{}^k\, \mathfrak{H}_{ijk}\,,
$$

the "blue" terms give precisely $\mathcal{L}_1^{\mathrm{NS}} + \mathcal{L}_2^{\mathrm{NS}}$.

DFT R-R action

DFT R-R action

One can put all R-R fields into the spinor representation of $O(D,D)$ as (Hohm, Kwak, Zwiebach, 1106.5452)

$$\mathcal{G} = \sum_n \frac{1}{n!} G^{(n)}_{i_1 \dots i_n} e_{a_1}{}^{i_1} \dots e_{a_n}{}^{i_n} \Gamma^{a_1 \dots a_n} |0\rangle \,,$$

Then, one can compactly define the R-R field strengths as

$$\mathcal{G} = \nabla\!\!\!/ \mathcal{C}$$

with the generalized fluxed Dirac operator defined as

$$\nabla\!\!\!/ = \Gamma^A D_A - \frac{1}{3}\Gamma^A \mathcal{F}_A - \frac{1}{6}\Gamma^{ABC}\mathcal{F}_{ABC} \,.$$

Consistent with the oxidized R-R action.

Summary and outlook

Summary and outlook

- Briefly reviewed some recent formal developments in the description of non-geometric fluxes.
- Further future studies of de Sitter vacua and non-geometric model building should be performed (no runaway directions and no flux/anti-brane, anti-flux/brane config.)
- Work in progress: Generalization of oxidation procedure to Calabi-Yau manifolds with non-geometric fluxes.

Lie-algebroids

Lie-algebroids

Differential geometry is usually based on the tangent bundle. This can be generalized by introducing the notion of a Lie-algebroid:

Definition: Let M be a manifold, $E \to M$ a vector bundle together with a bracket $[\cdot,\cdot]_E : \Gamma(E) \times \Gamma(E) \to \Gamma(E)$ satisfying the Jacobi identity, and a homomorphism $\rho : E \to TM$ called the anchor-map. Then $(E, [\cdot,\cdot]_E, \rho)$ is called Lie algebroid if the following Leibniz rule is satisfied

$$[s_1, f s_2]_E = f\,[s_1, s_2]_E + \rho(s_1)(f) s_2 \,,$$

for $f \in \mathcal{C}^\infty(M)$ and sections s_i of E.

Lie-algebroids

Lie-algebroids

- In a Lie algebroid vector fields and their Lie bracket $[\cdot,\cdot]_L$ are replaced by sections in E and the corresponding bracket.
- The relation between the different brackets is established by the anchor preserving the algebraic structure

$$\rho\big([s_1, s_2]_E\big) = \big[\rho(s_2), \rho(s_2)\big]_L \ ,$$