Heterotic String Compactification with Gauged Linear Sigma Models

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Based on:

[Lüdeling,FR,Wieck: 1203.5789], [Blaszczyk,Groot Nibbelink,FR: 1111.5852],

[Blaszczyk,Groot Nibbelink,FR: 1107.0320], [Work in progress]

Outline

1. Motivation
2. $\mathcal{N} = (2,2)$ GLSMs [cf. talk by Michael Blaszczyk]
3. $\mathcal{N} = (0,2)$ GLSMs
4. Example: $\mathbb{Z}_3$ Orbifold + Resolutions
   - GLSM construction
   - Study of (Kähler) moduli space
   - Study of discrete symmetries
5. Conclusion

Motivation

Study string **moduli space**

- Investigate different **GLSM phases**
- Study **discrete symmetries** in various phases

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Discrete symmetries on orbifold

- Non-R Symmetries from fixed point degeneracies
- R Symmetries from remnants of internal Lorentz group
  Lee *et al* found nice $\mathbb{Z}_4$ R symmetry

  [Lee,Raby,Ratz,Ross,Schieren,Schmidt-Hoberg,Vaudrevange]

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Discrete symmetries on CY

- Symmetries arise from isometries in GLSM
- Charges from graded cohomology

GLSM Action [Witten]

$S_{\text{GLSM}} = S_{\text{kin}} + S_{\text{FI}} + S_{\text{W}}$

- $S_{\text{kin}} = \int \mathrm{d}^2\sigma \mathrm{d}^4\theta \overline{\mathcal{Z}} e^{2qV} \mathcal{Z} + \frac{1}{e^2}\overline{\Sigma}\Sigma$
- $S_{\text{FI}} = \int \mathrm{d}^2\sigma \mathrm{d}\bar{\theta}^- \mathrm{d}\theta^+ \rho\Sigma + \text{h.c.}$ with $\rho = b + \mathrm{i}\beta$
- $S_{\text{W}} = \int \mathrm{d}^2\sigma \mathrm{d}^2\theta \mathcal{C} P(\mathcal{Z}) + \text{h.c.}$ with $q(\mathcal{C}) < 0, q(\mathcal{Z}) > 0$, $q(P(\mathcal{Z})) = q(\mathcal{C}) = \sum_i q(\mathcal{Z}_i)$

Algebraic EOMs

- ***D* term**: $\sum_i q_i |z_i|^2 + q_c |c|^2 = b$
- ***F* term**: $P(z) = 0,\ c\frac{\partial P(z)}{\partial z_i} = 0$

[cf. talk by Michael Blaszczyk]

superfield type	superfield notation	charge	bosonic DOF	fermionic DOF
chiral	Z^a	$(q_I)^a$	z^a	ψ^a
chiral–Fermi	Λ^α	$(Q_I)^\alpha$	$\widetilde{F}^\alpha$	λ^α
gauge	$(V, A)^I$	0	$a^I_\sigma, a^I_{\overline{\sigma}}, \widetilde{D}^I$	Φ^I
Fermi–gauge	Σ^i	0	s^i	φ^i
chiral	Φ^m	$(q_I)^m$	x^m	ψ^m
chiral–Fermi	Γ^μ	$(Q_I)^\mu$	$\widetilde{F}^\mu$	γ^μ

Think of 2D $\mathcal{N} = (0,2)$ **superspace** as $\mathcal{N} = (2,2)$ **superspace** and **dispense** of $\theta^+, \bar{\theta}^+$ [Dine, Seiberg]

Multiplets

- $(2,2)$ Chiral multiplet $\mathcal{Z}_{(2,2)} \Rightarrow (Z; \chi) =$(chiral; chiral-Fermi)
- $(2,2)$ Vector multiplet $V_{2,2} \Rightarrow (V, A; \Sigma) =$(gauge; Fermi-gauge)

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Geometry:

$\widetilde{D}$ **Term**:

$(q_I)^a|z^a|^2 + (q_I)^m|x^m|^2 - b_I = 0, \qquad b_I$: FI–parameter

$\widetilde{F}$ **Term**:

$W_{\text{geom}} = \Gamma^\mu P_\mu(Z) \Rightarrow P_\mu(Z) = 0$

Geometry

$\widetilde{D}$ **Terms** give SR ideal $\Rightarrow$ Kähler cone, GLSM phase

$\widetilde{F}$ **Terms** give restriction to hypersurface

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Gauge group:

Fermionic Transformation:

$\delta_\Theta \Lambda^\alpha = M^\alpha{}_i(Z)\Theta_i$

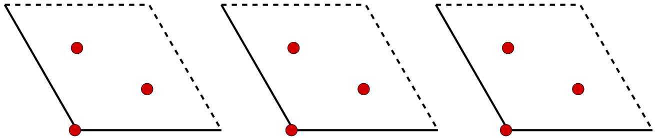
Superpotential:

$W_{\text{bundle}} = \Phi^m N_{m\alpha}(\Psi)\Lambda^\alpha \Rightarrow \Phi^m N_{m\alpha}(Z) = 0$

Gauge group

Gauge group and particle content given by (naturally arising) monad construction via $\ker(N)/\text{im}(M)$.

$\mathbb{Z}_3$ Orbifold



$$\theta : (z_1, z_2, z_3) \mapsto (e^{2\pi i/3} z_1, e^{2\pi i/3} z_2, e^{-2\pi i\, 2/3} z_3)$$

- **Orbifold action** given by **twist** vector $v = \frac{1}{3}(1, 1, -2)$
- **Gauge sector** given by **shift** V in the $\Lambda_{E_8 \times E_8}$
- Choose **standard embedding** $V = \frac{1}{3}(1, 1, -2, 0^5)(0^8)$ "$= v$"

$$\text{Gauge group:} \quad [E_6 \times SU(3)]_{\text{vis}} \times [E_8]_{\text{hidden}}$$

$$\text{Matter:} \quad 3(\mathbf{27}, \overline{\mathbf{3}}; \mathbf{1}) + 27[(\mathbf{27}, \mathbf{1}; \mathbf{1}) + 3(\mathbf{1}, \mathbf{3}; \mathbf{1})]$$

Let us study the moduli space of simplified $\mathbb{Z}_3$ GLSM

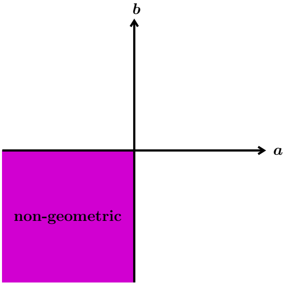
$U(1)$'s	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	c_1	c_2	c_3	x_1	x_2	x_3
R_1	1	1	1	0	0	0	0	0	0	-3	0	0	**0**	**0**	**0**
R_2	0	0	0	1	1	1	0	0	0	0	-3	0	**0**	**0**	**0**
R_3	0	0	0	0	0	0	1	1	1	0	0	-3	**0**	**0**	**0**
E_1	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**	**0**
E_2	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**	**0**
E_3	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**-3**

Superpotential:
$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:
$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a_i \,, \quad i = 1, 2, 3$
$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b_\rho \,, \quad \rho = 1, 2, 3$

For simplicity: Set $a_i = a$ and $b_\rho = b$.

Phase I: Non-Geometric Regime



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$a < 0,\ b < 0$:

$\langle c_i \rangle = \frac{\sqrt{a}}{3}, \qquad \langle x_\rho \rangle = \frac{\sqrt{b}}{3}, \qquad z_{i\rho} = 0$

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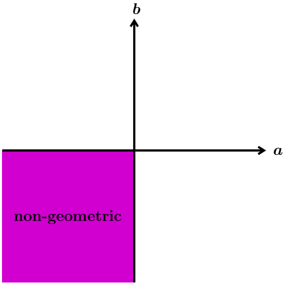

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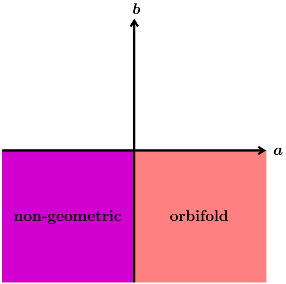
Target space is a point.



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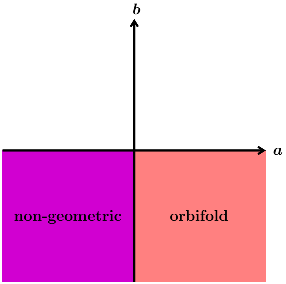
$a > 0,\ b < 0$:

$$c_i = 0, \qquad \langle x_\rho \rangle > 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Phase II: Orbifold



Superpotential:

$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:

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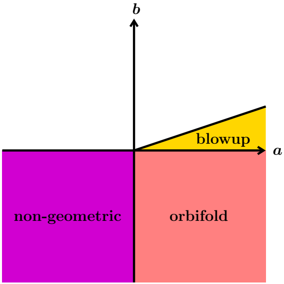
$c_i = 0, \qquad \langle x_\rho \rangle > 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space is the $T^6/\mathbb{Z}_3$ orbifold.

Phase III: Blowup



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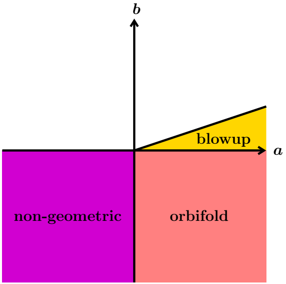
D–terms:

$$\sum_{\rho=1}^{3} |z_{i\rho}|^2 - 3|c_i|^2 = a$$
$$\sum_{i=1}^{3} |z_{i\rho}|^2 - 3|x_\rho|^2 = b$$

$a > 3b > 0$:

$$c_i = 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$$



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$a > 3b > 0$:

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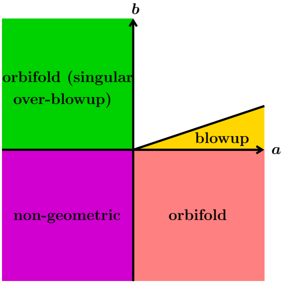
Target space is the resolution CY of the $T^6/\mathbb{Z}_3$ orbifold.



Superpotential:

$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$

D–terms:

$\sum_{\rho=1}^3 |z_{i\rho}|^2 - 3|c_i|^2 = a$
$\sum_{i=1}^3 |z_{i\rho}|^2 - 3|x_\rho|^2 = b$

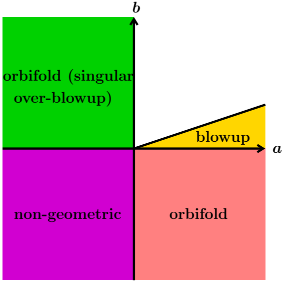
$a < 0, \; b > 0$:

Note complete symmetry of the model under

$x_\rho \leftrightarrow c_i, \;\; z_{i\rho} \leftrightarrow z_{\rho i}, \;\; a \leftrightarrow b$

$\langle c_i \rangle > 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Phase IV: Singular Over-Blowup



Superpotential:

$$\mathcal{W} = \sum_{i,\rho} c_i z_{i\rho}^3 x_\rho$$

D–terms:

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$a < 0,\ b > 0$:

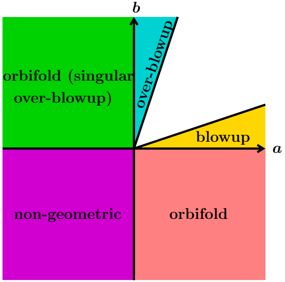
Note complete symmetry of the model under

$x_\rho \leftrightarrow c_i,\ \ z_{i\rho} \leftrightarrow z_{\rho i},\ \ a \leftrightarrow b$

$\langle c_i \rangle > 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$

Target space again **$T^6/\mathbb{Z}_3$ orbifold**, with x and c exchanged.

Phase V: Over-Blowup



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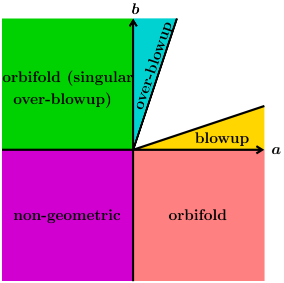
$b > 3a > 0$:

$$\langle c_i \rangle \geq 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Phase V: Over-Blowup



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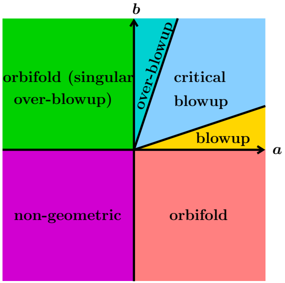
$b > 3a > 0$:

$$\langle c_i \rangle \geq 0, \qquad x_\rho = 0, \qquad \langle z_{i\rho} \rangle \geq 0$$

Target space is the **resolution CY** of the "other" $T^6/\mathbb{Z}_3$ orbifold.

Phase VI: Critical Blowup



Superpotential:

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D–terms:

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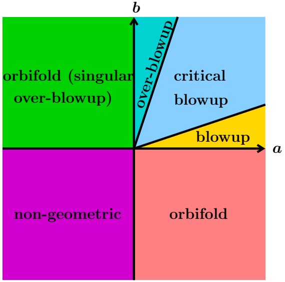
$a > 0,\ b \in \left[\frac{a}{3}, 3a\right]$:

$\langle c_{i\neq\rho} \rangle \geq 0, \qquad \langle x_\rho \rangle \geq 0, \qquad \langle z_{i\rho} \rangle \geq 0$



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Target space is **hybrid phase** with blowup limits for $b \downarrow \frac{a}{3}$ & $b \uparrow 3a$.

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Focus on orbifold and line bundle blowup now:

Procedure: [Blaszczyk,Groot Nibbelink,Ha,Klevers,FR,Trapletti,Vaudrevange,Walter,...]

- Introduce **exceptional divisors** $E_{\alpha\beta\gamma}$ at $x_{\alpha\beta\gamma} = 0$
- Introduce **gauge flux** $\mathcal{F} = E_{\alpha\beta\gamma} V^I_{\alpha\beta\gamma} H_I$
  - The H_I are the 16 Cartan generators of $E_8 \times E_8$
  - The 16×27 matrix $V^I_{\alpha\beta\gamma}$ describes the gauge line bundle at the 27 fixed points
- Note that in the **orbifold limit** the $E_{\alpha\beta\gamma}$ are shrunk to a point $\Rightarrow$ **flux** is located at fixed points

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To make contact with the **orbifold** description:

- Choose the $V_{\alpha\beta\gamma}$ to coincide with the internal $E_8 \times E_8$ **momentum** of some **twisted orbifold state** located at (α, β, γ)
- **Vev** of **orbifold state** generates the **blowup** of the $E_{\alpha\beta\gamma}$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Consistency requirements

Want to construct **smooth CY** with line bundles from orbifold using **toric** (algebraic) **geometry**. Impose

- **Bianchi identity** (ensures anomaly cancelation):

$$H = dB + \omega_{YM} - \omega_L \to \int_D dH = \int_D \mathrm{tr}\mathcal{R}^2 - \mathrm{tr}\mathcal{F}^2 \stackrel{!}{=} 0$$

- **Donaldson–Uhlenbeck–Yau** (ensures 4d $\mathcal{N} = 1$ SUSY)

$$\int_X J \wedge J \wedge \mathcal{F} = 0$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

Choose 3 different **bundle vectors** from $(\mathbf{27}, \mathbf{1})$ of $E_6 \times SU(3)$

- $V_1 = \frac{1}{3}(2, 2, 2, 0^5)(0^8)$ at 9 fixed points
- $V_2 = \frac{1}{3}(-1, -1, -1, 3, 0^4)(0^8)$ at 9 fixed points
- $V_3 = -(V_1 + V_2)$ at 9 fixed points

$\Rightarrow \mathcal{F} = \sum_{i=1}^{9} E_i V_1^I H_I + \sum_{j=10}^{18} E_j V_2^I H_I + \sum_{19}^{27} E_n V_3^I H_I$

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Bianchi identities

$\int_{E_{\alpha\beta\gamma}} \text{tr}\mathcal{F}^2 = \int_{E_{\alpha\beta\gamma}} \text{tr}\mathcal{R}^2 \quad \Rightarrow \quad V_1^2 = V_2^2 = V_3^2 = \frac{4}{3}$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

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DUY equations

$$\int J \wedge J \wedge \mathcal{F} = 0 \;\Rightarrow\; \sum_{i=1}^{9} V_1^I \mathrm{vol}(E_i) + \sum_{j=10}^{18} V_2^I \mathrm{vol}(E_j) + \sum_{k=19}^{27} V_3^I \mathrm{vol}(E_k) = 0$$

Resolution of $T^6/\mathbb{Z}^3$ with line bundles

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$$\int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{F}^2 = \int_{E_{\alpha\beta\gamma}} \mathrm{tr}\mathcal{R}^2 \quad \Rightarrow \quad V_1^2 = V_2^2 = V_3^2 = \frac{4}{3}$$

DUY equations

$$\int J \wedge J \wedge \mathcal{F} = 0 \;\Rightarrow\; \sum_{i=1}^{9} V_1^I \mathrm{vol}(E_i) + \sum_{j=10}^{18} V_2^I \mathrm{vol}(E_j) + \sum_{k=19}^{27} V_3^I \mathrm{vol}(E_k) = 0$$

Gauge group

$[\mathrm{E}_6 \times \mathrm{SU}(3)] \times [\mathrm{E}_8] \to [\mathrm{SO}(8) \times (\mathrm{U}(1)_A \times \mathrm{U}(1)_B) \times \mathrm{SU}(3)] \times [\mathrm{E}_8]$,
$\mathrm{U}(1)_{A,B}$ anomalous

Remnant non-R symmetries

Non-R symmetries arise as **discrete subgroups** of $U(1)_A$ and $U(1)_B$ which **leave vevs** of blowup modes **invariant**

$$\mathbf{27} \rightarrow \mathbf{8}_{s(1,-1)} + \mathbf{8}_{c(1,1)} + \mathbf{8}_{v(-2,0)} + \mathbf{1}_{(-2,-2)} + \mathbf{1}_{(-2,2)} + \mathbf{1}_{(4,0)}$$

Blowup modes:

$\mathbf{1}_{(-2,-2)}$, $\mathbf{1}_{(-2,2)}$, $\mathbf{1}_{(4,0)}$ corresponding to V_1, V_2, V_3

Leave **discrete $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry** generated by

$$T_\pm : \quad \phi_{(q_a,q_b)} \rightarrow e^{\frac{2\pi i}{2}(q_A \pm q_B)}\,\phi_{(q_A,q_B)}$$

Both **symmetries** are **non-anomalous**

Remnant R symmetries

Properties of R symmetries

- R symmetries do not commute with SUSY
- Grassmann coordinate θ transforms under R symmetries
- R symmetries only defined up to mixing with non-R symmetries

Remnant R symmetries

Properties of R **symmetries**

- R **symmetries** do **not commute** with **SUSY**
- Grassmann coordinate θ **transforms** under R **symmetries**
- R **symmetries** only defined up to **mixing** with **non-R symmetries**

R symmetries on orbifolds

R **charge** on **orbifold** defined via a combination of **right-moving momenta** q and **oscillator numbers** ΔN:
$R = q - \Delta N$ with $q = \frac{1}{3}(1,1,1)$ [Kobayashi,Raby,Zhang]

Remnant **symmetry** of internal space:
Sublattice rotations by $2\pi/3$ in each torus:

$$T_k^R : \phi \rightarrow e^{2\pi i/3 R_k} \phi \,, \qquad \text{k=1,2,3 labels tori}$$

Remnant R symmetries – Orbifold

Our orbifold **blowup modes** have

$$R = q - \Delta N = \frac{1}{3}(1,1,1)$$

To identify remnant **R-symmetries**, search for **invariant combinations** of T_k^R with $T_{U(1)_A}$ and $T_{U(1)_B}$:

$$\begin{aligned}
\mathbf{1}_{(-2,-2)} &\to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,-2)} \stackrel{!}{=} \mathbf{1}_{(-2,-2)} \\
\mathbf{1}_{(-2,2)} &\to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(-2,2)} \stackrel{!}{=} \mathbf{1}_{(-2,2)} \\
\mathbf{1}_{(4,0)} &\to (T_1^R)^a (T_2^R)^b (T_3^R)^c T_{U(1)_A} T_{U(1)_B} \mathbf{1}_{(4,0)} \stackrel{!}{=} \mathbf{1}_{(4,0)}
\end{aligned}$$

Result

One finds that $a + b + c = 3 \Rightarrow$ **only** a (trivial) **$\mathbb{Z}_2$ R symmetry** remains in **blowup**.

Look at simplified model with 3 exceptional divisors:

$$\begin{aligned} 0 &= z_{11}^3 x_1 + z_{12}^3 x_2 + z_{13}^3 x_3 \\ 0 &= z_{21}^3 x_1 x_2 x_3 + z_{22}^3 + z_{23}^3 \\ 0 &= z_{31}^3 x_1 x_2 x_3 + z_{32}^3 + z_{33}^3 \\ a_i &= |z_{i1}|^2 + |z_{i2}|^2 + |z_{i3}|^2 \\ b_\alpha &= |z_{1\alpha}|^2 + |z_{21}|^2 + |z_{31}|^2 - 3|x_\alpha|^2 \end{aligned}$$

Symmetries:

- $z_{i\alpha} \longrightarrow e^{2\pi i/3} z_{i\alpha}$
- $(x_1, x_2, x_3) \longrightarrow e^{2\pi i/3}(x_1, x_2, x_3)$
- $x_1 \longrightarrow x_2 \longrightarrow x_3 \longrightarrow x_1$ and $z_{11} \longrightarrow z_{12} \longrightarrow z_{13} \longrightarrow z_{11}$ if $b_1 = b_2 = b_3$
- ...

Origin of Symmetries

Note that the symmetries are inherited from the special choice of complex structure on the orbifold (absence of $\kappa\, z_{i1} z_{i2} z_{i3}$ term)

Remnant R symmetries – GLSM

How to check which of these symmetries are R symmetries?

***R* symmetries** will **transform** the holomorphic **$(3,0)$ form** Ω:

$\Omega \sim \eta\Gamma\eta dz^i dz^j dz^k \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W)$ [Witten]

Remnant R symmetries – GLSM

How to check which of these symmetries are R symmetries?

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How are the R symmetries broken in blowup?

(Presumably) via marginal deformations in Kähler potential under the presence of the gauge bundle:

$$\int d^2\theta^+ \phi_{4D}(x^\mu) N(z,x) \Lambda \overline{\Lambda}$$

- ϕ_{4D}: 4D modes
- $N(z,x)$: Polynomial in the geometry fields $z_{i\alpha}, x_\alpha$
- Λ: WS fermions describing the gauge bundle

$N(z,x)$ might not be compatible with rotational symmetries
$\Rightarrow$ **R symmetry broken**

Remnant R symmetries – GLSM

To check transformation of bundle under discrete symmetries:

- Find discrete transformations of coordinate fields z, x under symmetry in question
- Write down gauge bundle in ambient space
- Restrict bundle to toric hypersurface
- Find contributing monomials
- Check transformation of monomials under discrete symmetry

Conclusion

$\mathcal{N} = (2,2)$ GLSM

- Rich **moduli space** with interesting topology changes

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- Rich moduli space with interesting topology changes

$\mathcal{N} = (0,2)$ GLSM & Discrete Symmetries

- Find phenomenologically interesting discrete symmetries in GLSM
- Discrete symmetries appear at special points in moduli space
- Calculate matter charges from kinetic deformations

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- Rich **moduli space** with interesting topology changes

$\mathcal{N} = (0, 2)$ GLSM & Discrete Symmetries

- Find phenomenologically **interesting discrete symmetries** in GLSM
- Discrete **symmetries appear** at **special points** in moduli space
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Thank you for your attention!