String compactifications on smooth compact toric varieties

Magdalena Larfors

University of Oxford

16.07.2013

Based on M. L., D. Lüst, D. Tsimpis (1005.2194);
J. Gray, M. L., D. Lüst (1205.6208); M. L. (1307.XXXX)

$SU(3)$ structure compactifications

Motivation

String compactifications: standard model extension, cosmological models, gauge-gravity dual...
Compact geometry $\leftrightarrow$ low-dim phenomenology

4D SUSY theory $\Rightarrow$ nowhere vanishing spinor on $\mathcal{M}_6$
$\Rightarrow$ structure group restricted

$SU(3)$ structure

Nowhere vanishing spinor $\Leftrightarrow$
real two-form J and complex decomposable three-form Ω s.t.

$$\Omega \wedge J = 0, \qquad \frac{3i}{4}\Omega \wedge \overline{\Omega} = J \wedge J \wedge J \neq 0$$

$SU(3)$ structure compactifications

J, Ω closed $\Leftrightarrow$ $\mathcal{M}_6$ is Calabi–Yau.

Otherwise: non-zero torsion:

$$dJ = -\tfrac{3}{2}\mathrm{Im}(\mathcal{W}_1\overline{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$
$$d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}}_5 \wedge \Omega$$

Calabi–Yau ($\mathcal{W}_i = 0$)	Type II/Heterotic (no flux, SUSY Mkw)
Complex ($\mathcal{W}_1 = \mathcal{W}_2 = 0$)	Heterotic; Type IIB (SUSY Mkw)
Symplectic ($\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$)	Type IIA (SUSY Mkw)
...	...

Remark:
many Calabi–Yau $\rightarrow$ many fluxless compactifications (w. moduli).
In contrast: few $SU(3)$ structure manifolds with torsion known.

$SU(3)$ structure compactifications

Why so few explicit examples?

Non-complex manifolds: cannot use tools from algebraic geometry.

Tomasiello:07

$\mathbb{CP}_3$ and $\mathbb{CP}_1 \hookrightarrow \mathbb{CP}_2$ have two almost complex structures:

- $\mathcal{I}_m{}^p \sim J, \Omega$: not integrable
- $\tilde{\mathcal{I}}_m{}^p \sim$ Fubini-Study metric: integrable

LLT:10

Other manifolds with several almost complex structures?

$\mathbb{CP}_3$ and $\mathbb{CP}_1 \hookrightarrow \mathbb{CP}_2$ are smooth, compact, toric varieties

$\rightarrow$ look at other SCTVs!

Remark: No compact toric Calabi–Yau manifolds exist.

Toric geometry

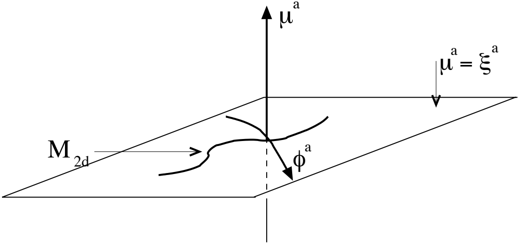
Symplectic quotient description (GLSM)

- $\{z^i,\ i = 1, \dots n\}$: holomorphic coordinates of $\mathbb{C}^n$
- $U(1)^s$ action: $z^i \longrightarrow e^{i\varphi_a Q_i^a} z^i$ generated by $V^a = \sum_i Q_i^a z^i \partial_{z^i}$.

Define moment maps $\mu^a := \sum_{i=1}^n Q_i^a |z^i|^2$ then

$$\mathcal{M}_{2d} = \{z^i \in \mathbb{C}^n | \mu^a = \xi^a\}/U(1)^s$$

is a toric variety (where $d = n - s$).



$SU(3)$ structure construction

$SU(n)$ structure on $\mathbb{C}^n \to$ local $SU(3)$ structure on $\mathcal{M}_{2d}$

On $\mathbb{C}^n$:

- $J_{\mathbb{C}^n} = dz^i \wedge d\bar{z}_i$
- $\Omega_{\mathbb{C}^n} = dz^1 \wedge \ldots \wedge dz^n$

On $\mathcal{M}_{2d}$:

- $\widetilde{J} = P(J_{\mathbb{C}^n}) = \mathcal{D}z^i \wedge \mathcal{D}\bar{z}_i$
- $\widetilde{\Omega} = A\, P(\Pi_a \iota_{V^a} \Omega_{\mathbb{C}^n})$

- $J_{\mathbb{C}^n} \wedge \Omega_{\mathbb{C}^n} = 0$
- $\Omega_{\mathbb{C}^n} \wedge \Omega^*_{\mathbb{C}^n} \propto \widetilde{J}^3_{\mathbb{C}^n}$
- $dJ_{\mathbb{C}^n} = d\Omega_{\mathbb{C}^n} = 0$

- $\widetilde{J} \wedge \widetilde{\Omega} = 0$
- $\widetilde{\Omega} \wedge \widetilde{\Omega}^* = \frac{4i}{3}\widetilde{J}^3$
- $d\widetilde{J} = 0,\ d\widetilde{\Omega} = dA \wedge \widetilde{\Omega}$ (integrable $\widetilde{\mathcal{I}}_m{}^p$)

$\widetilde{\Omega}$: complex decomposable **but not** gauge-invariant ($Q^a(\widetilde{\Omega}) \neq 0$).

$SU(3)$ structure construction

$SU(n)$ structure on $\mathbb{C}^n \to$ local $SU(3)$ structure on $\mathcal{M}_{2d}$

On $\mathbb{C}^n$:

- $J_{\mathbb{C}^n} = dz^i \wedge d\bar{z}_i$
- $\Omega_{\mathbb{C}^n} = dz^1 \wedge \ldots \wedge dz^n$

On $\mathcal{M}_{2d}$:

- $\widetilde{J} = P(J_{\mathbb{C}^n}) = \mathcal{D}z^i \wedge \mathcal{D}\bar{z}_i$
- $\widetilde{\Omega} = A\, P(\Pi_a \iota_{V^a} \Omega_{\mathbb{C}^n})$

- $J_{\mathbb{C}^n} \wedge \Omega_{\mathbb{C}^n} = 0$
- $\Omega_{\mathbb{C}^n} \wedge \Omega^*_{\mathbb{C}^n} \propto \widetilde{J}^3_{\mathbb{C}^n}$
- $dJ_{\mathbb{C}^n} = d\Omega_{\mathbb{C}^n} = 0$

- $\widetilde{J} \wedge \widetilde{\Omega} = 0$
- $\widetilde{\Omega} \wedge \widetilde{\Omega}^* = \frac{4i}{3} \widetilde{J}^3$
- $d\widetilde{J} = 0,\ d\widetilde{\Omega} = dA \wedge \widetilde{\Omega}$ (integrable $\widetilde{\mathcal{I}}_m{}^p$)

$\widetilde{\Omega}$: complex decomposable **but not** gauge-invariant ($Q^a(\widetilde{\Omega}) \neq 0$).

$SU(3)$ structure construction

$SU(n)$ structure on $\mathbb{C}^n \to$ local $SU(3)$ structure on $\mathcal{M}_{2d}$

On $\mathbb{C}^n$:

- $J_{\mathbb{C}^n} = dz^i \wedge d\bar{z}_i$
- $\Omega_{\mathbb{C}^n} = dz^1 \wedge \ldots \wedge dz^n$

On $\mathcal{M}_{2d}$:

- $\widetilde{J} = P(J_{\mathbb{C}^n}) = \mathcal{D}z^i \wedge \mathcal{D}\bar{z}_i$
- $\widetilde{\Omega} = A\, P(\Pi_a \iota_{V^a} \Omega_{\mathbb{C}^n})$

- $J_{\mathbb{C}^n} \wedge \Omega_{\mathbb{C}^n} = 0$
- $\Omega_{\mathbb{C}^n} \wedge \Omega^*_{\mathbb{C}^n} \propto \widetilde{J}^3_{\mathbb{C}^n}$
- $dJ_{\mathbb{C}^n} = d\Omega_{\mathbb{C}^n} = 0$

- $\widetilde{J} \wedge \widetilde{\Omega} = 0$
- $\widetilde{\Omega} \wedge \widetilde{\Omega}^* = \frac{4i}{3}\widetilde{J}^3$
- $d\widetilde{J} = 0$, $d\widetilde{\Omega} = dA \wedge \widetilde{\Omega}$ (integrable $\widetilde{\mathcal{I}_m}^p$)

$\widetilde{\Omega}$: complex decomposable **but not** gauge-invariant ($Q^a(\widetilde{\Omega}) \neq 0$).

$SU(3)$ structure construction

If we can rewrite $\widetilde{\Omega} = iK \wedge \omega$, where K:

1. is (1,0) (w.r.t. $\widetilde{\mathcal{I}}_m{}^p$) and vertical $P(K) = K$
2. has half the Q^a-charge of $\widetilde{\Omega}$.
3. can be normalized to $K \cdot K^* = 2$

then $\Omega = iK^* \wedge \omega$ has zero Q^a-charge and is well-defined.

$SU(3)$ structure construction

$$J := \widetilde{J} - iK \wedge K^* \quad \text{and} \quad \Omega := iK^* \wedge \omega$$

then define a global $SU(3)$ structure:

$$\Omega \wedge J = 0, \qquad \Omega \wedge \Omega^* = \frac{4i}{3} J^3$$

Ω is complex decomposable $\rightsquigarrow \mathcal{I}^2 = -1$

Extend: a family of $SU(3)$ structures
Let α, β, γ be nowhere vanishing real functions.

$$J := \alpha\widetilde{J} - \frac{i(\alpha+\beta^2)}{2} K \wedge K^* \quad \text{and} \quad \Omega := \alpha\beta e^{i\gamma} K^* \wedge \omega$$

Topological constraints

L:13, Dabholkar:13

Well-defined $SU(3)$ structure $\Leftrightarrow$ exists K s.t.

1. is (1,0) (w.r.t. $\widetilde{\mathcal{I}}_m{}^p$) and vertical $P(K) = K$
2. has half the Q^a-charge of $\widetilde{\Omega}$.
3. can be normalized to $K \cdot K^* = 2$

Condition 2: $\widetilde{\Omega}$ must have even Q^a-charge.

$\Leftrightarrow$ even first Chern class.

$\mathbb{CP}^3$ ✓	$\mathbb{CP}_1 \hookrightarrow$ 2D SCTV ✓	$\mathbb{CP}^2 \hookrightarrow \mathbb{CP}^1$ ✗
$c_1 = 4$	$c_1 = \sum_{a=1}^{n-2}(1+n^a)D_a + 2D_n$	$c_1 = (2+a+b)D_1 + 3D_5$

$SU(3)$ structure uniqueness

The $SU(3)$ structure is not unique.

Fixed K: LLT:10

$$J := \alpha j - \frac{i\beta^2}{2} K \wedge K^* \quad \text{and} \quad \Omega := e^{i\gamma} \alpha\beta K^* \wedge \omega$$

α, β, γ fixed in string vacuum

Several consistent choices of K possible (studied in L:13)

Example: $\mathbb{CP}^1 \hookrightarrow \mathbb{CP}^1 \times \mathbb{CP}^1$

$Q^1 = (0,1,0,1,n^1,0)$, $Q^2 = (1,0,1,0,0,-n^2)$, $Q^3 = (0,0,0,0,1,1)$

$$K_1 = -z^3\mathcal{D}z^1 + z^1\mathcal{D}z^3, \;\; K_2 = -z^4\mathcal{D}z^2 + z^2\mathcal{D}z^4:$$

(1,0) and vertical; $|K_i|^2 \neq 0$ if $n^1 < 0$, $n^2 > 0$.

$$K = \alpha_1 K_1 + \alpha_2 K_2$$

fulfills all constraints iff $\alpha_{1,2}$ not simultaneously zero and
$Q(\alpha_1) = \frac{1}{2}(2+n^1, -2-n^2, 2)$, $Q(\alpha_2) = \frac{1}{2}(-2+n^1, 2-n^2, 2)$

One solution: $n^1 = -2$; $n^2 = 2$; $\alpha_1 = z^6$; $\alpha_2 = z^5$

Torsion classes: always a symplectic limit

For constant α, β:

$\mathcal{W}_1, \mathcal{W}_3, \mathcal{W}_4 \propto (\alpha + \beta^2)$ and $\mathcal{W}_2, \mathcal{W}_5 \neq 0$.

UNIVERSITY OF OXFORD

Conclusions and outlook

Conclusions

- 3D SCTVs with even c_1 allow $SU(3)$ structures.
- $SU(3)$ structure constructed using one-form K.
- One SCTV can allow several $SU(3)$ structures.
- Can choose symplectic limit: $\mathcal{W}_1, \mathcal{W}_3, \mathcal{W}_4 = 0$

Outlook

- Construct new string vacua on SCTVs. AdS, Mkw, DW, dS.
- Classification of SCTV $SU(3)$ structures.
- Moduli of SCTV $SU(3)$ structures?

Conclusions and outlook

Conclusions

- 3D SCTVs with even c_1 allow $SU(3)$ structures.
- $SU(3)$ structure constructed using one-form K.
- One SCTV can allow several $SU(3)$ structures.
- Can choose symplectic limit: $\mathcal{W}_1, \mathcal{W}_3, \mathcal{W}_4 = 0$

Outlook

- Construct new string vacua on SCTVs.
  AdS, Mkw, DW, dS.
- Classification of SCTV $SU(3)$ structures.
- Moduli of SCTV $SU(3)$ structures?